Phonon zero sound in degenerate germanium in a quantizing magnetic field

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The modification of the optical oscillation spectrum for homopolar semiconductors analogous to degenerate germanium, caused by the interaction with electrons in a quantizing magnetic field, is considered. It is shown that the effective attraction between the electrons through the phonons leads to the appearance of a new branch of the zero sound type. The velocity of the phonon zero sound depends on the magnetic field strength and may vanish at certain threshold values of the field. Under such conditions, a phase transition involving a lowering of the lattice symmetry may occur.

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Electron-phonon interaction in degenerate homopolar semiconductors leads to an appreciable renormalization of the frequencies of the optical phonons.[1,2] The change in the frequency of the optical phonons with increase in concentration of the free carriers in germanium and silicon has been observed experimentally in Raman scattering.[3,4]

Modification of the phonon spectrum in homopolar semiconductors (of the n-Ge type) in a quantizing magnetic field is studied in the present work. It is shown that the electron-phonon interaction leads to an effective attraction between the electrons in this case, due to transitions with $\Delta n = 0$ (n is the number of the Landau level). A new undamped, unscreened branch of the zero-sound type appears in the vibrational spectrum, with a velocity less than the Fermi electron velocity. The velocity of this "zero sound" depends on the magnetic field and vanishes at certain values of the field. In the simple case of a single filled Landau level in n-Ge, this threshold value is equal to

$$H_{\text{th}} = 2mc\varepsilon_F / 3^{1/2} 2^{5/4} \zeta_0^{1/2} \hbar e,$$

where m is the mass of the electron, c is the velocity of light, ε_F is the Fermi energy in the absence of a magnetic field, ξ_0 is the constant of electron-phonon interaction, h is Planck's constant, and e is the charge on the electron.

At $H > H_{\text{th}}$, the cubic configuration becomes absolutely unstable and a redistribution of the electrons in the Brillouin zone takes place. This is accompanied by a shift of the sublattices. A phase transition of such a type has been studied phenomenologically by Kochelap and Sokolov.[5] This conclusion is valid when a homogeneous phase transition takes place.

1. CALCULATION OF THE POLARIZATION OPERATOR IN A MAGNETIC FIELD FOR n-Ge

The effect of electrons on the spectrum of long-wave optical vibrations has been studied previously by Demikhovskiĭ and Protogenov[6] for the case of polarization oscillations in an ionic crystal. The electron-phonon interaction in degenerate homopolar semiconductors differs essentially from the polarization interaction. It contains an important contribution of short-range forces.[7] As has been shown earlier,[1,4] it leads to such a redistribution of the electrons over the ellipsoids that oscillations of the electron density and current are absent in the mean over the Brillouin zone. It is important that, for long-wave optical oscillations, the matrix element of this interaction depends on the electron quasimomentum p. In the case of n-Ge, the electron ellipsoids are small in comparison with the distances between them in the Brillouin zone; therefore, we can neglect the dependence on the quasimomentum $\mathbf{p}_{0i} - \mathbf{p}$, where $\mathbf{p}_{0i}$ is the location of the center of the ellipsoid, in the limits of an individual ellipsoid. The diagonal matrix element of the electron-phonon interaction depends only on the direction in the given ellipsoid[1]:

$$V_j(\mathbf{p}_{0i}) = \left(\frac{\hbar}{2M\omega_0 N_0}\right)^{1/2} D \frac{1}{|p_{0i}|^2} [p_{0i}{}^x p_{0i}{}^y \xi_j{}^z + p_{0i}{}^x p_{0i}{}^z \xi_j{}^y + p_{0i}{}^y p_{0i}{}^z \xi_j{}^x]; \qquad (1)$$

where ξ_j^k is the polarization vector of the oscillations of the lattice branch j, M is the mass of the ion, ω_0 is the frequency of the optical phonon, N_0 is the number of atoms in a unit volume, D is the constant of the deformation potential. Account of quantization of motion of electrons in the magnetic field in this case cannot have an effect on the form of the matrix element of electron-phonon interaction.

