Nonlinear damping of helicons in metals

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A nonlinear theory of magnetic Landau damping of a helicon moving at a certain angle to a stationary magnetic field is developed. It is shown that trapping of particles by moving magnetic bottles formed by the force lines of the magnetic field and magnetic field of the wave is effective in the field of a strong wave. Trapping of the particles results in modulation of the velocity component along the stationary magnetic field. The conditions of resonant interaction between the particles and wave are then violated and the helicon damping coefficient decreases. In the case of strong nonlinearity the damping coefficient is proportional to $(\omega_0\tau)^{-1}\ll 1$, where ω_0 is a characteristic oscillation frequency of the trapped particles and τ^{-1} is the collision frequency. Estimates are presented which indicate the possibility of observing the effect experimentally.

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1. INTRODUCTION

In two metals with unequal electron and pole densities, at low temperatures and in the presence of a magnetic field, low-frequency electromagnetic excitations—helicons[1]—can propagate. These excitations were observed in many metals (see, e.g., the literature cited in the reviews[2,3]).

Helicons experience both collisionless and collision-induced damping. The latter is quite small in pure metals at a cyclotron frequency ω_c much higher than the collision frequency τ^{-1}. When helicons propagate along a magnetic field parallel to the symmetry axis of the crystal, there is a single mechanism of collisionless damping, namely cyclotron damping, which appears at $kR > 1$ (k is the wave vector and R is the Larmor radius). In the region $kR < 1$, which is of greatest interest from the point of view of experiment, helicons propagating at an angle θ to the magnetic field experience magnetic Landau damping. At not too small angles θ and at $|k_z l| \gg 1$ (k_z is the projection of $\mathbf{k}$ on the direction of the magnetic field $\mathbf{H}_0$ and l is the electron mean free path), the magnetic damping prevails over the collision damping.

The linear theory of the Landau magnetic damping was constructed by Kaner and Skobov.[4] The purpose of the present paper is to construct a nonlinear theory of the magnetic Landau damping. We shall show below that a helicon of sufficiently large amplitude can have a damping coefficient much smaller than the damping coefficient of a weak wave. Estimates that demonstrate the feasibility of experimentally observing this effect will be presented.

Let us stop to discuss the physical picture of the phenomenon. Let a helicon propagate at an angle θ to a magnetic field directed along the z axis, and let the wave vector $\mathbf{k}$ lie in the yz plane. The spectrum of the metal is assumed to be isotropic and quadratic. The magnetic field of the wave $\mathbf{H}$ is polarized here circularly in a plane perpendicular to $\mathbf{k}$, and has the components

$$H_x=H\sin(k_y y+k_z z-\omega t),\qquad H_y=H\cos\theta\cos(k_y y+k_z z-\omega t),\qquad H_z=-H_y \operatorname{tg}\theta, \tag{1}$$

where H is the amplitude of the magnetic field of the wave and ω is its frequency. The electric field lies in the xy plane, with

$$E_x=\frac{v_{\rm ph}}{c\cos\theta}H_y,\qquad E_y=-\frac{v_{\rm ph}}{c\cos\theta}H_x, \tag{2}$$

where $v_{\rm ph}=\omega/k$ is the phase velocity of the wave.

The longitudinal component E_z, as is well known, is small in comparison with E_x and E_y and will henceforth be disregarded. We shall assume that $kR\ll 1$; in addition, in metals we also have $\omega(\mathbf{k})\ll\omega_c$. This means that the motion of the electrons in the field of the wave in a constant magnetic field $\mathbf{H}_0$ constitutes rapid rotation on a Larmor orbit whose center moves in fields that vary slowly in space and in time.

It is easily seen that in the region $kR\ll 1$ the helicon interacts effectively with particles having orbits with centers that move in a constant-phase plane of the wave and for which the condition $k_z v_z=\omega$ is satisfied. As a result of the action exerted on these particles by the wave electric and magnetic field components E_x and H_y, the center of the orbit accelerates in the z direction and the particle energy changes. This leads to the magnetic Landau damping.

To consider the nonlinear effects it is necessary to take into account the fact that the magnetic field of a strong wave greatly distorts the motion of the resonant particles. This occurs in the following manner. The magnetic force lines of the field $\mathbf{H}_0+\mathbf{H}$ form a system of twisted moving magnetic "bottles." The planes of condensation and rarefaction of the force lines are perpendicular to $\mathbf{k}$. The magnetic bottles trap resonant particles. The centers of the Larmor orbits of the trapped electrons execute, besides translational motion with velocity ω/k_z, also oscillation along z with a characteristic frequency

$$\omega_0=k_z v_F\left(\frac{h\sin\theta}{2}\right)^{1/2}, \tag{3}$$

where v_F is the Fermi velocity, and $h=H/H_0$. In a weak-field wave the frequency of such oscillations is much lower than the collision frequency τ^{-1}; the trapping is then insignificant and the linear theory is valid. In the field of a sufficiently strong wave, the period of the oscillations is too short for the particle to be scattered. In this case, owing to the modulation of the velocity v_z, the conditions for resonant interaction of the particles with the wave are violated and the damping coefficient decreases. At the same time, since the helicon spectrum in the region $kR \ll 1$ is formed by all the electrons of the Fermi surface, and the trapped electrons lie in a narrow velocity region $|v_z - \omega/k_z| \sim \omega_0/k_z \ll v_F$, the real part of the helicon spectrum is not changed by the trapping.

