spheres (−0.39 and −0.38 respectively) correlate well both with each other and the values given above for the vacancy charge (−0.37± 0.01) obtained from variation of the IS.

CONCLUSION

The investigations of the high-temperature phase of the higher iron silicide with the aid of the Mössbauer effect indicate that a relation exists between the parameters of the spectrum and the distribution of the vacancies, and makes it possible to determine the short-range order parameters for the first two coordination spheres. This enables us, in turn, to calculate the number of iron atoms in the nearest surrounding of the resonant atoms and to determine the charges of the vacancies.

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On the Drude formula for semiconductors

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Intraband absorption of light in semiconductors is investigated. It is shown that the classical Drude formula for the mean rate of change of the energy of a conduction electron in a high-frequency electric field is valid for both one-photon and many-photon changes in the electron energy in individual scattering events. Formulas are obtained which describe the effect of the ordered, uniform translational and vibrational motions of the current carriers on the intraband absorption coefficient for different scattering mechanisms.

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1. It is well known that the processes of absorption and stimulated emission are possible in electron scattering in the presence of an electromagnetic wave. In recent years bremsstrahlung absorption and stimulated bremsstrahlung emission have been considered in connection with the problems of plasma heating and electronic cumulative ionization in gases and crystalline dielectrics under the action of high-power laser radiation. Of greatest interest for both of these problems is the derivation of a formula for the mean (over a large number of collisions) rate of change of the energy of an electron under the action of a field.

From the quantum-mechanical point of view, an electron can, in the course of a scattering process in the presence of an electromagnetic wave of angular frequency Ω, absorb or emit a whole number of the quanta $\hbar\Omega$. Zel'dovich and Raĭzer[1] have considered the variation of the energy of an electron only as a result of one-photon processes, which, for sufficiently fast electrons, occur in the case when

$$eEp/m\hbar\Omega^2 \ll 1, \qquad (1a)$$

(where E is the electric-field intensity, e and m are the electron charge and mass, and p is the electron momentum), and have shown that for

$$2\hbar\Omega m/p^2 \ll 1 \qquad (1b)$$

the rate of change of the energy of an electron is given by the classical Drude formula

$$\frac{d\varepsilon}{dt} = \frac{e^2E^2}{2m\Omega^2}\nu_{\text{eff}}(p), \quad \Omega \gg \nu_{\text{eff}}, \qquad (2)$$

where ν_{eff} is the effective collision rate. This result was of great importance, as it meant that, since in gases and crystalline dielectrics the ionization potential $I_{\text{ion}} \gg \hbar\Omega$, the problem of cumulative ionization can be solved classically.

On the other hand, according to the classical picture, the change in the energy of an electron in a single scattering event under the action of a field is, in order of magnitude, equal to $\pm eEp/m\Omega$. In the author's paper[2] it was shown that Drude's formula turns out to be also valid when

$$eEp/m\hbar\Omega^2 \gg 1, \qquad (3a)$$

$$eE/\Omega p \ll 1. \qquad (3b)$$

The inequality (3a) implies that the elementary processes of emission and absorption have a multiphoton character.

On the face of it, the result of the cited papers[1,2] contradict each other. Therefore, it is of interest to obtain, not restricting ourselves to the one-quantum or the classical approximation, exact conditions of applicability of the Drude formula. To the solution of this problem is devoted Sec. 2 of the present paper, where a double-series expansion in the small parameters (1b) and (3b) is obtained for the general quantum-mechanical expression for the rate of change of the energy of a conduction electron of a semiconductor located in a strong light field. Drude's formula corresponds to the zeroth term of the obtained expansion, and, therefore, the conditions of its applicability are the inequalities (1b) and (3b). On the other hand, the value of the parameter $eEp/m\hbar\Omega^2$ can be arbitrary. Physically, this means that Drude's formula is valid irrespective of whether the true change in the electron energy in individual scattering events is a one- or a many-photon change. For the formula to be applicable, it is necessary that the electron energy be sufficiently high (quantum condition) and the electric-field intensity be sufficiently low (classical condition). Let us note that the misunderstanding of these simple, in our opinion, circumstances has led to a number of attempts (see, for example,[3–5]) to generalize the Drude formula, as well as the classical formula for the coefficient of electron diffusion along the energy axis—a formula which is similar to the Drude formula—to the case of strong fields in the sense of (3a) or to the case of many-photon processes of absorption and emission.

