Relativistic effects in superradiance processes

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(Submitted March 18, 1975)
Zh. Eksp. Teor. Fiz. **70**, 435–444 (February 1976)

Two hydrogen-like atoms in an external radiation field are considered within the framework of third-order quantum electrodynamics effects with account taken of the interaction between the atoms via the virtual photon field. Allowance for the possibility of the atomic electrons exist in positron intermediate states yields an additional contribution to the interaction Hamiltonian of the atomic system and external field as compared to the Pauli Hamiltonian. Excitation of a system of N atoms by a coherent pulsed source is considered in the electric dipole approximation. It is shown that each atom "feels" the resultant field, which is the sum of three components, the external field, the so-called "dipole field" (electron polarization) and the field resulting from allowance for the positron intermediate states (positron polarization. In systems with inhomogeneous broadening of spectral lines in superradiance processes, the positron polarization may exceed the electron polarization.

PACS numbers: 31.50.+w, 32.10.Vc

1. INTRODUCTION

A theoretical description of the superradiance processes is based on an investigation of the evolution of a system of N atoms in a coherent radiation field. It is customary to use the model of noninteracting atoms, in which each of the atoms interacts with the external alternating field independently of the other.[1,2] From the quantum-electrodynamics point of view, this model corresponds to first-order processes, which can be represented by the diagrams shown in Fig. 1, where each solid line corresponds to a state of the atom, and a wavy line corresponds to emission (absorption) of a photon. The processes of photon emission (absorption), including the interaction between the atoms via the field of virtual photons, come into play in third-order perturbation theory and are represented by the diagrams of Fig. 2.[3] The traditional description of the processes of dynamic interaction of atoms in superradiance processes is based on introduction of phenomenological relaxation parameters.[4–6] On the other hand, the dynamic interaction between atoms situated in an external coherent field brings about polarization of the medium,[7] and becomes manifest, in particular, in superradiant effects of optical nutation[8] and self-induced transparency.[9] In the cited papers, however, no consistent account was taken of the relativistic effects. This pertains in particular to allowance for the possibility of finding the atomic electrons in intermediate positron states, a fact that manifests itself in the onset of additional terms in the Hamiltonian of the interaction of the system of atoms with the external radiation field in comparison with the Pauli Hamiltonian.[3] Such terms, without allowance for the spin variables, were obtained by a semiclassical method in our preceding paper.[10] Interaction-Hamiltonian terms of similar type were obtained by Drake[11] in an examination of relativistic corrections to the emission of a helium-like atom.

One of the main problems in the understanding of the processes of superradiance is the two-atom problem. It has been the subject of a whole number of studies. ies.[12–15] Regardless of the mathematical approach, the results of these studies reduce to the following: 1) the presence of the second atom can greatly change the lifetime of the excited state of the atom; 2) this change of the lifetime depends on the mutual orientation of the dipole moments of the transition of the atoms and on the distance between the atoms. Although the meanings of the processes considered in the cited papers are conveyed by the diagrams of Fig. 2, no account is taken in these papers of the positron states of the electrons in the atoms as intermediate states.

In the present paper we calculate the relativistic effects in the emission of two hydrogen-like atoms in third-order perturbation theory (Fig. 2). The nuclei of the atoms are at rest in the employed model. In contrast to[3,11] we consider a situation wherein the atoms (the radiating electrons) are at arbitrary distances from one another. We distinguish arbitrarily between two types of third-order effects due, first, to electronic intermediate states, and second, to positronic intermediate states. It is shown that effects of the first type, for nondegenerate atomic levels, lead to corrections to the unperturbed wave functions of the noninteracting atoms in the first order of perturbation theory of

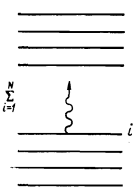

FIG. 1.

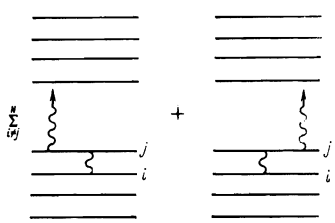

FIG. 2.

nonrelativistic quantum theory, where the perturbation is taken to be the operator of the interaction between the atoms. The second type of effects of third-order gives an additional contribution to the Hamiltonian of the interaction of the atoms with the external field in comparison with the Pauli Hamiltonian. The problem is then generalized to include the case of N atoms.

2. EFFECTIVE ENERGY OPERATOR OF THE INTERACTION OF TWO HYDROGEN-LIKE ATOMS WITH AN EXTERNAL RADIATION FIELD

We use the well-known expressions[3,11] for the S matrix of the considered third-order effects with the diagrams of Fig. 2 (at $N=2$). The effective interaction energy matrix U, defined by the formula

$$S^{(3)}_{mn,pr}=-2\pi i U^{(3)}_{mn,pr}\delta(\omega+\omega_p-\omega_m+\omega_r-\omega_n),$$

where ω is the frequency of the real photon and the subscripts m and n pertain to the initial states of the atoms, and p and r to the final ones, takes the form

