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Non-linear stabilization of the modulational instability

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We consider the non-linear stabilization process for the modulational instability. We obtain more exact dynamical equations which take into account electron non-linearities and higher-order non-linearities. We use these equations to find the limitations to the development of modulational perturbations which indicate the prohibition of the Langmuir collapse. We show that it is possible that fast Langmuir solitons (spikons) can exist.

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1. The development of the modulational instability[1,2] of three-dimensional Langmuir turbulence is in principle possible: 1) either up to the formation of a system of weakly interacting solitons[3]; or "without limits" down to a region where Landau damping is important[4] (the so-called Langmuir collapse); 3) or up to a state of interacting non-stationary perturbations in which the non-linear stabilization guarantees stationarity only on average.[5] To describe the latter possibility we[6] developed a statistical theory of the Langmuir condensate. The aim of the present paper is to analyze within the framework of the dynamical approach the role of various non-linearities in the stabilization processes of the modulational instability and to determine the limits of the development of modulational perturbations. This enables us, in particular, to estimate the possibilities for the realization of the Langmuir collapse which earlier has been analyzed both theoretically and numerically (for a number of selected initial conditions) in the framework of the simplest system of equations[4] in which the non-linear processes which we consider below were neglected.

From the definition of a collapse it follows that the non-linear dynamic motions corresponding to a collapse must reach dimensions of the order of r_d (r_d is the Debye radius) so that if non-linear effects limit the process for $r \gg r_d$, this indicates the impossibility of the collapse. We show in the present paper that such limitations exist. We start by showing that the simplest equations used for describing the collapse and the formation of solitons[3,4] follow directly from the well known non-linear equations from plasma theory[7] when we restrict ourselves to quadratic and cubic non-linearities. However, even in the approximation of the quadratic and cubic non-linearities the equations used in the non-linear plasma theory are more general and take into account not only the non-linear Landau damping and the breakdown of the quasi-neutrality of the perturbations, but also the electron non-linearities which are of the same order of magnitude. Therefore, even in the framework of the simplest equations of the non-linear plasma theory, which take into account non-linearities only up to cubic terms, there are a whole number of effects which restrict the region of applicability of the hydrodynamic equations (HE in what follows) used by Rudakov[3] and Zakharov.[4] This leads to well defined criteria which are obtained below. We obtain in the present paper exact equations which take into account effects neglected in the HE. These equations are written in the coordinate representation which is normally used for numerical simulations. We evaluate higher-order non-linear effects and give an estimate of the limitations connected with them, and also obtain the corresponding dynamic equations.

2. We show how the HE are obtained from the well known plasma theory equations. We write the Fourier component of the non-linear change density in the form

$$\rho_k = \int S_{k,k_1,k_2} E_{k_1} E_{k_2} \delta(k-k_1-k_2)\, dk_1\, dk_2 + \int \Sigma_{k,k_1,k_2,k_3} E_{k_1} E_{k_2} E_{k_3} \delta(k-k_1-k_2-k_3)$$
$$\times dk_1\, dk_2\, dk_3 + \int S'_{k,k_1,k_2,k_3,k_4} E_{k_1} E_{k_2} E_{k_3} E_{k_4} \delta(k-k_1-k_2-k_3-k_4)\, dk_1\, dk_2\, dk_3\, dk_4$$
$$+ \int \Sigma'_{k,k_1,k_2,k_3,k_4,k_5} E_{k_1} E_{k_2} E_{k_3} E_{k_4} E_{k_5} \delta(k-k_1-k_2-k_3-k_4-k_5)\, dk_1\, dk_2\, dk_3\, dk_4\, dk_5,$$
$$k=\{\mathbf{k}, \omega\}, \qquad dk = d\mathbf{k}\, d\omega. \tag{1}$$

For obtaining the HE it is sufficient to use the first two terms of the expansion (1); the next two terms we use to obtain corrections to the HE. Both modulated high-frequency fields of Langmuir oscillations and low-frequency fields take part in modulational perturbations. It is convenient to split in the high-frequency field the positive-frequency (+) and the negative-frequency (–) parts $E_k^{hf} = E_k^+ + E_k^-$. In what follows we shall deal with only either the positive-frequency or the negative-frequency part of the high-frequency field. We shall therefore distinguish the high-frequency fields by upper indexes, but we shall write the low-frequency fields without upper indexes. The Poisson equation for the high- and low-frequency fields then becomes, if we take into account only the first two terms of (1)

$$(4\pi)^{-1} ik\varepsilon_k E_k^+ = 2\int S_{k,k_1,k_2} E_{k_1}^+ E_{k_2} \delta(k-k_1-k_2)\, dk_1\, dk_2$$
$$+2\int \Sigma_{k,k_1,k_2,k_3} E_{k_1}^+ E_{k_2}^+ E_{k_3}^- \delta(k-k_1-k_2-k_3)\, dk_1\, dk_2\, dk_3, \tag{2}$$
$$(4\pi)^{-1} ik\varepsilon_k E_k = 2\int S_{k,k_1,k_2} E_{k_1}^+ E_{k_2}^- \delta(k-k_1-k_2)\, dk_1\, dk_2. \tag{3}$$