An interesting effect, connected with the processes of stimulated bremsstrahlung emission and absorption by free electrons, is the negative absorption of a linearly polarized electromagnetic wave in the presence of a uniform, directed—along the polarization vector—and sufficiently-fast electron motion relative to the scattering Coulomb centers. This effect was first proposed by Marcuse,[6] and has subsequently been considered by a number of authors.[7–10] It has been shown that negative absorption arises as a result of the fact that under the indicated conditions the processes of stimulated bremsstrahlung emission predominate over the absorption processes. In[6–10] the effect of the ordered motion of electrons was investigated only for the case of electron scattering by the Coulomb potential. It is of interest to also consider other possible scattering mechanisms. Such a problem is solved in Secs. 3 and 4, where, besides Coulomb scattering, the scattering of the semiconductor-current carriers by polar optical and deformation acoustic phonons is considered. It is shown that negative absorption arises only in scattering by the Coulomb potential, it being possible for both linearly and circularly polarized waves. Scattering by polar optical phonons under conditions analogous to the Marcuse effect leads to the vanishing of the classical part of the absorption coefficient; the residual absorption coefficient is of a quantum nature, and decreases with increasing velocity of the ordered motion (drift) of the current carriers like v^{-5}.

In Sec. 5 we consider the problem of the effect of a strong electromagnetic wave on the absorption of a weak wave of another frequency. It is evident that a strong linearly polarized wave establishes a distinct direction in a plasma, forcing the electrons to oscillate along the direction of polarization. If the oscillation velocity is sufficiently high, then there are created, upon the coincidence of the directions of polarization of the weak and strong waves, conditions which are close to the conditions in the Marcuse effect and which are distinguished only by the vibrational nature of the electron motion. The computations carried out in Sec. 5 show that in the case of scattering by the Coulomb potential the coefficient of absorption of the weak wave turns out, under the indicated conditions, to be negative, i.e., the weak wave is intensified at the expense of the strong wave. Notice that an analogous problem has been considered before,[11,12] but the possibility of negative absorption was not discovered, which is due, in our opinion, to the nature of the approximations used in the analyses, i.e., in[11,12].

2. The general expression for the power dissipated on conduction electrons in a high-frequency electric field can be represented in the form

$$\frac{dW}{dt}=\sum_{\mathbf{p}}\frac{d\varepsilon_{\mathbf{p}}}{dt}g(\mathbf{p}), \tag{4}$$

where $g(\mathbf{p})$ is the distribution function, while $d\varepsilon_{\mathbf{p}}/dt$ is the mean rate of change of the energy of an electron with momentum $\mathbf{p}$. Let us take the electric field in the form

$$\mathbf{E}(t)={}^1/_2(\mathbf{E}e^{i\Omega t}+\mathbf{E}^*e^{-i\Omega t}). \tag{5}$$

Then for $d\varepsilon_{\mathbf{p}}/dt$ there is the following quantum-mechanical formula[13,2]:

$$\frac{d\varepsilon_{\mathbf{p}}}{dt}=\frac{2\pi}{\hbar}\sum_{\mathbf{k}}\frac{|C_k|^2}{V}(2\bar{n}_k+1)\sum_{n=-\infty}^{n=+\infty}n\hbar\Omega J_n^2\left(\frac{e\bar{E}k}{m\hbar\Omega^2}\right)\delta(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}}-n\hbar\Omega), \tag{6}$$

where

$$\bar{E}=[(\mathbf{E}'\mathbf{k})^2+(\mathbf{E}''\mathbf{k})^2]^{1/2}/k,\quad \mathbf{E}'=(\mathbf{E}+\mathbf{E}^*)/2,$$
$$\mathbf{E}''=(\mathbf{E}-\mathbf{E}^*)/2i,\quad \varepsilon_{\mathbf{p}}=p^2/2m,$$

$C_k/V^{1/2}$ is the matrix element of the electron-phonon interaction, the $\bar{n}_k$ are the equilibrium occupation numbers of the phonons, V is the volume of the system under consideration, and J_n is a Bessel function. In the formula (6) it is assumed that $\Omega\gg\nu_{\text{eff}}$, ω_0, where ω_0 is the maximum frequency of the phonon spectrum.