$$U^{(3)}_{mn,pr}=\frac{ie^3}{4\pi}\int\psi_r^+(\mathbf{r}')\psi_p^+(\mathbf{r}''')\sum_{l_\pm}\left\{\gamma_\delta'\gamma_\delta' A_\delta'\frac{\psi_l(\mathbf{r}')\psi_l^+(\mathbf{r}'')}{\omega_l(1-i0)-\omega-\omega_r}\right.$$
$$\times\frac{1-\boldsymbol{\alpha}''\boldsymbol{\alpha}'''}{|\mathbf{r}''-\mathbf{r}'''|}\exp\{i|\omega_p-\omega_n|\mathbf{r}''-\mathbf{r}'''|\}+\frac{1-\boldsymbol{\alpha}'\boldsymbol{\alpha}'''}{|\mathbf{r}'-\mathbf{r}'''|}\exp\{i|\omega_p-\omega_n|\mathbf{r}'-\mathbf{r}'''|\}$$
$$\left.\times\frac{\psi_l(\mathbf{r}')\psi_l^+(\mathbf{r}'')}{\omega_l(1-i0)+\omega-\omega_m}\gamma_\delta''\gamma_\delta'' A_\delta''\right\}\psi_m(\mathbf{r}'')\psi_n(\mathbf{r}''')\,d\mathbf{r}'\,d\mathbf{r}''\,d\mathbf{r}'''. \quad (2.1)$$

Here $\boldsymbol{\alpha}'$, $\boldsymbol{\alpha}''$, and $\boldsymbol{\alpha}'''$ are Dirac matrices, $\delta=1,2,3$; γ_δ are γ matrices, $\psi_{m(n)}$ are the solutions of the Dirac equation for the electron in the field of its nucleus without time factors, A_δ are the components of the vector potential, also without time-dependent factors, while $\mathbf{r}'$, $\mathbf{r}''$, and $\mathbf{r}'''$ are radius vectors of the electrons. The summation in (2.1) is over all the intermediate states of the electrons with both positive and negative frequencies.

Let the coordinates $\mathbf{r}'$ and $\mathbf{r}''$ pertain to the first atom, and $\mathbf{r}'''$ to the second. The effects of Fig. 2 were considered in[11] for two electrons in the helium atom. It was taken into account there, that the distance between the electrons is smaller than the radiated wavelength. In the case when the distances between the electrons and the atoms are arbitrary, we shall proceed in the following manner. We assume the nuclei of the atoms to be immobile. The radius vectors of the electrons are represented in the form of a sum of two vectors $\mathbf{a}_i+\boldsymbol{\xi}_i=\mathbf{r}_i$ $(i=1,2)$, where $\mathbf{a}_i$ is the radius vector of the nucleus of the i-th atom and $\boldsymbol{\xi}_i$ is the displacement of the electron of the i-th atom relative to its nucleus. Then

$$|\mathbf{r}''-\mathbf{r}'''|\approx a\left(1+\frac{a(\boldsymbol{\xi}''-\boldsymbol{\xi}''')}{a^2}+\frac{(\boldsymbol{\xi}''-\boldsymbol{\xi}''')^2}{2a^2}+\ldots\right), \quad (2.2)$$

where a is the distance between nuclei. Taking (2.2) into account, we can write down the following equality:

$$\frac{1-\boldsymbol{\alpha}_1''\boldsymbol{\alpha}_2'''}{|\mathbf{r}_1''-\mathbf{r}_2'''|}\exp\left\{\frac{i}{c}|\omega_p-\omega_n|\mathbf{r}_1''-\mathbf{r}_2'''|\right\}=(1-\boldsymbol{\alpha}_1''\boldsymbol{\alpha}_2''')\exp\left\{\frac{i}{c}|\omega_p-\omega_n|a\right\}$$
$$\times\left[\frac{1}{|\mathbf{r}_1''-\mathbf{r}_2'''|}+\frac{i}{c}|\omega_p-\omega_n|f_1(\mathbf{r}_1'',\mathbf{r}_2''')-\frac{1}{2c^2}(\omega_p-\omega_n)^2 f_2(\mathbf{r}_1'',\mathbf{r}_2''')\right], \quad (2.3)$$

where the quantities f_1 and f_2 are given by

$$f_1(\mathbf{r}_1'',\mathbf{r}_2''')=1-\frac{a}{|\mathbf{r}_1''-\mathbf{r}_2'''|},\quad f_2(\mathbf{r}_1'',\mathbf{r}_2''')=|\mathbf{r}_1-\mathbf{r}_2|+\frac{a^2}{|\mathbf{r}_1''-\mathbf{r}_2'''|}-2a.$$

We replace the frequencies in (2.3) by operators, in analogy with the procedure used in[3]. Then

$$\frac{1-\boldsymbol{\alpha}_1''\boldsymbol{\alpha}_2'''}{|\mathbf{r}_1''-\mathbf{r}_2'''|}\exp\left\{\frac{i}{c}|\omega_p-\omega_n|\,|\mathbf{r}_1''-\mathbf{r}_2'''|\right\}\to\exp\left\{\frac{i}{c}|\omega_p-\omega_n|a\right\}$$
$$\times\left\{\frac{1-\boldsymbol{\alpha}_1''\boldsymbol{\alpha}_2'''}{|\mathbf{r}_1''-\mathbf{r}_2'''|}\pm\frac{i}{c}[\mathcal{H}_2''',f_1(\mathbf{r}_1'',\mathbf{r}_2''')]+\frac{1}{2c^2}[\mathcal{H}_1'',[\mathcal{H}_2''',f_2(\mathbf{r}_1'',\mathbf{r}_2''')]]\right.$$
$$\left.-\frac{1}{2c^2}(\omega-\omega_r+\omega_l)[\mathcal{H}_2''',f_2(\mathbf{r}_1'',\mathbf{r}_2''')]\right\}, \quad (2.4)$$

where

$$\mathcal{H}_2'''=c\boldsymbol{\alpha}_2'''\mathbf{p}_2'''+\gamma_{i2}'''mc^2-Ze^2/\xi_2''',\quad \mathcal{H}_1''=c\boldsymbol{\alpha}_1''\mathbf{p}_1''+\gamma_{i1}''mc^2-Ze^2/\xi_1'',$$