The mathematical formulation of the theory is based on a solution of the Boltzmann kinetic equation by the method of characteristics, which are the trajectories of the particles in a constant magnetic field and the wave fields. These trajectories will be considered in the next section.

It should be noted that our problem is analogous to a certain degree to the problem of nonlinear absorption of longitudinal sound in conductors, which was considered in the interesting papers of Gal'perin, Gurevich, Kagan, and Kozub.[5,6] In both cases, the decrease of the damping coefficient is connected with the trapping of the particles, but in one case the trapping is due to the action of the longitudinal sound-wave field, and in the other to the presence of magnetic bottles. The problem of nonlinear cyclotron damping of helicons in a collisionless ionospheric plasma was considered by Bud'ko, Karpman, and Pokhotelov.[7]

2. PARTICLE TRAJECTORY

The nonlinear increment to the equilibrium distribution function is conveniently sought in a coordinate system that moves along the z axis with velocity $v_0 = v_{ph}/\cos\theta$. It is easy to verify that in this coordinate system there is no electric field (we neglect the component E_z) and the magnetic field of the wave, accurate to terms of order $(v_{ph}/c)^2$, is equal to its value in the laboratory frame.

Let us consider the particle trajectories. Their motion is described by the equation

$$m\frac{d\mathbf{v}}{dt} = \frac{e}{c}[\mathbf{v}\times(\mathbf{H}_0+\mathbf{H})], \tag{4}$$

where $\mathbf{H}$ is given by formulas (1), and e and m are the charge and effective mass of the particle. We introduce the variable $\xi = k_y y + k_z z$ and express (4) in the cylindrical coordinates v_z, $v_\perp$, Φ, where $v_x = v_\perp\cos\Phi$, $v_y = v_\perp\sin\Phi$:

$$\dot\Phi = \omega_c\left\{1 - h\frac{v_z}{v_\perp}(\cos\xi\cos\theta\sin\Phi + \sin\xi\cos\Phi) - h\cos\xi\sin\theta\right\},$$
$$\dot v_z = -\omega_c h v_\perp(\cos\xi\cos\Phi\cos\theta - \sin\Phi\sin\xi), \tag{5}$$
$$\dot v_\perp = -(v_z/v_\perp)\dot v_z.$$

In order for the system (5) to be closed, it must be supplemented by an equation for ξ, which takes the form

$$\frac{d\xi}{dt} = k_y v_\perp\sin\Phi + k_z v_z = \omega_c\left(k_y R\sin\Phi + k_z R\frac{v_z}{v_\perp}\right), \tag{6}$$

where $R = v_\perp/\omega_c$. The energy conservation law $v^2 = v_\perp^2 + v_z^2 = \text{const}$ follows from (4). Inasmuch as in the considered coordinate system the waves interact effectively with particles in the velocity region $|v_z| \lesssim \max\{(k_z\tau)^{-1}, \omega_0/k_z\} \ll v_\perp \approx v_F$, when considering the equations of motion it is assumed that $v_z \ll v_\perp$. In view of the smallness of h, kR, and $v_z/v_\perp$, Eqs. (5) and (6) can be investigated by the method of averaging over the fast variations of the phase Φ ([8], Sec. 25). In our case, however, the analysis becomes simpler. In view of the smallness of $v_z/v_\perp$, we can neglect the variation of $v_\perp$ and put $v_\perp \approx v$. Taking into account also the smallness of the parameter h, we can neglect the small modulation of the angular velocity and assume that $\Phi = \omega_c t$. Allowance for the small modulation of the angular velocity would lead to inessential corrections in the equations of the characteristics (12).

We shall seek the solution of equations (5) and (6) in the form $\xi = \bar\xi - k_y R\cos\Phi$, $v_z = \bar v_z$, where $\bar\xi$ and $\bar v_z$ describe the motion of the center of the Larmor orbit, and the term $-k_y R\cos\Phi$ describes the rapid variation of the coordinate y of the electron as a result of the rotation. Averaging with respect to Φ in (5) and (6), confining ourselves to terms of first order in $k_y R \ll 1$, and changing over to the dimensionless variable

$$s = \frac{\bar v_z}{v_F(\frac{1}{2}h\sin\theta)^{1/2}},$$

we obtain

$$\frac{ds}{dt} = -\omega_0\sin\bar\xi, \quad \frac{d\bar\xi}{dt} = \omega_0 s, \tag{7}$$

where ω_0 is defined by formula (3). The integral $\mathscr{H} = \frac{1}{2}s^2 - \cos\bar\xi = \text{const}$ follows from (7).

