Investigation of the thermomagnetic effect and its possible use for the study of orientational interaction of molecules with a surface

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We present the results of an experimental and theoretical investigation of the thermomagnetic effect (TME) in a strongly rarefied molecular gas (the change of the heat flux in a gas between two plates in a magnetic field). The effect is due to the dependence of the interaction between the gas molecules and the surface of the solid, and on the orientation of the molecules. Oscillations of the heat flux were observed with increasing field intensity in the gases N_2, CO_2, and CO interacting with a sputtered layer of gold. A theory of the TME is developed, in which the orientational interaction is described by the nonspherical part of the probability of molecule scattering by the surface. The experimentally observed decrease of the heat flux in the field is ascribed to a definite symmetry of the scattering probability. It is shown that comparison of the theoretical dependences of the heat flux on the field with the experimental ones makes it possible to reconstruct the explicit form of the nonspherical part of the scattering probability. A model-dependent expression is proposed for the nonspherical part of the probability of scattering of N_2 or CO_2 molecules by a gold surface.

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1. INTRODUCTION

This paper is devoted to an investigation of the predicted[1] and previously observed[2,3] thermomagnetic effect (TME) ($\mathrm{Kn} \gg 1$; $\mathrm{Kn} = \lambda/L$, λ is the mean free path of the molecules and L is a characteristic dimension) molecular gas, and to an assessment of the possibility of using this effect to determine how the probability of molecule scattering from a surface depends on the molecule orientation. This possibility is obvious in principle, since the phenomenon under consideration, which consists of the influence of a magnetic field (**H**) on the heat transfer in a collision-free gas situated between two plates, is due to the dependence of the interaction of the gas molecules with the surface of the solid on their orientation. This dependence has so far hardly been investigated. At the same time, allowance for the orientational interaction of the gas molecules with the wall may turn out to be appreciable when it comes to describing many phenomena (adsorption, catalytic reactions, relaxation of internal degrees of freedom of the molecule, etc.).

A characteristic feature of the TME) are the theoretically predicted[1,4] oscillations of the heat flux in a magnetic field. These heat-flux oscillations were observed[2] in the gases N_2, NF_3, and CO at $\mathrm{Kn} \sim 1$, i.e., under conditions when it was necessary to take into account both the collisions of the molecules with the walls and the collisions of the molecules with one another. However, in experiments[3] in the collisionless regime, only a monotonic dependence of the heat flux on the field was observed. The possible causes of the absence of oscillations have been discussed in[4]. In particular, the oscillations might have been absent because the plates in these experiments were made of different materials.

In this paper (Sec. 2) we present the results of an experimental investigation of the TME in the gases N_2, CO_2, and CO in the case when both plates are coated with an evaporated layer of gold. Oscillations of the heat flux were observed in all these gasses.

In the previosly proposed theory of the TME, [1,4] the dependence on the interaction of the molecules with the surface on their orientation was taken into account by generalizing the well-known Maxwellian boundary condition, [5,6] which connects the distribution functions of the incident and reflected molecules, to include the case when the coefficient of diffuse reflection (α) is a function of the velocity vector (**v**), of the angular momentum (**M**) of the molecules, and of the normal to the surface (**k**). It turned out that this theory can not describe (see Sec. 4 of the present paper) the observed amplitudes of the oscillations of the heat flux in the investigated gases.

In Sec. 3 we develop a theory of the TME, using for the boundary condition of the distribution function a general integral form that makes it possible to describe the experimental data.

In Sec. 4, are compared the theoretical and experimental field dependences of the variation of the heat flux. It is shown that it is possible to derive an expression for the nonspherical part of the scattering probability, and a model-dependent expression is proposed for the scattering probability; this expression describes the orientational interaction of the N_2 and CO_2 molecules with the gold surface. As a result it becomes possible to indicate the most probable orientation of the molecules emitted from the wall.

2. EXPERIMENTAL INVESTIGATION OF TME IN THE GASES N_2, CO_2, AND CO

An experimental setup analogous to that described earlier in[3] was designed and constructed for the investigation of the TME. The thermal elements (TE) of

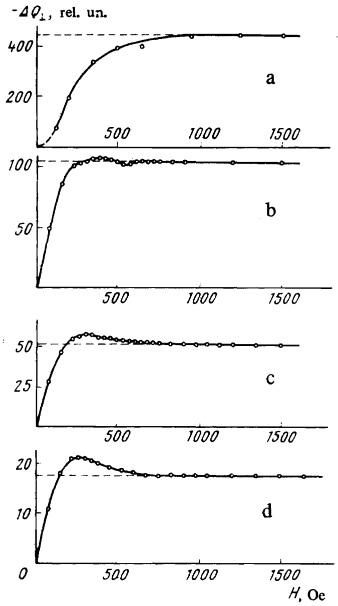

FIG. 1. Plots of $\Delta Q_\perp(H)$ for N_2 gas at $\mathbf{H} \perp \mathbf{k}$: a) $p = 24\times10^{-3}$; b) $p = 6\times10^{-3}$; c) $p = 3\times10^{-3}$; d) $p = 1\times10^{-3}$ mm Hg.

the pickups (mica plates ~ 5 μ thick coated with gold on both sides) measured 40×50 mm and the resistance of the evaporated layer was ~ 40 Ω. The sensitivity of a pickup with such a thermal element is approximately one order of magnitude higher than that described in[3]. Instead of the previously employed indium soldering of the wire conductors to the gold layer, we used clamp contacts, with the aid of which a platinum strip (~ 10 μ thick) was clamped to the surface of the evaporated layer. This made it possible to increase the working temperatures of the TE to 250–300 °C. To ensure sufficient rigidity, the TE was glued between two mica frames ~ 20 μ thick each. A stack made up of the TE and of the mica frames was freely placed in a gap (~ 0.1 mm) between the bases of the "cold" plates at a distance 2 mm from each plate. Such a construction makes it possible to avoid deformation of the TE at temperatures up to 300 °C. In all the experiments, the cold surface was a gold layer evaporated on a polished brass surface.

To investigate the dependence of the effect on the field orientation (the anisotropy of the TME) we used a rotating induction mechanism, which made it possible to rotate the pickup in the magnet gap and to fix it in two positions ($\mathbf{H} \perp \mathbf{k}$ and $\mathbf{H} \parallel \mathbf{k}$, where $\mathbf{k}$ is the normal to the surface).