The Hamiltonian of the electron phonon interaction has the form

$$H_{\text{th}} = \sum_{s;s';\mathbf{q};j} \langle s | V_j(\mathbf{r}) e^{i\mathbf{q}\mathbf{r}} | s' \rangle a_s^+ a_{s'} (b_{\mathbf{q}j} + b_{-\mathbf{q}j}^+), \qquad (2)$$

where a_s^+ and a_s are the creation and annihilation operators of the electrons, b_{qj}^+ and b_{qj} are the creation and annihilation operators of phonons, and the diagonal matrix element $\langle s | V_j(\mathbf{r}) | s' \rangle$ is calculated in terms of the wave functions of the electrons in the magnetic field and is identical with $V_j(\mathbf{p}_{0i})$ from (1) in our approximation.

The spectrum of optical phonons is determined from Dyson's equation, which has the form

$$D_j^{-1}(\mathbf{q}, \omega) = D_{0j}^{-1}(\mathbf{q}, \omega) - \Pi_j(\mathbf{q}, \omega) \qquad (3)$$

for an isolated branch j. Here $D_{0j}(\mathbf{q}, \omega)$ is the Green's function of the free phonon, and the polarization operator $\Pi_j(\mathbf{q}; \omega)$ is calculated by perturbation theory for the case of weak interaction of the type (2).

The frequencies of the phonon spectrum are obtained from the equation $D_j^{-1}=0$, i.e.,

$$D_{0j}^{-1}(\mathbf{q},\ \omega)=\Pi_j(\mathbf{q},\ \omega), \tag{4}$$

where $\Pi_j(\mathbf{q}, \omega)$ is expressed in terms of the electron Green's function G_0 with account of (2):

$$\Pi_j(\mathbf{q},\omega)\,\delta(\mathbf{q}-\mathbf{q}')=-\frac{16i}{(2\pi\hbar)^4\hbar^2\omega_0}\int d\mathbf{p}_1\,d\mathbf{p}_2\,d\omega_2$$
$$\times V_j(\mathbf{p}_1)G_0(\mathbf{p}_1+\mathbf{q},\ \mathbf{p}_2,\ \omega_2+\omega)\,V_j(\mathbf{p}_2)G_0(\mathbf{p}_2-\mathbf{q}',\mathbf{p}_1,\ \omega_2). \tag{5}$$

For an isotropic electron spectrum of metals located in a magnetic field, $\Pi_j(\mathbf{q}, \omega)$ was calculated by Blank and Kaner.[8] In this case, $\mathrm{Re}\Pi_j$ determines the modification of the phonon frequencies in the presence of electrons, and $\mathrm{Im}\Pi_j$ gives the Landau damping of the corresponding excitations. In Ref. 6, regions in the (ω, q) plane were established in which the Landau damping was different from zero. The results were shown in Figs. 1 and 2 of Ref. 9, where the region of existence of the Landau damping is shaded.

We are interested in the modification of the phonon spectrum of long-wave optical phonons in a degenerate semiconductor of the type n-Ge, where the electron spectrum is essentially anisotropic and consists of four ellipsoids, located along the diagonals of a cube. We shall consider optical phonons with wave vector $\mathbf{q}$, directed along a fourfold axis. As was shown in Ref. 1, for phonons propagating along symmetry directions there is no screening of the effects from the interaction (1), since there are groups of carriers that are symmetric relative to this direction and whose electron-current-density oscillations add up to zero.

