persion laws can be easily derived by noting that approximately (for $\omega \approx \omega_{AR}$) all of them except those treated here coincide with zeros of the denominator of $\sigma_k(\omega)$. In the present investigation we are assuming that either there are no such waves, or their length is much greater than the length of a spin wave (see below)—although this assumption cannot be justified by approach of ω to ω_{AR}.

[5)]This does not limit the use of the method of successive approximations applied here; the next term of the expansion in powers of $\alpha^{1/2}$ will be small in comparison with the second term. For specular and for diffuse reflection of the electrons, with an isotropic dispersion law, an exact calculation of the impedance was carried out, for an arbitrary value of the frequency ω. The expression obtained was expanded in powers of $\alpha^{1/2}/\delta$. The result obtained of course coincided with formula (24).

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Translated by W. F. Brown, Jr.

Magnetic structure of thin films of a "ferromagnetic" metal

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(Submitted October 16, 1975)
Zh. Eksp. Teor. Fiz. **70**, 1121–1129 (March 1976)

It is shown that interaction of spin waves with conduction electrons in a quantizing film (at temperature $T = 0$) may lead to destruction of the ferromagnetic order and to a transition to an antiferromagnetic state, with a period of the order of the film thickness.

PACS numbers: 75.70.+j, 75.30.Nc, 75.30.Fv

1. INTRODUCTION

Interaction of spin waves with a degenerate gas of conduction electrons leads, as is well known,[1] to occurrence of a singularity of the Migdal–Kohn[2] type in the magnon spectrum. Because of the separation of the Fermi surfaces resulting from the presence of magnetization, singularities should be observed not only at $k = p_F^+ + p_F^-$ ($\mathbf{k}$ is the quasimomentum of a magnon, $p_F^\pm$ are the Fermi momenta of electrons with spin projections $\pm\frac{1}{2}$ respectively), but also at $k = p_F^+ - p_F^- \equiv \Delta$, $p_F^+ > p_F^-$. The singularities at $k = p_F^+ + p_F^-$ are located in the range $k \sim \hbar/a$ (a is the lattice constant), while the singularity at $k = \Delta$ is located in the long-wave part of the spin-wave spectrum. Hereafter, only this latter singularity will be of interest to us.

The separation is of the order of magnitude[3]

$$\Delta \approx p_F J/\varepsilon_F \ll p_F \approx \hbar/a, \tag{1}$$

where p_F and ε_F are the Fermi momentum and energy in the paramagnetic phase, and where J is a quantity with the dimensions of energy, describing the coupling between the conduction electrons and the magnetization and equal to the energy "separation" of the Fermi steps. In f-metals (such as Gd and Dy), the Curie temperature $\Theta_C \approx J^2/\varepsilon_F$; in d-metals, Θ_C is somewhat larger than J^2/ε_F, since there is direct exchange interaction between d-electrons (rather than via s-electrons).

In nonferromagnetic metals, quantization of the motion of the electrons in a magnetic field H leads to enhancement of the Migdal–Kohn singularity in the phonon spectrum.[4] Blank and Kondratenko[5] showed that similar enhancement of the singularity in the magnon spectrum is not observed because of the large value of the

separation ($J \gg \hbar\omega_C$, where $\omega_C = eH/m^*c$; m^* is the effective mass of an electron).

Quantization of the motion of the electrons in a plate leads to enhancement of the singularity in the long-wave magnon spectrum[6] and, as will be shown in the present paper, may lead to a complete reconstruction of the spin-wave spectrum and even to loss of stability of the ferromagnetic state. Furthermore, it will be shown that because of the interaction of spin waves with conduction electrons, there may occur a stable periodic "antiferromagnetic" structure, with a period of the order of the plate thickness d, in a metal that in the bulk state is uniformly magnetized. The nature of the initiation of the periodic magnetic structure is the same as in metals with a complicated Fermi surface (see the paper of Dzyaloshinskiĭ[3]).

We shall assume that the thickness d of the plate is much smaller than the free path $l = v_F\tau$ (v_F is the Fermi velocity, τ is the free passage time of the electrons); ansiotropy of the dispersion law of the electrons is disregarded. We treat the case of specular reflection of the electrons by the specimen boundary; for the spin waves, we use Rado's[7] condition, that the normal derivative of the magnetic moment vanishes at the plate boundaries, $z = 0$ and d (Rado's condition corresponds to an ideally clean plate surface, with a normal that coincides with one of the principal crystallographic directions[8]).

