Possible use of the Borrmann effect in the gamma laser

A. V. Andreev and Yu. A. Il'inskiĭ

Moscow State University
(Submitted November 5, 1975)
Zh. Eksp. Teor. Fiz. 70, 1713–1719 (May 1976)

The efficiency of using Bragg diffraction to amplify finite beams in finite crystals is discussed. The conditions under which the plane-wave approximation begins to lose its validity are determined. It is shown that the appearance of modes propagating with weak attenuation is possible in finite crystals. Numerical estimates are given.

PACS numbers: 42.60.—v, 61.10.Dp

The energy dependence of the gamma-ray absorption cross section of solids is satisfactorily described by a well-known curve with a minimum in the region of a few hundred keV in which the absorption process is largely determined by the photoelectric effect. It is therefore natural that studies of the generation of gamma rays in solids should be concerned with this region of energy, and that attempts should be made to reduce the interaction between gamma rays and electrons in some way. This can, in fact, be done because of the existence of the Borrmann effect[1] which involves a sharp reduction in the x-ray absorption coefficient when the Bragg conditions are satisfied for x rays propagating through a crystal. The Borrmann effect is closely analogous to the suppression of inelastic nuclear-reaction channels discovered by Afanas'ev and Kagan[2] in which the collective character of the interaction between nuclei and the electromagnetic field in crystals produces a field distribution, when Bragg diffraction conditions are satisfied, such that there is a sharp reduction in the amplitude for the formation of the nuclear excited states, which ensures that even resonance gamma rays propagate in these directions in the crystal practically without absorption.

It is clear that, in the gamma laser, we must retain the interaction between gamma rays and nuclei, and remove only their interaction with electrons, i.e., while retaining the Borrmann effect, we must try to eliminate the suppression effect. It is shown in[3,4] that this can, in fact, be achieved by using nuclear transitions of high multipole order as the working laser transitions.

The dynamic theory describing all the above effects has been developed for the case of plane waves and an unbounded entrance surface of the crystal. However, it has frequently been noted in the literature that a needle-shaped working body is preferred for the gamma laser. This shape has a number of advantages in the case of the gamma laser. Firstly, the single-transit gamma laser is the most promising, and the needle-shaped body then ensures a highly directional flux of the stimulated coherent gamma radiation.[5] Secondly, a needle-shaped body enables us to reduce the heating of the medium due to the presence of cascade gamma rays.[6] Thirdly and finally, needle-shaped bodies have the advantage that single crystals can be rapidly grown in this form ("whiskers") and this is particularly important in the case of the gamma laser using long-lived nuclear isomers.

Thus, the dynamic theory must be extended to the case of finite beams propagating in finite crystals before the question of the use of anomalous propagation in the gamma laser can be properly resolved.

In this paper, we consider two separate cases. In the first section, we shall discuss the propagation of finite beams in semi-infinite crystals[7] in order to establish the ratios of longitudinal and transverse crystal dimensions for which the influence of boundaries can still be neglected. A general solution of the dynamic problem for this case is given in an integral form in[8]. Unfortunately, an analytic form of solution cannot be obtained from this integral solution even in the simplest cases, and this complicates the analysis of the various properties in which we are interested. However, it is shown in[8] that a simple description of the process is possible in a case that is of interest for the gamma laser.

In the second section of the present paper, we shall consider the propagation in crystals of beams whose width is approximately equal to the transverse dimensions of the crystal. This will enable us to exhibit a number of new properties of the process.

1. DIFFRACTION OF WAVE PACKETS

Consider a monochromatic gamma-ray beam incident at a Bragg angle on a perfect crystal. We shall assume that the reflecting plane of the crystal is perpendicular to its boundary. Diffraction of the electromagnetic wave by the crystal atoms produces two waves in the plane of scattering that propagate at a considerable angle to one another (of the order of 10°). In the plane perpendicular to the plane of scattering, on the other hand, we have only the usual diffraction spreading of the packet which is much smaller than the first. We shall therefore confine our attention to processes which occur in the plane of scattering, which are more important and interesting from our point of view, i.e., we shall suppose that the beams are bounded in one direction only.