Introducing

$$\varkappa = \operatorname{sign} s[2/(\mathscr{H}+1)]^{1/2},$$

we obtain an equation for $\bar\xi$:

$$\frac{d\bar\xi}{dt} = \frac{2\omega_0}{\varkappa}\sqrt{1-\varkappa^2\sin^2\frac{\bar\xi}{2}}, \tag{8}$$

the solution of which is expressed in terms of the Jacobi amplitude. It is easily seen that $|\varkappa|$ ranges from zero to infinity. If $0 < |\varkappa| < 1$, then $\bar\xi$ can assume arbitrary values and the motion of the particle is infinite with respect to $\bar\xi$. These particles will be called untrapped. At $|\varkappa| > 1$, the motion of the particle is finite and the turning points are determined from the condition that the right-hand side of (8) vanish. These particles will be called trapped. In the next section we shall find the distribution functions of the trapped and untrapped particles.

3. DISTRIBUTION FUNCTION

In a coordinate system that moves along the z axis with velocity v_0, the Boltzmann kinetic equation takes the form

$$v_y\frac{\partial f}{\partial y}+v_z\frac{\partial f}{\partial z}+\frac{e}{c}[\mathbf{v}\times(\mathbf{H}_0+\mathbf{H})]\frac{\partial f}{\partial \mathbf{p}}+\hat{I}\{f\}=0, \quad (9)$$

where $\hat{I}\{f\}$ is the collision integral. (We assume that the temperature is low enough and that the electrons are scattered primarily by impurities. In this case the collision integral is linear in f.) We have left out from (9) the derivative $\partial f/\partial t$, which is proportional to the small damping coefficient of the wave.

We seek the solution of (9) in the form

$$f=F_0(\mathbf{v}+\mathbf{v}_0)+g_{\mathbf{p}}(y,z),$$

where $F_0(\mathbf{v})$ is the equilibrium distribution function. Changing over to the variables

$$\xi=k_y y+k_z z,\quad v_z,\quad \varepsilon={}^1/_2 m(v_z^2+v_\perp^2),\quad \Phi,$$

we obtain

$$(k_z v_z+k_y v_\perp(\varepsilon,v_z)\sin\Phi)\frac{\partial g}{\partial \xi}-h\omega_c v_\perp(\varepsilon,v_z)$$
$$\times[\cos\Phi\cos\xi\cos\theta-\sin\Phi\sin\xi]\frac{\partial g}{\partial v_z}+\omega_c\frac{\partial g}{\partial \Phi}+\hat{I}\{g\}$$
$$=-F_0'(\mathbf{v}+\mathbf{v}_0)\frac{v_{\mathrm{ph}}eH}{c}v_\perp(\varepsilon,v_z)\left[\cos\Phi\cos\xi-\frac{\sin\Phi\sin\xi}{\cos\theta}\right]. \quad (10)$$

Here

$$v_\perp(\varepsilon,v_z)=(2m\varepsilon-v_z^2)^{1/2},$$

F_0' is the derivative of the distribution function with respect to energy. In the coefficient of the derivative $\partial g/\partial\Phi$ we have omitted the inessential terms of order $h\ll 1$ (see Sec. 2).

Instead of ξ it is convenient to change over in (10) to the variable

$$\bar{\xi}=\xi+k_y\frac{v_\perp}{\omega_c}\cos\Phi,$$

which describes the motion of the center of the Larmor orbit. We next expand in terms of $k_y v_\perp(\varepsilon,v_z)/\omega_c\ll 1$ up to terms of first order. The resultant equation, just as (10), is linear in g, so that we can separate in it that part of the distribution function (which we designate g_1), which describes in the region $kR<1$, $\omega\ll\omega_c$ the resonant interaction of the particles with the wave, and consequently also the Landau absorption. It is easy to verify that the equation for g_1 should contain in the right-hand side only the expression

$$-F_0'(\mathbf{v}+\mathbf{v}_0)\frac{v_{\mathrm{ph}}eH}{2c\omega_c}k_y v_\perp^2(\varepsilon,v_z)\sin\bar{\xi},$$

which is connected with the component H_y. Recognizing that the particles effectively interacting with the waves are those from the narrow region

$$|v_z|\lesssim\max\{(k_z\tau)^{-1},\ \omega_0/k_z\}\ll v_F,$$

i.e., the distribution function g_1 differs significantly from zero only in a narrow velocity region v_z, we can neglect in the collision integral the arrival terms and write this integral in the form $\hat{I}\{g_1\}\approx\tau^{-1}g_1$, where τ is the time of departure from the state $\mathbf{p}$ to the resonance region. In view of the smallness of v_z, we can also put $v_\perp(\varepsilon,v_z)\approx v_\perp(\varepsilon,0)\equiv v_\perp(\varepsilon)=(2m\varepsilon)^{1/2}$.

