A study of cyclotron absorption of electromagnetic waves in a plasma located in an inhomogeneous magnetic field

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Results are presented of an experimental study of cyclotron absorption of electromagnetic waves in a plasma located in an inhomogeneous magnetic field. The experimental data are compared with the theoretical calculations in which electron-neutral collisions are taken into account. The comparison shows that the phenomenon can be described quite satisfactorily by the theory.

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It is known that cyclotron oscillations are used to heat plasma in magnetic traps. We shall show that this heating method can be investigated in a gas-discharge plasma. We have determined experimentally the absorption, transmission, and reflection coefficients of the cyclotron oscillations at cyclotron resonance in an inhomogeneous magnetic field. Their values agree with those calculated theoretically.

It is known that if circularly-polarized electromagnetic oscillations propagate in a plasma placed in an inhomogeneous magnetic field, then the oscillations are absorbed at the point where the frequency ω becomes equal to the electron cyclotron frequency ω_c. Near this point is located a region opaque to electromagnetic waves. Cyclotron absorption of electromagnetic waves propagating in a plasma along an inhomogeneous magnetic field was studied theoretically in[1–3]. It was shown that the coefficients of the absorption and reflection of the wave from the resonant region depend on whether the wave propagates in the plasma in the direction where the magnetic field decreases or in the opposite direction.

In the former case (with decreasing magnetic field) in a collisionless plasma, for a wave with circular polarization (the electric vector of the field rotates in the same direction as the electron in the magnetic field) and for a magnetic field that varies linearly, the reflection and absorption coefficients ξ and η are given by the expressions

$$\eta_H=1-e^{-\beta},\quad \xi_H=0. \tag{1}$$

In the second case (with increasing magnetic field)

$$\eta_L=(1-e^{-\beta})e^{-\beta},\quad \xi_L=(1-e^{-\beta})^2, \tag{2}$$

where $\beta=\pi\omega_{pe}^2L/\omega c$, ω_{pe} is the electron plasma frequency, $L=B(dB/dz)^{-1}$, B is the induction of the magnetic field, the $0z$ axis is directed along the magnetic field. It is assumed here that the frequency of the collisions of the electrons with the neutral atoms is $\nu_e \ll c/L$.

Although the absorption of electromagnetic waves in a plasma under cyclotron resonance conditions was investigated experimentally not only in a homogeneous magnetic field[4] but also in an inhomogeneous one, [5,6] expressions analogous to (1) and (2) have heretofore not been verified by direct experiment. Yet expressions such as (1) and (2) are frequently used in the analysis of processes that occur in magnetic traps with mirror geometry. [7,8]

This is apparently due to the fact that under magnetic-bottle conditions it is difficult to vary independently the quantities that enter in (1) and (2). However, this verification is possible under gas-discharge conditions, where the concentration of the charged particles and the gradient of the magnetic field are much easier to vary.

Besides the study of cyclotron absorption of oscillations under conditions of a collisionless plasma ($c/L > \nu_e$), we took into account the effect exerted on this phenomenon by electron-neutral collisions and have generalized formulas (1)–(2) to include this case. We used to this end an equation of the form

$$\frac{d^2E}{dz^2}+\frac{\omega^2}{c^2}\left\{1+\frac{\omega_{pe}^2}{\omega(\omega_e(z)-\omega-i\nu_e)}\right\}E=0, \tag{3}$$

where $\omega_e(z)=eB(z)/m_ec$ is the electron cyclotron frequency. From the solution of (3) we can obtain an expression for the coefficients of reflection and absorption of the electromagnetic waves. For the case of a magnetic field that varies linearly, when $\omega_e(z)=\omega(1-z/L)$, the substitution

$$\Phi=-2i\frac{\omega}{c}\left(z+i\frac{\nu_eL}{\omega}\right)$$

transforms (3) into

$$\frac{d^2E}{d\Phi^2}+\left(\frac{k}{\Phi}-\frac{1}{4}\right)E=0, \tag{4}$$

where $k=\beta/2\pi i$.

Equation (4) is solved in terms of Whittaker functions. Calculations show that allowance for the collisions of the electrons with the neutral atoms does not lead to a change in the coefficients η and ξ in the case when the wave propagates towards the weaker magnetic field.

When the wave propagates in the opposite direction, these coefficients are given by

$$\eta_{Lc}=(1-e^{-\beta})[1-(1-e^{-\beta})e^{-4\nu_e L/c}],$$
$$\xi_{Lc}=(1-e^{-\beta})^2 e^{-4\nu_e L/c}. \tag{5}$$

It is assumed here that ν_e is constant.