The energy parameters encountered in the work satisfy a chain of strong inequalities:

$$\omega \ll \varepsilon_d \ll J \ll \varepsilon_F. \tag{2}$$

Here $\varepsilon_d = (\pi\hbar/d)^2/2m^*$ is the first surface energy level of an electron with $\mathbf{p}_\perp = 0$ ($\mathbf{p}_\perp$ is the two-dimensional momentum of the electron; $\mathbf{p}_\perp^2 = p_x^2 + p_y^2$; the z axis coincides with the normal to the plate); ω is the energy of a magnon with momentum $\mathbf{k}$. Without allowance for interaction of the magnons with the electrons,

$$\omega = \omega(\mathbf{k}) = \Theta k^2/p_F^2 + \omega_0, \qquad p_F = \hbar/a, \tag{3}$$

where $\Theta \sim \Theta_C$, and where ω_0 is the energy gap in the spin-wave spectrum, a consequence of the anisotropy; by hypothesis, $\lambda = 2\pi\hbar/k \gg a$. The renormalized (by interactions with the electrons) dispersion law of the spin waves is denoted by $\tilde{\omega}(\mathbf{k})$.

2. INSTABILITY OF THE STATE OF UNIFORM MAGNETIZATION OF A METALLIC PLATE

Allowance for the interaction of spin waves with conduction electrons and for quantization of the momentum of the plate boundaries leads to the following equation for the magnon energy ω:

$$\omega = \omega_\eta + \Pi_\eta(\omega), \tag{4}$$

$$\Pi_\eta(\omega) = -\frac{J^2\mu}{M_0 d(2\pi\hbar)^2}\sum\!\!\!\!\!\!\int' d^2p_\perp \left\{\frac{n_{\lambda+\eta\uparrow} - n_{\lambda\downarrow}}{\omega + \varepsilon_{\lambda\downarrow} - \varepsilon_{\lambda+\eta\uparrow} + i\delta} - \frac{n_{\lambda\uparrow} - n_{\lambda\downarrow}}{J+\omega}\right\}, \tag{5}$$

$$\begin{aligned} &\omega_\eta = \omega_0 + (\Theta/p_F^2)[(\pi\hbar n/d)^2 + k_\perp^2], \qquad n = 0, \pm1, \pm2, \ldots \\ &\varepsilon_{\lambda\sigma} = [(\pi\hbar\nu/d)^2 + p_\perp^2]/2m^* - \sigma J, \qquad \sigma = \pm{}^1\!/_2, \qquad \nu = \pm1, \pm2, \ldots . \end{aligned} \tag{6}$$

The stroke on the summation sign means that the terms with $\nu = 0$ and $\nu + n = 0$ are omitted; $\lambda \equiv \{\nu, \mathbf{p}_\perp\}$, $\eta \equiv \{n, \mathbf{k}_\perp\}$; $n_{\lambda\sigma}$ are the Fermi steps at $T = 0$; $M_0 \approx \mu/a^3$ is the magnetization of a bulk body at $T = 0$; $k_\perp^2 = k_x^2 + k_y^2$. The expression (4) can be derived by starting from a Fermi-liquid treatment.[4,6] The boundary conditions at $z = 0$ and $z = d$ (for electrons, vanishing of the wave function; for magnons, vanishing of the normal derivative of the magnetic moment) permit the derivation of "selection rules," which have been taken into account in writing $\Pi_\eta(\omega)$. The expression (4) can be used not only when the change of the magnon spectrum is small, but also in those most interesting cases in which the renormalization of the magnon energy is significant. But in order that it may be possible to neglect damping of the spin waves, it is necessary that the denominators in $\Pi_\eta(\omega)$ be not too small (not less than $\varepsilon_d \gg \omega$). This fact will be taken into account below.

We shall be interested in the spectrum of standing spin waves (magnons with $\mathbf{k}_\perp = 0$) for $n \neq 0$; and we shall show that at certain thicknesses d of the plate, the polarization operator $\Pi_\eta(\omega) < 0$, while $|\Pi_\eta(\omega)| \gg \omega$. Hence according to (4), $\tilde{\omega} < 0$, and consequently the ferromagnetic state is unstable.

The denominator in the first of the integrals in the formula, for $\mathbf{k}_\perp = 0$, has the form

$$\omega + J - \varepsilon_d(n^2 + 2n\nu) = \omega + J - (2\nu+1)\varepsilon_d \quad \text{at} \quad n = 1. \tag{7}$$

We shall discuss the case $n = 1$. We shall be interested in the situation in which the denominator (7) is small. It is clear that here we must consider the largest ν's, since to them correspond the smallest ε_d's, and the phenomenon can be observed at comparatively large thicknesses. But the Fermi steps bound the sum over ν. We introduce

$$N = E[(\varepsilon_F^+/\varepsilon_d)^{1/2}] \gg 1, \qquad \varepsilon_F^\pm = \varepsilon_F \pm J/2 \tag{8}$$

($E[x]$ is the interger part of x). In order of magnitude, $2N$ determines the number of subbands below the Fermi energy.