Inside the crystal, the slowly varying wave amplitude is described by the following set of equations[4]:

$$\begin{aligned} \sin\theta\frac{\partial E_0}{\partial x}+\cos\theta\frac{\partial E_0}{\partial z}&=g_{00}E_0+g_{01}E_1,\\ -\sin\theta\frac{\partial E_1}{\partial x}+\cos\theta\frac{\partial E_1}{\partial z}&=g_{10}E_0+g_{11}E_1, \end{aligned} \tag{1}$$

where θ is the Bragg diffraction angle, E_0 is the electric field in the transmitted wave, and E_1 is the electric field in the diffracted wave. The coefficients $g_{\alpha\beta}$ are proportional to the scattering amplitudes of the crystal atoms and contain both the electron and nuclear components. Under the conditions of our problem, we have the following properties: $g_{00}=g_{11}$ whereas $g_{01}=g_{10}$ when the complex polarizability $\chi(\mathbf{r})$ satisfies the condition $\chi(\mathbf{r})=\chi(-\mathbf{r})$, which we shall assume henceforth.

Since we are interested in diffraction in the Laue geometry, the initial conditions that determine the shape of the incident beam are as follows:

$$E_0(x,0)=\varphi(x),\quad E_1(x,0)=0. \tag{2}$$

If we resolve the incident beam into plane-wave components on the entrance boundary, we obtain for each of these components a set of equations similar to that investigated previously in[4]. Thus, if we take into account the exponential dependence of each of the plane components on z, the slowly-varying amplitudes of the diffracted and transmitted waves inside the crystal can be written in the form

$$E_\alpha(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\nu\, E_\alpha(\nu)\exp[i(\nu x+\mu z)], \tag{3}$$

where $\mu\equiv\mu(\nu)$ and $\alpha=0.1$.

The condition that (1) has a solution yields the following dependence of μ on ν:

$$i\mu=[g_{00}\pm\sqrt{g_{01}^2-\nu^2\sin^2\theta}]\cos^{-1}\theta, \tag{4}$$

and if we use the inequality

$$|g_{01}|\gg\nu\sin\theta \tag{5}$$

this takes the quadratic form

$$i\mu=\frac{g_{00}-g_{01}}{\cos\theta}+\frac{\sin^2\theta}{2g_{01}\cos\theta}\nu^2, \tag{6}$$

so that, in the (x, z) plane, we obtain the parabolic equation

$$\frac{\partial E}{\partial z}=\frac{g_{00}-g_{01}}{\cos\theta}E-\frac{\sin^2\theta}{2g_{01}\cos\theta}\frac{\partial^2 E}{\partial x^2}. \tag{7}$$

This equation has a simpler solution than the hyperbolic equation obtained directly from (1), and also has a clearer physical interpretation. However, at first sight, it might seem that this equation has an identically zero solution for the scattered wave because the initial condition for E_1 is zero since, to obtain a definite solution of the parabolic equation, it is sufficient to know only the values of the function itself on $z=0$ without specifying its derivative (in contrast to the hyperbolic equation). In point of fact, Eq. (7) does not describe separately the transmitted and diffracted waves, but the strongly and weakly attenuated modes corresponding to the two signs in (4). Equation (7) was obtained by taking the negative sign in (4) and describes the weakly attenuated Borrmann wave. The corresponding initial conditions are:

$$E_0^{\mathrm{I}}(x,0)=\varphi(x)/2,\quad E_1^{\mathrm{I}}(x,0)=-\varphi(x)/2.$$

For the strongly attenuated wave, they become

$$E_0^{\mathrm{II}}(x,0)=\varphi(x)/2,\quad E_1^{\mathrm{II}}(x,0)=\varphi(x)/2.$$

We can see from (6) that the amplification coefficient obtained in[4] now acquires a further term which represents the effects of the finite width of the beam and, as expected, this is associated with a reduction in the amplification coefficient for the side components:

$$\mathrm{Re}(i\mu)=\frac{g_{00}'-g_{01}'}{\cos\theta}-\frac{|g_{01}'|}{g_{01}''^2}\frac{\nu^2\sin^2\theta}{2\cos\theta}, \tag{8}$$

where $g_{\alpha\beta}=g'_{\alpha\beta}+ig''_{\alpha\beta}$.

The parabolic equation given by (7) has a complex diffusion coefficient, the imaginary part of which is greater than the real part (see[4]); thus, the broadening of the beam in the course of its propagation in the crystal is largely due to transverse diffusion of the slowly-varying amplitude, which is the analog of diffraction problems in quasioptics.[9] Thus, for example, for a Gaussian beam $[\varphi(x)=\mathscr{E}_0\exp(-x^2/a_0^2)]$, the increase in the radius of the beam with increasing distance from the entrance boundary occurs in accordance with the formula

$$a(z)=\left[a_0^2+\left(\frac{z}{a_0}b\right)^2\right]^{1/2}, \tag{9}$$

where $b=2\sin^2\theta/g''_{01}\cos\theta$. For 25 keV gamma rays in the crystal lattice of aluminum, we have $b=5\times10^{-6}$ cm.[10] Thus, the distance over which the entering beam, having a width of the order one-tenth of a millimeter, expands by a factor of two amounts to about 50 cm.