The vacuum system of the installation ensured a minimum pressure ~ 1×10^{-6} mm Hg. The rate of gas inflow to the system $\lesssim 5\times10^{-5}$ mm Hg/hr. The experimental setup made it possible to monitor the energy accommodation coefficient simultaneously with the measurement of the TME.

The experimental investigations were performed in the gases N_2, CO_2, and CO at pressures $p \approx (0.5–25)\times10^{-3}$ mm Hg and at magnetic-field intensities up to 4 kOe. The distance $L = 2$ mm between the cold and hot surfaces in the pickup, and these pressures (for example of nitrogen), correspond to Knudsen numbers **Kn** ≃ 1–50. The temperatures of the cold and hot surfaces were maintained at 30 and 120 °C, respectively.

Plots of the heat flux ($\Delta Q_\perp$) in nitrogen as a function of the magnetic field intensity (H) are shown in Fig. 1 for the case $\mathbf{H} \perp \mathbf{k}$. The relative measurement error ΔQ was $\lesssim 5\%$. It is seen from Fig. 1 that in the investigated region of pressures the heat flux decreases in the magnetic field. At a pressure $p = 24\times10^{-3}$ mm Hg (**Kn** ≃ 1), the heat transfer is due mainly to collisions of the molecules with one another, therefore the $\Delta Q_\perp(H)$ plot (Fig. 1a) coincides with the known monotonic variation of the heat-conductivity coefficient of N_2 in a magnetic field.[7] When the pressure is decreased, the fraction of molecules transferring energy from the hot to the cold wall without colliding with one another increases. At heat-transfers corresponding to the transition to the free-molecular regime ($p = 6\times10^{-3}$ mm Hg, **Kn** ≈ 4—Fig. 1b; $p = 3\times10^{-3}$ mm Hg, **Kn** ≈ 24—Fig. 1d), the $\Delta Q_\perp(H)$ plots differ significantly from the corresponding plot on Fig. 1a. With decreasing pressure, a characteristic maximum appears in the $\Delta Q_\perp(H)$ plots. In all cases, $\Delta Q_\perp$ tends to a limiting value $(\Delta Q_\perp)_{\text{sat}}$ at large H. With decreasing pressure, the characteristic value of the field ($H^{\perp}_{1/2}$), at which $\Delta Q_\perp = (1/2)\,(\Delta Q_\perp)_{\text{sat}}$, decreases from 220 ± 10 Oe at $p = 24\times10^{-3}$ mm Hg to 73 ± 3 Oe at $p = 1\times10^{-3}$ mm Hg. With increasing **Kn** number, the relative magnitude of the maximum increases, and at **Kn** ≈ 24 we have the ratio $(\Delta Q_\perp)_{\max}/(\Delta Q_\perp)_{\text{sat}} \approx 1.24 \pm 0.08$.

It can be assumed that the heat-transfer regime at $p = 1\times10^{-3}$ mm Hg (**Kn** ≈ 24) for nitrogen is close enough to the free-molecular regime. Indeed, as can be seen from Fig. 2, starting with a pressure $p = 2\times10^{-3}$ mm Hg, we have $(\Delta Q_\perp/\Delta Q_\parallel)_{\text{sat}} \approx 2.04 \pm 0.10$ and this value is independent of the pressure, in agreement with the TME theory. At high pressures the ratio $(\Delta Q_\perp/\Delta Q_\parallel)_{\text{sat}}$ is likewise independent of p and is determined by the quantity $(\Delta\varkappa_\perp/\Delta\varkappa_\parallel)_{\text{sat}}$ (the Senftleben effect).[7] Similar relations $(\Delta Q_\perp/\Delta Q_\parallel)_{\text{sat}} = f(p)$ were obtained also for the gases CO_2 and CO, and this has made it possible to choose for these gases working pressures at which the free-molecular heat-transfer regime is realized.

Figures 3, 4, and 5 show plots of the heat flux (ΔQ) against the field (H) for the gases N_2, CO_2, and CO at low pressures (**Kn** ≈ 24). It is seen from the figures that the $\Delta Q_\parallel(H)$ plots also have an oscillatory character. $\Delta Q_\parallel$ tends to a limiting value $(\Delta Q_\parallel)_{\text{sat}}$ with increasing H. The numerical parameters of the curves are listed in Table 1.

3. THEORY OF TME

A. Boundary condition

A natural approach to the description of the TME is to solve the Boltzmann equation with the boundary con-

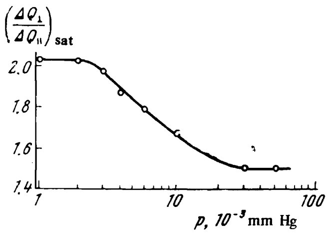

FIG. 2. Plot of $(\Delta Q_\perp/\Delta Q_\parallel)_{\text{sat}}$ against pressure for nitrogen.

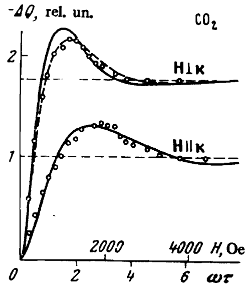

FIG. 3. Plot of $\Delta Q(H)$ for CO_2 gas at a pressure $p = 5\times10^{-4}$ mm Hg. (**Kn** ≈ 24). The solid lines correspond to the theoretical relations (4.2) and (4.3); the dashed line corresponds to (4.6).

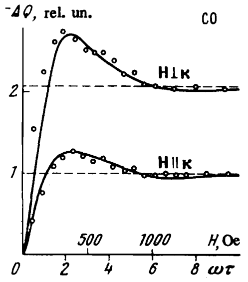

FIG. 5. Plot of $\Delta Q(H)$ for CO gas at a pressure $p = 1\times10^{-3}$ mm Hg (**Kn** ≈ 24). The solid lines correspond to the theoretical relations (4.2) and (4.9).

ditions for the distribution function on the surfaces of the plates. The general form of the boundary condition, which connects the distribution functions of the incident molecules and those reflected from the surface, is that of an integral equation, the kernel of which is the probability density of the molecule scattering by the surface.[5,6,8] For molecules with rotational degrees of freedom, this relation can be written in the form

$$|\mathbf{vk}|f_{\text{ref}}(\mathbf{v},\mathbf{M}) = \int_{\mathbf{v'k}<0} |\mathbf{v'k}|W(\mathbf{v'},\mathbf{M'}\to\mathbf{v},\mathbf{M};\mathbf{k})f_{\text{inc}}(\mathbf{v'},\mathbf{M'})d\mathbf{v'}\,d\mathbf{M'}. \tag{3.1}$$

Here $\mathbf{v'}$ and $\mathbf{M'}$ are respectively the velocity and the angular momentum of the molecules incident on the surface, $\mathbf{v}$ and $\mathbf{M}$ pertain to the reflected molecules, and $\mathbf{k}$ is the normal to the surface.