Z_e is the charge of the nucleus, $\mathbf{p}_2'''$ and $\mathbf{p}_1''$ are the momentum operators, the plus sign in (2.4) corresponds to the case $\omega_p>\omega_n$, and the minus sign to the case $\omega_p<\omega_n$. We transform in the same manner the operator

$$\frac{1-\boldsymbol{\alpha}_1'\boldsymbol{\alpha}_2'''}{|\mathbf{r}_1'-\mathbf{r}_2'''|}\exp\left\{\frac{i}{c}|\omega_p-\omega_n|\,|\mathbf{r}_1'-\mathbf{r}_2'''|\right\}\to\exp\left\{\frac{i}{c}|\omega_p-\omega_n|a\right\}$$
$$\times\left\{\frac{1-\boldsymbol{\alpha}_1'\boldsymbol{\alpha}_2'''}{|\mathbf{r}_1'-\mathbf{r}_2'''|}\pm\frac{i}{c}[\mathcal{H}_2''',j_1(\mathbf{r}_1',\mathbf{r}_2''')]+\frac{1}{2c^2}[\mathcal{H}_1'[\mathcal{H}_2''',j_2(\mathbf{r}_1',\mathbf{r}_2''')]\right.$$
$$\left.-\frac{1}{2c^2}(\omega+\omega_m-\omega_l)[\mathcal{H}_2''',j_2(\mathbf{r}_1',\mathbf{r}_2''')]\right\}. \quad (2.5)$$

Let us calculate the commutators that enter in (2.4) and (2.5). Further, omitting the primes, we obtain in place of (2.1),

$$U^{(3)}_{mn,pr}=\frac{ie^3}{4\pi}\exp\left\{\frac{i}{c}|\omega_p-\omega_n|a\right\}\int\psi_r^+(\xi_1)\psi_p^+(\xi_2)$$
$$\times\sum_{l_\pm}\left\{\gamma_{\delta_1}\gamma_{\delta_1}A_{\delta_1}\frac{\psi_l(\xi_1)\psi_l^+(\xi_1)}{\omega_l(1-i0)-\omega-\omega_r}\mathcal{B}+\mathcal{B}\frac{\psi_l(\xi_1)\psi_l^+(\xi_1)}{\omega_l(1-i0)+\omega-\omega_m}\right.$$
$$\left.\times\gamma_{\delta_1}\gamma_{\delta_1}A_{\delta_1}\right\}\psi_m(\xi_1)\psi_n(\xi_2)\,d\xi_1\,d\xi_2, \quad (2.6)$$

where the operator $\mathcal{B}$ takes the form

$$\mathcal{B}=\frac{1}{r}\pm\frac{a(\boldsymbol{\alpha}_2\mathbf{n})}{r^2}-\frac{1}{2}\frac{(\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2)+(\boldsymbol{\alpha}_1\mathbf{n}_{12})(\boldsymbol{\alpha}_2\mathbf{n}_{12})}{r}$$
$$-\frac{a^2}{2}\frac{(\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2)-3(\boldsymbol{\alpha}_1\mathbf{n}_{12})(\boldsymbol{\alpha}_2\mathbf{n}_{12})}{r^3},\qquad r=|\mathbf{r}_1-\mathbf{r}_2|,\quad \mathbf{n}_{12}=\frac{\mathbf{r}_1-\mathbf{r}_2}{r}. \quad (2.7)$$

We consider in (2.6) that part of the sum which pertains to the intermediate states with negative frequencies l_-. We take into account here the fact that the electron energies differ little from the rest energy mc^2. Introducing the operator

$$\Lambda_-=\frac{mc^2-\mathcal{H}_1}{2mc^2}\approx\frac{1}{2}\left(1-\gamma_{i1}-\frac{\boldsymbol{\alpha}_1\mathbf{p}_1}{mc}\right), \quad (2.8)$$

we obtain the following effective-energy operator corresponding to the chosen type of intermediate states:

$$U^{(3)-}=\frac{ie^3}{8\pi mc^2 a}\exp\left\{\frac{i}{c}\omega_0 a\right\}\{\gamma_{a1}\gamma_{b1}A_{b1}\Lambda_-\mathscr{B}+\mathscr{B}\Lambda_-\gamma_{a1}\gamma_{b1}A_{b1}\},\quad \omega_0=|\omega_p-\omega_n|. \tag{2.9}$$