It is seen from (5) that even in the case when $c/L < \nu_e \ll \omega$ and the damping of the electromagnetic wave propagating in the plasma outside the resonance region is negligibly small, the collisions can greatly influence the wave reflection conditions. With increasing collision frequency, an ever decreasing part of the signal is reflected, and the absorption coefficient ceases to depend on the wave propagation direction in the plasma.

The validity of formulas (1), (2), and (5) above was verified by us experimentally. The experimental setup is illustrated in Fig. 1. The plasma was produced in cylindrical glass tubes 6 with diameters 3 and 5.6 cm and length 85 cm and with conical ends. The inert gases He, Ar, and Kr were used. In the pressure range from 0.5 to 0.05 mm Hg, the plasma was produced with the aid of either a dc discharge or with the aid of a high-frequency discharge. At lower pressures, down to $p=1\times10^{-3}$ mm Hg, the measurements were performed only in a high-frequency discharge plasma maintained with a 3.3-MHz generator of 400 W power, with an output symmetrical with respect to the ground. The voltage from the generator was applied to thin copper rings 5 of 1 cm width, which encircled the discharge tube on the outside and were placed at its ends.[9] Since the diameter of the employed ring electrodes differs little from the waveguide diameter, these electrodes introduce practically no distortion to the waveguide channel. The source of the electromagnetic oscillations was generator 1 of type G4-9 of frequency 3-GHz and maximum power 0.5 mW. We used both a plane-polarized wave and a wave with circular polarization. In the latter case, the rectangular waveguide was joined to the circular one with the aid of the "tourniquet" junction[10] 4. The wave polarization was monitored either against the azimuthal rotation of the detector current, or against Lissajous figures with the aid of an S7-8 stroboscopic oscilloscope, to the vertical and horizontal inputs of which were fed signals from probes located in the same cross section of the waveguide and displaced relative to each other in azimuth. To eliminate stray pickup when working with the high-frequency discharge, we used amplitude-pulsed modulation of the probing microwave signal at a frequency 1 kHz. The dismountable waveguide sec-

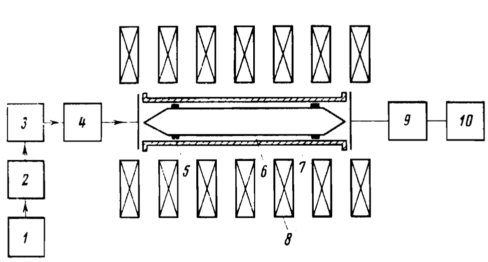

FIG. 1. Experimental setup: 1—generator, 2—decoupler, 3—measuring line, 4—"tourniquet" junction, 5—electrodes, 6—discharge tube, 7—dismountable waveguide section, 8—multisector solenoid, 9—detector section, 10—matched load.

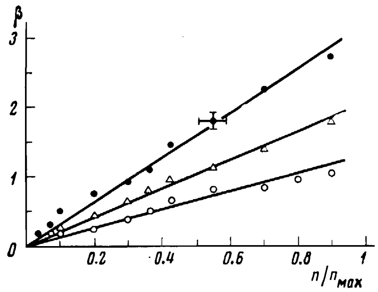

FIG. 2. Dependence of the quantity $\beta=-\ln(1-\eta_H)$ on the relative electron density n/n_{max}, obtained by experiment. Argon, pressure $p=0.15$ nm Hg, dc discharge, the values of the parameter L are 35 (●), 25 (△), or 15 (○) cm.

tion 7 in which the discharge tube placed was installed in the interior of a multisector solenoid 8 with overall length 100 cm. By series-parallel connection of the individual sections it was possible to produce a magnetic field that varied linearly along the tube axis. Deviations from linearity in the cyclotron-resonance region did not exceed 5%. The homogeneity of the magnetic field over the tube cross section was of the order of 2%. The maximum value of the magnetic field was 1.8 kG. The parameter L could range under the experimental conditions from 12 to 50 cm. To find the concentration of the charged particles in the plasma, a double electric probe oriented perpendicular to the force lines of the magnetic field was placed on the axis of the column. In the measurements we used the ion part of the probe current-voltage characteristic. The average electron density and frequency ν_e were determined by using short-circuited plungers and the method described in[11]. The electron temperature range in the experiments from 2 to 5 eV. The maximum value of the electron density reached 2×10^{10} cm^{-3}, i.e., the plasma was weakly ionized.

Figure 2 shows plots of $\beta=-\ln(1-\eta_H)$ against the relative charged-particle density n/n_{max} for an electromagnetic wave propagating in the direction of the decrease of the magnetic field, at three different values of the field gradient B. n_{max} was assumed here to be the electron density in the plasma at a discharge current 50 mA. The data were obtained in an argon plasma at a pressure 0.15 mm Hg, for which $\nu_e/\omega\approx0.1$. It is seen from Fig. 2 that the character of the variation of the experimentally obtained absorption coefficient η_H agrees with that predicted by formula (1). Indeed, β increases with increase of the parameter L, which characterizes the magnetic-field gradient, and with increasing charged-particle density, as follows from (1), since $\beta\propto Ln$.