The scattering probability density W can be regarded as independent of the gas distribution function if[8,9]

$$n\bar{v}r_0^2t \ll 1, \tag{3.2}$$

where n is the density of the gas molecules $\bar{v}$ is their average velocity, r_0 is the radius of the intermolecular forces, and t is the average adsorption time period. The quantity t can be estimated at[9]

$$t \sim t_0 \exp(E_a/kT),$$

where $t_0 \sim 10^{-13}$ sec (it is determined by the surface properties of the body), E_a is the adsorption energy, and T is the surface temperature. Relation (3.2) is satisfied in the described TME experiments, inasmuch as under the experimental conditions (at a pressure $\sim 10^{-3}$ mm Hg and at an average temperature ~340 °K) and at a physical-adsorption energy $E_a \sim 10$ kcal/mole we have $n\bar{v}r_0^2t \sim 10^{-5}$.

The scattering kernel W in (3.1) is[5,6] a non-negative function normalized to

$$\int_{\mathbf{vk}>0} W(\mathbf{v'},\mathbf{M'}\to\mathbf{v},\mathbf{M};\mathbf{k})d\mathbf{v}\,d\mathbf{M} = 1, \tag{3.3}$$

and also satisfies the reciprocity relation (the detailed balancing principle)[10,6,8]

$$\begin{gathered}|\mathbf{v'k}|\exp\left(-\frac{E'}{kT}\right)W(\mathbf{v'},\mathbf{M'}\to\mathbf{v},\mathbf{M};\mathbf{k})\\ =|\mathbf{vk}|\exp\left(-\frac{E}{kT}\right)W(-\mathbf{v},-\mathbf{M}\to-\mathbf{v'},-\mathbf{M'};\mathbf{k}),\end{gathered} \tag{3.4}$$

where E' and E are the energies of the incident molecules and those reflected from the surface, respectively.

It is clear from the precession mechanism of the influence of the field on the distribution function of the gas molecules[1,4] that the change of the heat flux in the field is quadratic in the parameter that characterizes the degree of nonsphericity of the interaction of the molecules with the walls. The observed change of the flux in the field is small (for nitrogen, for example, we have $\Delta Q/Q \sim 10^{-3}$[3]). We shall therefore assume that the scattering probability depends little on the orientation of the molecules and can be represented in the form

$$W(\mathbf{v'},\ \mathbf{M'}\to\mathbf{v},\ \mathbf{M};\ \mathbf{k}) = W_0 + \varepsilon W_1(\mathbf{v'},\ \mathbf{M'}\to\mathbf{v},\ \mathbf{M};\ \mathbf{k}). \tag{3.5}$$

where ε is a small nonsphericity parameter. The value of W_0 does not depend on the orientation of the molecule angular momentum. It describes primarily elastic specular reflection, diffuse reflection that is isotropic with respect to the orientations and directions of the molecule emission, and also reflection with a probability that depends on the direction of only the vectors $\mathbf{v}$, $\mathbf{v'}$, and $\mathbf{k}$. We assume for simplicity that the part of the scattering probability corresponding to the processes of the third type is small and can be included in W_1. We note that this assumption is inessential for the description of the TME, since it is the dependence of the interaction of the gas molecules with the walls on just their orientation (on the directions of the vectors $\mathbf{M'}$ and $\mathbf{M}$) which governs the influcence of

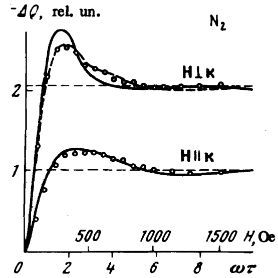

FIG. 4. Plot of $\Delta Q(H)$ for N_2 gas at a pressure $p = 1\times10^{-3}$ mm Hg (**Kn** ≈ 24). The solid line corresponds to the theoretical relations (4.2) and (4.3); the dashed line corresponds to (4.6).

TABLE 1.

Gas	factor m.m.	$\frac{(\Delta Q_\perp)_{\max}}{(\Delta Q_\perp)_{\text{sat}}}$	$\frac{(\Delta Q_\parallel)_{\max}}{(\Delta Q_\parallel)_{\text{sat}}}$	$H_{1/2}^{\perp}$, Oe	$H_{1/2}^{\parallel}$, Oe	$H_{\max}^{\perp}$, Oe	$H_{\max}^{\parallel}$, Oe	$\left(\frac{\Delta Q_\perp}{\Delta Q_\parallel}\right)_{\text{sat}}$
N_2	0.2593	1.24±0.08	1.24±0.08	73±3	115±5	285±20	475±30	2.04±0.10
CO_2	0.0551	1.24±0.12	1.34±0.12	270±10	350±20	1095±110	1770±200	1.75±0.10
CO	0.2691	1.34±0.12	1.28±0.12	64±3	115±5	270±30	435±45	2.07±0.10

the magnetic field on the heat flux.

The first two processes are described by the following scattering kernel:

$$W_0=(1-\alpha)\delta(\mathbf{v}'-\mathbf{v}_R)\delta(\mathbf{M}'-\mathbf{M})+(\alpha-\varepsilon)f_T(E)|\mathbf{vk}|\Big(\int\limits_{\mathbf{vk}>0}|\mathbf{vk}|f_T(E)\,d\mathbf{v}\,d\mathbf{M}\Big)^{-1},\tag{3.6}$$

where $\mathbf{v}_R=\mathbf{v}-2\mathbf{k}(\mathbf{k}\cdot\mathbf{v})$, and $f_T(E)$ is a Maxwellian distribution with the same temperature as the wall. Expression (3.6) takes into account the relation (3.3) for the normalization of the total probability, and we assume (without loss of generality) that the function W_1 is also normalized to unity.

