tion ($a \gg 1$)—the coefficients of proportionality in the latter case are different, and the ratio of the expression (8.5b) to (8.5a) is equal to $\sqrt{\pi}/2 \approx 0.887$.

It can be shown that the range of applicability of the result (8.5) is not limited to the condition $\nu T_2 \ll 1$. Moreover, the indicated result is valid independently of the degree of correlation (γ) of the random process.

[1)]An exception is the previously mentioned real normal process, strictly resonant to the atomic transition.

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Acceleration of atoms by a strong resonance field

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A consistent investigation is carried out of the acceleration of an atom in the field of a traveling monochromatic wave, taking into account the quantization of the translational coordinates of the atom. It is shown that in the case of metastable working levels the momentum distribution arising in the process of acceleration is exponential and, consequently, its width Δp is of the order of the average momentum $\langle p \rangle$ transferred to the atom. Thus, the usually adopted description by means of an average force is incorrect in the general case. But if the lower working level is the ground level, then in the case of a large number of photons scattered by the atom $n \gg 1$ the momentum distribution is Gaussian, with $\Delta p \ll \langle p \rangle$. The origin of the uncertainty Δp is determined by two causes: recoil on spontaneous emission of photons and the uncertainty Δn in the number of photons scattered by the atom. It is shown that for $n \gg 1$ the first cause always leads to a small uncertainty $\Delta p \ll \langle p \rangle$, while it is specifically the second cause that leads to a large uncertainty $\Delta p \sim \langle p \rangle$ in the case of metastable levels.

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1. INTRODUCTION

Acceleration of atoms by a resonance field has been investigated in a number of papers[1–4] on the assumption that both the field and the translational motion of the atom can be treated as classical. The effect of acceleration was described by means of an average force

$$\mathbf{F}(t) = \nabla (\mathbf{E}(\mathbf{r}, t) \langle \mathbf{d}(t) \rangle), \qquad (1)$$

where ∇ operates only on the intensity $\mathbf{E}(\mathbf{r}, t)$ of the electric field, and $\langle \mathbf{d}(t) \rangle$ is the quantum average of the dipole moment of the atom.[2–4]

The quantum fluctuations in this force were taken into

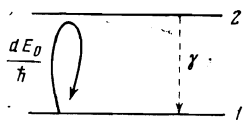

FIG. 1.

account by Kazantsev[5] in calculating the passage of an atomic beam across the boundary of separation between vacuum and the resonance field. In this case $\langle \mathbf{d}(t) \rangle = 0$, but as a result of fluctuations in the dipole moment the atoms experience the influence of the resonance field, and their motion has quantum features: double refraction of the atomic beam occurs.

In this paper we have investigated the quantum features of the acceleration of an atom in the homogeneous field of a traveling monochromatic wave. The atom is accelerated as a result of absorption of photons of the traveling wave and of spontaneous emission of a spherical wave (as a result of each induced absorption and a subsequent spontaneous emission of a photon the atomic acquires a momentum equal to the momentum of the photon of the traveling wave $\hbar \mathbf{k}_0$).

The effect of acceleration in such a field was investigated by Kazantsev.[2,4][1)] The quantum fluctuations of the force (1) were not taken into account in this investigation. However, their magnitude can be of the order of the force itself in view of the fact that the behavior of the dipole moment of an atom is essentially a quantum phenomenon. In this case the uncertainty in the momentum Δp becomes of the order of the momentum p transferred from the field, and therefore the translational motion of the atom must be treated in a quantum manner.

We have calculated the acceleration of an atom on the basis of a complete density matrix which contains both the internal coordinates of the atom and its translational degrees of freedom. In doing so we assumed that the coherent field acting on the atom is sufficiently intense so that the classical description of it remains valid.

Thus, the improvement of the methodology utilized by Kazantsev[2,4] consists of the replacement of the classical description of the translational motion of the atom by a consistent quantum mechanical description. It is shown that along with the situation in which such a replacement is not mandatory ($\Delta p \ll p$, and consequently one can assume that the atom moves along a classical trajectory under the action of the force (1))there exists a situation in which this replacement is mandatory ($\Delta p \sim p$, and the concept of the classical trajectory of motion becomes meaningless even for very heavy atoms).

In Sec. 2 it is established that the former situation ($\Delta p \ll p$) occurs if the lower working level is the ground level and $\sqrt{\langle n \rangle} \gg 1$ ($\langle n \rangle$ is the average number of absorbed photons). In this case the momentum distribution is Gaussian. In Sec. 3 the acceleration is calculated for the case of metastable working levels and it is shown that in this case $\Delta p \sim p$, while the momentum distribution is exponential.