Let us expand in (6) the δ-functions in a Taylor series in $n\hbar\Omega$. The formula (6) assumes the form

$$\frac{d\varepsilon_{\mathbf{p}}}{dt}=-\frac{4\pi}{\hbar}\sum_{\mathbf{k}}\frac{|C_k|^2}{V}(2\bar{n}_k+1)\sum_{l=0}^{\infty}\frac{1}{(2l+1)!}$$
$$\times\,\delta^{(2l+1)}(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}})\sum_{n=1}^{\infty}(n\hbar\Omega)^{2l+2}J_n^2\left(\frac{e\bar{E}k}{m\hbar\Omega^2}\right). \tag{7}$$

The series in n in (7) can be summed. Let us use the Neumann formula for $J_n^2(a)$ ([14], p. 31). Then

$$\sum_{n=1}^{\infty}n^{2l+2}J_n^2(a)=\frac{2}{\pi}\int_0^{\pi/2}d\theta\sum_{n=1}^{\infty}n^{2l+2}J_{2n}(2a\sin\theta)$$

$$= \frac{2}{\pi} \int_0^{\pi/2} d\theta \cdot 2^{-2(l+1)} \sum_{m=0}^{l+1} (2a \sin\theta)^{2m} Q_{2m}^{2l+2}$$

$$= \sum_{m=0}^{l+1} 2^{2(m-l-1)} \frac{(2m-1)!!}{2m!!} Q_{2m}^{2l+2} a^{2m}, \tag{8}$$

where the Q_{2m}^{2l+2} are coefficients, given by Schlömilch ([14], p. 35):

$$Q_m{}^l = \sum_{k=0}^{\leq m/2} (-1)^k \binom{m}{k} \frac{(m-2k)^l}{2^m m!}. \tag{9}$$

Thus, the series (8) sums into a polynomial in a of order $2l+2$:

$$\sum_{n=1}^{\infty} n^{2l+2} J_n{}^2(a) = T_{2l+2}(a). \tag{10}$$

In the particular cases of $l=0, 1, 2$, we have[1)]

$$T_2(a) = a^2/4, \tag{11a}$$

$$T_4(a) = {}^1/_4 a^2 + {}^3/_{16} a^4, \tag{11b}$$

$$T_6(a) = {}^1/_4 a^2 + {}^{15}/_{16} a^4 + {}^5/_{32} a^6. \tag{11c}$$

Substituting (10) into (7), we obtain

$$\frac{d\varepsilon_p}{dt} = -\frac{4\pi}{\hbar} \sum_k \frac{|C_k|^2}{V} (2\bar{n}_k + 1) \sum_{l=0}^{\infty} (\hbar\Omega)^{2l+2}$$

$$\times T_{2l+2}\left(\frac{eEk}{m\hbar\Omega^2}\right) \frac{1}{(2l+1)!} \delta^{(2l+1)}(\varepsilon_{p+k} - \varepsilon_p). \tag{12}$$

The formula (12) gets significantly simplified in two limiting cases:

$$eEk/m\hbar\Omega^2 \gg 1, \tag{13}$$

$$eEk/m\hbar\Omega^2 \ll 1. \tag{14}$$

If (13) is fulfilled, then in (12) we can in each polynomial $T_{2l+2}(a)$ retain only the term with the maximum power of a, i.e., assume that $T_{2l+2}(a) \sim a^{2l+2}$. In this case $\hbar$ drops out from the series in l in (12), and the series goes over into the classical formula[2]:

$$\frac{d\varepsilon_p}{dt} = \frac{2}{\hbar} \sum_k \frac{|C_k|^2}{V} (2\bar{n}_k + 1)(\varepsilon_{p+k} - \varepsilon_p) \left[\left(\frac{eEk}{m\Omega}\right)^2 - (\varepsilon_{p+k} - \varepsilon_p)^2\right]^{-1/2}. \tag{15}$$

To prove, (15), let us use the formula

$$b[a^2 - b^2]^{-1/2} = a \int_0^{\infty} J_1(at) \sin bt\, dt, \quad a^2 > b^2. \tag{16}$$

If we expand $J_1(at)$ in (16) in a series and integrate term by term over t, then we can easily verify that the series arising in (15) is identical to the series in (12) in the approximation $T_{2l+2}(a) \sim a^{2l+2}$.

In the case of weak fields, when the inequality (14) is fulfilled, we can use the approximation $T_{2l+2}(a) = a^2/4$. The formula (12) then assumes the form

$$\frac{d\varepsilon_p}{dt} = \frac{\pi}{2\hbar} \sum_k \frac{|C_k|^2}{V} (2\bar{n}_k + 1) \left(\frac{eEk}{m\Omega}\right)^2 \frac{1}{\hbar\Omega}$$

$$\times [\delta(\varepsilon_{p+k} - \varepsilon_p - \hbar\Omega) - \delta(\varepsilon_{p+k} - \varepsilon_p + \hbar\Omega)]. \tag{17}$$