We write down the matrix of the operator (2.9) with the wave functions

$$\psi=\begin{pmatrix}\varphi\\ \chi\end{pmatrix},\quad \chi=\frac{\boldsymbol{\sigma}\mathbf{p}}{2mc}\varphi,\quad \varphi=\left(1-\frac{p^2}{8m^2c^2}\right)\Phi, \tag{2.10}$$

in analogy with the procedure used in[11]. Here $\boldsymbol{\sigma}$ are Pauli matrices. We discard terms containing $1/c$ of degree higher than the third. Further, the coordinates $\mathbf{r}'$, $\mathbf{r}''$, and $\mathbf{r}'''$ in (2.1) can pertain, respectively, to the second and first atoms. Taking all the foregoing into account, we obtain in the quasirelativistic approximation

$$\begin{aligned}U_{QR}^{(3)-}=&\frac{e^3}{8\pi m^2c^3}\exp\left\{\frac{i}{c}\omega_0 a\right\}\left\{-\sum_{i=1}^{2}\sigma_i\left[\mathbf{A}_i\frac{\mathbf{n}_{ij}}{r^2}\right]\right.\\&+\sum_{i\neq j}\left(\frac{1}{r}(\mathbf{A}_i\mathbf{p}_j)+\sigma_j\frac{1}{r^2}[\mathbf{n}_{ij}\mathbf{A}_i]+\frac{1}{r}(\mathbf{A}_i\mathbf{n}_{ij})(\mathbf{n}_{ij}\mathbf{p}_j)\right)\\&\left.+a^2\sum_{i\neq j}\left(\frac{1}{r^3}(\mathbf{A}_i\mathbf{p}_j)+\frac{3}{r^5}\sigma_j[\mathbf{n}_{ij}\mathbf{A}_i]-\frac{3}{r^5}(\mathbf{A}_i\mathbf{n}_{ij})(\mathbf{n}_{ij}\mathbf{p}_j)\right)\right\}.\end{aligned} \tag{2.11}$$

On going to the case one helium-like atom we have $a\to 0$ and (2.11) coincides with the corresponding operator obtained by Drake.[11]

We consider now in (2.6) the part of the sum pertaining to the intermediate states with positive frequencies l_-. We obtain

$$\begin{aligned}U_{mn,pr}^{(3)+}=&\sum_{i,}\sum_{i\neq j}\left[\frac{\langle\Phi_{pr}{}^{ij}|U_{QRj}^{(1)}|\Phi_{pl}{}^{ij}\rangle\langle\Phi_{pl}{}^{ij}|U_{QR}^{(2)}|\Phi_{nm}{}^{ij}\rangle}{-\omega_l(1-i0)+\omega+\omega_r}\right.\\&\left.+\frac{\langle\Phi_{pr}{}^{ij}|U_{QR}^{(2)}|\Phi_{nl}{}^{ij}\rangle\langle\Phi_{nl}{}^{ij}|U_{QR}^{(1)}|\Phi_{nm}{}^{ij}\rangle}{-\omega_l(1-i0)-\omega+\omega_m}\right]\\=&\sum_{i\neq j}[\langle\Phi_{pr}{}^{ij}|U_{QR}^{(1)}|\Delta\Phi_{nm}{}^{ij}\rangle+\langle\Delta\Phi_{pr}{}^{ij}|U_{QR}^{(1)}|\Phi_{nm}{}^{ij}\rangle].\end{aligned} \tag{2.12}$$

where $U_{QRj}^{(1)}$ is the operator of the effective interaction energy of the j-th atom and corresponds to the diagram of Fig. 1 in the quasirelativistic approximation.[3,11] The operator $U_{QR}^{(2)}$ is a generalization of the Breit operator[3] to the case of two hydrogen-like atoms situated at arbitrary distances from one another, under the condition that the atomic dimensions are small in comparison with the internuclear distance and with the characteristic wavelength in the spectra of the interacting atoms:

$$\begin{aligned}U_{QR}^{(2)}=&\exp\left\{\frac{i}{c}\omega_0 a\right\}\left\{\frac{\alpha}{r}-\frac{\alpha}{4m^2c^2}\frac{1}{r^2}([\mathbf{n}_{12}\times\mathbf{p}_1]\sigma_1-[\mathbf{n}_{12}\times\mathbf{p}_2]\sigma_2+2[\mathbf{n}_{12}\mathbf{p}_1]\sigma_2\right.\\&-2[\mathbf{n}_{12}\mathbf{p}_2]\sigma_2]-\frac{\alpha}{2m^2c^2r}[(\mathbf{p}_1\mathbf{p}_2)+\mathbf{n}_{12}(\mathbf{n}_{12}\mathbf{p}_1)\mathbf{p}_2]\\&+\frac{\alpha}{4m^2c^2}\frac{1}{r^3}[(\sigma_1\sigma_2)-3(\mathbf{n}_{12}\sigma_1)(\mathbf{n}_{12}\sigma_2)]\pm\alpha a\frac{1}{mc}\frac{1}{r^2}(\mathbf{p}_2\mathbf{n}_{12})\\&\left.-\frac{\alpha a^2}{2m^2c^2}\frac{1}{r^3}[(\mathbf{p}_1\mathbf{p}_2)-3\mathbf{n}_{12}(\mathbf{n}_{12}\mathbf{p}_1)\mathbf{p}_2]\right\},\\&\alpha=e^2/4\pi.\end{aligned} \tag{2.13}$$

The wave functions $\Phi_{pr}^{ij}=\Phi_p^i\Phi_r^j$ etc. are constructed with the aid of the two-component normalized functions Φ (2.10) of the individual atoms.