2. ACCELERATION OF TWO-LEVEL ATOMS

We consider the acceleration of an atom by a traveling wave of constant amplitude whose frequency ω_0 coincides with ω_{21}—the frequency of transition between two nondegenerate levels of the atom. In doing so we assume that the lower working level of the atom is the ground level. The energy levels of the atom and the transitions between them are shown in Fig. 1.

We describe the state of the atom by means of the complete density matrix $\rho_{ik}(\mathbf{r}_1, \mathbf{r}_2)$, which depends both on the internal coordinates (i, k), and on the coordinates of the centre of mass $(\mathbf{r}_1, \mathbf{r}_2)$. In order to obtain equations which determine the variation of this matrix with time, we utilize Eqs. (A.3) and (A.6), which describe the spontaneous emission of photons (cf., Appendix A), and the Hamiltonian for the atom whose matrix elements are

$$H_{11}=0,\ H_{22}=\hbar\omega_{21},\ H_{12}=-dE_0 \exp\{i(\omega_0 t-\mathbf{k}_0\mathbf{r})\}, \tag{2}$$

where E_0 is the amplitude of the electric field, d is the matrix element of the dipole moment, and $\mathbf{k}_0$ is the propagation vector. The Hamiltonian (2) contains the internal energy of the atom and the energy of interaction with the classical electromagnetic field (it is assumed that the kinetic energy of the atom $T \ll dE_0$ and therefore one can neglect the effect of the motion of the atom on the process of interaction). Thus, the equations for $p_{ik}(\mathbf{r}_1, \mathbf{r}_2)$ have the form

$$\begin{aligned} \dot\rho_{11}&=-Va_0^*\sigma+\gamma R\rho_{22}, \quad \dot\rho_{22}=V\sigma-\gamma\rho_{22}, \\ \dot\sigma&=2V(a_0\rho_{11}-\rho_{22})-{}^1/_2\gamma\sigma, \end{aligned} \tag{3}$$

where R is defined in accordance with (A.7), $V = dE_0/\hbar$,

$$\sigma=i\{\rho_{12}\exp[i(\mathbf{k}_0\mathbf{r}_1-\omega_0 t)]-\rho_{21}\exp[-i(\mathbf{k}_0\mathbf{r}_2-\omega_0 t)]\}, \tag{4}$$

$$a_0=\exp[i\mathbf{k}_0(\mathbf{r}_1-\mathbf{r}_2)] \tag{5}$$

We adopt the following initial conditions: the atom is in the ground state, and its translational motion is described by the density matrix $P_0(\mathbf{r}_1, \mathbf{r}_2)$. The matrix P_0 can, in particular, correspond to the state of the atom which has a distribution both in coordinate space, and in velocity space sufficiently narrow so that the initial motion of the atom can be described classically.

We trace the evolution in time of the translational motion of the atom (the process of acceleration) by utilizing the density matrix $P(t) = \rho_{11}(t) + \rho_{23}(t)$, obtained by taking the trace of the complete density matrix with respect to the internal variables. For this we take the Laplace transform

$$\tilde f=\int_0^\infty e^{-st}f(t)\,dt$$

of the system (3). Solving the equations for the Laplace transforms $\tilde\rho_{11}$, $\tilde\rho_{22}$ and $\tilde\sigma$, we obtain

$$\tilde P=\frac{(s+\gamma)(s+\gamma/2)+2V^2(1+a_0)}{s(s+\gamma/2)(s+\gamma)+4V^2 s+2V^2\gamma(1-a_0R)}. \tag{6}$$

Thus, the solution has the form

$$P(t)=\sum_{i=1}^{3} A_i e^{s_i t}, \tag{7}$$

where s_i are the roots of the denominator of expression (6).

We continue the further investigation in two limiting cases—of rapid and slow spontaneous emission:

A. Rapid spontaneous emission ($\gamma >> V$)

In this case the roots s_i are determined by the expressions

$$s_1=-u(1-a_0R),\ s_2=-\gamma/2-2ua_0R,\ s_3=-\gamma+u(1+a_0R), \tag{8}$$

where $u=4V^2/\gamma$. Since

$$|A_2|\sim|A_3|\sim\frac{u}{\gamma}|P_0|\ll|A_1|=|P_0|,$$

the density matrix is given by

$$P(t)=\exp[u(a_0R-1)t]P_0=e^{-ut}\sum_{n=0}^{\infty}\frac{(uta_0R)^n}{n!}P_0. \tag{9}$$

To what state of the atom does the matrix (9) correspond? We note first of all that $P(\mathbf{r},\mathbf{r})=P_0(\mathbf{r},\mathbf{r})$, i.e., the atom is at rest (we recall that we have not included the kinetic energy in the Hamiltonian and thereby have neglected the displacements of the atom during the time of acceleration).