The electron concentrations in the semiconductors are smaller by 2–3 orders of magnitude than in metals, and therefore the quantum limit $\varepsilon_F\approx\hbar\Omega$, where Ω is the cyclotron frequency, is easily achieved in a magnetic field. Only several lowest Landau levels are filled here. Since the distance between the ellipsoids is much greater than the Fermi momentum in a semiconductor and correspondingly larger than the magnetic momentum $p\gg p_F\approx(m\hbar\Omega)^{1/2}$, then we can neglect the interellipsoidal transitions in (5) and write down the polarization operators in the form of a sum over the ellipsoids:

$$\Pi_j(\mathbf{q},\omega)\,\delta(\mathbf{q}-\mathbf{q}')=-\frac{16i}{(2\pi\hbar)^4\hbar^2\omega_0}\sum_i\int d\mathbf{p}_1{}^i\,d\mathbf{p}_2{}^i\,d\omega_2$$
$$\times V_j(\mathbf{p}_1{}^i)G_0(\mathbf{p}_1{}^i+\mathbf{q},\mathbf{p}_2{}^i,\omega_2+\omega)\,V_j(\mathbf{p}_2{}^i)G_0(\mathbf{p}_2{}^i-\mathbf{q}',\mathbf{p}_1{}^i,\omega_2), \tag{6}$$

where the index i denotes the number of the ellipsoid.

We can also carry out such a transformation of coordinates in momentum space in Eq. (6) that the problem reduces to the case of the isotropic electron spectrum described by Landau oscillators. In each component we can choose the coordinate axes along the principal axes of the llipsoid and deform the reciprocal space in the following fashion:

$$(p_\alpha{}^i)'=p_\alpha{}^i(m/m_\alpha)^{1/2},\qquad \alpha=x,\,y,\,z, \tag{7}$$

where $m=(m_xm_ym_z)^{1/2}$. Correspondingly, the magnetic field in coordinate space will be transformed as follows (see, for example, Ref. 7):

$$(H_\alpha{}^i)'=H_\alpha{}^i(m_\alpha/m)^{1/2}. \tag{8}$$

It is seen from (7) and (8) that if $\mathbf{q}\parallel\mathbf{H}$ in the initial system of coordinates, then they will be directed at an angle with respect to one another in the deformed system. In this case, generally speaking, electron transitions not only with $\Delta n=0$, but also with $\Delta n\neq 0$ make a contribution to the renormalization of the phonon spectrum. However, as is shown in Ref. 10, in the long-wave limit, the principal role in the renormalization of the phonon spectrum in the case of low frequencies will be played by transitions with $\Delta n=0$ as before.

We now carry out a rotation of the deformed set of coordinates about the X axis so that the Z' axis coincides with the direction of $\mathbf{H}'$. Then, in the new coordinate system,

$$H_{x''}=H_{y''}=0,$$
$$H_{z''}=|H|\left[\frac{m_y}{m}\sin^2\alpha+\frac{m_z}{m}\cos^2\alpha\right]^{1/2}; \tag{9}$$

$$q_{x''}=q_x(m/m_x)^{1/2},\qquad q_{y''}=\frac{|q|\,(m_y/m_z)^{1/2}\sin 2\alpha\,(1-m_z/m_y)}{2(m_y\sin^2\alpha/m+m_z\cos^2\alpha/m)^{1/2}},$$
$$q_{z''}=|q|\left(\frac{m_y}{m}\sin^2\alpha+\frac{m_z}{m}\cos^2\alpha\right)^{1/2}, \tag{10}$$

where α is the angle between the directions [111] and [100]. For n-Ge,

$$\cos\alpha=1/\sqrt{3}. \tag{11}$$

We now make use of the expression for $\Pi_j(q, \omega)$ obtained by Blank and Kaner[8] for the isotropic electron spectrum, substituting H_z and q_z in it in the form (9) and (10). We then get

$$\mathrm{Re}\,\Pi_j(q,\omega)=\frac{1}{2\pi^2\hbar^2}\frac{emH}{cq}\sum_i\frac{2V_j^2(p_{0i})}{\hbar\omega_0}$$
$$\times\sum_{n,m}M_{nm}{}^2(\rho)\ln\left|\frac{[(q-p_m)^2-p_n{}^2]^2-(2m\hbar\omega)^2}{[(q+p_m)^2-p_n{}^2]^2-(2m\hbar\omega)^2}\right|, \tag{12}$$