For $\nu \approx N$ the denominator is small, if $\varepsilon_d \approx J/2N$. By use of (8) we see that the thicknesses of interest to us satisfy the condition

$$\varepsilon_d \approx J^2/4\varepsilon_F, \qquad d = \pi\hbar(2\varepsilon_F/m^*J^2)^{1/2}. \tag{9}$$

We shall define more accurately the occupation of subbands by electrons at thicknesses d satisfying the condition (9). Various situations are possible (see Fig. 1).

I. The last sub-band under the Fermi level belongs to electrons with $\sigma = +\frac{1}{2}$ (Fig. 1a):

$$\varepsilon_F^- < \varepsilon_d N^2 < \varepsilon_F^+, \qquad \varepsilon_d(N+1)^2 > \varepsilon_F^+. \tag{10}$$

II. The last occupied sub-band has $\sigma = -\frac{1}{2}$ (Fig. 1b):

$$\varepsilon_d N^2 < \varepsilon_F^-, \qquad \varepsilon_d(N+1)^2 > \varepsilon_F^+. \tag{11}$$

This case is possible only when $\varepsilon_d > J^2/4\varepsilon_F$.

III. The last two sub-bands belong to electrons with $\sigma = \frac{1}{2}$ (Fig. 1c):

$$\varepsilon_d(N-1)^2 > \varepsilon_F^-, \qquad \varepsilon_d N^2 < \varepsilon_F^+, \qquad \varepsilon_d(N+1)^2 > \varepsilon_F^+. \tag{12}$$

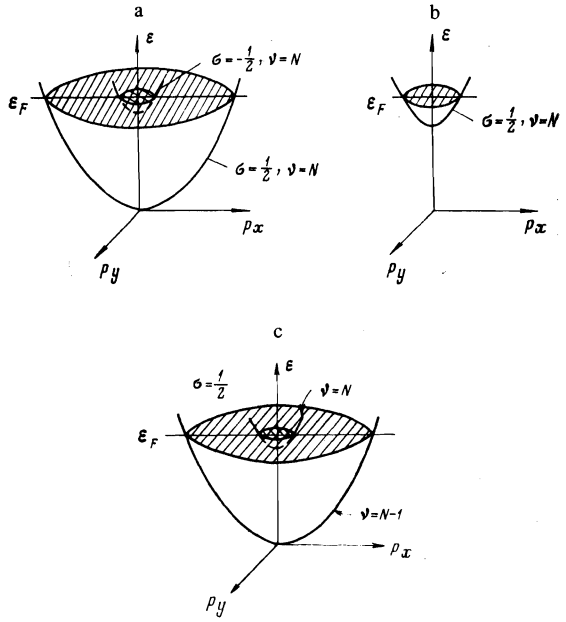

FIG. 1.

The last case is possible only if $\varepsilon_d < J^2/4\varepsilon_F$.

With increase of plate thickness, there is a possibility of cases in which the last (third, fourth, etc.) subbands belong to $\sigma=\frac{1}{2}$. We shall not consider these cases.

Since $\mathbf{k}_\perp = 0$, the integrand in formula (4) is independent of $\mathbf{p}_\perp$, and the integration presents no difficulty:

$$\Pi_{1,0}(\omega) = -g^2 \sum\nolimits' \left\{ \frac{[\varepsilon_F^+ - \varepsilon_d(\nu+1)^2] - [\varepsilon_F^- - \varepsilon_d\nu^2]}{\omega + J - (2\nu+1)\varepsilon_d} - \frac{[\varepsilon_F^+ - \varepsilon_d\nu^2] - [\varepsilon_F^- - \varepsilon_d\nu^2]}{\omega + J} \right\} \tag{13}$$

$$g^2 = J^2 \mu m^* / 2\pi\hbar^2 I_0 d \approx (\varepsilon_d/\varepsilon_F)^{1/2} J^2/\varepsilon_F.$$

As we have already said, for f-metals $J^2/\varepsilon_F \approx \Theta$. We shall use this relation hereafter; that is, we shall suppose that $g^2 \approx \Theta(\varepsilon_d/\varepsilon_F)^{1/2}$. The stroke on the summation sign means that the summation is so carried out that the expression in square brackets is positive; terms with $\nu = 0$ and $\nu + 1 = 0$, as before, are absent; the index on Π means:

$$n=1, \quad \mathbf{k}_\perp = 0.$$

We shall treat the three cases in succession.