3. THE SCHRÖDINGER WAVE EQUATION

For an illustrative interpretation of the obtained expressions it is convenient to construct the corresponding Schrödinger wave equation that takes into account both the interaction between the atoms and their interaction with the external radiation field. As seen from (2.12), the expressions for $\Delta\Phi$ coincide exactly with the first-order corrections of the nonrelativistic perturbation theory to the wave functions for the system of atoms with nondegenerate energy levels, where the perturbation is chosen to be the interaction between the atoms and the operator $U_{QR}^{(2)}$.[16] If it is taken into account that the matrix elements of the type $\langle\Delta\Phi_{pr}^{ij}|U_{QR}^{(3)-}|\Phi_{nm}^{ij}\rangle$, $\langle\Delta\Phi_{pr}^{ij}|U_{QR}^{(1)}|\Delta\Phi_{nm}^{ij}\rangle$ are quantities of higher degree of smallness in comparison with (2.12), then the complete expression for the matrix element of the operator of the effective interaction energy of the pair of atoms with the radiation field, including effects of first and third order, can be represented in the form

$$U_{mn,pr}=\sum_{i\neq j}\langle\Phi_{pr}{}^{ij}+\Delta\Phi_{pr}{}^{ij}|U_{QRj}^{(3)-}+U_{QR}^{(1)}|\Phi_{nm}{}^{ij}+\Delta\Phi_{nm}{}^{ij}\rangle. \tag{3.1}$$

Expression (3.1) can be interpreted in the following manner: The emission (absorption) of a real photon by a system of two hydrogen-like atoms interacting with one another via the field of virtual photons, accurate to third-order effects, can be described arbitrarily as a direct process if the pair of atoms is regarded as a unified dynamic system. This is equivalent to the case when the stationary energy states of the system of interacting atoms are obtained from a solution of the Schrödinger equation in perturbation theory of first order in the interaction between the atoms:

$$\begin{aligned}&\mathscr{H}\Phi^{(1,2)}=(\mathscr{H}_{01}+\mathscr{H}_{02}+U_{QR}^{(2)})\Phi^{(1,2)}=E\Phi^{(1,2)},\\&\mathscr{H}_{01}=\sum_i\left(\frac{p_i^2}{2m}+eA_{0i}\right),\quad A_{0i}=-\frac{ze}{\xi_i},\\&\mathscr{H}_{02}=-\sum_i\left(\frac{p_i^4}{8m^3c^2}-\frac{e}{4m^2c^2}\sigma_i[\nabla A_{0i}\times\mathbf{p}_i]+\frac{e}{8m^2c^2}\Delta A_{0i}\right),\end{aligned} \tag{3.2}$$

where $\mathscr{H}_{01}+\mathscr{H}_{02}$ is the unperturbed Hamiltonian. The probabilities of the transition between these stationary states are sought in perturbation theory of first order in the interaction of the atoms with the external field, described by the operator $\mathscr{H}_1+\mathscr{H}_2$, where $\mathscr{H}_1$ coincides with the known Pauli operator,[3,11] and $\mathscr{H}_2$ is none other than the operator $U_{QR}^{(3)-}$ (2.11), in which the vector-potential operators contain time-dependent factors. Those terms of the Hamiltonian $\mathscr{H}_2$ which do not include spin variables for the distances between atoms smaller than the radiation wavelength, were obtained by us in[10] from the Darwin Lagrange functions of a system of interacting charges by changing over to quantum-mechanical operators and by simply replacing the charge momentum operators $\mathbf{p}_i$ by $\mathbf{p}_i-e\mathbf{A}_i/c$.

To change over to the system of N atoms, we sum the operators (2.13), (3.2), and (2.11) over the indices i and j from 1 to N, replacing at the same time a by a_{ij} and r by r_{ij}, where a_{ij} and r_{ij} are respectively the distance between the nuclei of the i-th and j-th atoms and the distance between the electrons of the same atoms.

4. ELECTRIC DIPOLE TRANSITIONS IN COHERENT EXCITATION

To explain the physical meaning and to estimate the value of the obtained relativistic terms we shall seek in the electric dipole approximation, for the system of two atoms, the matrix element of the transition whereby one atom (the i-th) changes its state. To this end, we retain in (2.11) and (2.13) only the part that depends on the operators of the electric dipole moments of the atoms. We consider next the case $\omega_0 a_{ij}/c \gg 1$, i.e., distances between atoms much larger than the wavelength. This enables us to consider in (2.13) only the retarded part of the interaction.

Assume that a plane monochromatic wave

$$\mathbf{A}_j = {}^1/_2 A_0 \mathbf{e}_{fk} \exp(-i\mathbf{k}\mathbf{a}_j) \exp(i\omega_k t) + \text{c.c.} \tag{4.1}$$

is incident on the system, where A_0 is the amplitude, $\mathbf{e}_{fk}$ is the polarization vector, $\mathbf{k}$ is the wave vector, and ω_k is the frequency. Then, by making simple transformations in (2.12), we can show that allowance for the interaction between the atoms via the electronic intermediate states, leading to a distortion of the atomic wave functions, is equivalent to allowance for a certain additional internal field acting on the i-th atom. The magnitude of this field coincides with the so-called "dipole field" calculated in the quantum theory of dispersion.[7,17] Thus, allowance for only the electronic intermediate states of the electrons of the atoms accounts fully for the usually considered effects of polarization of the medium.