$$\mathrm{Im}\,\Pi_j(q,\omega)=-\frac{\pi}{2\pi^2\hbar^2}\frac{emH}{cq}\sum_i\frac{2V_j^2(p_{0i})}{\hbar\omega_0}$$
$$\times\sum_{n,m}M_{nm}{}^2\,|f_0[\varepsilon_m(p_z^{(0)}-q_z)]-f_0[\varepsilon_n(p_z^{(0)})]|;$$
$$\varepsilon_n(p_z^{(0)})-\varepsilon_m(p_z^{(0)}-q)=\omega, \tag{13}$$

where

$$p_n=[2m[\varepsilon_F(H)-(n+{}^1/_2)\hbar\Omega]]^{1/2}.$$

Here

$$M_{nm}(\rho)=e^{-\rho/2}\rho^{|m-n|/2}L_{\min(m,n)}^{|m-n|}(\rho), \tag{14}$$
$$\rho=c(q_x{}^2+q_y{}^2)/2eH\hbar. \tag{15}$$

Using the explicit expression (1) for $V_j(\mathbf{p})$, we get $\mathrm{Re}\Pi_j(q, \omega)$ in the form of a sum of three contributions, corresponding to three possible polarizations of the optical phonon. The cross terms in the polarization vanish:

$$\mathrm{Re}\,\Pi_j(q,\omega)=4\xi_0\frac{\hbar\Omega}{v_Fq}\sum_{n,m}M_{nm}{}^2(\rho)\ln\left|\frac{[(q-p_m)^2-p_n{}^2]^2-(2m\hbar\omega)^2}{[(q+p_m)^2-p_n{}^2]^2-(2m\hbar\omega)^2}\right|$$
$$\times|p_{0i}|^{-4}[(p_{0i}{}^xp_{0i}{}^y)^2+(p_{0i}{}^xp_{0i}{}^z)^2+(p_{0i}{}^yp_{0i}{}^z)^2], \tag{16}$$

$$\zeta_0=\left(\frac{D_j a}{\hbar\omega_{0j}}\right)^2\frac{m^2}{M}\frac{v_F a}{2\pi^2\hbar}. \tag{16a}$$

2. VIBRATION SPECTRUM IN A QUANTIZING MAGNETIC FIELD

Substituting (16) in (4), we get the equation for the determination of the excitation spectrum:

$$\sum_{n,m=0}^{N} M_{nm}^2(\rho)\ln\left|\frac{[(q-p_m)^2-p_n^2]^2-(2m\hbar\omega)^2}{[(q+p_m)^2-p_n^2]^2-(2m\hbar\omega)^2}\right| =\frac{1}{4\zeta_0}\frac{v_F q}{\hbar\Omega}\frac{\omega^2(q)-\omega_0^2}{\omega_0^2} \tag{17}$$

Here N is the number of the upper filled Landau level.

For an isotropic electron spectrum $\rho=0$ and $M_{nm}^2(\rho)=\delta_{nm}$, and there are no transitions with $\Delta n\neq 0$.[9] In the case of n-Ge we have $m_x=m_y\neq m_z$; consequently $\rho\neq 0$ and M_{nm}^2 differs from zero at $n\neq m$. Transitions with $\Delta n\neq 0$ take place because of the anisotropy. However, as Protogenov and Demikhovskiĭ have shown,[10] the nondiagonal elements M_{nm} are small in terms of the parameter $q\ll(eH/c)^{1/2}$ in the case of long-wave phonons, and can be neglected. Equation (17) differs from the corresponding equation of Ref. 10 in that, because of the non-Coulomb character of our electron-phonon interaction, the right side contains only the first degree of q and does not contain poles at the frequency of the transverse optical phonon. Account of the acoustical phonons, expressed in terms of an addition to the Green's function of the acoustic phonons on the right side of Eq. (17) does not change the character of the equation in the low-frequency region.