According to (10) and (8) we have, taking into account the inequality $\omega \ll J$ (we omit the stroke on the summation sign),

$$\Pi_{1,0}^{(I)}(\omega) = g^2 \left\{ \omega \sum_{\nu=-(N-1)}^{N-1} \frac{1}{\omega + J - (2\nu+1)\varepsilon_d} + 2\frac{\varepsilon_{FN}^+}{J} - \frac{2\varepsilon_{FN}^+ + 2N\varepsilon_d}{J + 2N\varepsilon_d} \right\}. \tag{14}$$

We have separated out the terms with $\nu = \pm N, -(N+1)$.

Similarly we have

$$\Pi_{1,0}^{(II)}(\omega) = g^2 \left\{ \omega \sum_{\nu=-N}^{N-1} \frac{1}{\omega + J - (2\nu+1)\varepsilon_d} + \frac{\varepsilon_{FN}^-}{\omega + J - (2N+1)\varepsilon_d} + 1 - \frac{\varepsilon_{FN}^+}{J + 2N\varepsilon_d} \right\}, \tag{15}$$

$$\Pi_{1,0}^{(III)}(\omega) = g^2 \left\{ \omega \sum_{\nu=-N+2} \frac{1}{\omega + J - (2\nu+1)\varepsilon_d} + \frac{4\varepsilon_{FN}^+ + 4N\varepsilon_d}{J} - \frac{\varepsilon_{FN}^+}{J - (2N-1)\varepsilon_d} - \frac{3\varepsilon_{FN}^+ + 6N\varepsilon_d}{J + 2N\varepsilon_d} \right\}. \tag{16}$$

In formula (15) the terms with $\nu = N$ and $\nu = -(N+1)$ have been separated out, and in formula (16) those with $\nu = -(N+1)$, $\nu = \pm N$, and $\nu = \pm(N-1)$. We denote $\varepsilon_F^\pm - N^2\varepsilon_d$ by $\varepsilon_{FN}^\pm$.

We shall investigate formula (14) for $\varepsilon_d \lesssim J^2/4\varepsilon_F$ (we shall be interested in the onset of instability at large thicknesses; see the expression for ε_d after Eq. (2)). On going over from summation to integration and noting that the denominator nowhere vanishes, we get, with logarithmic accuracy,

$$\Pi_{1,0}^{(I)}(\omega) \approx g^2 \left\{ \frac{\omega}{2\varepsilon_d} \ln\frac{J}{\varepsilon_d} - \frac{2N\varepsilon_d}{J + 2N\varepsilon_d} + \frac{4N\varepsilon_d}{J(J+2N\varepsilon_d)} \varepsilon_{FN}^+ \right\}.$$

Since $2N\varepsilon_d \approx J$, the last expression takes the form

$$\Pi_{1,0}^{(I)}(\omega) \approx g^2 \left\{ \frac{\omega}{2\varepsilon_d} \ln\frac{J}{\varepsilon_d} - \frac{1}{2} + \frac{1}{J}\varepsilon_{FN}^+ \right\}. \tag{14a}$$

It is possible to choose the value of the plate thickness in such a way that the third term is much smaller than unity, and because of the term $-\frac{1}{2}$ the whole polarization operator is less than zero. On comparing $\Pi_{1,0}^{(I)}(\omega)$ with the value of $\omega_{1,0} = \omega_0 + \varepsilon_d\Theta/\varepsilon_F$ (see (5)), we see that with approach to the thickness for which $(\varepsilon_F^+/\varepsilon_d)^{1/2}$ is an integer, $\tilde{\omega}$ changes sign, and consequently the system of spin waves proves to be unstable.

In analogous fashion, for the third case (see (11) and (8))

$$\Pi_{1,0}^{(III)}(\omega) = g^2 \left\{ \frac{\omega}{2\varepsilon_d} \ln\frac{J}{\varepsilon_d} - \frac{1}{2} - \frac{\varepsilon_{F,N-1}^-}{J - 2N\varepsilon_d + \varepsilon_d} \right\}. \tag{16a}$$

It is evident that by choice of the thickness one can make the third term arbitrarily small (according to (12) it is positive) and, in complete agreement with the preceding case, observe instability.