Taking for foregoing into account, we write down the sought matrix element in the following form:

$$M_{mn,pn} = -\frac{e}{mc} \langle \Phi_p{}^i \Phi_n{}^j | \mathbf{p}_i (\mathbf{A}_i + \mathbf{A}_i^{(e)} + \mathbf{A}_i^{(p)}) | \Phi_m{}^i \Phi_n{}^j \rangle, \tag{4.2}$$

where $\mathbf{A}_i$ is the vector potential of the field of the external sources, $\mathbf{A}_i^{(e)}$ and $\mathbf{A}_i^{(p)}$ are the components of the internal field at the location of the i-th atom, with $\mathbf{A}_i^{(e)}$ the "dipole field"[7] and $\mathbf{A}_i^{(p)}$ the component due to the allowance for the positron intermediate states of the electron in the atom, i.e., the states with negative energy. The quantities $\mathbf{A}_i^{(e)}$ and $\mathbf{A}_i^{(p)}$ take the following form

$$\mathbf{A}_i^{(e)} = \frac{e^2}{8\pi m^2 c^2 \hbar} \exp\left\{\frac{i}{c}\omega_0 a\right\} \frac{1}{a_{ij}} \exp\{-i\mathbf{k}\mathbf{a}_j\}$$
$$\times A_0 \sum_{l_*} \left\{ \frac{1}{\omega_{ln} + \omega_k} [(\mathbf{p}_j \mathbf{e}_{fk})_{ln} (\mathbf{p}_j)_{nl} - (\mathbf{p}_j \mathbf{e}_{fk})_{ln} \mathbf{n}_{ij} (\mathbf{p}_j \mathbf{n}_{ij})_{nl}] \right.$$
$$\left. + \frac{1}{\omega_{ln} - \omega_k} [(\mathbf{p}_j \mathbf{e}_{fk})_{nl} (\mathbf{p}_j)_{ln} - (\mathbf{p}_j \mathbf{e}_{fk})_{nl} \mathbf{n}_{ij} (\mathbf{p}_j \mathbf{n}_{ij})_{ln}] \right\} \exp\{i\omega_k t\}, \tag{4.3}$$

$$\mathbf{A}_i^{(p)} = -\frac{e^2}{8\pi m c^2} \exp\left\{\frac{i}{c}\omega_0 a_{ij}\right\} \frac{1}{a_{ij}} \exp\{-i\mathbf{k}\mathbf{a}_j\} A_0 [\mathbf{e}_{fk} - (\mathbf{e}_{fk}\mathbf{n}_{ij})\mathbf{n}_{ij}] \exp\{i\omega_k t\}. \tag{4.4}$$

As seen from (4.3), the quantity $\mathbf{A}_i^{(e)}$ is determined by the magnitude and orientation of the dipole moment of the transition of the j-th atom. If $\mathbf{p}_j \perp \mathbf{e}_{fk}$, then the field $\mathbf{A}_i^{(e)}$ is equal to zero. The interaction of the i-th dipole with the field $\mathbf{A}_i^{(e)}$ also vanishes if $\mathbf{p}_j \perp \mathbf{p}_i$. At the same time, the magnitude of the field $\mathbf{A}_i^{(p)}$ does not depend on the orientation and on the value of the j-th dipole moment of the transition.

Let us estimate the ratio of $|\mathbf{A}_i^{(e)}|/|\mathbf{A}_i^{(p)}|$ in the two-level approximation, assuming that the main contribution to $\mathbf{A}_i^{(e)}$ is made by the "resonant" term, which is inversely proportional to the natural width γ_0 of the line of the atom, equal to $4\omega_0^3 d^2/3c^3\hbar$, $(d = |d_{nm}|)$. We then obtain at $\mathbf{p}_j \parallel \mathbf{e}_{fk}$

$$\frac{|\mathbf{A}_i^{(e)}|}{|\mathbf{A}_i^{(p)}|} \approx \frac{m\omega_0^2 d^2}{\hbar\gamma_0 e^2} = \frac{3c^3 m}{4\omega_0 e^2}. \tag{4.5}$$

At optical frequencies ($\omega = 10^{16}$–10^{13} Hz) this ratio is approximately equal to 6×10^6–10^9. Thus, in the case two atoms, the value of $\mathbf{A}_i^{(e)}$ at $\mathbf{p}_j \parallel \mathbf{e}_{fk}$ is much larger than $\mathbf{A}_i^{(p)}$ in the optical band.

In systems of N identical atoms, the components of the internal field (4.3) and (4.4) are summed over all the indices j from 1 to $N-1$. What is the external field acting on each of the atoms in the superradiant processes? Assume that resonant radiation ($\omega_k = \omega_0$) described by the plane wave (4.1) acts on a system of N atoms. As is well known,[1] this excitation leads the system to a superradiant state if the duration Δt of the action of the radiation is much shorter than all the relaxation times T_1, T_2, or T_2^*, where T_1 is the time of the energy (longitudinal) relaxation, T_2 and T_2^* are the times of the phase (transverse) irreversible and reversible relaxation, respectively. After the termination of the action of the pulsed excitation, we record the spontaneous superradiant response of the system during the "phase memory" time characterized by the time T_2^*. For simplicity we assume that the direction of the superradiance $\mathbf{k}$ coincides with the z axis ($\mathbf{k} \parallel z$).