Taking the foregoing into account, we write the equation for g_1 in the form

$$k_z v_z\frac{\partial g_1}{\partial\bar{\xi}}-h\omega_c v_\perp(\varepsilon)\Big[\cos\Phi\cos\bar{\xi}\cos\theta-\sin\Phi\sin\bar{\xi}$$
$$+k_y\frac{v_\perp(\varepsilon)}{\omega_c}(\cos^2\Phi\sin\bar{\xi}\cos\theta+\sin\Phi\cos\Phi\cos\bar{\xi})\Big]\frac{\partial g_1}{\partial v_z}$$
$$+\omega_c\frac{\partial g_1}{\partial\Phi}+\tau^{-1}g_1=-F_0'(\mathbf{v}+\mathbf{v}_0)\frac{v_{\mathrm{ph}}eH}{2c\omega_c}k_y v_\perp^2(\varepsilon)\sin\bar{\xi}. \quad (11)$$

Neglecting in the coefficient of the derivative $\partial g_1/\partial v_z$ the terms that oscillate rapidly in Φ (see Sec. 2), we obtain the following equations of the characteristics:

$$\frac{d\bar{\xi}}{k_z v_z}=\frac{dv_z}{-{}^1/_2 hk_y v_\perp^2(\varepsilon)\cos\theta\sin\bar{\xi}}=\frac{d\Phi}{\omega_c}=\frac{dg_1}{-\tau^{-1}g_1+U}, \quad (12)$$

where U denotes the right-hand side of (11).

From (12) follows the integral of motion

$$\mathscr{H}=v_z^2/v_F^2h\sin\theta-\cos\bar{\xi}={}^1/_2 s^2-\cos\bar{\xi}$$

(see Sec. 2). The equation for g_1 has a solution periodic in Φ, with a period 2π in the form

$$g_1(\bar{\xi},\Phi,\varepsilon,\mathscr{H})=-F_0'(\mathbf{v}+\mathbf{v}_0)\frac{v_{\mathrm{ph}}eH}{2c}k_yR^2\int_{-\infty}^{\Phi}\exp\left(\frac{\Phi'-\Phi}{\omega_c\tau}\right)\sin\bar{\xi}(\Phi')d\Phi' \quad (13)$$

where $\bar{\xi}(\Phi')$ is the solution of the equation

$$\frac{d\bar{\xi}(\Phi')}{d\Phi'}=\frac{2\omega_0}{\omega_c\varkappa}\left(1-\varkappa^2\sin^2\frac{\bar{\xi}(\Phi')}{2}\right)^{1/2}, \quad (14)$$

which passes through the point $\bar{\xi}(\Phi'=\Phi)=\xi$.

We consider separately the trapped and untrapped particles. For the trapped particles ($|\varkappa|<1$) the solution (14), as noted above, is expressed in terms of the Jacobi amplitude

$$\bar{\xi}(\Phi')=2\,\mathrm{am}\left\{\frac{\omega_0}{\omega_c\varkappa}(\Phi'-\Phi_0(\bar{\xi},\Phi)),\varkappa\right\}. \quad (15)$$

Here $\Phi_0(\bar{\xi},\Phi)$ is the value of Φ' at which the curve $\bar{\xi}(\Phi')$ arriving at the point $(\bar{\xi},\Phi)$ passes through the point $\bar{\xi}=0$.

Substituting (15) in (13), we obtain the distribution function $g_{1\,ut}$ of the untrapped particles. To calculate the integral we use the expansion of $\sin\bar{\xi}(\Phi')$ in a Fourier series[9]:

$$\sin\bar{\xi}(\Phi')=2\,\mathrm{sn}\left\{\frac{\omega_0}{\omega_c\varkappa}(\Phi'-\Phi_0(\bar{\xi},\Phi)),\varkappa\right\}\mathrm{cn}\left\{\frac{\omega_0}{\omega_c\varkappa}(\Phi'-\Phi_0(\bar{\xi},\Phi)),\varkappa\right\}$$
$$=\left(\frac{2\pi}{\varkappa K(\varkappa)}\right)^2\sum_{n=1}^{\infty}\frac{nq^n(\varkappa)}{1+q^{2n}(\varkappa)}\sin\left(\frac{\pi n\omega_0}{\omega_c\varkappa K(\varkappa)}(\Phi'-\Phi_0(\bar{\xi},\Phi))\right),$$

where $K(\varkappa)$ is a complete elliptic integral of the first kind

$$q(\varkappa)=\exp\left(-\frac{\pi K(\sqrt{1-\varkappa^2})}{K(\varkappa)}\right).$$

Elementary integration yields

$$g_{1ut}=-F_0'(v+v_0)\frac{v_\Phi eH}{2c}k_yR^2\left(\frac{2\pi}{\varkappa K(\varkappa)}\right)^2\sum_{n=1}^{\infty}\frac{nq^n(\varkappa)}{1+q^{2n}(\varkappa)}$$
$$\times\frac{1}{(\omega_c\tau)^{-2}+(\pi n\omega_0/\omega_c\varkappa K(\varkappa))^2}\left\{(\omega_c\tau)^{-1}\sin\left(\frac{\pi nF(\bar\xi/2,\varkappa)}{K(\varkappa)}\right)\right.$$
$$\left.-\frac{\pi n\omega_0}{\omega_c\varkappa K(\varkappa)}\cos\left(\frac{\pi nF(\bar\xi/2,\varkappa)}{K(\varkappa)}\right)\right\}. \qquad (16)$$

Here $F(\bar\xi/2,\varkappa)$ is an elliptic integral of the first kind.