Estimates of the minimum widths of beams satisfying (5) shows that this condition is not really very stringent for our purposes and, for the above values of energy and lattice constant, it is equivalent to $a_0\gg10^{-5}$ cm.

2. PROPAGATION OF FINITE BEAMS IN FINITE CRYSTALS

We may conclude from the foregoing results that, in crystals whose length is of the order of a few centimeters, beams whose width is smaller by a factor two or three than the width of the entrance boundary of the crystal will propagate through it without being affected by the boundaries. In this section, therefore, we shall investigate those cases in which the width of the incident beam is approximately equal to the width of the entrance boundary.

In this case, the set of equations given by (1), subject to the initial conditions given by (2), must be augmented by the boundary conditions which, in our problem, take the form

$$E_0(-l,z)=0,\quad E_1(l,z)=0, \tag{10}$$

where $2l$ is the width of the crystal. It is readily seen that (1), with the boundary conditions given by (10), is invariant under the transformations $E_0(x,z)\to E_1(-x,z)$,

$E_1(x, z) \to E_0(-x, z)$. It is therefore sufficient to consider the symmetric and antisymmetric solutions $E_0(x, z) = qE_1(-x, z)$, where $q = \pm 1$ for the symmetric and antisymmetric solutions, respectively.

The boundary conditions then reduce to the single condition $E_0(-l, z) = 0$, and the set of equations given by (1) reduces to the equation

$$\sin\theta \frac{\partial}{\partial x} E_0(x, z) + \cos\theta \frac{\partial}{\partial z} E_0(x, z) = g_{00} E_0(x, z) + q g_{01} E_0(-x, z). \quad (11)$$

We shall seek the solutions of this equation in the form of damped waves: $E_0(x, z) = \tilde{E}_0(x) e^{\lambda z}$. For $\tilde{E}_0(x)$, we then have the equation

$$\sin\theta \frac{d\tilde{E}_0(x)}{dx} = (g_{00} - \lambda\cos\theta)\tilde{E}_0(x) + q g_{01} \tilde{E}_0(-x). \quad (12)$$

The solution of this is

$$\tilde{E}_0(x) = A e^{ikx} + B e^{-ikx}.$$

Substituting this solution in (12), and using the boundary conditions, we obtain the following equations for k and λ:

$$\lambda = \frac{g_{00}}{\cos\theta} - \frac{q g_{01}}{2\cos\theta}(e^{2ikl} + e^{-2ikl}), \quad (13)$$

$$k = \frac{q g_{01}}{2i\sin\theta}(e^{2ikl} - e^{-2ikl}). \quad (14)$$

We thus have a transcendental equation for the complex variable k, which we can use to determine the eigenvalues of our problem. The first N roots of this equation can be found relatively simply. They have the form

$$k_n = (\pi n/2l) + \varepsilon_n, \quad (15)$$

where ε_n satisfy the condition $|\varepsilon_n| 2l \ll 1$ and are given by

$$\varepsilon_n = \frac{\pi n}{2l}\left[\frac{q(-1)^n g_{01}}{\sin\theta} 2l - 1\right]^{-1}. \quad (16)$$

It is clear from (16) that the condition $|\varepsilon_n| 2l \ll 1$ is equivalent for small n to the condition $|g_{01}| 2l/\sin\theta \gg 1$, which, in turn, is identical with the condition given by (5). This condition has a clear physical interpretation: the quantity $2l/\sin\theta$ is, roughly speaking, the distance that must be traversed by the gamma ray in the crystal if it is not reflected, and $1/|g_{01}|$ is the characteristic length for reflection from the interatomic planes.

Borrmann solutions will exist if $E_0(x, z) \approx -E_1(x, z)$; on the other hand, $E_0(x, z) \approx -(-1)^n E_0(-x, z) = -q(-1)^n \times E_1(x, z)$ and, consequently, for quasi-Borrmann solutions, we must have $q(-1)^n = 1$.