We recall once again that both the cases considered (formulas (14a) and (16a)) correspond to comparatively thick plates ($\varepsilon_d \lesssim J^2/4\varepsilon_F$). When $\varepsilon_d \gtrsim J^2/4\varepsilon_F$, in case I formula (14a) is valid, while in case II, in the same approximation,

$$\Pi_{1,0}^{(II)}(\omega) \approx g^2 \left\{ \frac{\omega}{2\varepsilon_d} \ln\frac{J}{\varepsilon_d} - \frac{\varepsilon_{FN}^+}{J + 2N\varepsilon_d} - \frac{\varepsilon_{F,N+1}^+}{(2N+1)\varepsilon_d - J + \omega} \right\}. \tag{15a}$$

Within the bounds of the inequalities (11), $\frac{1}{2} \lesssim \varepsilon_{FN}^+/(J + 2N\varepsilon_d) \lesssim 1$ for $J \sim 2N\varepsilon_d$, but $\varepsilon_{F,N-1}^- < 0$. For observation of instability, it is advantageous to make it vanish. Obviously, according to the considerations presented above, on approach to a certain thickness ($(\varepsilon_F^+/\varepsilon_d)^{1/2}$ an integer) instability should be observed. We remark that for $\varepsilon_d > J^2/4\varepsilon_F$, cases are possible in which one of the denominators in formula (15) may be of order ω. Then, of course, perturbation theory is inapplicable (see the beginning of Sec. 2); but the vanishing occurs only for isolated values of the plate thickness. This fact justifies us in ignoring such cases.

Thus when $\varepsilon_d \approx J^2/4\varepsilon_F$, in certain narrow thickness intervals ($\Delta d \sim \hbar/p_F$) the system of spin waves is unstable, since

$$\tilde{\omega}_{1,0}=\omega_0+\Theta\varepsilon_d/\varepsilon_F-\Theta(\varepsilon_d/\varepsilon_F)^{1/2}<0. \quad (17)$$

By considering the following modes of standing spin waves ($n = 2, 3, \ldots, k_\perp = 0$), it can be demonstrated that observed instability sets in at large plate thicknesses:

$$\varepsilon_d \approx J^2/4n^2\varepsilon_F, \quad d=\pi\hbar n(2\varepsilon_F/Jm^*)^{1/2}, \quad (18)$$

while $\tilde{\omega}_{n,0}$ has the form

$$\tilde{\omega}_{n,0}=\omega_0+\Theta\varepsilon_d n^2/\varepsilon_F-\Theta(\varepsilon_d/\varepsilon_F)^{1/2}. \quad (19)$$

We emphasize that the value of the polarization operator $\Pi_{n,0}(\omega)$ in the region of maximum instability is independent of the mode number.

Formally, there will be no instability for $n \gtrsim (\varepsilon_F/\varepsilon_d)^{1/2}$ at thicknesses that satisfy the condition

$$\varepsilon_d \lesssim J^4/16\varepsilon_F^3. \quad (20)$$

Considerably earlier, however (at smaller thicknesses), dissipative mechanisms (finiteness of the free path of the electrons and nonspecularity of their reflection by the boundary) destroy the quantization of the electron spectrum. Therefore the chief role is apparently played by the first modes ($n = 1, 2, 3$), whose instability should have real physical meaning.

3. PERIODIC "ANTIFERROMAGNETIC" STRUCTURE OF THE GROUND STATE

The instability of the uniformly magnetized state discovered in the preceding section shows that at certain thicknesses the spin structure of a film should differ significantly from the spin structure of the bulk metal.

We shall show that with a periodic dependence of the mean magnetic moment, the energy of the system can be lowered by choice of the minimizing period $2\pi/q$. We shall. Consider two magnetic structures that vary periodically with the coordinate x:

$$M_z(x)=M_0\cos qx, \quad M_y(x)=M_0\sin qx, \quad M_x=0, \quad (21a)$$

$$M_z(x)=M_0\cos qx, \quad M_x=M_y=0. \quad (21b)$$

Since we have chosen the simplest $\mathbf{M}(\rho)$ dependence and have disregarded the role of demagnetizing factors, we shall not pretend to predict the true most stable magnetic structure and its exact parameters. Our aim is to show the possibility of liquidation of the instability by a transition from the uniform ferromagnetic structure to a periodic antiferromagnetic structure, and also to find the order of magnitude of the pitch of the spiral and of the depth of the minimum of the stable state.

The Hamiltonian of the interaction of the conduction electrons with the magnetic moment of the ground state is

$$\hat{\mathcal{H}}_{\text{int}}=\sum_{\sigma\sigma'}\int dV\,\psi_\sigma^*(\mathbf{r})\frac{J}{M_0}\hat{\sigma}\mathbf{M}(\mathbf{r})\psi_{\sigma'}(\mathbf{r}).$$

Here $\psi_\sigma(\mathbf{r})$ is the wave function and $\hat{\sigma}$ the spin operator of an electron. The idea of the subsequent calculation goes back to the work of Dzyaloshinskiĭ,[3] according to whom the appearance of a periodic magnetic structure leads to a change of the electronic spectrum that decreases the total energy of the magnet, despite the appearance of a positive exchange energy E_{ex} due to the inhomogeneity:

$$E_{ex}=(\Theta a^2/\mu M_0)\int dV(\nabla M)^2. \quad (22)$$

In our case, the change of the electronic spectrum leads to the result that the last (unsafe) sub-bands ($\nu \approx N$) are located above the Fermi level. Since it is to them that the instability was due, a new state is energetically advantageous. Expulsion of the sub-bands from below the Fermi level may be called, in the language used in the theory of phase transitions of the $2\frac{1}{2}$-th kind (I. M. Lifshitz[9]), a change of topology of the Fermi surface, which in the present case is due to the disappearance of one Fermi circle (Fig. 1 a–c).