We note that, in agreement with the theory, the experiments have revealed no reflection of electromagnetic waves. Nor was wave reflection observed for the reverse wave propagation in the plasma (in the direction of the magnetic-field gradient). The latter follows also from (5), since ξ_{Lc} is close to zero in this case. In addition, the experimentally obtained absorption coefficients do not depend noticeably on the wave propagation direction in the plasma.

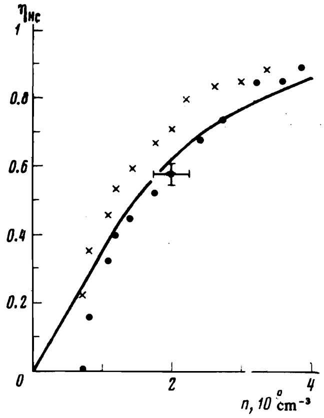

FIG. 3. Dependence of the absorption coefficient η_{Hc} on the electron density n. Argon, pressure $p = 0.3$ mm Hg, $L = 25$ cm. Dc discharge. ●—wave propagates in the direction of the decrease of the magnetic field, ×—wave propagates in the direction of the increase of the magnetic field.

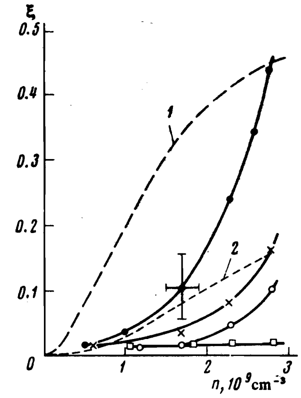

FIG. 5. Dependence of the reflection coefficient on the density. Ar, $p = 5\times10^{-3}$ mm Hg, HF discharge, Dashed lines—calculation from formula (5): 1—$L = 50$ cm, 2—$L = 12$ cm. Symbols: ●—$L = 50$ cm, ×—$L = 12$ cm, wave propagating towards increasing magnetic field; o—$L = 50$ cm, □—$L = 12$ cm, wave propagating towards decreasing magnetic field.

The data shown in Fig. 2 were obtained in experiments in which rectangular waveguides were used. The subsequent measurements were performed already under conditions when the wave was circularly polarized.

Figure 3 shows the dependence of the absorption coefficient on the charge-particle density, obtained in argon plasma at a pressure 0.3 mm Hg in a dc discharge. The crosses mark here the result obtained with the wave propagating along the magnetic field gradients, while the points correspond to the opposite direction. The solid line was obtained by calculation from formula (5) at $L = 25$ cm, and in fact coincides under these conditions with that calculated from (1).

Figure 4 shows plots of the absorption coefficient against the charge-particle density when the wave propagates in the plasma in the direction of the decreasing magnetic field. The data were obtained at pressures $p = 0.15$ and 0.005 mm Hg for $L = 50$ cm in argon and krypton dc discharge and a high-frequency discharge plasmas. It is seen from Fig. 4 that an appreciable change of the pressure (of the frequency of the electron-neutral collisions) causes no noticeable change in the absorption coefficient within the limits of the measurement errors. We note that the absorption coefficient does not depend noticeably on the method used to produce the plasma.

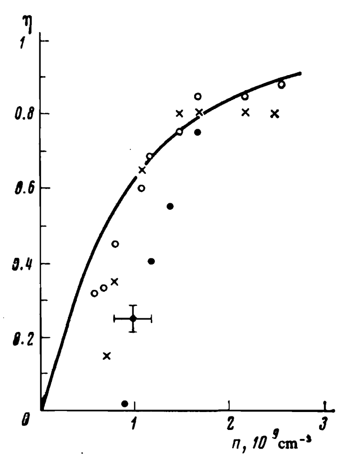

FIG. 4. Dependence of the absorption coefficient η on the density n at different pressures. $L = 50$ cm. Wave propagates towards decreasing magnetic field. Solid line—calculation by formula (1). ●—Kr, $p = 5\times10^{-3}$ mm Hg, HF discharge; o—Ar, $p = 1.5\times10^{-1}$ mm Hg, dc discharge; ×—Ar, $p = 5\times10^{-3}$ mm Hg, HF discharge.

We investigated also the dependence of the reflection coefficient of an electromagnetic wave on the electron density in an argon plasma at a pressure 5×10^{-3} mm Hg ($\nu_e/\omega \approx 5\cdot10^{-3}$). The results of these measurements are shown in Fig. 5. It is seen from the figure that, as concluded from the theory, that the wave reflection coefficient is larger when the wave propagates towards the decreasing magnetic field than for the opposite direction. The quantity ξ increases with increasing parameter L and with increasing charge-particle density. The dashed lines shown the frequency as given by formula (5). We note that in the collision regime, at pressures 0.15 mm Hg ($\nu_e/\omega = 0.1$) no wave reflection was observed, as follows indeed from (5).