After substituting the scattering kernel (3.5), with allowance for (3.6), into the general relation (3.1), we can reduce the boundary condition to the form

$$\begin{aligned}f_{\text{ref}}(\mathbf{v},\mathbf{M})&=(1-\alpha)f_{\text{inc}}(\mathbf{v}_R,\mathbf{M})+\alpha f_T(E)-\varepsilon f_T(E)G\{f_{\text{inc}}\}\\&+\varepsilon\int\limits_{\mathbf{v}'\mathbf{k}<0}\frac{|\mathbf{v}'\mathbf{k}|}{|\mathbf{vk}|}W_1(\mathbf{v}',\mathbf{M}'\to\mathbf{v},\mathbf{M};\mathbf{k})f_{\text{inc}}(\mathbf{v}',\mathbf{M}')\,d\mathbf{v}'\,d\mathbf{M}'.\end{aligned}\tag{3.7}$$

where

$$G\{f_{\text{inc}}\}=\frac{2}{n}\int\limits_{\mathbf{vk}>0}\int\limits_{\mathbf{v}'\mathbf{k}<0}\frac{|\mathbf{v}'\mathbf{k}|}{|\mathbf{vk}|}W_1(\mathbf{v}',\mathbf{M}'\to\mathbf{v},\mathbf{M};\mathbf{k})f_{\text{inc}}(\mathbf{v}',\mathbf{M}')\,d\mathbf{v}'\,d\mathbf{M}'\,d\mathbf{v}\,d\mathbf{M}.$$

We represent the nonspherical part of the scattering probability W_1 in the form of an expansion in the spherical functions of the vectors $\mathbf{v}'$, $\mathbf{M}'$, $\mathbf{v}$, $\mathbf{M}'$ and $\mathbf{k}$. Recognizing that the function W_1 is scalar, we write down this expansion in the form

$$\begin{gathered}W_1(\mathbf{v}',\mathbf{M}'\to\mathbf{v},\mathbf{M};\mathbf{k})=|\mathbf{vk}|\exp\Big(-\frac{E}{kT}\Big)\sum_L\beta_L(v'^2,M'^2,v^2,M^2)A_L,\\L=\{(l_1'l_1)j_1,\ (l_2'l_2)j_2,\ l_3\},\\A_L=\sum_{\text{over all }m}\begin{pmatrix}j_1&j_2&l_3\\m&-m&0\end{pmatrix}\langle l_1'm_1'l_1m_1|j_1m\rangle\langle l_2'm_2'l_2m_2|j_2-m\rangle\\\times Y_{l_1'm_1'}(\mathbf{v}')Y_{l_1m_1}(\mathbf{v})Y_{l_2'm_2'}(\mathbf{M}')Y_{l_2m_2}(\mathbf{M})Y_{l_30}(\mathbf{k}).\end{gathered}\tag{3.8}$$

Expression (3.8) was written under the assumption that the z axis is parallel to $\mathbf{k}$. We note that according to the normalization condition (3.3) the parameters β_L with $l_2=0$ should depend on v^2 or M^2. It is assumed below for simplicity that the β_L do not depend on v^2 or v'^2. The dependence of β_L on M^2 and M'^2 has turned out to have no effect on the behavior of the heat flux in the field.

The developed semi-phenomenological approach implies the use of only a finite number of terms of the expansion (3.8) to describe the experimental data. The quantity W_1 can in principle have any sign. However, the total scattering probability (3.5) remains non-negative if $\varepsilon\ll\alpha$. We note that for polyatomic gases, with the exception of hydrogen, the accomodation coefficient α is close to unity,[11] and this inequality should hold true.

B. Heat flux in a magnetic field

We consider the problem of the influence of a constant homogeneous magnetic field H, of arbitrary orientation, on the heat transfer in a collisionless (**Kn** ≫ 1) molecular gas between two parallel surfaces having different temperatures (T_0 and T_L) and separated by a distance L. The coordinate system is such that the xy plane coincides with one of the walls, while the second wall is defined by the equation $z=L$. The vector $\mathbf{k}$ is taken to mean the normal to the lower wall.

To find the heat flux it is necessary to determine the gas distribution function in the gap between the plates. This function can be obtained by solving the stationary Boltzmann kinetic equation with zero right-hand side, simultaneously with a system of boundary conditions written for the upper ($z=L$) and the lower ($z=0$) surfaces in the form (3.7). The problem is solved by perturbation theory in terms of the small parameter ε. The dependence of the heat flux on the field appears in the second-order approximation in ε. Assuming that $|\Delta T|=|T_0-T_L|\ll T_0$, we can obtain the following expression for the change of the heat flux in a magnetic field:

$$\begin{aligned}\Delta Q(\mathbf{H})=\frac{8\pi\varepsilon^2}{(2-\alpha)^2}\frac{\Delta T}{kT_0^2}\iiint\limits_{v_z,v_z',v_z''>0}\frac{\zeta'E'f_0(E')\zeta''E''f_0(E'')}{\zeta f_0(E)}\sum_{l_2m_2}(-1)^{m_2}\\\times\Big\{[B_{l_2-m_2}(\Gamma_R',\Gamma)B_{l_2m_2}(\Gamma_R'',\Gamma)-C_{l_2-m_2}(\Gamma_R',\Gamma)C_{l_2m_2}(\Gamma_R'',\Gamma)]\\\times\Big[\sum_n d_{nm_2}^{(l_2)^2}\Psi_n(\eta)-\Psi_0\Big]\\+[K_{l_2-m_2}(\Gamma_R',\Gamma)K_{l_2m_2}(\Gamma_R'',\Gamma)-H_{l_2-m_2}(\Gamma_R',\Gamma)H_{l_2m_2}(\Gamma_R'',\Gamma)]\\\times\Big[\sum_n d_{nm_2}^{(l_2)^2}\Phi_n(\eta)-\Phi_0\Big]\Big\}d\Gamma\,d\Gamma'\,d\Gamma''.\end{aligned}\tag{3.9}$$