In contrast to the distribution of the coordinates of the atom, the momentum distribution undergoes essential changes, for the determination of which we write the matrix (9) in the momentum representation (in doing so we replace R by the operator $\hat{R}$, defined in (A.9), and we take into account the fact that multiplication by a_0^n describes the increase in the momentum of the atom by the amount $\hbar\mathbf{k}_0 n$[8]):

$$P(\mathbf{p}_1,\mathbf{p}_2)=e^{-ut}\sum_{n=0}^{\infty}\frac{(ut)^n}{n!}\hat{R}^n P_0(\mathbf{p}_1-n\hbar\mathbf{k}_0,\mathbf{p}_2-n\hbar\mathbf{k}_0), \tag{10}$$

where $P_0(\mathbf{p}_1,\mathbf{p}_2)$ is the initial density matrix in the momentum representation. From (10) it follows that the probability for the scattering of n photons is equal to

$$\frac{(ut)^n}{n!}e^{-ut},$$

i.e., n is characterized by a Poisson distribution with the average value $\langle n\rangle=ut$, where u is the probability of photon absorption per unit time.

The average momentum transferred to the atom is given by

$$\langle\mathbf{p}\rangle=\hbar\mathbf{k}_0\langle n\rangle=\hbar\mathbf{k}_0 ut \tag{11}$$

and together with the displacement of the center of the distribution a smearing out of it occurs due to two causes: spontaneous emission (this smearing out is described by the operator $\hat{R}$, cf., Appendix B) and by the increase in the uncertainty

$$\Delta n=\sqrt{\langle n^2\rangle-\langle n\rangle^2}=\sqrt{ut}.$$

The former cause leads to an increase in the uncertainty in all the components of the momentum, while the latter leads to an increase in the uncertainty only in the component of the momentum along an axis parallel to $\mathbf{k}_0$ (the z axis). We obtain the shape of the momentum distribution. Since for $\langle n\rangle\gg1$ the distribution of n is well approximated by a Gaussian, while the distribution to which the emission of photons gives rise in the case $n\gg1$ (Appendix B) is also Gaussian, then the distribution of all the components of momentum turns out to be of the same kind, with

$$\begin{aligned}\Delta p_x^2&=\alpha_x(\hbar k_0)^2\langle n\rangle=\alpha_x(\hbar k_0)^2 ut,\\ \Delta p_y^2&=\alpha_y(\hbar k_0)^2\langle n\rangle=\alpha_y(\hbar k_0)^2 ut,\\ \Delta p_z^2&=\alpha_z(\hbar k_0)^2\langle n\rangle+(\hbar k_0)^2(\Delta n)^2=(1+\alpha_z)(\hbar k_0)^2 ut.\end{aligned} \tag{12}$$

The quantities α_i are characterized by $\alpha_i\sim1$. Their values are given in Appendix B in expressions for the isotropic scattering of photons (B.7), for the scattering by the atoms of a wave linearly polarized along the x axis (B.8), and of a circularly polarized wave (B.9).

Thus since in the case discussed here

$$\Delta p_i\sim\hbar k_0\sqrt{\langle n\rangle},\quad a\quad|\langle\mathbf{p}\rangle|=\hbar k_0\langle n\rangle,$$

then for $\sqrt{\langle n\rangle}\gg1$ we have $\Delta p_i\ll|\langle\mathbf{p}\rangle|$, i.e., the approximation of a classical trajectory is acceptable.

B. Slow spontaneous emission ($\gamma << V$)

In this case the roots of the demonimator of expression (6) will be given by:

$$\begin{aligned}s_1&=-\frac{\gamma}{2}(1-a_0R),\quad s_2=2iV-\frac{\gamma}{2}\left(1+\frac{a_0R}{2}\right),\\ s_3&=-2iV-\frac{\gamma}{2}\left(1+\frac{a_0R}{2}\right),\end{aligned} \tag{13}$$

and for large times $t\gg\gamma^{-1}$ we obtain from (6) and (7)

$$P(t)=A_1e^{s_1t}\doteq e^{-\gamma t/2}\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{\gamma t}{2}\right)^n a_0^n R^{n-1}\left(\frac{R}{2}+\frac{n}{\gamma t}\right)P_0. \tag{14}$$

In formula (14) $A_2e^{s_2t}$ and $A_3e^{s_3t}$ are omitted because the momentum distribution is determined largely by terms for which $n\approx\gamma t/2\gg1$, while taking e^{s_2t} and e^{s_3t} into account corrects these terms only by a quantity of the order of smallness of $(\frac{1}{2})^{\gamma t/2}$.