To calculate the distribution function of the trapped particles $g_{1t}(|\varkappa|>1)$ it is convenient to break up the integration region into intervals of length $\Phi(\varkappa)$, where $\Phi(\varkappa)$ is the angle through which the particle rotates on the Larmor orbit with "energy" $\mathscr{H}=2/\varkappa^2-1$, executing a complete oscillation in the "potential" well. If $\bar\xi_1$ and $\bar\xi_2$ are respectively the left-hand and right-hand turning points, determined from the condition $\mathscr{H}=-\cos\bar\xi$, then it is easily seen that

$$\Phi(\varkappa)=\frac{2\omega_c|\varkappa|}{\omega_0}\left[F\left(\frac{\bar\xi_2}{2},\varkappa\right)-F\left(\frac{\bar\xi_1}{2},\varkappa\right)\right].$$

Breaking up the integral in (13) in integrals over the segments $(\Phi,\Phi-\Phi(\varkappa))$, $(\Phi-\Phi(\varkappa),\Phi-2\Phi(\varkappa))\ldots$ and making in each integral a change of the integration variable of the type $\Phi'\to\Phi'-n\Phi(\varkappa)$ $(n=1,2,\ldots)$, we obtain

$$g_{1t}(\bar\xi,\Phi,\varepsilon,\varkappa)=-F_0'(v+v_0)\frac{v_{\rm ph}eH}{2c}k_yR^2$$
$$\times\frac{1}{\exp(\Phi(\varkappa)/\omega_c\tau)-1}\int_{\Phi}^{\Phi+\Phi(\varkappa)}\exp\left(\frac{\Phi-\Phi'}{\omega_c\tau}\right)\sin\bar\xi(\Phi')d\Phi'. \qquad (13a)$$

Without loss of generality, we can assume that the trapped particles oscillate in a well containing the point $\bar\xi=0$. It is obvious that $\bar\xi_1=-\bar\xi_2$. We denote by $\Phi_0(\bar\xi,\Phi)$ the value of Φ' closest to Φ at which the particle passes through the point $\bar\xi=0$ as it moves along a trajectory passing through the point $(\bar\xi,\Phi)$. Then, as can be easily verified, the equation of the trajectory takes the form

$$\frac{\bar\xi(\Phi')}{2}=\mathrm{am}\left\{\frac{\omega_0}{\omega_c\varkappa}(\Phi'-\Phi_0(\bar\xi,\Phi)),\varkappa\right\},\quad \Phi<\Phi'<\Phi_0(\bar\xi,\Phi)+\frac{1}{4}\Phi(\varkappa);$$
$$\frac{\bar\xi(\Phi')}{2}=-\mathrm{am}\left\{\frac{\omega_0}{\omega_c\varkappa}\left(\Phi'-\Phi_0(\bar\xi,\Phi)-\frac{1}{2}\Phi(\varkappa)\right),\varkappa\right\},$$
$$\Phi_0(\bar\xi,\Phi)+{}^1/_4\Phi(\varkappa)<\Phi'<\Phi_0(\bar\xi,\Phi)+{}^3/_4\Phi(\varkappa); \qquad (17)$$
$$\frac{\bar\xi(\Phi')}{2}=\mathrm{am}\left\{\frac{\omega_0}{\omega_c\varkappa}(\Phi'-\Phi_0(\bar\xi,\Phi)-\Phi(\varkappa)),\varkappa\right\},$$
$$\Phi_0(\bar\xi,\Phi)+{}^3/_4\Phi(\varkappa)<\Phi'<\Phi+\Phi(\varkappa).$$

We next expand $\sin\bar\xi(\Phi')$ in a Fourier series. For concreteness we assume

$$\Phi<\Phi'<\Phi_0(\bar\xi,\Phi)+{}^1/_4\Phi(\varkappa)$$

(the generalization to other regions of Φ' is obvious). According to[9], we have

$$\sin\bar\xi(\Phi')=\frac{2}{|\varkappa|}\mathrm{sn}\left\{\frac{\omega_0\varkappa}{\omega_c|\varkappa|}(\Phi'-\Phi_0(\bar\xi,\Phi)),\frac{1}{\varkappa}\right\}$$
$$\times\mathrm{dn}\left\{\frac{\omega_0\varkappa}{\omega_c|\varkappa|}(\Phi'-\Phi_0(\bar\xi,\Phi)),\frac{1}{\varkappa}\right\}=\left(\frac{2\pi}{K(\varkappa^{-1})}\right)^2\sum_{n=1}^{\infty}\frac{(n-{}^1/_2)q^{n-1/2}(\varkappa^{-1})}{1+q^{2n-1}(\varkappa^{-1})}$$
$$\times\sin\left(\frac{\pi\omega_0(n-{}^1/_2)\varkappa}{\omega_c|\varkappa|K(\varkappa^{-1})}(\Phi'-\Phi_0(\bar\xi,\Phi))\right).$$