Let us now consider the approximation in which we can restrict ourselves in (12) to the first few terms of the series in l. In the general case $d\varepsilon_p/dt$ is a function of two variables: p and the angle χ between **p** and **E** (linear polarization). Let us, for simplicity, restrict ourselves to the study of the general expansion only for the symmetric part of $d\varepsilon_p/dt$, obtainable from (12) by averaging over the angle χ. The extraction of the symmetric part from (12) can be effectively carried out by averaging (12) over the angular variables of either **p** or **E**. We shall use the second method. It is then clear that we should make in (12) the substitution

$$T_{2l+2}(e\mathbf{E}\mathbf{k}/m\hbar\Omega^2) \to \bar{T}_{2l+2}(eEk\cos\theta/m\hbar\Omega^2),$$

where the bar over T_{2l+2} denotes averaging over θ.

Notice that the δ-functions in (12) have a singularity at the point $\mathbf{k}=0$, at which their effect is not defined. Therefore, let us, for the purposes of carrying out specific computations with the formula (12), go over from integration over **k** to integration over $\mathbf{p}' = \mathbf{p} + \mathbf{k}$.

$$\xi = \frac{\hbar\Omega}{\varepsilon_p}, \quad \mu = \frac{eEp}{m\hbar\Omega^2}, \quad \eta = \frac{1}{2}\mu\xi = \frac{eE}{\Omega p} \tag{18}$$

and assume that

$$|C_k|^2 (2\bar{n}_k + 1) = A_s k^s, \quad s = 0, -2, -4. \tag{19}$$

The values of s indicated in (19) correspond, in the same order, to electron scattering by deformation acoustic phonons, polar optical phonons, and Coulomb centers.

Going over in (12) from summation to integration, and introducing the dimensionless variable $x = p'^2/p^2$, let us represent the formula (12) in the form

$$\frac{d\varepsilon}{dt} = \frac{e^2E^2}{2m\Omega^2} \nu_s(p) I_s(\xi, \eta), \tag{20}$$

where

$$\nu_s(p) = 2A_s m (2p)^{s+1}/3\pi\hbar^4,$$

$$I_s(\xi, \eta) = -\frac{3}{2^s \mu^2} \sum_{l=0}^{\infty} \frac{\xi^{2l}}{(2l+1)!} \int_0^{\infty} dx\, x^{1/2} \delta^{(2l+1)}(x-1) \tag{21}$$

$$\times \int_{-1}^{+1} dy\, (x + 1 - 2x^{1/2} y)^{s/2} \bar{T}_{2l+2}(\mu(x + 1 - 2x^{1/2} y)^{1/2} \cos\theta). \tag{22}$$

Let us define the effect of the δ-functions entering into (22) according to the formula

$$-\int f(x) \delta^{(2l+1)}(x-1)\, dx = {}^1/_2 \lim [f^{(2l+1)}(1+\rho) + f^{(2l+1)}(1-\rho)], \quad \rho \to +0. \tag{23}$$

Restricting ourselves in (22) to the first three terms of the series in l, and performing the integration, we obtain for the scattering mechanisms under consideration the expressions

$$I_0(\xi, \eta) = 1 + {}^3/_{20}\eta^2 + {}^1/_{128}\xi^4 + \ldots, \tag{24a}$$

$$I_{-2}(\xi, \eta) = 1 + {}^1/_8\xi^2 + {}^7/_{128}\xi^4 + {}^9/_{32}\xi^2\eta^2 + \ldots, \tag{24b}$$

$$I_{-4}(\xi, \eta) = 2 + {}^5/_{24}\xi^2 + {}^9/_{10}\eta^2 + {}^{63}/_{640}\xi^4 + {}^{63}/_{32}\xi^2\eta^2 + {}^{15}/_{88}\eta^4 + \ldots. \tag{24c}$$

It can be seen directly from the formulas (20)–(24) that the approximation under discussion is connected with the expansion of $d\varepsilon/dt$ in a double series in the small parameters ξ^2 and η^2. When $\xi \ll 1$ and $\eta \ll 1$, we can have $\xi \ll \eta$, or $\eta \ll \xi$. The inequalities $\xi \ll \eta$, $\eta \ll \xi$ are equivalent to the inequalities (13), (14), since for $\xi \ll 1$ and $\eta \ll 1$ we have $k \sim p$. Therefore, when $\xi \ll \eta$, the scattering processes in which the true change in the energy of an electron is a many-photon change predominate. In the opposite limiting case, when $\xi \gg \eta$, the change in the electron energy in individual scattering events is a one-phonon change. The classical Drude formula corresponds to the zeroth term of the expansion and, consequently, turns out to be valid for a one-photon, as well as for a many-photon, change in the energy of an electron in individual scattering events.