Comparing (9) and (14) we reach the conclusion that (14) describes the same physical situation as (9). Just as in the case $\gamma\gg V$, in the case under consideration the number of absorbed photons is distributed in accordance with the Poisson distribution, but $\langle n\rangle=\gamma t/2$. From the last expression it follows that the rate of absorption of photons is equal to $\gamma/2$, and not to u, as in the preceding case. Such a value for the rate has a simple explanation: the rate is equal to the product of the probability of spontaneous emission of a photon γ by the average population of the upper working level, which is equal to $\frac{1}{2}$ due to the fact that the intense field gives rise to frequent transitions (during a time $V^{-1}\ll\gamma^{-1}$) between levels.

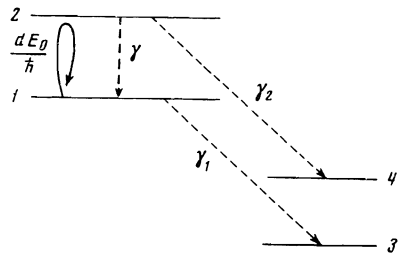

FIG. 2.

The distributions of the components of the momentum are Gaussian, and their average values and widths are determined by Eqs. (11) and (12), if in them we replace u by $\gamma/2$. Thus, in the case $\gamma \ll V$, and also in the case of rapid spontaneous emission, $\Delta p_i \ll |\langle \mathbf{p} \rangle|$ when the number of absorbed photons is large and the concept of the average accelerating force is both appropriate and useful. The situation is quite different in the case of excited states of an atom to the investigation of which we now proceed.

3. METASTABLE LEVELS

We calculate the acceleration of atoms which have metastable working levels. The level scheme and the transitions between the levels are shown in Fig. 2. In order to carry out the calculation we utilize the system of equations (3), having added to it the spontaneous transitions to levels 3 and 4. Moreover, we assume the existence of detuning $\Delta = \omega_{21} - \omega_0$. Then the initial equations will be of the following form:

$$\begin{gathered}\dot\rho_{11}=-Va_0^*\sigma+\gamma R\rho_{22}-\gamma_1\rho_{11}, \quad \dot\rho_{22}=V\sigma-(\gamma+\gamma_2)\rho_{22},\\ \dot\rho_{33}=\gamma_1R_1\rho_{11}, \quad \dot\rho_{44}=\gamma_2R_2\rho_{22},\\ \dot\sigma=2V(a_0\rho_{11}-\rho_{22})-\xi\Delta-{}^1/_2(\gamma+\gamma_1+\gamma_2)\sigma,\\ \dot\xi=\sigma\Delta-{}^1/_2(\gamma+\gamma_1+\gamma_2)\xi,\end{gathered} \tag{15}$$

where σ and a_0 are determined by expressions (4) and (5),

$$\xi=\rho_{12}\exp[i(\mathbf{k}_0\mathbf{r}_1-\omega_0t)]+\rho_{21}\exp[-i(\mathbf{k}_0\mathbf{r}_2-\omega_0t)],$$

while R_1 and R_2 describe the uncertainty in the momentum originating in the spontaneous transitions $1 \to 3$ and $2 \to 4$. The quantities R_1 and R_2 are determined by the frequencies of the corresponding transitions similarly to the definition of R in Appendix A.

The atom will be accelerated under the action of the field only in the case if at the initial moment of time the population of at least a single working level differs from 0. We assume that for $t=0$

$$\rho_{11}=P_0(\mathbf{r}_1,\ \mathbf{r}_2), \quad \rho_{22}=\rho_{33}=\rho_{44}=0. \tag{16}$$

Taking into the account the initial conditions (16) we calculate the momentum transferred to the atom during a time which is greater than the lifetime in the metastable levels 1 and 2. During such a time the atom goes over into the states 3 and 4 and, consequently, the process of acceleration ceases, while the momentum distribution is determined by the sum $\rho_{33}+\rho_{44}$ for $t=\infty$, while in accordance with (15), we have

$$\rho_{33}(\infty)=\gamma_1R_1\bar\rho_{11}, \quad \rho_{44}(\infty)=\gamma_2R_2\bar\rho_{22}, \tag{17}$$

where

$$\bar\rho_{ii}=\int_0^\infty\rho_{ii}(t)\,dt$$

is the zeroth component of the Laplace transform. Solving the equations for $\bar\rho_{11}$, $\bar\rho_{22}$, $\bar\sigma$ and $\bar\xi$ obtained from (15) we obtain

$$\begin{gathered}\rho_{33}(\infty)=\left(1-w-\frac{\gamma_2w}{\gamma}\right)\frac{R_1P_0}{1-wa_0R}=\left(1-w-\frac{\gamma_2w}{\gamma}\right)\sum_{n=0}^{\infty}(wa_0R)^nR_1P_0,\\ \rho_{44}(\infty)=\frac{\gamma_2wa_0R_2P_0}{\gamma(1-wa_0R)}=\frac{\gamma_2w}{\gamma}\sum_{n=0}^{\infty}(wa_0R)^na_0R_2P_0,\end{gathered} \tag{18}$$

where

$$w=\frac{\gamma u_1}{u_1(\gamma+\gamma_1+\gamma_2)+\gamma_1(\gamma+\gamma_2)}, \tag{19}$$

$$u_1=\frac{4(\gamma+\gamma_1+\gamma_2)V^2}{4\Delta^2+(\gamma+\gamma_1+\gamma_2)^2}. \tag{20}$$