Carrying out the integration in (13a), we obtain the distribution function of the trapped particles

$$g_{1t}=-F_0'(v+v_0)\frac{v_\Phi eH}{2c}k_yR^2\left(\frac{2\pi}{K(\varkappa^{-1})}\right)^2$$
$$\times\sum_{n=1}^{\infty}\frac{(n-{}^1/_2)q^{n-1/2}(\varkappa^{-1})}{1+q^{2n-1}(\varkappa^{-1})}\frac{1}{(\omega_c\tau)^{-2}+(\pi\omega_0(n-{}^1/_2)/\omega_cK(\varkappa^{-1}))^2}$$
$$\times\left\{(\omega_c\tau)^{-1}\sin\left(\frac{\pi(n-{}^1/_2)|\varkappa|}{K(\varkappa^{-1})}F\left(\frac{\bar\xi}{2},\varkappa\right)\right)\right.$$
$$\left.-\frac{\pi\omega_0(n-{}^1/_2)\varkappa}{\omega_c|\varkappa|K(\varkappa^{-1})}\cos\left(\frac{\pi(n-{}^1/_2)|\varkappa|}{K(\varkappa^{-1})}F\left(\frac{\bar\xi}{2},\varkappa\right)\right)\right\}. \qquad (18)$$

The derived formulas (16) and (18) for the distribution function enable us to obtain the nonlinear damping coefficient.

4. NONLINEAR DAMPING COEFFICIENT

As noted in the Introduction, the magnetic Landau damping is connected with the action exerted on the electrons by the field components E_x and H_y. In the linear theory, the damping is determined by a single component σ_{xx} of the conductivity tensor.[4] In the nonlinear regime, when the field of the wave changes the trajectories of the resonant particles, we shall calculate the damping by using a relation that connects the work done by the wave on the particles with the change of the wave energy. This relation is written in the form

$$\frac{\partial}{\partial t}\langle W\rangle=2\,\mathrm{Im}\,\omega\langle W\rangle=-\langle\mathbf{jE}(\mathbf{r}t)\rangle. \qquad (19)$$

The angle brackets denote averaging over the period of the wave,

$$\langle W\rangle=\frac{1}{16\pi}\left\{\frac{d}{d\omega}(\omega\varepsilon_{ik})E_i^*E_k+|\mathbf{H}|^2\right\}$$

is the electromagnetic-wave energy averaged over the period,[10] and $\mathbf{E}$ and $\mathbf{H}$ are the complex amplitudes of the wave field and are defined by relations of the type

$$\mathbf{H}(\mathbf{r}t)=\mathrm{Re}(\mathbf{H}e^{-i\omega t+i\mathbf{kr}}).$$

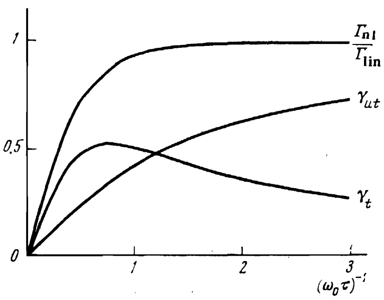
In our case, the helicon energy is in essence magnetic, i.e., $\langle W\rangle\approx|\mathbf{H}|^2/16\pi$. As follows from the foregoing

$$\langle\mathbf{jE}(\mathbf{r}t)\rangle=\langle j_xE_x(\mathbf{r}t)\rangle.$$

The energy absorbed by the particles will be calculated in an immobile coordinate system by using the formula

$$\langle j_xE_x\rangle=\frac{\omega}{2\pi}\int_t^{t+2\pi/\omega}dt'j_x\frac{v_\Phi H}{c}\cos(k_yy+k_zz-\omega t')=\frac{1}{2\pi}\int_{-\pi}^{\pi}d\xi\,j_x\frac{v_\Phi H}{c}\cos\xi, \qquad (20)$$

where the current j_x is given by



$$j_x=\frac{2em^2}{(2\pi)^3}\int_0^\infty d\varepsilon \int_{-(2m\varepsilon)^{1/2}}^{(2m\varepsilon)^{1/2}} dv_z \int_0^{2\pi} d\Phi\, v_\perp(\varepsilon, v_z)\, g_1 \cos\Phi \tag{21}$$