The value of $\mathbf{A}_i^{(p)}$ obtained from (4.4) by summation over j from 1 to $N-1$ is proportional to lattice sums of the form

$$S_{1i} = \sum_j \frac{1}{a_{ij}} \exp\{i\mathbf{k}_0 \mathbf{a}_{ij}\} \exp\{-i\mathbf{k}\mathbf{a}_j\},$$
$$S_{2i} = \sum_j \frac{1}{a_{ij}} \exp\{i\mathbf{k}_0 \mathbf{a}_{ij}\} \exp\{-i\mathbf{k}\mathbf{a}_j\} (\mathbf{e}_{fk}\mathbf{n}_{ij})\mathbf{n}_{ij}. \tag{4.6}$$

If we assume that the surrounding atoms are uniformly distributed inside a sphere with a center at the point $\mathbf{a}_i$, with a radius r_0 and a volume V, then we can go over in (4.6) from summation to integration. Then

$$S_{1i} \approx -\frac{2\pi N}{V} [\exp\{i(k_0 + k_z) r_0\} - 1] \frac{1}{k_z(k_0 + k_z)} \exp\{-i\mathbf{k}\mathbf{a}_i\}. \tag{4.7}$$

The "lattice sum" S_{2i} is smaller than S_{1i} in absolute magnitude by at least a factor of three.[10] Let us estimate the component of the internal field $\mathbf{A}_i^{(p)}$ due to allowance for the positronic intermediate states of the electrons in the atom, and let us compare it with the external field at the location of the i-th atom. We obtain

$$\frac{|\mathbf{A}_i^{(p)}|}{|\mathbf{A}_i|} \approx \frac{e^2N}{8mc^2V} |e^{2ik_0r_0}-1| \frac{1}{k_z^2}. \quad (4.8)$$

We see therefore that the internal-field component $\mathbf{A}_i^{(p)}$ vanishes if $2k_0r_0 = 2\pi n$ ($n = 0, 1, 2\ldots$). In all other cases, at $k_z = 10^4$ cm^{-1} and $N/V = 10^{19}-10^{20}$ cm^{-3} this quantity ranges approximately from 1 to 0.1. Thus, the internal-field component at any one of the atoms, due to allowance for the intermediate positronic states of the electrons of the atom, can be of the same order of magnitude in superradiant processes as the external field at the same atom. This circumstance has enabled us to account for the experimental plot of the photon-echo intensity vs. the concentration.[18]

Calculation of the internal-field component $\mathbf{A}_i^{(e)}$ due to allowance for only the electronic intermediate states in the atom in superradiation processes is also connected with calculation of "lattice sums" (4.6), if we put $\mathbf{p}_j \parallel \mathbf{e}_{fk}$. In addition, it must be recognized that in real systems we have a spread of the natural frequencies of the atoms, leading to an inhomogeneous broadening of the spectral lines. We shall consider samples such that

$$c^{-1}|\Delta\omega|a_{ij} \ll 1, \quad (4.9)$$

where $\Delta\omega$ is the line half-width. Then the internal-field component at the i-th atom, due to allowance for only the electronic intermediate states of the atoms, takes the form[9]

$$\tilde{\mathbf{A}}_i^{(e)} = \int_{\omega_0-\delta}^{\omega_0+\delta} g(\omega)\mathbf{A}_i^{(e)}\, d\omega, \quad (4.10)$$

where the summation in $\mathbf{A}_i^{(e)}$ is over all the indices $j \neq i$, $g(\omega)$ is the line shape, ω_0 is the frequency of the maximum of $g(\omega)$, and δ is a quantity characterizing the integration region. Taking (4.9) into account, we find in the case of a Lorentz line shape that the ratio of the internal-field components $\tilde{\mathbf{A}}_i^{(e)}$ and $\mathbf{A}_i^{(p)}$ produced by $N-1$ surrounding atoms at the location of the i-th atom takes the form

$$\frac{|\tilde{\mathbf{A}}_i^{(e)}|}{|\mathbf{A}_i^{(p)}|} \approx \frac{m\omega_0^2d^2}{\hbar e^2\Delta\omega}. \quad (4.11)$$

If $d \sim 10^{-18}$ cgs esu and $\omega_0 = 10^{15}-10^{13}$ Hz, then the ratio (4.11) is approximately equal to $(10^{13}-10^9)$ sec$^{-1} \cdot \Delta\omega^{-1}$.

At sufficiently large $\Delta\omega$, the ratio (4.11) can then be of the order of unity, and the contribution to the internal field, due to allowance for the positronic intermediate states in the atom, is comparable with the contribution from the electronic intermediate states, which is usually taken into account.