The expansion that has been carried out in terms of (wa_0R) is correct, since $w<1$, $|a_0|=1$ and, in accordance with (A.7), $|R| \le 1$. For the convenience of further analysis we obtain from (18) expressions for the probability of scattering of n photons $\mathscr{W}(n)$ and $\langle n \rangle$:

$$\mathscr{W}(n)=\begin{cases}1-w-\gamma_2w/\gamma & \text{for} \quad n=0,\\ (1-w)(1+\gamma_2/\gamma)w^n & \text{for} \quad n=1,2,\ldots,\end{cases} \tag{21}$$

$$\langle n\rangle=\frac{w}{1-w}\left(1+\frac{\gamma_2}{\gamma}\right). \tag{22}$$

Utilizing the solution (18)–(22) we determine the shape and the width of the momentum distribution. First of all we note that since the uncertainty due to spontaneous emission is $\Delta p_i \sim \hbar k_0\sqrt{n}$, then one can expect that the condition $\Delta p_i \ll |\langle \mathbf{p} \rangle|$ will be satisfied only in the case $\sqrt{\langle n\rangle} \gg 1$. In this case the uncertainty associated with spontaneous emission is known to be small and it is necessary to investigate only the uncertainty associated with the distribution of the number of scattered photons n. In accordance with (21) we have

$$\mathscr{W}(n) \propto w^n=\exp(-n\ln w^{-1})$$

(we recall that $w<1$), i.e., the distribution of n is exponential; the width of such a distribution is of the order of the average value, and therefore $\Delta p_z \sim \langle p_z \rangle$ (the photon momentum is directed along the z axis).

Thus, the distribution of the component of the momentum of the atom p_z (in the case of $\langle n\rangle \gg 1$, when one can neglect the uncertainty due to spontaneous emission) is well described by the exponential law

$$dW(p_z)=\exp\left(-\frac{p_z}{\langle p_z\rangle}\right)\frac{dp_z}{\langle p_z\rangle},\quad p_z\geqslant 0. \tag{23}$$

Since for $\langle n\rangle \gg 1$ it follows from (19) and (22) that $1-w \ll 1$, $\gamma_1+\gamma_2 \ll \gamma$ and $\gamma_1 \ll u_1$, we have

$$\langle p_z\rangle=\hbar k_0\frac{\gamma u_1}{\gamma_1(u_1+\gamma)+\gamma_2 u_1}, \tag{24}$$

while the width is given by

$$\Delta p_z=\sqrt{\langle p_z^2\rangle-\langle p_z\rangle^2}=\langle p_z\rangle.$$

The exponential nature of the distribution is a consequence of the exponential character of the spontaneous deactivation of metastable working levels. In order to illustrate this we consider the case $\langle n\rangle \gg 1$ the necessary and sufficient conditions for the realization of which are $u_1 \gg \gamma_1$ and $\gamma_1+\gamma_2 \ll \gamma$. The first condition (in accordance with (20), $u_1 \propto E_0^2$) indicates that the rate of induced transitions between working levels dominates the rate of spontaneous deactivation of the lower level γ_1. The second condition is also necessary in order to guarantee multiple scattering of photons during the time that an atom spends in the working levels, since the rate of scattering of photons by an atom can not exceed γ. Since γ_1 and γ_2 are small the acceleration process occurs in the manner described in Sec. 2: the atom acquires a momentum, and $p_z=Ma_z t$ (M is the mass of the atom, a_z is the acceleration). The probability of transition to levels 3 and 4 in a state with momentum p_z is proportional to

$$e^{-t/\tau}=\exp(-p_z/Ma_z\tau)$$

as a result of the exponential deactivation of working levels (τ is the characteristic deactivation time). Such is the physical mechanism accounting for the origin of the exponential momentum distribution in states 3 and 4.

Thus, in the case of metastable levels and in the presence of a large number of scattered photons $\langle n\rangle \gg 1$ only the uncertainty in the transverse components of the momentum is small:

$$\Delta p_x=\Delta p_y\sim\hbar k_0\langle n\rangle^{1/2}\ll|\langle \mathbf{p}\rangle|.$$

But the uncertainty in the longitudinal component $\Delta p_z=\langle p_z\rangle$ and, consequently, the acceleration of the atom can not be described by means of an average force (1), and one can not introduce the concept of a classical trajectory for the motion of an accelerated atom. This circumstance should also be kept in mind when discussing the problem of separating out from a multicomponent atomic beam the atoms interacting with the field in a resonant manner, since the exponential smearing out of the beam hinders the selection of accelerated particles.