The change of topology of the Fermi surface leads to an abrupt change of the energy of the Fermi electrons with $\nu \approx N$ (we shall call it the "singular" energy and shall denote it by $E_{\text{sing}}(q)$). At the same time the energy of the other electrons changes smoothly. This (in accordance with[3]) enables us to consider $E_{\text{sing}}(q)$ alone.

In order to calculate $E_{\text{sing}}(q)$, it is necessary to investigate the energy spectrum of the electrons with allowance for their interaction with the nonuniform magnetic moment (21).

Determination of the energy spectrum of the electrons reduces to solution of Pauli's equation. In the case (21a) it has the form

$$\left\{\hat{\mathcal{H}}_0\hat{I}-\frac{J}{2}\begin{pmatrix}0 & e^{-iqx}\\ e^{iqx} & 0\end{pmatrix}\right\}\psi=\varepsilon\psi, \quad (23a)$$

I is the unit matrix, and ψ is a column of two functions. In the case (21b) equation (23a) gives

$$\hat{\mathcal{H}}_0\psi_\pm\pm\frac{J}{2}\cos qx\,\psi_\pm=\varepsilon_\pm\psi_\pm. \quad (23b)$$

We shall first consider case (21a)—a helical structure, which permits exact solution of equation (23a) (see[10]). The energy of an electron can have the following values:

$$\varepsilon=\varepsilon_d N^2+\frac{p_y^2}{2m^*}+\frac{p_x^2}{2m^*}+\frac{1}{4}\frac{(\hbar q)^2}{2m^*}\pm\left[\left(\frac{p_x\hbar q}{2m^*}\right)^2+\frac{J^2}{4}\right]^{1/2}. \quad (24)$$

The two signs before the square root correspond to the two spin states.[1)]

In the calculation of $E_{\text{sing}}(q)$, the integration over $\mathbf{p}_\perp$ extends over a small region, and therefore the expression (24) can be simplified by expanding the square root as a series in $(p_x\hbar q/m^*)^2/J^2$ (the final result confirms the possibility of the expansion).

For $\nu = \pm N$ we have from (24)

$$\varepsilon=\varepsilon_d N^2+\frac{p_y^2}{2m^*}+\frac{p_x^2}{2m^*}\left(1\pm\frac{\hbar^2q^2}{2m^*J}\right)\pm\frac{J}{2}+\frac{1}{4}\frac{\hbar^2q^2}{2m^*}. \quad (25)$$

Depending on the relation between $\varepsilon_d N^2$ and ε_F (see (14)–(16)), the contribution to $E_{\text{sing}}(q)$ is made either by those electrons in whose energy expression there is a plus before the square root (see (24)), or by those for which there is a minus before the square root:

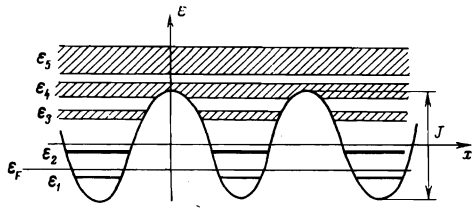

FIG. 2.

$$E^{\pm}_{\text{sing}}(q) \approx \frac{4V(2m^*)}{(2\pi\hbar)^3}\varepsilon_F \frac{\pi\hbar}{d}\left(\varepsilon_{FN}^{\pm} - \frac{1}{4}\frac{\hbar^2q^2}{2m^*}\right)\Theta\left(\varepsilon_{FN}^{\pm} - \frac{1}{4}\frac{\hbar^2q^2}{2m^*}\right),$$

$$\Theta(x) = \begin{cases} 1 & \text{at } x>0 \\ 0 & \text{at } x<0 \end{cases}.$$