(the current j_x has the same values in the moving and in the immobile coordinate systems). It is next necessary to substitute (21) in (20), change the order of integration with respect to Φ and ξ, and change over from integration with respect to the variables ξ and v_z to integration with respect to the variables $\bar{\xi}$ and $\varkappa$. It is also necessary to take into account the fact that, in view of the smallness of v_0/v_F, we can put $F_0'(\mathbf{v}+\mathbf{v}_0) \approx -\delta(\varepsilon-\varepsilon_F)$. In the integration with respect to Φ it is necessary to use the fact that for any function $f(\bar{\xi})$ which is periodic with a period 2π, the following relation holds at $k_yR \ll 1$:

$$\int_0^{2\pi} d\Phi \cos\Phi \int_{-\pi+k_yR\cos\Phi}^{\pi+k_yR\cos\Phi} d\bar{\xi}\, f(\bar{\xi})\cos(\bar{\xi}-k_yR\cos\Phi)=\pi k_yR\int_{-\pi}^{\pi} d\bar{\xi}\, f(\bar{\xi})\sin\bar{\xi}.$$

The integration with respect to $\bar{\xi}$ can then be easily carried out by making the change of variable $\bar{\xi} = 2\,\mathrm{am}\{\tau, \varkappa\}$ and subsequently expanding $\sin(2\,\mathrm{am}(\tau,\varkappa))$ in a Fourier series (in analogy with the procedure used in Sec. 3).

As the result of the transformations, we obtain from (19) and (20) the nonlinear damping coefficient

$$\Gamma_{nl}=-\mathrm{Im}\,\omega/\omega=\Gamma_{lin}(\gamma_t+\gamma_{ut}), \text{ where } \Gamma_{lin}={}^3/_{16}\pi kR\sin^2\theta$$

is the linear damping coefficient of the helicon, γ_t and γ_{ut} determine the contributions of the trapped and untrapped particles, respectively, with

$$\gamma_t=128\pi^2\int_1^{+\infty}\frac{d\varkappa}{\varkappa^3K^3(\varkappa^{-1})}\sum_{n=1}^{\infty}\frac{(n-{}^1/_2)^2q^{2n-1}(\varkappa^{-1})}{(1+q^{2n-1}(\varkappa^{-1}))^2}\,\frac{(\omega_0\tau)^{-1}}{(\omega_0\tau)^{-2}+(\pi(n-{}^1/_2)/K(\varkappa^{-1}))^2}, \tag{22}$$

$$\gamma_{ut}=128\pi^2\int_0\frac{d\varkappa}{\varkappa^6K^3(\varkappa)}\sum_{n=1}\frac{n^2q^{2n}(\varkappa)}{(1+q^{2n}(\varkappa))^2}\,\frac{(\omega_0\tau)^{-1}}{(\omega_0\tau)^{-2}+(\pi n/\varkappa K(\varkappa))^2}. \tag{23}$$

At $\omega_0\tau \ll 1$ the trapping is negligible and the linear theory is valid. Indeed, in that case the particles contributing to the absorption are those from the region $|v_z-v_0| \sim (\tau k_z)^{-1}$, which corresponds to the values

$$s=\frac{|v_z-v_0|}{v_F({}^1/_2 h\sin\theta)^{1/2}}\sim\frac{1}{\omega_0\tau}\gg 1$$

(untrapped particles). Replacing in (23)

$$\frac{\omega_0\tau}{(\omega_0\tau)^2+(\varkappa K(\varkappa)/\pi n)^2}$$

by $2\pi n\delta(\varkappa)$ and using the fact that as $\varkappa\to 0$

$$K(\sqrt{1-\varkappa^2})\approx\ln\frac{4}{\varkappa},\quad \frac{q^{2n}(\varkappa)}{(1+q^{2n}(\varkappa))^2}\approx\left(\frac{\varkappa}{4}\right)^{4n},$$

we obtain $\gamma_{ut}\approx 1$. As can be easily seen, $\gamma_t \sim \omega_0\tau \ll 1$.

The results of the numerical calculation of γ_t and γ_{ut} are shown in the figure. It is seen from the figure that in the region $\omega_0\tau \ll 1$ the quantity $\Gamma_{nl}/\Gamma_{lin}=\gamma_t+\gamma_{ut}$ tends to unity. It is interesting to note that as $\omega_0\tau \lesssim 1$ the ratio Γ_{nl}/Γ_{lin} is close to unity, even though the contribution of the trapped particles to the damping, as seen from the figure, is comparable with the contribution of the untrapped particles. At $\omega_0\tau \gg 1$ it follows from (22) and (23) that $\gamma_t+\gamma_{ut}$ is proportional to $(\omega_0\tau)^{-1}$. A numerical calculation (see the figure) yields $\gamma_t+\gamma_{ut}\approx 2/\omega_0\tau$. The contribution of the trapped particles is here approximately three times larger than the contribution of the untrapped particles.