$\mathrm{Im}\Pi_j$ from (13) determines the damping of the excitations whose spectrum is obtained from (17). The expression (13) for $\mathrm{Im}\Pi_j$ differs from that obtained by Blank and Kaner[8] only in the anisotropy, which can be removed by the transformation (9), (10). Therefore, the regions of (ω, q) with non-vanishing Landau damping are the same in our case as those found by Demikhovskiĭ and Protogenov.[6] These regions are shown in Ref. 9 in Fig. 1 by the cross-hatched region (for the case of a single filled Landau level) and by solid lines (for the case of two filled Landau levels).

We investigate the undamped solutions of Eq. (17) in the regions I, II, IV. In region I ($\omega\gtrsim\omega_0$), we get the renormalized optical phonon:

$$\omega^2(q)=\frac{\omega_0^2}{2}+\left(\frac{\omega_0^4}{4}+8\zeta_0\frac{\Omega\omega_0^2\beta_N}{\hbar m^2 v}q^2\right)^{1/2},\quad \beta_N=\sum_{0}^{N}p_n. \tag{18}$$

In the case of a single filled Landau level, this expression reduces to

$$\omega^2=\frac{\omega_0^2}{2}+\left\{\frac{\omega_0^4}{4}+8\zeta_0\frac{\Omega\omega_0^2}{\hbar m^2 v_F}q^2\left[2m\left(\varepsilon_F(H)-\frac{1}{2}\hbar\Omega\right)\right]^{1/2}\right\}^{1/2}. \tag{19}$$

In Fig. 1 of Ref. 9, the branch (19) is indicated by thick lines in region I.

In region of type IV ("petal-shaped") in Fig. 2 of Ref. 9, the solution in the case of two filled Landau levels has the form of a sound wave with velocity

$$s^2=\frac{u_0^2+u_1^2}{2}-\frac{4\zeta_0\hbar\Omega}{mv_F}(u_0+u_1)\pm\left\{\frac{(u_0^2-u_1^2)^2}{4}+\left(\frac{4\zeta_0\hbar\Omega}{mv_F}\right)\times(u_0+u_1)\left[\frac{4\zeta_0\hbar\Omega}{mv_F}(u_0+u_1)-(u_0-u_1)^2\right]\right\}^{1/2}, \tag{20}$$

where the electron velocity is $u_n=p_n/m$. The two solutions in (20) correspond to two acoustic branches. One solution is similar to the acoustic plasmons, found earlier in metals by Konstantinov and Perel',[11] and the second to second sound, which we shall discuss below for a single filled Landau level.

We now consider the region II ("bell-shaped"). In the quantum limit $\varepsilon_F(0)\ll\hbar\Omega$ only a single Landau level is filled; region II is small and extends from $q=0$ to $2q=4\sqrt{2}\,\varepsilon_F(0)p_F(0)/3\hbar\Omega$. A solution of Eq. (17) at $\omega\ll\omega_0$ can be obtained in this case at any q within the limits of the bell-shaped II (see Fig. 1 in Ref. 9):

$$\omega^2=q^2\left\{\left(u_0^2+\frac{q^2}{4m^2}\right)-u_0\frac{q}{m}\operatorname{cth}\left(-\frac{qv_F}{8\zeta_0\hbar\Omega}\right)\right\}. \tag{21}$$

In the limit of long waves, at $q\ll\hbar\Omega\cdot 8\xi_0/v_F$, this solution corresponds to zero-sound oscillations of the electrons interacting through the optical phonons. The velocity of this phonon zero sound is equal to

$$s^2=\frac{2}{9}\left(\frac{mv_F^2}{\hbar\Omega}\right)^2 v_F^2\left(1-3\sqrt{2}\cdot 4\zeta_0\left(\frac{\hbar\Omega}{mv_F^2}\right)^2\right). \tag{22}$$

It is then seen that a threshold value of the magnetic field exists:

$$H_{\rm th}=2mc\varepsilon_F/(3\sqrt{2}\cdot 4\zeta_0)^{1/2}\hbar e, \tag{23}$$

For this value, s^2 vanishes. At $H>H_{\rm th}$ the velocity s^2 is negative and corresponds to absolute instability of the cubic configuration of the lattice, which leads to a phase transition with a lowering of the symmetry of the crystal.