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APPENDIX A

We obtain the equations for the density matrix $\rho_{ik}(\mathbf{r}_1, \mathbf{r}_2)$, which describe spontaneous emission and which take into account the change in the momentum of an atom accompanying the emission of photons (such equations have been earlier utilized by Vorobe'ev, Rautian and Sokolovskiĭ[9]). We start with the Schrödinger equation for the total wave function containing the coordinates of the atom and of the electromagnetic field.[10] We write this equation for the total wave function containing the coordinates of the atom and of the electromagnetic field.[10] We write this equation in the interaction representation (in doing so we assume that the kinetic energy of the atom is small and that therefore it can be left out from the Hamiltonian of the system):

$$\dot{c}_2(t,\mathbf{r})=-\frac{i}{\hbar}\sum_{\mathbf{k}}H_{2\mathbf{k}}\exp[i(\omega_{21}-\omega)t+i\mathbf{kr}]c_{\mathbf{k}}(t,\mathbf{r}), \tag{A.1a}$$

$$\dot{c}_{\mathbf{k}}(t,\mathbf{r})=-\frac{i}{\hbar}H_{\mathbf{k}2}\exp[i(\omega-\omega_{21})t-i\mathbf{kr}]c_2(t,\mathbf{r}), \tag{A.1b}$$

$$\dot{c}_1(t,\mathbf{r})=0. \tag{A.1c}$$

Here ω_{21} is the frequency of transition between the states of the atom $|2\rangle$ (excited state) and $|1\rangle$ (ground state) $i\omega$ and $\mathbf{k}$ are the frequency and the propagation vector of the photon; $c_2(t,\mathbf{r})$ is the probability amplitude of the state which contains no photons, and the atom is situated at the point $\mathbf{r}$ and in the level $|2\rangle$; $c_1(t,\mathbf{r})$ corresponds to an atom in level $|1\rangle$; $c_{\mathbf{k}}(t,\mathbf{r})$ corresponds to the case when there is an atom in level $|1\rangle$ and there exists a single photon of momentum $\hbar\mathbf{k}$; $H_{2\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}}$ is the matrix element of the Hamiltonian.

Assuming that for $t=0$ there are no light quanta (i.e., only $c_1(0,\mathbf{r})$ differ from zero), and utilizing the solution of the system (A.1) proposed by Weisskopf and Wigner (it is given in [10]) we derive an equation for the matrix elements $\rho_{ik}(\mathbf{r}_1, \mathbf{r}_2)$ which can be expressed in terms of the amplitudes which we have introduced;

$$\rho_{22}(\mathbf{r}_1,\mathbf{r}_2)=c_2(t,\mathbf{r}_1)c_2^*(t,\mathbf{r}_2), \tag{A.2a}$$

$$\rho_{11}(\mathbf{r}_1,\mathbf{r}_2)=c_1(t,\mathbf{r}_1)c_1^*(t,\mathbf{r}_2)+\int c_{\mathbf{k}}(t,\mathbf{r}_1)c_{\mathbf{k}}^*(t,\mathbf{r}_2)\rho_{\mathbf{k}}\,d\omega\,d\Omega, \tag{A.2b}$$

$$\rho_{12}(\mathbf{r}_1,\mathbf{r}_2)=c_1(t,\mathbf{r}_1)c_2^*(t,\mathbf{r}_2), \tag{A.2c}$$

where $\rho_{\mathbf{k}}$ is the number of oscillators per unit volume, the frequency of which lies in the interval $d\omega$, while the direction of the vector $\mathbf{k}$ is restricted within $d\Omega$. We obtain the equations

$$\dot{\rho}_{12}(\mathbf{r}_1,\mathbf{r}_2)=-\frac{\gamma}{2}\rho_{12}(\mathbf{r}_1,\mathbf{r}_2),\quad \dot{\rho}_{22}(\mathbf{r}_1,\mathbf{r}_2)=-\gamma\rho_{22}(\mathbf{r}_1,\mathbf{r}_2) \tag{A.3}$$

by differentiating (A.2c) and (A.2a) taking into account (A.1c) and the exponential law for the decay (with a lifetime γ^{-1}) of the amplitude c_2[10]:

$$c_2(t,\mathbf{r})=c_2(0,\mathbf{r})\exp(-\gamma t/2). \tag{A.4}$$

For $\dot{\rho}_{11}$, differentiating (A.2b) and substituting into the expression so obtained (A.1b) and

$$c_{\mathbf{k}}(t,\mathbf{r})=-\frac{1}{\hbar}H_{\mathbf{k}2}e^{-i\mathbf{k}\mathbf{r}}\frac{\exp(\gamma t/2)-\exp[i(\omega-\omega_{21})t]}{\omega_{21}-\omega-i\gamma/2}c_2(t,\mathbf{r})$$