If we take into account the above conditions, we find that the expressions for ε_0 and $\Delta\lambda_n = \lambda_n - \lambda_0$ [where $\lambda_0 = (g_{00} - g_{01})/\cos\theta$] assume the form

$$\varepsilon_n = \frac{\pi n}{2l}\frac{\sin\theta}{2lg_{01}}\left(1 + \frac{\sin\theta}{2lg_{01}}\right), \quad (17)$$

$$\Delta\lambda_n = \left(\frac{\pi n}{2l}\right)^2 \frac{\sin^2\theta}{2g_{01}\cos\theta}\left(1 + \frac{\sin\theta}{g_{01} l}\right). \quad (18)$$

If we use the fact that the imaginary part of the coefficients g_{01} is much greater than the real part, the real part of $\Delta\lambda_n$, which characterizes the attenuation along the crystal axis, is given by

$$\mathrm{Re}(\Delta\lambda_n) = -\left(\frac{\pi n}{2l}\right)^2 \frac{\sin^2\theta}{2\cos\theta}\left[\frac{|g'_{01}|}{g''^2_{01}} + \frac{\sin\theta}{g''^2_{01} l}\right], \quad (19)$$

from which it is clear that the principal modes will be those with the smallest values of n.

We have thus obtained a set of quasiorthogonal eigenfunctions

$$E_0^{(n)}(x, z) = E_n (e^{ik_n(x+l)} - e^{-ik_n(x+l)}) e^{\lambda_n z} \quad (20)$$

with eigenvalues

$$k_n = \pi n/2l + \varepsilon_n, \quad \lambda_n = \lambda_0 + \Delta\lambda_n,$$

where ε_n and $\Delta\lambda_n$ are given by (17) and (18). Although the set of eigenfunctions given by (20) is not complete because $|n| \leq N$, we can use it to resolve, with sufficient accuracy, any form of incident beam because the first two or three harmonics will predominate during the propagation of the beam in the crystal. The influence of the remaining harmonics is reduced by the strong exponential dependence on n, namely, $\exp(-\alpha n^2)$. It is precisely for these harmonics that the eigenvalues given by (16) and (17) are most accurate.

The quantity N can be estimated for the same characteristic parameters of the problem that were used in Sec. 1 and the result is $N \ll (2l/\mathrm{cm}) \times 10^5$.

CONCLUSIONS

Similar information on the Borrmann effect can be deduced from the character of the growth of the field with depth in the crystal, which is determined by the real part of the coefficients in front of the coordinate z. The following are the explicit expressions for these coefficients:

$$\mu_0 = \frac{g_{00}' - g_{01}'}{\cos\theta}, \quad (21)$$

$$\mu_\nu = \mu_0 - \nu^2 \frac{\sin^2\theta}{2\cos\theta}\frac{|g_{01}'|}{g''^2_{01}}, \quad (22)$$

$$\mu_n = \mu_0 - \left(\frac{\pi n}{2l}\right)^2 \frac{\sin^2\theta}{2\cos\theta}\left[\frac{|g_{01}'|}{g''^2_{01}} + \frac{\sin\theta}{g''^2_{01} l}\right], \quad (23)$$

where μ_0 is the amplication coefficient for the case of plane waves in an infinite crystal, μ_ν is the corresponding coefficient for a wave packet in a semi-infinite crystal, and μ_n is the amplication coefficient for modes present in a finite crystal when an arbitrary wave packet is incident upon it. It is important to note that the formulas for μ_ν and μ_n were obtained for the only case that was of interest for the gamma laser, namely, $|g_{01}| 2l/\sin\theta \gg 1$.

Comparison of the last two formulas shows the first term in the brackets in the expression for μ_n is due to aperture effects which are always present during the propagation of waves in bounded crystals and which reduce the amplification coefficient because the Bragg condition is not satisfied exactly for all the components of the packet. Let us denote by l_1 the characteristic attenuation length. The physical interpretation of the second term in the brackets in the expression for μ_n is also readily understood if we recall that, when $g_{\alpha\beta} = ig''_{\alpha\beta}$, volume losses become unimportant and there are only losses due to radiation from the boundaries. We shall denote by l_2 the corresponding characteristic length. The above formulas then take the form

$$\mu_0=1/l_0,\quad \mu_\nu=1/l_0-1/l_1,\quad \mu_n=1/l_0-1/l_1-1/l_2.$$

The two lengths l_1 and l_2 can readily be estimated by introducing one further characteristic length. This length, l will be referred to as the transverse diffusion length and will be defined as the length over which the radius of the beam, given by (9), increases by a factor of $\sqrt{2}$. Assuming that $a_0 \approx 2l$, we obtain

$$l_1/l\approx|g_{01}''/g_{01}'|\gg1,\quad l_2/l_1\approx|g_{01}'|l/\sin\theta\gg1.$$

We note that $l_0 \sim 1$ cm[11] and, for the parameter values used above, $l=20$ cm.