We have taken into account the contribution of the two sub-bands, with $\nu = N$ and with $\nu = -N$. Per electron, the singular part of the energy is ($\mathfrak{N}$ is the number of electrons)

$$\frac{E_{\text{sing}}(q)}{\mathfrak{N}} \approx \left(\frac{\varepsilon_d}{\varepsilon_F}\right)^{1/2}\left(\varepsilon_{FN}^{\pm} - \frac{1}{4}\frac{\hbar^2q^2}{2m^*}\right)\Theta\left(\varepsilon_{FN}^{\pm} - \frac{1}{4}\frac{\hbar^2q^2}{2m^*}\right). \quad (26)$$

Per particle, the nonuniform exchange energy (22) has the order of magnitude $\Theta(aq)^2 \approx \Theta(\hbar q)^2/2m^*\varepsilon_F$. (We note that $\hbar/a \approx p_F = (2m^*\varepsilon_F)^{1/2}$.) Consequently

$$\frac{E^{\pm}(q)}{\mathfrak{N}} = \frac{\Theta(\hbar q)^2}{\varepsilon_F 2m^*} + \left(\frac{\varepsilon_d}{\varepsilon_F}\right)^{1/2}\left(\varepsilon_{FN}^{\pm} - \frac{1}{4}\frac{(\hbar q)^2}{2m^*}\right)\Theta\left(\varepsilon_{FN}^{\pm} - \frac{1}{4}\frac{(\hbar q)^2}{2m^*}\right). \quad (27)$$

Analysis of this expression shows that $E^{\pm}(q)$ takes its smallest value at $\hbar q = \hbar q_{\pm} = 2(2m^*\varepsilon^{\pm}_{FN})^{1/2}$, when $E_{\text{sing}}(q)$ vanishes. We recall that this means expulsion of the last electronic sub-bands from below the Fermi level, while the remaining sub-bands ($\nu < N$) are located far from the Fermi level. Thus the instability caused by the sub-bands with $\nu = \pm N$ is liquidated (see (14)–(16)).

On comparing the value of $E^{\pm}(q)/\mathfrak{N}$ with the value of the energy in the ferromagnetic phase, we see that the gain in energy is approximately $(\varepsilon_d/\varepsilon_F)^{1/2}\varepsilon^{\pm}_{FN}$.

In the case described by formula (14), there are two unsafe sub-bands (the corresponding Fermi energies are $\varepsilon^{+}_{FN}, \varepsilon^{-}_{F,N-1} < J$) that must be taken into account in the calculation of $E_{\text{sing}}(q)$. In this case also, q may be so chosen that both sub-bands are above the Fermi level, and the gain in energy $\sim (\varepsilon_d/\varepsilon_F)^{1/2}(\varepsilon^{+}_{FN} + \varepsilon^{-}_{F,N-1})$.

The case of the sinusoidal structure (23b) reduces to the solution of Mathieu's equation. Unfortunately there are no simple formulas for the spectrum. We shall therefore restrict ourselves to a few remarks. It is easy to analyze the case depicted in Fig. 2, in which $\varepsilon^{+}_{FN} \ll J$. In the unsafe sub-band the electrons are in deep potential wells, almost unconnected with each other, and therefore the p_x-width of the band is exponentially small. Calculation of $E_{\text{sing}}(q)$ gives the following result:

$$\frac{E_{\text{sing}}(q)}{\mathfrak{N}} = \frac{3}{\pi}\left(\frac{\hbar^2q^2}{2m^*}\right)^{1/2}\left(\frac{\varepsilon_d}{\varepsilon_F}\right)^{1/2}\left[\varepsilon_{FN}^{+} - \frac{\hbar q}{2}\left(\frac{J}{2m^*}\right)\right]^{1/2}, \quad (28)$$

and analysis of $E(q)$ shows that in this case also, $q \approx q_{\pm}$, and the energy gain due to expulsion of the unsafe sub-band is of the same order as for the helical structure.

We must point out the more abrupt singularity of $E_{\text{sing}}(q)$ in this case (in comparison with the helical); this may lead to the actual occurrence of a sinusoidal structure. Apparently the final choice of the most stable antiferromagnetic structure can be made by taking into account demagnetizing factors, temperature, etc.

From the point of view of experimental detection of the effect treated above, special interest attaches to the compound $ZrZn_2$, which is ferromagnetic at a temperature below 20 K. Wayne and Edwards[11] showed experimentally that the temperature of degeneracy of the electrons in $ZrZn_2$ ~ 300 K. At present there are no data on the effective mass of the charge carriers in this compound, but from the Mott–Arkhipov criterion[9] (see, for example, Sec. 11) it follows that m^* is of the same order in it as in bismuth. At the same time, it is well known[12] that the quantum dimensional effect in metals of the Bi type is already observable at thicknesses of order 10^{-4} cm.