(the last equation follows from (A.1b) and (A.4)), we obtain

$$\dot{\rho}_{11}=\rho_{22}\int\frac{2}{\hbar^2}\operatorname{Im}\left\{\frac{\exp[i(\omega_{21}-\omega)t+\gamma t/2]-1}{\omega_{21}-\omega-i\gamma/2}\right\}\rho_{\mathbf{k}}\,d\omega\int a(-\mathbf{k})\,|H_{\mathbf{k}2}|^2\,d\Omega, \tag{A.5}$$

where $a(\mathbf{k})=\exp[i\mathbf{k}\cdot(\mathbf{r}_1-\mathbf{r}_2)]$. Since $|\mathbf{k}|\approx\omega_{21}/c$ the right hand side of (A.5) is a product of integrals over $d\omega$ and $d\Omega$. Then utilizing the equation

$$\frac{2}{\hbar^2}\int\operatorname{Im}\left\{\frac{\exp[i(\omega_{21}-\omega)t+\gamma t/2]-1}{\omega_{21}-\omega-i\gamma/2}\right\}\rho_{\mathbf{k}}\,d\omega\int|H_{\mathbf{k}2}|^2\,d\Omega=\gamma$$

(cf., [10] formula (18, 8)), we obtain from (A.5)

$$\dot{\rho}_{11}(\mathbf{r}_1,\mathbf{r}_2)=\gamma R\rho_{22}(\mathbf{r}_1,\mathbf{r}_2), \tag{A.6}$$

where

$$R=\int a(-\mathbf{k})\,\Phi(\mathbf{k}/|\mathbf{k}|)\,d\Omega,\qquad |\mathbf{k}|=\omega_{21}/c, \tag{A.7}$$

while the "direction function"

$$\Phi(\mathbf{k}/|\mathbf{k}|)=|H_{\mathbf{k}2}|^2\left[\int|H_{\mathbf{k}2}|^2\,d\Omega\right]^{-1} \tag{A.8}$$

is the probability for the emission of a photon within $d\Omega$.

Thus, the sought equations describing the evolution of the state of the atom in the case of spontaneous emission are (A.3) and (A.6). The change in the momentum of the atom as a result of recoil is taken into account in these equations by the factor R. We examine how the momentum distribution varies when the density matrix is multiplied by R, i.e., as a result of the emission of a single photon. Since in accordance with[8], multiplication by $a(\mathbf{k})$ corresponds to an increase in the momentum of the atom by an amount $\hbar\mathbf{k}$, then the product $R\rho(\mathbf{r}_1,\mathbf{r}_2)$ corresponds in the momentum representation to

$$\int\rho(\mathbf{p}_1+\hbar\mathbf{k},\mathbf{p}_2+\hbar\mathbf{k})\,\Phi(\mathbf{k}/|\mathbf{k}|)\,d\Omega=\hat{R}\rho(\mathbf{p}_1,\mathbf{p}_2), \tag{A.9}$$

where the integration is carried out over all the directions of the vector $\mathbf{k}$. Consequently, the "coordinate" factor R in going over to the momentum representation should be replaced by the operator $\hat{R}$, the definition of which (A.9) specifies the nature of the variation of the distribution function $\rho(\mathbf{p},\mathbf{p})$. According to this law the width of the momentum distribution increases as a result of spontaneous emission of photons. The regularities to which this broadening is subject are examined in Appendix B.

APPENDIX B

We consider the variation in the momentum distribution to which spontaneous emission gives rise. If after the emission of n photons we have a distribution for the component of the momentum

$$dW(p_i)=\rho(p_i,\ p_i)\,dp_i=f_n(k)\,dk,\quad k=p_i/\hbar, \tag{B.1}$$

then in view of the fact that the variation of this distribution brought about by the omission of a single photon is described (cf., Appendix A) by multiplying the density matrix by $\hat{R}$:

$$\hat{R}\rho(p_i,p_i)=\int\rho(p_i+\hbar k_0\mu_i,\ p_i+\hbar k_0\mu_i)\,\Phi(\boldsymbol{\mu})\,d\Omega=\frac{1}{\hbar}\int j_n(k+k_0\mu_i)\,\Phi(\boldsymbol{\mu})\,a\Omega$$