3. Let us restrict ourselves in (12) to the first term of the series in l. Then

$$\frac{d\varepsilon_{\mathbf{p}}}{dt}=-\frac{\pi}{\hbar}\sum_{\mathbf{k}}\frac{|C_k|^2}{V}(2\bar{n}_k+1)\left(\frac{e\tilde{E}k}{m\Omega}\right)^2\delta^{(1)}(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_{\mathbf{p}}). \tag{25}$$

Let us analyze (25) for the particular cases of linear and circular polarizations.

a) The case of linear polarization. For linear polarization $\tilde{E}k=\mathbf{E}\cdot\mathbf{k}$. Let us go over in (25) from the variable $\mathbf{k}$ to the variable $\mathbf{p}'=\mathbf{p}+\mathbf{k}$ and introduce spherical coordinates with the polar axis coinciding with the direction of $\mathbf{p}$. Then

$$\mathbf{E}\mathbf{k}=E[-p\cos\chi+p'(\cos\chi\cos\theta+\sin\theta\sin\chi\sin\varphi)], \tag{26}$$

where χ is the angle between $\mathbf{p}$ and $\mathbf{E}$.

Let us substitute (19) and (26) into (25) and perform the integration over φ:

$$\frac{d\varepsilon_{\mathbf{p}}}{dt}=\frac{e^2E^2}{2m\Omega^2}\nu_s(p)I_s(\chi); \tag{27}$$

$$I_s(\chi)=-\frac{3}{2^{s+2}}\int\limits_0^\infty dx\,x^{1/2}\delta^{(1)}(x-1)\int\limits_{-1}^{+1}dy\,(x+1-2x^{1/2}y)^{(s+2)/2}\left\{\frac{1}{3}+P_2(\cos\chi)\left[\frac{(x^{1/2}y-1)^2}{(x+1-2x^{1/2}y)}-\frac{1}{3}\right]\right\}, \tag{28}$$
$$P_2(\cos\chi)={}^1/_2(3\cos^2\chi-1).$$

Performing the integration in (28) for $s>-4$, we obtain

$$I_s(\chi)=1+\frac{2(s+1)}{s+4}P_2(\cos\chi),\quad s>-4. \tag{29}$$

The case $s=-4$ requires a special investigation. Performing the integration in (28) for $s=-4$ in accordance with (23), we obtain

$$I_{-4}(\chi)=2+P_2(\cos\chi)[4-3\lim_{\rho\to+0}\ln(4/\rho)]. \tag{30}$$

Thus, for $s=-4$ the formulas (27) and (28) are inapplicable, since there arises in (28) a divergent integral. The divergence of (30) is connected with the fact that when $s=-4$ the expression for $d\varepsilon_{\mathbf{p}}/dt$, defined according to (6), is not an analytic function with respect to the parameter ξ. This divergence can be eliminated by introducing a screened Coulomb potential, i.e., by subsituting into (25) in place of (19) the expression

$$A_{-4}/(\hbar^2r_D^{-2}+k^2)^2, \tag{31}$$

where r_D is the Debye screening distance. Assuming that $\hbar/r_Dp\ll 1$, and neglecting the terms of the order of $(\hbar/r_Dp)^2$, we obtain for $I_{-4}(\chi)$ the expression

$$I_{-4}(\chi)=2+P_2(\cos\chi)[4-6\ln(2pr_D/\hbar)]. \tag{32}$$

The divergence at $s=-4$ does not arise if we use in place of (25) the more general formula (17). It is not difficult to see that the minimum value that k can assume in (17) when $\xi\ll 1$ is $m\hbar\Omega/p$. Therefore, for $\xi\ll 1$ we obtain the correct expression for $I_{-4}(\chi)$ by replacing $\hbar/r_D$ in (32) by $m\hbar\Omega/p$.