From the data presented it is evident that because of the small degeneracy temperature and, apparently, the small effective mass of the electrons in $ZrZn_2$, it is comparatively simple to satisfy the quantum condition at thicknesses ~ 10^{-4} cm, and consequently to observe an antiferromagnetic state in films of this metal. Similar deductions are apparently correct for other metals in which there are small groups of electrons with small effective masses.[2)] For example, in iron and nickel there are groups of electrons—"pockets"—with $\varepsilon_F \sim 0.1$ eV and $m^* \sim 0.1\, m_0$. There is also a possibility of observing this effect in ferromagnetic rare-earth metals (REM), whose Fermi surface is very complicated: in the presence of dimensional quantization in REM films, there may appear unsafe sub-bands that make the existence of ferromagnetism in them disadvantageous. But at present the obtaining of pure REM presents great difficulties.

We take this opportunity to thank I. M. Lifshitz for stimulating discussions.

[1)] In the present case the spin motion is entangled with the orbital: the spin of an electron is reoriented during motion along the helicon.

[2)] As was remarked in[12], the quantum dimensional effect is observed not only in semimetals, but also on electrons of small groups in ordinary metals.

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Translated by W. F. Brown, Jr.

Phase diagram of an excitonic ferromagnet at finite temperatures

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(Submitted October 21, 1975)
Zh. Eksp. Teor. Fiz. **70**, 1130–1141 (March 1976)

The phase diagram of an excitonic ferromagnet is investigated at finite temperatures. It is shown that near the ferromagnetic-transition temperature, the Curie law holds for the static magnetic susceptibility. The conditions under which a nondegenerate electron system can go over into the ferromagnetic state are determined. The possible existence of two Curie temperatures, outside of which the substance is paramagnetic, is observed. The character of the phase transition is investigated using the two-parameter Landau expansion. Certain experimental data are discussed qualitatively on the basis of the results.

PACS numbers: 75.30.Jy, 75.30.La

1. INTRODUCTION

This is a continuation of our investigation of excitonic ferromagnets.[1] We investigate here the behavior of excitonic ferromagnets at finite temperatures.

We obtain first an expression for the longitudinal static magnetic susceptibility χ of the excitonic-insulator phase at $T \neq 0$. This expression is used to determine the limits of the region of the instability of the dielectric phase with respect to its transition to the ferromagnetic state at different values of the singlet (g_s) and triplet (g_t) coupling constants. We show that near the instability region the susceptibility χ obeys the Curie law $\chi^{-1} \sim T - T_c$. We then obtain analytic expressions for the phase-transition line in the low-temperature region. It turns out that at low concentrations $2n$ of the excess electrons, the dielectric phase, in which the chemical potential μ of the electrons lies inside the dielectric gap $2\Delta_s(\Delta_s > \mu)$, exhibits ferromagnetic instability. Inasmuch as at close values of the singlet and triplet coupling constants the temperatures T_s and T_c of the dielectric and ferromagnetic transitions are close to each other in a considerable region on the (n, T) plane, it becomes possible to construct in this region a two-parameter Landau expansion for the free energy F in powers of the singlet (Δ_s) and triplet (Δ_t) order parameters. An investigation of this expansion shows that the ferromagnetic transformation proceeds always (except for the case $g_s = g_t$) by a second-order phase transition. At strictly equal constants, there exists a section where the entropy S and the order parameters change jumpwise.

In conclusion, we present results of numerical computer calculations, in the self-consistent field approximation, of the complete phase diagram of an excitonic ferromagnet and its susceptibility in the paramagnetic region.

2. FUNDAMENTAL EQUATIONS

A model for an excitonic ferromagnet was proposed in[2] and was considered in detail for $T = 0$ in[1] in the high-density approximation ($e^2/\hbar v_F \ll 1$, where e is the electron charge and v_F is its Fermi velocity). In this paper we investigate an excitonic ferromagnet at finite temperatures in the same approximation, using the Hamiltonian and the notation of[1], i.e., we consider a semimetal having one electron and one hole Fermi surface, the centers of which are either at one point of the Brillouin zone, or are shifted relative to each other by one-half the reciprocal lattice vector, the electron density being larger than the hole density by an amount $2n$ (for example, owing to doping). We retain in the interaction Hamiltonian only the terms that determine the instability of the semimetal with respect to formation of charge density and spin-density waves (CDW and SDW, respectively). These instabilities correspond to singlet (g_s) and triplet (g_t) coupling constants, expressions for which in terms of the initial interaction constants are given in[1] (see (1–21)[1)]). The development of these instabilities is accompanied by appearance of a singlet Δ_s (CDW) and triplet Δ_t (SDW) order parameters.

To obtain a self-consistent system of equations in the high-density approximation we use the method of temperature Green's functions[3] defined by the formulas