(here $k_0=|\mathbf{k}|=\omega_{21}/c$ and $\boldsymbol{\mu}=\mathbf{k}/k_0$), we obtain the distribution function arising after the emission of $(n+1)$ photons in the following form:

$$f_{n+1}(k)=\int f_n(k+k_0\mu_i)\,\Phi(\boldsymbol{\mu})\,d\Omega. \tag{B.2}$$

The expression (B.2) in principle allows us to determine the distribution function $f_n(k)$ in terms of its initial form $f_0(k)$. However, it is more convenient to study $f_n(k)$ by starting with the moments of the distribution

$$\langle k^{2m}\rangle_n=\int_{-\infty}^{+\infty}k^{2m}f_n(k)\,dk \tag{B.3}$$

(it is sufficient to take the even moments, if $f_0(k)=f_0(-k)$). We express these moments in terms of $\langle k^{2l}\rangle_{n-1}$, $l=1,2,\ldots,m$. Utilizing (B.2) we obtain

$$\langle k^{2m}\rangle_n=\int_{-\infty}^{+\infty}f_{n-1}(k)\,dk\int(k-k_0\mu_i)^{2m}\Phi(\boldsymbol{\mu})\,d\Omega,$$

from which, taking into account $\Phi(\boldsymbol{\mu})=\Phi(-\boldsymbol{\mu})$, we obtain the expression

$$\langle k^{2m}\rangle_n=\sum_{l=0}^{m}\langle k^{2l}\rangle_{n-1}C_{2m}^{2l}k_0^{2(m-l)}\int\mu_i^{2(m-l)}\,\Phi(\boldsymbol{\mu})\,d\Omega, \tag{B.4}$$

which will serve as the basis for the subsequent analysis.

First of all, we obtain from (B.4) the width of the distribution $\sqrt{\langle k^2\rangle_n}$:

$$\langle k^2\rangle_n=\alpha_i k_0^2+\langle k^2\rangle_{n-1}=n\alpha_i k_0^2+\langle k^2\rangle_0,$$

where

$$\alpha_i=\int\mu_i^2\Phi(\boldsymbol{\mu})\,d\Omega. \tag{B.5}$$

Thus, in the case of a small initial width we have

$$\Delta p_i=\hbar\sqrt{\langle k^2\rangle_n}=\hbar k_0\sqrt{n\alpha_i}. \tag{B.6}$$

The quantities α_i are defined by means of formula (B.5).

For example, in the case $\Phi(\mu)=1/4\pi$ (isotropic spontaneous emission)

$$\alpha_x=\alpha_y=\alpha_z={}^1/_3; \tag{B.7}$$

in the case of a linear dipole (along the x axis)

$$\Phi(\mu)=\frac{3}{8\pi}(1-\mu_x^2),\quad \alpha_x=\frac{1}{5},\quad \alpha_y=\alpha_z=\frac{2}{5}, \tag{B.8}$$

and in the case of a dipole rotating in the xy plane

$$\Phi(\mu)=\frac{3}{16\pi}(1+\mu_z^2),\quad \alpha_x=\alpha_y=\frac{3}{10},\quad \alpha_z=\frac{2}{5}. \tag{B.9}$$

We note that $\alpha_x+\alpha_y+\alpha_z=1$ and therefore $\Delta\mathbf{p}^2=n(\hbar k_0)^2$.

Finally we show that for $n\gg 1$ the distribution $f_n(k)$ (with the exception of its wings) is Gaussian. In order to do this we prove that the moments (B.3) have for $m\ll n$ the properties of the moments of a Gauss distribution

$$\langle k^{2m}\rangle=(2m-1)!!(\langle k^2\rangle)^m. \tag{B.10}$$

The restriction $m\ll n$ means that only the centre of the distribution $f_n(k)$ is Gaussian.

We carry out the proof by the method of mathematical induction. It is evident that (B.10) is satisfied for $m=1$. Assuming that (B.10) is satisfied for $m=2, 3, \ldots, l-1$, we obtain from (B.4) and (B.6) for $m=l\ll n$

$$\langle k^{2l}\rangle_n=\langle k^{2l}\rangle_{n-1}+(2l-1)!!(\alpha_1 k_0^2)^l n^{l-1},$$

and from this (B.10) follows for $m=l$ (in order to obtain this it is necessary to sum over n the difference $\langle k^{2l}\rangle_n-\langle k^{2l}\rangle_{n-1}$).

Thus, we have proved that spontaneous emission leads to a Gauss distribution with width (B.6) if the initial distribution is sufficiently narrow (we can assume that $f_0(k)=\delta(k)$). At the same time in the wings $|k|\gtrsim nk_0$, $f_n(k)$ is not of the Gaussian form in view of the fact that when n photons are emitted in accordance with (B.2) the distribution is smeared out only over a distance $\Delta k=nk_0$.

[1)] References [1, 6, 7] are devoted to the experimental investigation of the acceleration of atoms in the field of a traveling wave.

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