b) The case of circular polarization. In the case of circular polarization

$$\tilde{E}k=E[k_x^2+(k_y\cos\psi-k_z\sin\psi)^2]^{1/2}$$
$$=E[(p'\sin\theta\sin\varphi)^2+(p'\sin\theta\sin\varphi\cos\psi-p'\cos\theta\sin\psi-p\sin\psi)^2]^{1/2}, \tag{33}$$

where ψ is the angle between the normal to the plane of polarization and $\mathbf{p}$. We obtain in analogy to (27)–(32)

$$\frac{d\varepsilon_{\mathbf{p}}}{dt}=\frac{e^2E^2}{2m\Omega^2}\nu_s(p)I_s(\psi), \tag{34}$$

$$I_s(\psi)=2-\frac{2(s+1)}{s+4}P_2(\cos\psi),\quad s>-4, \tag{35}$$

$$I_{-4}(\psi)=4-\left(4-6\ln\frac{2pr_D}{\hbar}\right)P_2(\cos\psi). \tag{36}$$

It follows from the formulas (27), (29), (32), and (34)–(36) that in the case of linear polarization the rate, defined according to (25), of change of the energy of an electron is positive for $s>-2$ and any χ. For $s=-2$ and $\chi=0$, π the quantity $d\varepsilon_{\mathbf{p}}/dt=0$. For $-4\leq s<-2$ there exists a region of χ values near $\chi=0$, π in which $d\varepsilon_{\mathbf{p}}/dt<0$. In the case of circular polarization $d\varepsilon_{\mathbf{p}}/dt<0$ when $s>-3$. For $s=-3$ and $\psi=\pm\pi/2$ we obtain $d\varepsilon_{\mathbf{p}}/dt=0$.[2)] For $-4\leq s<-3$ there exists a region of ψ values near $\psi=\pm\pi/2$ in which $d\varepsilon_{\mathbf{p}}/dt<0$. Notice that, since the formula (25) is applicable under conditions when $\xi\ll 1$ and $\eta\ll 1$, the results obtained in this section are valid only for sufficiently fast electrons. Therefore, to obtain exact conditions for the anisotropy of $d\varepsilon_{\mathbf{p}}/dt$, we should use the formulas (15) and (17).

4. Let us investigate in greater detail the case of scattering by polar optical phonons, restricting ourselves to the consideration of only the case of linear polarization. We shall assume that the field is weak in the sense of (13). Then, performing the integration in the formula (17), we obtain

$$\frac{d\varepsilon_{\mathbf{p}}}{dt}=\frac{e^2E^2}{2m\Omega^2}\nu_{-2}(p)[I_{-2}^+(\xi,\chi)-I_{-2}^-(\xi,\chi)], \tag{37}$$

$$I^\pm(\xi,\chi)=\frac{(1\pm\xi)^{1/2}}{\xi}+\frac{3}{2}P_2(\cos\chi)\left[\frac{\xi}{4}\ln\frac{1+(1\pm\xi)^{1/2}}{\mp1\pm(1\pm\xi)^{1/2}}\mp\frac{1}{2}(1\pm\xi)^{1/2}+\frac{1}{3}\frac{(1\pm\xi)^{3/2}}{\xi}\right], \tag{38}$$

where

$$I_{-2}^-(\xi,\chi)=0\quad\text{for}\quad\xi>1. \tag{39}$$

For $\xi\gg 1$

$$I_{-2}^{+}(\xi,\chi)=\xi^{-1/2}+{}^{1}/_{2}\xi^{-1/2}[1+{}^{5}/_{6}P_2(\cos\chi)]. \quad (40)$$

It can be seen from the formula (40) that the dependence of $d\varepsilon_p/dt$ on the angle χ vanishes for slow electrons. For $\xi \ll 1$ we have up to terms of second order in ξ that

$$I_{-2}(\xi,\chi)=1-P_2(\cos\chi)+{}^{1}/_{6}\xi^2[1+5P_2(\cos\chi)]. \quad (41)$$

Thus, for $\chi=0$, π, only the classical part of $d\varepsilon_p/dt$ vanishes in the case of scattering by polar optical phonons. For the rate of change of the energy during the motion along the direction of polarization, we have

$$\frac{d\varepsilon_\parallel}{dt}=\frac{3}{4}\frac{e^2E^2\hbar^2m}{p^4}\nu_{-2}(p). \quad (42)$$

Let us now consider the case, (13), of strong fields. It is not possible to perform the integration in the formula (15) for an arbitrary value of χ. Therefore, let us perform the integration for $\chi=0$ and $\chi=\pi/2$, i.e., for electron motion along and perpendicular to the direction of polarization:

$$\frac{d\varepsilon_\parallel}{dt}=\frac{e^2E^2}{2m\Omega^2}\nu_{-2}(p)6\theta(\eta-1)\eta^{-2}[\eta^2-1]^{1/2}, \quad (43)$$

$$\theta(x)={}^{1}/_{2}(1+\operatorname{sign}x);$$

$$\frac{d\varepsilon_\perp}{dt}=\frac{e^2E^2}{2m\Omega^2}\nu_{-2}(p)\cdot 6\int_0^{\pi/2}d\varphi\sin^2\varphi(1+\eta^2\sin^2\varphi)^{-1/2}. \quad (44)$$