It can be shown that formulas (22) and (23) and the plots in the figure describe also the behavior of the damping coefficient of a strong longitudinal sound wave in metals, if ω_0 is taken to mean the characteristic frequency of the oscillations of electrons trapped by the sound wave.[5,6]

Experimental observation of the decrease of the helicon absorption coefficient with increasing amplitudes should be carried out in a region where the frequencies and the magnetic fields are such as to ensure satisfaction of the conditions $|k_z|l \gg 1$ and $kR \ll 1$. An appreciable decrease of the damping coefficient, as follows from the results of this paper, takes place at $\omega_0\tau \gtrsim 1$. Assuming by way of estimate $\tau = 5\times 10^{-10}$ sec, $H_0 = 4\times 10^4$ Oe, $kR = 0.3$, and assuming that the Fermi velocity and the carrier density n have values typical of metals, namely $v_F \sim 10^8$ cm/sec and $n \sim 10^{22}$ cm^{-3}, we find that at a power of flux density 1 W/cm^2 delivered to the sample the parameter $\omega_0\tau$ is equal to three, and the damping coefficient is decreased by approximately one-half. At the indicated values of the parameters, the condition $|k_z|l \gg 1$ is well satisfied. Simple estimates show that at a flux density 1 W/cm^2 the heating of the electron system by the helicon is negligible.

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Raman scattering spectra and structural phase transitions in the improper ferroelastics Hg_2Cl_2 and Hg_2Br_2

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The Raman scattering (RS) spectra of crystals of the homologous calomel series, possessing at room temperature a tetragonal structure with a single linear molecule Hg_2X_2 (X = Cl, Br, I) in the primitive cell (space group $D_{4h}{}^{17}$) are investigated in the 10 to 300°K temperature range. When the crystals are cooled below $T_c = 185$°K (Hg_2Cl_2) or $T_c = 143$°K (Hg_2Br_2) the RS spectra undergo a number of qualitative changes (appearance of new lines and splitting of degenerate oscillations), which point to a structural transition of the lattice to the orthorombic phase $D_{2h}{}^{17}$ with a double unit cell. Polarization of the RS spectral lines of the low-temperature phase is measured in samples made monodomain by uniaxial compression. The structural transition is analyzed within the framework of the phenomenological Landau theory of second-order phase transitions. It is shown that in Hg_2X_2 crystals a transition of the displacement type is due to lattice instability with respect to oscillations from the acoustic transverse branch (soft mode) at two non-equivalent X-points on the boundary of the Brillouin zone of the tetragonal phase. The transition is characterized by a two-component order parameter and is accompanied by a spontaneous strain in the basal plane of the $D_{4h}{}^{17}$ lattice ("improper" ferroelastic). Five of the six new RS-spectrum lines predicted by the theory are found below T_c. The intensities of the new lines (normalized by taking into account the temperature dependence of the phonon occupation numbers) and widths of the doublet splitting are linear functions of the squared frequency of the soft mode. The parameters of the model thermodynamic potential for calomel are determined from data on the dependence of the soft-mode frequency on the temperature, on the uniaxial compression, on the magnitude of spontaneous strain, and on the monodomainization threshold stress. The jumps in the specific heat and elastic constants at the transition point are estimated.

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A new interesting group of materials, halides of monovalent mercury, Hg_2X_2, X = Cl, Br, I, was recently synthesized[1] in the form of synthetic single crystals. These isomorphic compounds have a unique crystal structure at 20 °C, consisting of parallel chains of linear molecules Hg_2X_2, which are relatively weakly coupled to one another. The molecules form a body-centered tetragonal lattice D_{4h}^{17} with one molecule per unit cell.[2] The chain structure of the Hg_2X_2 crystals leads to an extraordinarily strong anisotropy of their physical properties. Thus, the crystals of calomel (Hg_2Cl_2) have a very large elastic anisotropy (one of the sound velocities is the lowest of the velocities known in the condensed phase and is comparable with v_s in air[3]), and has a record value of optical birefringence ($\Delta n = +0.65$[4]). In[5,6], investigations were made also of the spectroscopic properties of Hg_2X_2 single crystals, namely the IR spectra and the Raman scattering (RS) spectra.

The study of the RS spectra has revealed[7] that at temperatures lower than $T_c = 185$ K (Hg_2Cl_2) and $T_c = 143$ K (Hg_2Br_2) they undergo a number of qualitative changes that point to a phase transition. The main effect consists in the appearance, in the first-order RS spectra, of additional weak lines at $T \leqslant T_c$, which are missing at $T > T_c$.[7] The existence of a phase transition was directly confirmed by observation of the domain structure of Hg_2Cl_2 and Hg_2Br_2 at $T < T_c$.[8] According to[8], at $T \leqslant T_c$ the tetragonal point group of the crystal D_{4h} is lowered to the centrosymmetrical orthorhombic group D_{2h}, with onset of spontaneous deformation; the samples can become single-domain by uniaxial compression (a "pure" ferroelastic).

In this paper, to explain the microscopic nature of the phase transition in Hg_2X_2, we report a detailed investigation of the RS spectra of the compounds Hg_2Cl_2 and Hg_2Br_2 at $T \leqslant T_c$. The clear-cut manifestation of