Here

$$\begin{gathered}\Psi_n(\eta)=\frac{1-(1-\alpha)e^{in\eta}}{e^{-in\eta}-(1-\alpha)^2e^{in\eta}},\qquad\Phi_n(\eta)=\frac{1+(1-\alpha)e^{in\eta}}{e^{-in\eta}-(1-\alpha)^2e^{in\eta}},\\\eta=\frac{\omega\tau}{\zeta}\Big(\frac{2kT_0}{m}\Big)^{1/2},\qquad\zeta=|v_z|,\\\Gamma=(\mathbf{v},M^2),\quad\Gamma_R=(\mathbf{v}_R,M^2),\quad d\Gamma=d\mathbf{v}M^2dM.\end{gathered}\tag{3.10}$$

f_0 is a Maxwellian distribution with a temperature T_0, and the functions $d_{nm}^{(l)}(\theta_H)$[12] determine the dependence of ΔQ on the magnetic field orientation angle (θ_H). The quantities $B_{l_2m_2}$, $C_{l_2m_2}$, $H_{l_2m_2}$, and $K_{l_2m_2}$ in (3.9) are determined by those terms of the expansion (3.8) which depend on the vector $\mathbf{M}$ but do not depend on the orientation of the vector $\mathbf{M}'$. The corresponding part ($W^{(1)}$) of the probability W_1 takes the form

$$\begin{gathered}W_1^{(-1)}(\mathbf{v}',M'^2;\mathbf{v},\mathbf{M};\mathbf{k})=|\mathbf{vk}|\exp\Big(-\frac{E}{kT}\Big)\sum_s\beta_sA_s,\\s=\{(l_1'l_1)j_1,\ (0l_2)l_2\ l_3\},\\A_s=(4\pi)^{-1/2}\sum_{\text{over all }m}\begin{pmatrix}j_1&l_2&l_3\\-m_2&m_2&0\end{pmatrix}\langle l_1'm_1'l_1m_1|j_1-m_2\rangle\\\times Y_{l_1'm_1'}(\mathbf{v}')Y_{l_1m_1}(\mathbf{v})Y_{l_2m_2}(\mathbf{M})Y_{l_30}(\mathbf{k}).\end{gathered}\tag{3.11}$$

With the aid of (3.11) the functions $B_{l_2m_2}$, $C_{l_2m_2}$, $H_{l_2m_2}$ can be determined from the following general expression (the quantities B, C, H, and K are designated by a single symbol X):

$$W_1^{(1)}=(4\pi)^{-1/2}\sum_{l_2m_2}X_{l_2m_2}Y_{l_2m_2}\Big(\frac{\mathbf{M}}{M}\Big),$$

TABLE 2.

$X_{l_2m_2}$	Values $l_1'+l_1+l_3$	Values $m_1'+m_1$ $=-m_2$
$B_{l_2m_2}$	even	even
$C_{l_2m_2}$	odd	odd
$H_{l_2m_2}$	odd	even
$K_{l_2m_2}$	even	odd

the indices l_1', l_1, l_3, m_1', and m_1 assuming only the values indicated in Table 2.

The dependence of the change of the heat flux on the field intensity (see (3.9) and (3.10)) is determined by a dimensionless parameter equal to the product of the molecule precession frequency $\omega=\gamma H$ in the field (γ is the gyromagnetic ratio of the molecule) by the characteristic time of flight $\tau=L(m/2kT_0)^{1/2}$ of the molecules from wall to wall. From (3.9) follows an expression for Q at large values of the field:

$$(\Delta Q)_{\omega\tau\to\infty}\approx\frac{8\pi\varepsilon^2}{(2-\alpha)^2}\frac{\Delta T}{kT_0^2}\iiint\limits_{v_z,v_z',v_z''>0}\frac{\zeta'E'f_0(E')\zeta''E''f_0(E'')}{\zeta f_0(E)}$$
$$\times\sum_{l_2m_2}(-1)^{m_2+1}[1-d_{0m_2}^{(l_2)^2}]\{[B_{l_2-m_2}(\Gamma_R',\Gamma)B_{l_2m_2}(\Gamma_R'',\Gamma)$$
$$-C_{l_2-m_2}(\Gamma_R',\Gamma)C_{l_2m_2}(\Gamma_R'',\Gamma)]\Psi_0+[K_{l_2-m_2}(\Gamma_R',\Gamma)K_{l_2m_2}(\Gamma_R'',\Gamma)$$
$$-H_{l_2-m_2}(\Gamma_R',\Gamma)H_{l_2m_2}(\Gamma_R'',\Gamma)]\Phi_0\}d\Gamma\,d\Gamma'\,d\Gamma''. \qquad (3.12)$$

We consider the contributions of the individual terms of the expansion (3.11) to the expression for the heat flux in the field (there are no crossing combinations in (3.9)). We denote by $[\Delta Q(\omega\tau)]_{s\gamma}$ the contribution to ΔQ $(\omega\tau)$ corresponding to a term of the expansion (3.11) having definite values of the set of indices s and m_2 (the summation is over m_1' and m_1). Retaining in (3.9) the corresponding term and integrating over all the variables except v_z, we obtain for the change of the heat flux in a field parallel and perpendicular to the normal **k**, respectively,

$$\begin{aligned}&[\Delta Q_{\parallel}(\omega\tau)]_{s\nu}=K(J_0-J_\nu),\\&[\Delta Q_{\perp}(\omega\tau)]_{s\nu}=K_0J_0-K_1J_1-\ldots-K_{l_2}J_{l_2}.\end{aligned} \qquad (3.13)$$

Here K and K_k ($k=0,\ 1,\ldots,l_2$) are quantities that do not depend on the field. The functions $J_k(\omega\tau)$ in the cases when $l_1'+l_1+l_3+m_2$ is even and odd are respectively given by

$$J_k^{(1)}(\omega\tau,\alpha)=-\int\limits_0^\infty P(x)e^{-x^2}$$
$$\times\frac{(1-\alpha)^3+\alpha(2-\alpha)\cos(k\omega\tau/x)-(1-\alpha)\cos(2k\omega\tau/x)}{1-2(1-\alpha)^2\cos(2k\omega\tau/x)+(1-\alpha)^4}dx,$$
$$J_k^{(2)}(\omega\tau,\alpha)=\int\limits_0^\infty P(x)e^{-x^2} \qquad (3.14)$$
$$\times\frac{(1-\alpha)^3-\alpha(2-\alpha)\cos(k\omega\tau/x)-(1-\alpha)\cos(2k\omega\tau/x)}{1-2(1-\alpha)^2\cos(2k\omega\tau/x)+(1-\alpha)^4}dx,$$

where $x=\zeta(m/2kT_0)^{1/2}$, and $P(x)$ is a polynomial whose coefficients can be easily calculated in each concrete case. We designate the quantities $J_k(\omega\tau,\ \alpha)$ (3.14) at $P(x)=x^n$ by $J_k(\omega\tau,\ \alpha,\ n)$. Plots of $J_k(\omega\tau,\ \alpha,\ n)$ against $\omega\tau$ for different values of k and n are shown in Fig. 6.