The integral in (44) can be expressed in terms of elliptic integrals of the first and second kinds.[15b]

Notice that the properties of $d\varepsilon_p/dt$ considered in Secs. 3 and 4 may manifest themselves in the coefficient of absorption of light if by chance the distribution function is anisotropic. The anisotropy of the distribution function can be caused by the application of a strong constant electric field. It is clear that the formulas obtained for $d\varepsilon_p/dt$ allow us to estimate only the maximum magnitude of the effect of the ordered motion (drift) of the current carriers on the absorption coefficient, a magnitude which corresponds to a distribution function of the type

$$g(\mathbf{p})\sim\delta(\mathbf{p}-m\mathbf{v}). \quad (45)$$

where $\mathbf{v}$ is the velocity of the ordered motion.

5. In the framework of quantum mechanics, the solution to the problem of the effect of one electromagnetic wave on the intraband coefficient of absorption of another is given in Malevich and Éps̆teĭn's paper.[12] In the general formula obtained there for the absorption coefficient the field of both waves is taken into account exactly, in the same way as is done in (6). However, the specific computations are carried out in[12] in lowest-order perturbation theory in the field of the first, as well as in the field of the second, wave, i.e., under the condition that both waves are weak. It is clear that of greatest interest is the case when the field of one of the waves is strong, while the field of the other is weak. But then the effect of the strong wave can be considered classically.

Let $\mathbf{E}_1$ and Ω_1 be the field intensity and the angular frequency of the weak wave; $\mathbf{E}_2$ and Ω_2 the corresponding quantities for the strong wave. As a result of a rigorous solution, based on the quantum-kinetic equation,[13] of the problem, in which the field $\mathbf{E}_2$ is taken into account exactly in the framework of the classical approximation, while an expansion is carried out in the field $\mathbf{E}_1$ (the one-photon approximation), we obtain for the absorption coefficient for the weak wave the expression

$$\alpha_1=\frac{4\pi}{VncE_1^2\hbar}\sum_{\mathbf{p},\mathbf{k}}\frac{|C_\mathbf{k}|^2}{V}(2\bar{n}_\mathbf{k}+1)g(\mathbf{p})\left(\frac{e\mathbf{E}_1\mathbf{k}}{m\Omega_1}\right)^2$$
$$\times\frac{1}{\hbar\Omega_1}\left\{\left[\left(\frac{e\mathbf{E}_2\mathbf{k}}{m\Omega_2}\right)^2-(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_\mathbf{p}-\hbar\Omega_1)^2\right]^{-1/2}\right.$$
$$\left.-\left[\left(\frac{e\mathbf{E}_2\mathbf{k}}{m\Omega_2}\right)^2-(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_\mathbf{p}+\hbar\Omega_1)^2\right]^{-1/2}\right\}, \quad (46)$$

where n is the refractive index and c is the velocity of light.

The formula (46) becomes almost obvious if we compare it with the formula (17) after replacing $\mathbf{p}$ in the latter by $\mathbf{p}+(e\mathbf{E}_2/\Omega_2)\sin\Omega_2t$ and averaging over t. In fact,

$$\left\langle\delta\left(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_\mathbf{p}\pm\hbar\Omega_1+\frac{e\mathbf{E}_2\mathbf{k}}{m\Omega_2}\sin\Omega_2t\right)\right\rangle$$
$$=\frac{1}{\pi}\left[\left(\frac{e\mathbf{E}_2\mathbf{k}}{m\Omega_2}\right)^2-(\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_\mathbf{p}\pm\hbar\Omega_1)^2\right]^{-1/2}. \quad (47)$$

Let us carry out the subsequent computations under the assumption[17], typical of such type of problems, that the initial thermal velocity of the carriers is low compared to the oscillation velocity of the carriers in the field of the strong wave and that this relation does not change during the time of action of the strong field. This assumption allows us to replace $\varepsilon_{\mathbf{p}+\mathbf{k}}-\varepsilon_\mathbf{p}$ in (46) by $\varepsilon_\mathbf{k}$ and give the following conditions of applicability of the formula (46):

$$2e^2E^2/m\hbar\Omega_2^3\gg1, \quad (48)$$

$$2e^2E_2E_1/m\hbar\Omega_1^2\Omega_2\ll1. \quad (49)$$

Let us introduce the parameter

$$\zeta=(2m\hbar\Omega_1)^{1/2}\Omega_2/eE_2, \quad (50)$$

which we shall assume to be small. Then, going over in (46) from summation to integration, assuming that $\mathbf{E}_1 \parallel \mathbf{E}_2$ (the case, which, according to the results of the preceding sections, is of greatest interest), and using (19), let us represent (46) in the form