Thus, the internal-field component due to the positron intermediate states of the electrons in the atom is negligibly small in the case of two-atom systems in comparison with the external field at the location of one of the atoms. In coherent processes, in systems consisting of many atoms the picture can be significantly altered because of summation of the contributions made to this internal field by a large number of atoms. Estimates show that in media with large content of active centers, even in the optical band, situations are possible when the influence of the external field in the medium is noticeably weakened by this internal field. Its value, as can be seen from (4.8), depends strongly here on the frequency of the radiated photons. On the other hand, despite the fact that in two-atom systems the internal field $\mathbf{A}_i^{(e)}$ resulting from the electronic intermediate states of the electrons in the atom greatly exceeds, at the corresponding orientations of the electric dipole moments of the atoms, the internal field $\mathbf{A}_i^{(p)}$ resulting from the positronic intermediate states of the electrons, and in coherent processes in many-atom systems they can be comparable in magnitude. The reason is that the frequency of the transition from the electronic state of the atom to an intermediate positronic state is incomparably larger than the frequency of the transition from an electronic state to an intermediate electronic state. Therefore, as seen from (2.6), under the condition (4.9) we can neglect, in the calculation of $\mathbf{A}_i^{(p)}$ of a system of N atoms, the random spread of the natural frequencies of the atoms, which leads to the inhomogeneous broadening of the spectral lines. In the calculation of $\mathbf{A}_i^{(e)}$ however, such a neglect is incorrect. Thus, even though $\mathbf{A}_i^{(e)}$ is proportional to the number of surrounding atoms, the inhomogeneous broadening of the spectral lines greatly decreases the internal field resulting from the electronic intermediate states of the electrons in the atom.

Consequently, in superradiant processes any atom of matter is acted upon by a resultant field consisting of three components: the external field, the field produced when account is taken of only the electronic intermediate states in the atoms (polarization of the medium[7]), and the field produced when account is taken of the positronic intermediate states of the electrons of the atoms. The last contribution can be naturally called the positronic polarization of the medium. In the case of sufficiently large inhomogeneous broadening, as follows from (4.11), the positron polarization can exceed the ordinary (electron) polarization and can be observed in investigations of superradiant processes such as photon induction and photon echo.

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Translated by J. G. Adashko

Investigation of the absorption spectrum of a two-level system in intense nonmonochromatic radiation fields

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(Submitted June 29, 1975)
Zh. Eksp. Teor. Fiz. **70**, 445–457 (February 1976)

The change in the absorption line shape of weak radiation from two-level systems (Zeeman splitting levels of Cd^{113} atoms) under the action of Gaussian noise emission is investigated. Under strong nonresonance action, increase in the noise field power resulted in a shift of the absorption line peak and its broadening, and also in a strong growth in the fluctuations of the absorption index. In the region of low saturation, the line broadening is proportional to the square of the power, whereas it is proportional to the noise field power in the case of high saturation. A theory of the effect of nonresonance noise fields is developed. The results of the theory are in good agreement with the experimental results, and are used to predict the nature of the change in the atomic energy structure in nonresonance radiation fields of multimode lasers. An amplification of the weak radiation is observed in the action of a resonance or quasiresonance strong noise field. The maximum attainable value of the field decreases with increase in the spectral width of the noise field line.

PACS numbers: 32.10.Vc, 32.10.Dk, 42.50.+q

1. INTRODUCTION

We have communicated previously[1] on the observation of the change of the absorption spectrum of weak probing radiation of a two-level system (the levels of the Zeeman splitting of atoms of cadmium) under the action of an intense rotating monochromatic radiation field. For strongly nonresonant action, a shift of the frequency of the single phonon absorption line of atoms without change in its width is observed. In cases of the coincidence of the frequency of the strong field and the frequency of the atomic transition or of an insignificant difference between them, splitting of the absorption lines is observed, proportional to the field amplitude, and a reversal of the sign of the absorption index in some parts of the line, without creation of inversion of population levels. The width of the components of the splitting was close to the width of the initial absorption line. It was established that the usual course of observed changes in the shape of the absorption spectrum, the values of the line shift and splitting, and also the values of the absorption peaks agree with predictions of existing theory.

The investigation of the change in the absorption spectrum of a two-level system under the action of irregular nonmonochromatic radiation fields is of considerable interest. Such fields are as a rule generated by multimode lasers in regimes without mode locking. Since the instantaneous values of the shifts and splitting of the absorption line depend on the amplitude of the existing field, it follows that in the case of its irregular change with time, one can expect not only splitting and a shift of the absorption line, but also its broadening and the generation of significant fluctuations of the absorption index relative to its mean value. It is clear that the dynamics of these fluctuations and the shape of the absorption line will be determined in principle by the character of the statistics of the operating field.

The theoretical analysis of the excitation of the spectrum of even the simplest two-level system by an irregular intense radiation field presents a very complicated problem, general methods of the solution of which for an arbitrary statistics of the operating field do not exist at the present time. A more complete theoretical investigation of the dynamics of two-level systems has been carried out for the case in which the radiation field is a purely discontinuous random Markov process.[2,3] The use of such a type of radiation field enables us to write down a closed set of equations for the averaged density matrix elements and to compute the dynamics of the change in the mean level populations[2] and the spectrum of mean values of the absorption coefficient of of weak probing radiation.[3] The most important results of these researches lie in the prediction of the field broadening of the lines of atomic absorption and the possibility, as in the case of a monochromatic field, of reversal of the sign of the absorption coefficient without inversion of the average level population under the action of resonance radiation. The model of the