C. Symmetry property of scattering probability

As will be shown below, it becomes possible to connect the observed sign of the effect ($(\Delta Q)_{\omega\tau\to\infty}<0$) with a definite symmetry of the nonspherical part W_1 of the scattering kernel. We consider the quantity

$$X_{l_2}=\sum_{m_2}X_{l_2m_2}(\Gamma_R',\Gamma)Y_{l_2m_2}\left(\frac{\mathbf{M}}{M}\right) \qquad (3.15)$$

and write down the obvious inequality

$$\int\limits_{v_z>0}\frac{1}{\zeta f_0(E)}\left(\int\limits_{v_z'>0}\zeta'E'f_0(E')X_{l_2}d\Gamma'\right)^2 d\mathbf{v}\,d\mathbf{M}\geqslant 0. \qquad (3.16)$$

Substituting (3.15) in (3.16) and taking into account the orthogonality of the spherical functions, we obtain

$$\iiint\limits_{v_z,v_z',v_z''>0}\frac{\zeta'E'f_0(E')\zeta''E''f_0(E'')}{\zeta f_0(E)}$$
$$\times\sum_{m_2}(-1)^{l_2+m_2}X_{l_2-m_2}(\Gamma_R',\Gamma)X_{l_2m_2}(\Gamma_R'',\Gamma)$$
$$\times d\Gamma\,d\Gamma'\,d\Gamma''\geqslant 0. \qquad (3.17)$$

Comparison of (3.12) and (3.17) shows that the B and K terms in (3.9) decrease the heat flux in the field at even l_2 and increase it at odd l_2, whereas the C and H terms lead to an increase at even l_2 and to a decrease of the heat flux at odd l_2. Consequently (see Table 2), the heat flux is decreased by those terms of the expansion $W_1^{(1)}$ (3.11) which have even $l_1'+l_1+l_2+l_3$, i.e., by those terms that are invariant to inversion of the vectors **v′**, **v**, **M**, and **k**. Thus, the experimentally observed sign of the effect can be explained by assuming that the nonspherical part W_1 of the scattering probability, and consequently the total probability W (3.5), does not change when the vectors **v′**, **v**, **M′**, **M** and **k** are simultaneously inverted.

The assumption that the scattering probability W is invariant under the inversion of the vectors **v′**, **v**, **M′**, **M**, and **k** is a reasonable one. Indeed, since W is a scalar function, it follows that invariance of W to the inversion of the vectors **v′**, **v**, **M′**, **M**, and **k** is equival-

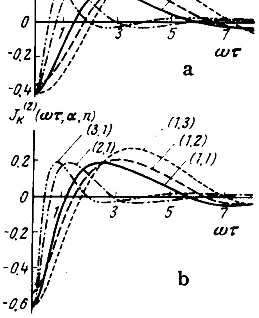

FIG. 6. Dependence of the integrals $J_k(\omega\tau,\ \alpha,\ n)$ (3.14) on the parameter $\omega\tau$ at $\alpha=0.8$ and at different (kn): a—for $J_k^{(1)}$; b—$J_k^{(2)}$.

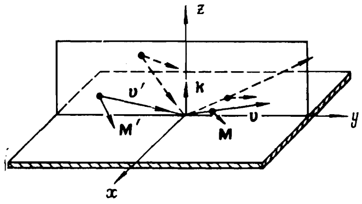

FIG. 7.

ent to equality of the probabilities of processes that are symmetrical about a vertical plane (see Fig. 7). It appears that such scattering processes should be equally probable if there is no physically preferred direction in the plane of the surface.

4. COMPARISON OF THEORY WITH EXPERIMENT

The expression (3.9) for the change of the heat flux in a magnetic field contains $n+1$ parameters, provided that we confine ourselves to n terms in the expansion (3.11) for the scattering probability $W_1^{(1)}$. One of these parameters is α, which has the meaning of the energy accommodation coefficient. Since we shall confine ourselves henceforth to only the relative magnitudes of the change of the heat flux, the determination of the absolute value of the nonsphericity parameter ε is no longer necessary. The remaining $n-1$ parameters in the expression for the heat flux in the field can be determined from a comparison of the theoretical and experimental relations.

Such a comparison of the functions $\Delta Q(\omega\tau)$ (see (3.13) and Fig. 6) with the experimental relations for the gases N_2, CO_2, and CO (Figs. 3–5) shows, in particular, that the functions $\Delta Q_{\parallel}(\omega\tau)$ for these gases can be described by the expressions $J_k(\omega\tau, \alpha)$ (3.14) only with $k=0.1$. It is then seen from (3.13) that it is necessary to retain in (3.11) only the first terms of the expansion ($|m_2|\leqslant 1$).

If we use also the experimental values of the amplitude of the first maximum of the oscillating relations $\Delta Q_{\parallel}(\omega\tau)$ and $\Delta Q_{\perp}(\omega\tau)$ for N_2 and CO_2 and take the sign of the effect into account, then it turns out to be possible to determine the principal terms of the expansion of the scattering probability $W_1^{(1)}$ (3.11) for these gases. A detailed comparison of the theoretical and experimental relations has shown that we can confine ourselves in (3.11) to a minimum of two expansion terms. The best approximation is then reached by using a linear combination of two terms in the following forms: $s=\{(l_1'1)l_1',\ (02)2,\ l_3\}$ if $l_1'+l_3$ is odd and $l_1'\neq 0$, and $s=\{(l_1'0)\,l_1',\ (02)2,\ l_3\}$ if $l_1'+l_3$ is even (the values of the indices l_1' and l_3 do not affect here the form of the function $\Delta Q(\omega\tau)$). Taking the foregoing into account, we write down a model-dependent expression for the scattering probability $W_1^{(1)}$ in the form

$$W_1^{(1)}=|\mathbf{kv}|\exp\left(-\frac{E}{kT}\right)[\beta_{s_1}A_{s_1}+\beta_{s_2}A_{s_2}],$$
$$s_1=\{(11)1,\ (02)2{,}2\},\quad s_2=\{(00)0,\ (02)2{,}2\}. \tag{4.1}$$