$$\alpha_1=\alpha_1^0\frac{3}{2\pi}(2\zeta)^{-s-1}\Phi_s(\zeta), \quad (51)$$

$$\alpha_1^0=\frac{4\pi Ne^2}{ncm\Omega_1^2}\nu_s[(2m\hbar\Omega_1)^{1/2}], \quad (52)$$

where N is the concentration of the carriers,

$$\Phi_s(\zeta)=\frac{1}{2\zeta^2}\left\{\int_{y_-}^{y_+}dx\,x^{s+3}\left[\left[1-\frac{(x^2-\zeta^2)^2}{4x^2}\right]^{1/2}+\frac{(x^2-\zeta^2)^2}{4x^2}\operatorname{arch}\left|\frac{2x}{x^2-\zeta^2}\right|\right]\right.$$
$$\left.-\int_{z_-}^{z_+}dx\,x^{s+3}\left[\left[1-\frac{(x^2-\zeta^2)^2}{4x^2}\right]^{1/2}+\frac{(x^2-\zeta^2)^2}{4x^2}\operatorname{arch}\frac{2x}{x^2+\zeta^2}\right]\right\},$$
$$y_\pm=\pm1+(1+\zeta^2)^{1/2},\quad z_\pm=1\pm(1-\zeta^2)^{1/2}. \quad (53)$$

For $\zeta \to 0$ the expression for $\Phi_s(\zeta)$ is an indeterminacy of the type 0/0. Expanding this indeterminacy, we obtain

$$\Phi_s(0) = 2^{s+2} \int_0^1 dx\, x^{s+3} \left[(1-x^2)^{-1/2} - \operatorname{arch} \frac{1}{x} \right]. \qquad (54)$$

In particular,

$$\Phi_0(0) = 4, \quad \Phi_{-2}(0) = 1, \quad \Phi_{-2}(0) = 0.$$

For $s = -4$ the integral in (54) diverges at the lower limit. The divergence in (54) implies that the first term of the asymptotic expansion of $\Phi_{-4}(\zeta)$ in $\zeta \ll 1$ is a nonanalytic function of ζ. Carrying out the asymptotic expansion of $\Phi_{-4}(\zeta)$ up to terms of zeroth order in ζ^2, we obtain

$$\Phi_{-4} = -{}^1/_2 \ln^2 \zeta + (2 \ln 2 - 1) \ln \zeta + C_1,$$

$$C_1 = {}^1/_2 + {}^3/_2 \ln 2 - \ln^2 2 - \frac{1}{2} \int_0^1 dx \left\{ x^{-1} \operatorname{arsh} x + x^{-1} \ln \frac{1}{2} [1 + (1+x^2)^{1/2}] \right.$$

$$\left. + 3x^{-1} \ln \frac{1}{2} [1 + (1-x^2)^{1/2}] + 2x^{-3} \left[(1-x^2)^{1/2} - 1 + \frac{1}{2} x^2 \right] \right\} \approx 0.78.$$

It follows from (55) that $\Phi_{-4}(\zeta) < 0$ when $\zeta < \zeta_0 \approx 0.4$.

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[1)] The formula (11a) is contained in the handbook.[15a] Its connection with the problem under consideration was first pointed out in[16].

[2)] The value $s = -3$ does not correspond to any real scattering mechanism.

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Low-temperature luminescence study of generation of normal acoustic waves in pure germanium under double injection conditions

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An investigation of the spatial and temporal distributions of low-temperature luminescence emitted from rectangular pure germanium resonators under double injection conditions revealed generation of normal finite-amplitude acoustic waves. A strong interaction was observed between various modes of these normal waves, which resulted in their coupling. The interaction of the excited vibrations with the injected plasma deformed strongly the carrier energy spectrum, as manifested by a long-wavelength shift (by up to 6 meV) of exciton luminescence lines. A special feature of this generation mechanism was the accumulation of gain from pulse to pulse because of weak attenuation of sound at the generation frequencies. A limiting value of the gain was reached because of nonlinear effects.

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Properties of semiconductors at high excitation rates are attracting considerable interest. The present investigation was intended to investigate, by the radiative recombination method, the properties of excitons created in high densities in pure germanium by double injection of carriers and the properties of the nonequilibrium state of the phonon system. Injection of nonequilibrium carriers in high densities was accompanied by the