The threshold values of the magnetic field, at which $s=0$ and phase transitions of the specified type take place, also exist in the case of two, three, and more filled Landau levels. The equations for the threshold values of the field can be simply obtained by setting $s=0$ in (17) in the long-wave limit. We then have

$$\frac{1}{2}\sum_{n=0}^{N}\hbar\Omega\left[\varepsilon_F(H)-\left(n+\frac{1}{2}\right)\hbar\Omega\right]^{-1/2}\frac{\zeta_0\sqrt{2}}{m^{1/2}v_F}=\frac{1}{8}. \tag{24}$$

We note that the condition of instability of the lattice of type (24) was obtained by Migdal,[12] in the form

$$\zeta_0={}^1/_2. \tag{25}$$

For a Fermi surface consisting of four ellipsoids, $\zeta=4\zeta_0$, and this condition is transformed into $\zeta_0=1/8$. As is seen from the definition (16a), ζ_0 is proportional to the electron density of states at the Fermi level $\zeta_0=Am^2v_F=An(\varepsilon_F)$. Then, in place of (25), we have

$$An(\varepsilon_F)={}^1/_8. \tag{26}$$

The condition (24) that we have obtained has exactly the same structure, but in it, $n(\varepsilon_F)$ signifies the electron density of states at the Fermi level in a magnetic field.

A phenomenological theory of the phase transition of such a type, due to the interaction of electrons with acoustic phonons, was constructed for an arbitrary number of electronic Landau levels by Kochelaev and Sokolov.[5] The difference in the electron-phonon interaction with acoustic and optical phonons, which is important for the determination of the renormalization of the phonon spectrum, does not appear in the case of establishment of the transition point. Correspondingly, Eq. (24) for the determination of the threshold values of the field, and in particular, H_{th} from (23) for a single filled Landau level, coincide with the values found in Ref. 5, with accuracy to replacement of the constant of the deformation potential for the optical oscillations by a similar constant for acoustic oscillations (see Eq. (21) in Ref. 5).

At $H<H_{th}$ the quantity s^2 increases to the value

$$s^2=\frac{4}{3^{4/3}}v_F^2\left[1-\frac{4\sqrt{2}}{3^{1/3}}\zeta_0\right],$$

which corresponds to initial filling of the second Landau level and a modification of the regions of damping from those of Fig. 1 to those of Fig. 2 in Ref. 9. Our solution of the zero-sound type in this case transforms into the solution in region IV (20). Here the positive solution for s^2 is initially absent in region II (which corresponds to the second region of instability in Ref. 5). Upon further decrease in H, the zero-sound solution reappears. These oscillations of the velocity of zero sound have the same nature as the giant oscillations in the absorption of sound by metals.[13]

It should be noted that solution (21) vanishes at finite values of q close to $2p_0$ for the case $H=H_{th}$, which corresponds to an instability of the Peierls type. However, in the case of not too low temperatures, the maximum value of Π close to the threshold of Landau damping is finite. Therefore, the minimum value of the frequency ω is close to $q\approx 2p_0$ is positive and vanishes only at $q=0$.

In a magnetic field of $H\approx 10^5$ Oe, the cyclotron frequency $\Omega\approx 10^{13}$ sec^{-1} is greater than the Fermi energy in semiconductors with $n\approx 10^{17}$ cm^{-3} and $\varepsilon_F/\hbar\approx 10^{12}$ sec^{-1}. In strongly doped semiconductor, the collision frequency can be of the order of $\varepsilon_F/\hbar$ and the obtained branches will be diffuse. Therefore it is desirable to carry out experiments, for example, in pure germanium with optical pumping of electrons to $n\approx 10^{17}$ cm^{-3}.

Thus, there is a possibility in principle of observation of the branches mentioned. Such an observation is made difficult by the fact that the corresponding oscillations are not accompanied by oscillations of the electron density and interact weakly with the external electromagnetic field. However, such oscillations can be found in light scattering and should give a picture similar to Brillouin scattering, but with a strong dependence on the magnetic field H.

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