From (4.1) and (3.9)–(3.10) follow expressions for the changes in the heat flux in a field parallel and perpendicular to the normal k to the wall

$$\Delta Q_{\parallel}(\omega\tau)=N\{J_0^{(2)}(\omega\tau,\alpha,1)-J_1^{(2)}(\omega\tau,\alpha,1)\}, \tag{4.2}$$

$$\Delta Q_{\perp}(\omega\tau)=N\{J_0^{(2)}(\omega\tau,\alpha,1)-{}^1/{}_2J_1^{(2)}(\omega\tau,\alpha,1)-{}^1/{}_2J_2^{(2)}(\omega\tau,\alpha,1)$$
$$+c[J_0^{(1)}(\omega\tau,\alpha,1)-J_2^{(1)}(\omega\tau,\alpha,1]\}, \tag{4.3}$$

where N is a quantity proportional to the square of the nonsphericity parameter ε. The parameter c is chosen such as to make the quantity $(\Delta Q_{\perp}/\Delta Q_{\parallel})_{\omega\tau\to\infty}$ equal to the experimental value (see Table 1). As the result we get $c\approx 1.13$ for carbon dioxide and $c\approx 1.94$ for nitrogen. The obtained values for the energy accommodation coefficient α in the case of interaction of CO_2 or N_2 with the gold surface are respectively $\alpha=0.8\pm 0.1$ and $\alpha=0.72\pm 0.07$. Plots of $\Delta Q_{\parallel}(\omega\tau)$ (4.2) and $\Delta Q_{\perp}(\omega\tau)$ (4.3) for the gases CO_2 and N_2 are shown by solid lines in Figs. 3 and 4. As seen from these figures, the theoretical plots agree with the experimental ones within $\leqslant 10\%$.

The obtained model expression for the scattering probability $W_1^{(1)}$ (4.1) may have different forms of the functions β_s for CO_2 and N_2, but has the same angular dependence on the vectors **v′**, **v**, **M′**, **M**, and **k**. This result is natural for molecules with identical symmetry group (in this case $D_{\infty h}$).

According to (3.1), part of the scattering probability $W_1^{(1)}(\mathbf{v}', M'^2;\ \mathbf{v},\ \mathbf{M};\ \mathbf{k})$ determines the angular dependence of the distribution function of the molecules emitted from the surface, if the distribution function of the molecules incident on the surface is isotropic with respect to the orientations. Let $f_{\text{inc}}=f_{\text{inc}}(\mathbf{v}', M'^2)$. According to (3.7) and (4.1) we obtain

$$f_{\text{ref}}=f_1(\mathbf{v},\ M^2)+f_2(\mathbf{v},\ \mathbf{M}),$$
$$f_2(\mathbf{v},\mathbf{M})=\varepsilon F_1\exp\left(-\frac{E}{kT}\right)\sin\theta_v\sin\theta_M\cos\theta_M\cos(\varphi_M-\varphi_v)$$
$$+\varepsilon F_2\exp\left(-\frac{E}{kT}\right)(1-3\cos^2\theta_M), \tag{4.4}$$

where F_1 and F_2 are integrals of f_{inc} and do not depend on the orientation angles of the vectors **v** and **M**. The anisotropic part f_2 of the distribution function of the reflected molecules is a small increment ($\sim\varepsilon$) to the molecule distribution f_1 which is isotropic in the orientations of **M**. It follows from (4.4) that at $F_2>0$ the probability of emission of a molecule with an orientation such that the molecule axis is perpendicular to the surface is larger than the probability of emission of a molecule with an axis parallel to the surface. The situation is reversed if $F_2<0$. Unfortunately, the signs of F_1 and F_2 can not be determined with the aid of the TME. This question calls for research with direct experiments on the scattering of molecules by a surface, or else for a study of kinetic phenomena that are odd in the nonsphericity parameter ε, for example, thermal polarization of the molecules.[1]

The theoretical description of the experimental functions $\Delta Q_{\perp}(\omega\tau)$ for CO_2 and N_2 (Figs. 3 and 4) can be made more precise by including in the model-dependent expression for the probability $W_1^{(1)}$ an additional third term in the form

$$s_3=\{(l_1'0)l_1',\ (01)1,\ l_3\},\quad l_1'+l_3=\text{odd}. \tag{4.5}$$

The expression for $\Delta Q_{\parallel}(\omega\tau)$ then coincides with (4.2), and the expression for $\Delta Q_{\perp}(\omega\tau)$ takes the form

$$\Delta Q_{\perp}(\omega\tau)=N\{J_0^{(2)}(\omega\tau,\alpha,1)-{}^1/_2J_1^{(2)}(\omega\tau,\alpha,1)-{}^1/_2J_2^{(2)}(\omega\tau,\alpha,1)$$
$$+c_1[J_0^{(1)}(\omega\tau,\alpha,1)-J_2^{(1)}(\omega\tau,\alpha,1)]+c_2[J_0^{(2)}(\omega\tau,\alpha,1)-J_1^{(2)}(\omega\tau,\alpha,1)]\}. \tag{4.6}$$

The quantity $(\Delta Q_{\perp}/\Delta Q_{\parallel})_{\omega\tau\to\infty}$ becomes equal to the experimental value (see Table 1) at

$$c_2\approx 0.75-0.66c_1$$

in the case of CO_2 and at

$$c_2\approx 1.04-0.54c_1$$

in the case of N_2. A change in the parameter c_1 leads to a change in the shape of the $\Delta Q_{\perp}(\omega\tau)$ curve. The best approximation to the experimental relations is reached at $c_1\approx 0.76$ for CO_2 and $c_1\approx 0.85$ for N_2. The corresponding plots of $\Delta Q_{\perp}(\omega\tau)$ (dashed lines in Figs. 3 and 4) coincide with the experimental ones within the limits of errors.

We note that the use of any other three terms of the expansion (3.11) in place of (4.1) and (4.5) does not describe the experimental data within the limits of the experimental error.

Allowance for (4.5) introduces into the expression for the anisotropic part of the distribution function (4.4) of the molecules reflected from the surface an additional term

$$\varepsilon F_3\exp(-E/kT)\cos\theta_M. \tag{4.7}$$

It is seen from (4.7) that the probability of molecule emission from the wall becomes dependent on the direction of the rotation. The possible existence of a scattering process described by the probability term (4.5) does not contradict the known general normalization requirement (3.3) and reciprocity requirement (3.4), nor the requirement that the function W be a scalar.