We may therefore conclude that, for the above numerical values of the parameters of our problem, the plane-wave approximation for the incident waves gives a correct representation of the efficacy of the Borrmann effect as a means of reducing the absorption coefficient provided the width of the beam (crystal) is $a_0 \gg 10^{-2}$ cm, whereas, for $a_0 \leq 10^{-3}$ cm, the aperture effect and the effect connected with radiation from boundaries begin to play an appreciable role.

In general, the plane-wave approximation begins to lose its validity when the characteristic length l approaches l_0.

1. J. Borrmann, Phys. Z. 42, 157 (1941).
2. A. M. Afanas'ev and Yu. M. Kagan, Zh. Eksp. Teor. Fiz. 48, 327 (1965) [Sov. Phys. JETP 21, 215 (1965)].
3. Yu. M. Kagan, Pis'ma Zh. Eksp. Teor. Fiz. 20, 27 (1974) [JETP Lett. 20, 11 (1974)].
4. A. V. Andreev and Yu. A. Il'inskiĭ, Zh. Eksp. Teor. Fiz. 68, 811 (1975) [Sov. Phys. JETP 41, 403 (1975)].
5. G. S. Baldwin, J. P. Neissel, and L. Tonks, Proc. IEEE 51, 1247 (1963).
6. V. I. Gol'danskiĭ and Yu. M. Kagan, Zh. Eksp. Teor. Fiz. 64, 90 (1973) [Sov. Phys. JETP 37, 49 (1973)].
7. A. V. Andreev and Yu. A. Il'inskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. 22, 462 (1975) [JETP Lett. 22, 223 (1975)].
8. I. Sh. Slobodetskiĭ, F. N. Chukhovskiĭ, and V. L. Indenbom, Pis'ma Zh. Eksp. Teor. Fiz. 8, 90 (1968) [JETP Lett. 8, 55 (1968)].
9. G. D. Malyuzhinets, Usp. Fiz. Nauk 69, 321 (1959) [Sov. Phys. Usp. 2, 749 (1959)].
10. Z. G. Pinsker, Dixamicheskoe rasseyanie rentgenovskikh luchei v ideal'nykh kristallakh (Dynamic Scattering of X rays in Perfect-Crystals), Nauka, 1974.
11. R. V. Khokhlov, Pis'ma Zh. Eksp. Teor. Fiz. 15, 580 (1972) [JETP Lett. 15, 414 (1972)].

Translated by S. Chomet

Polarization effects in a strong field in two-mode lasing

I. V. Evseev, V. M. Ermachenko, and V. K. Matskevich

Moscow Engineering-Physics Institute

(Submitted November 12, 1975)
Zh. Eksp. Teor. Fiz. 70, 1720–1728 (May 1976)

The possibility is investigated of two-mode lasing at a large relative excitation of the active medium for an atomic transition with level moments 0 or 1. It is shown that in the case of parallel polarizations of the generated modes a two-mode regime takes place, providing the intermode distance ω_{21} exceeds a certain critical value ω_{21cr}; in the case of orthogonal polarizations the regime sets in at arbitrary intermode distances. The dependence of the size of the two-mode generation region on various parameters is investigated for parallel and orthogonal mode polarizations.

PACS numbers: 42.50.+q

Many recent studies, both experimental and theoretical, have been devoted to problems of nonlinear spectroscopy. A recent book by Letokhov and Chebotaev[1] contains an extensive bibliography on this subject. One of the methods of investigating spectroscopic characteristics is to sound a resonant medium saturated by a strong field with a weak probing wave. We note that the results of the probing depend significantly on the mutual direction of the polarizations of the weak and strong fields, as was first pointed out by Alekseev.[2] The latter considered, for an atomic transition with total angular momentum change $1 \to 0$, the waveform of the weak signal as it passes through a glass laser operating in the single-mode generation regime. The analysis took into account the depolarizing atomic collisions, when the strong field in the laser and the field of the transmitted signal had parallel and orthogonal polarizations.

One more method of spectroscopy of resonant media is to study the two-mode generation regime in them. The principles of theoretical analysis of the two-mode regime were developed in a paper by Lamb.[3] No account was taken of the degeneracy of the resonance levels, and the case was considered when the mode intensities could be regarded as weak (small relative