Thus, our results allow us to conclude that the theoretically obtained formulas can be used to calculate the reflection and the absorption coefficients of electromagnetic waves in a plasma in an inhomogeneous magnetic field. Some difference between the calculated and experimental data is apparently due to the fact that the theoretically derived formulas (1), (2), and (5) do not take into account the transverse inhomogeneity of the charge-particle density, or the fact that the plasma and the magnetic field are spatially bounded in the direction of electromagnetic wave propagation.

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Non-linear stabilization of the modulational instability

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We consider the non-linear stabilization process for the modulational instability. We obtain more exact dynamical equations which take into account electron non-linearities and higher-order non-linearities. We use these equations to find the limitations to the development of modulational perturbations which indicate the prohibition of the Langmuir collapse. We show that it is possible that fast Langmuir solitons (spikons) can exist.

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1. The development of the modulational instability[1,2] of three-dimensional Langmuir turbulence is in principle possible: 1) either up to the formation of a system of weakly interacting solitons[3]; or "without limits" down to a region where Landau damping is important[4] (the so-called Langmuir collapse); 3) or up to a state of interacting non-stationary perturbations in which the non-linear stabilization guarantees stationarity only on average.[5] To describe the latter possibility we[6] developed a statistical theory of the Langmuir condensate. The aim of the present paper is to analyze within the framework of the dynamical approach the role of various non-linearities in the stabilization processes of the modulational instability and to determine the limits of the development of modulational perturbations. This enables us, in particular, to estimate the possibilities for the realization of the Langmuir collapse which earlier has been analyzed both theoretically and numerically (for a number of selected initial conditions) in the framework of the simplest system of equations[4] in which the non-linear processes which we consider below were neglected.

From the definition of a collapse it follows that the non-linear dynamic motions corresponding to a collapse must reach dimensions of the order of r_d (r_d is the Debye radius) so that if non-linear effects limit the process for $r \gg r_d$, this indicates the impossibility of the collapse. We show in the present paper that such limitations exist. We start by showing that the simplest equations used for describing the collapse and the formation of solitons[3,4] follow directly from the well known non-linear equations from plasma theory[7] when we restrict ourselves to quadratic and cubic non-linearities. However, even in the approximation of the quadratic and cubic non-linearities the equations used in the non-linear plasma theory are more general and take into account not only the non-linear Landau damping and the breakdown of the quasi-neutrality of the perturbations, but also the electron non-linearities which are of the same order of magnitude. Therefore, even in the framework of the simplest equations of the non-linear plasma theory, which take into account non-linearities only up to cubic terms, there are a whole number of effects which restrict the region of applicability of the hydrodynamic equations (HE in what follows) used by Rudakov[3] and Zakharov.[4] This leads to well defined criteria which are obtained below. We obtain in the present paper exact equations which take into account effects neglected in the HE. These equations are written in the coordinate representation which is normally used for numerical simulations. We evaluate higher-order non-linear effects and give an estimate of the limitations connected with them, and also obtain the corresponding dynamic equations.

2. We show how the HE are obtained from the well known plasma theory equations. We write the Fourier component of the non-linear change density in the form

$$\rho_k = \int S_{k,k_1,k_2} E_{k_1} E_{k_2} \delta(k-k_1-k_2)\, dk_1\, dk_2 + \int \Sigma_{k,k_1,k_2,k_3} E_{k_1} E_{k_2} E_{k_3} \delta(k-k_1-k_2-k_3)$$
$$\times dk_1\, dk_2\, dk_3 + \int S'_{k,k_1,k_2,k_3,k_4} E_{k_1} E_{k_2} E_{k_3} E_{k_4} \delta(k-k_1-k_2-k_3-k_4)\, dk_1\, dk_2\, dk_3\, dk_4$$
$$+ \int \Sigma'_{k,k_1,k_2,k_3,k_4,k_5} E_{k_1} E_{k_2} E_{k_3} E_{k_4} E_{k_5} \delta(k-k_1-k_2-k_3-k_4-k_5)\, dk_1\, dk_2\, dk_3\, dk_4\, dk_5,$$
$$k=\{\mathbf{k}, \omega\}, \qquad dk = d\mathbf{k}\, d\omega. \tag{1}$$

For obtaining the HE it is sufficient to use the first two terms of the expansion (1); the next two terms we use to obtain corrections to the HE. Both modulated high-frequency fields of Langmuir oscillations and low-fre-