Consider the TME in CO gas (Fig. 5). The CO molecule has a lower symmetry group ($C_{\infty v}$) than the molecules N_2 and CO_2, and the situation turns out to be more complicated in this case. An analysis of the possible theoretical relations shows that if we confine ourselves to terms of the expansion (3.11) with $l_2\leq 2$, then the best approximation to the experimental results is reached by using the model

$$W_1^{(1)}=|\mathbf{vk}|\exp(-E/kT)[\beta_{s_1}A_{s_1}+\beta_{s_4}A_{s_4}+\beta_{s_5}A_{s_5}],$$
$$s_4=\{(11)0,\ (02)2,\ 2\},\quad s_5=\{(11)0,\ (01)1,\ 1\}. \tag{4.8}$$

The expression for $\Delta Q_{\parallel}(\omega\tau)$ coincides in this case with (4.2), and $\Delta Q_{\perp}(\omega\tau)$ takes the form

$$\Delta Q_{\perp}(\omega\tau)=N\{J_0^{(2)}(\omega\tau,\alpha,1)-{}^1/_2J_1^{(2)}(\omega\tau,\alpha,1)-{}^1/_2J_2^{(2)}(\omega\tau,\alpha,1)$$
$$+c_3[J_0^{(1)}(\omega\tau,\alpha,3)-J_2^{(1)}(\omega\tau,\alpha,3)]+c_4[J_0^{(2)}(\omega\tau,\alpha,3)-J_1^{(2)}(\omega\tau,\alpha,3)]\}. \tag{4.9}$$

The parameters c_3 and c_4 are determined in the same manner as in the cases of N_2 and CO_2 above. As a result we have $c_3\approx 0.75$ and $c_4\approx 0.57$. The corresponding functions $\Delta Q(\omega\tau)$ at $\alpha=0.8$ are shown in Fig. 5. The discrepancy between the theoretical and experimental relations in the region $0<\omega\tau<2$ leads to the conclusion (see Fig. 6 and expression (3.13)) that in the expression for the probability $W_1^{(1)}$ it is necessary to take into account the terms of the expansion (3.11) with $l_2>2$. It appears that additional experimental data are needed to obtain an unambiguous result for example, the dependence of the change of the heat flux on the field orientation angle.

We have used above an expansion in the form (3.8) for the nonspherical part of the scattering probability. It can be shown that other expansion methods, differing from (3.8) in the order in which the spherical harmonics are added, do not make it possible to describe the experiment with a smaller number of terms than used above. The choice of (3.8) is governed by convenience considerations.

It is seen from the foregoing that the study of the TME makes it possible to solve the so-called inverse problem—to obtain an expression for the nonspherical part of the probability of molecule scattering by a surface from a comparison of the theoretical dependence of the heat flux on the magnetic field with experiment.

In conclusion, let us examine the possibility of describing the experimental results presented above with the aid of the previously proposed TME theory.[1,4] A model description within the framework of this theory implies allowance for some particular finite number of terms in the expansion of the sticking (evaporation) coefficient $\alpha(\mathbf{v},\mathbf{M},\mathbf{k})$ in spherical functions of the vectors $\mathbf{v}$, $\mathbf{M}$, and $\mathbf{K}$. If we use any such model that leads to a decrease of the heat flux in the field, then the expressions for $\Delta Q_{\parallel}$ and $\Delta Q_{\perp}$ contain only the functions $J_k^{(1)}(\omega\tau)$ (3.14). These functions (see Fig. 6a) have much larger heat-flux oscillation amplitudes than the observed ones (Figs. 3–5). Thus, the theory of[4] is incapable of explaining the oscillations of the heat flux as functions of the increasing field in the gases N_2, CO_2, and CO.

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Sound oscillations in a plasma with "magnetic filaments"

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In laboratory and astrophysical conditions one meets with a situation when the magnetic flux is concentrated in the plasma in narrow tubes ("magnetic filaments") which are lying far from one another. We study on the basis of the magnetohydrodynamic equations the long-wavelength sound oscillations of such a system. We show that even if there are no dissipative processes (viscosity, thermal conductivity, ohmic losses) the sound oscillations are absorbed because of an effect that is in a certain sense analogous to Landau damping in a rarefied plasma and which consists in the resonance excitation of flexure waves that move along the magnetic filament. We find the contribution to the damping from the scattering of the sound wave by the filaments and we indicate the conditions under which the scattering is unimportant. We consider the damping of a small-(but finite-) amplitude monochromatic sound wave.

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1. INTRODUCTION

In the present paper we derive equations describing the propagation of waves in a plasma which contains a system of "magnetic filaments" (tubes which include inside them magnetic flux). Apparently, such a kind of structure exists in the solar chromosphere.[1,2] They can also occur near the boundary between a plasma and a magnetic field when flute instability develops.

We shall assume that the tube radius a is small compared to the average distance between tubes l (small tube concentration). We also assume that all characteristic dimensions of the problem are sufficiently large so that we can use single-fluid magnetohydrodynamics. The magnetic field outside the tubes is assumed to be small. Just such a situation exists apparently in some regions of the solar chromosphere.

It was noted in[3] that this kind of system has an interesting property: long-wavelength ($\lambda \gg l$) sound oscillations are damped in it even if there are no dissipative processes (such as viscosity, thermal conductivity or ohmic losses) whatever, due to an effect which is similar to Landau damping and which consists of the following.

Bending oscillations with phase velocity u which is equal to (see below) $H/[4\pi(\rho_e+\rho_i)]^{1/2}$, where H is the magnetic field strength inside the tube, while ρ_i and ρ_e are, respectively, the density of matter inside and outside the tube, can propagate along a separate filament. If a sound wave propagates in the plasma at an angle θ to the direction of the tubes, when

$$v_s = u\cos\theta, \tag{1}$$

where v_s is the sound velocity in the liquid, there occurs a resonance transfer of wave energy into the energy of the oscillations of the filament. Since, generally speaking, the density of matter and the magnetic field strength are different inside different filaments, the velocity u varies from filament to filament. Correspondingly one can find for each angle of propagation θ filaments for which the condition (1) is satisfied[1)] and which, therefore, remove energy from the sound wave.

The plan for solving the problem is the following. First of all, we consider the motion of a separate tube relative to the liquid and find the force of the interaction between the tube and the liquid. After that, by averaging over a volume through which many tubes pass (but which is small compared to the wavelength), we get an expression for the volume force acting on the liquid due to the tubes and, accordingly we can write down macroscopic equations of the liquid. We also study the dispersion characteristics of the system and find the damping rate of the sound wave.

2. EQUATIONS OF MOTION

We take the initial direction of the tube as the z axis. We shall characterize the displacement of the tube from its equilibrium position by a vector $\xi(z,t)$ which is at right-angles to the z-axis. Since the relative velocity