We assume here that the matrix elements S are symmetric in the indexes ε_1, k_1, k_2 and Σ correspondingly in the indexes k_2 and k_3. Thus we have

$$S_{k,k_1,k_2} = -\frac{e^3}{2m_e^2 k_1 k_2}\int \frac{1}{(\omega-\mathbf{kv})}\left\{\left(\mathbf{k}_1\frac{\partial}{\partial \mathbf{v}}\right)\frac{1}{\omega_2-\mathbf{k}_2\mathbf{v}}\left(\mathbf{k}_2\frac{\partial}{\partial \mathbf{v}}\right)\right.$$
$$\left.+\left(\mathbf{k}_2\frac{\partial}{\partial \mathbf{v}}\right)\frac{1}{\omega_1-\mathbf{k}_1\mathbf{v}}\left(\mathbf{k}_1\frac{\partial}{\partial \mathbf{v}}\right)\right\} f\frac{d\mathbf{p}}{(2\pi)^3}, \quad n_0 = \int f_{\mathbf{p}} \frac{d\mathbf{p}}{(2\pi)^3}. \tag{4}$$

The quantity S occurs in (2) which in the first approximation equals

$$S_{k,k_1,k_2} = S_1 \approx \frac{e(\mathbf{kk}_1)}{8\pi m_e \omega_{pe}^2}\frac{k_2}{k_1}(\varepsilon_{k_2}^e - 1) \approx \frac{e(\mathbf{kk}_1)}{8\pi k_1 k_2 T_e},$$
$$\omega \gg k v_{Te}, \quad \omega_1 \gg k_1 v_{Te}; \quad v_{Te} k_2 \ll \omega_{pe}, \quad \omega_2 \ll k_2 v_{Te}, \tag{5}$$
$$v_{Te}^2 = T_e/m_e, \quad \omega_{pe}^2 = 4\pi e^2 n_0/m_e.$$

Here ε_k ($k = \{\mathbf{k}, \omega\}$) is the linear plasma dielectric permittivity. On the other hand, the quantity S in (3) has the approximate form:

$$S_{k,k_1,k_2} = S_2 \approx -\frac{e(\mathbf{kk}_2)}{8\pi m_e k_1 k_2}\frac{k^2}{\omega_{pe}^4}(\varepsilon_k^e - 1) \approx -\frac{e(\mathbf{k}_1\mathbf{k}_2)}{8\pi k_1 k_2 T_e},$$
$$\omega \ll k v_{Te}, \quad \omega_1 \gg k_1 v_{Te}, \quad \omega_2 \gg k_2 v_{Te}. \tag{6}$$

Exactly in the same way we have an approximate expression for Σ which occurs in (2):

$$\Sigma_{k,k_1,k_2,k_3} = -\frac{e^4}{2m_e^3 i k_1 k_2 k_3}\int \frac{1}{(\omega-\mathbf{kv})}\left(\mathbf{k}_1\frac{\partial}{\partial \mathbf{v}}\right)\frac{1}{\omega-\omega_1-(\mathbf{k}-\mathbf{k}_1)\mathbf{v}}$$
$$\times\left\{\left(\mathbf{k}_2\frac{\partial}{\partial \mathbf{v}}\right)\frac{1}{\omega_3-\mathbf{k}_3\mathbf{v}}\left(\mathbf{k}_3\frac{\partial}{\partial \mathbf{v}}\right)+\left(\mathbf{k}_3\frac{\partial}{\partial \mathbf{v}}\right)\frac{1}{\omega_2-\mathbf{k}_2\mathbf{v}}\left(\mathbf{k}_2\frac{\partial}{\partial \mathbf{v}}\right)\right\} f\frac{d\mathbf{p}}{(2\pi)^3} \tag{7}$$
$$\approx \frac{e^2}{8\pi i m_e^2 \omega_{pe}^4}\frac{(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)}{k_1 k_2 k_3}|\mathbf{k}-\mathbf{k}_1|^2(\varepsilon_{k-k_1}^e - 1) \approx \frac{e^2(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)}{8\pi i m_e T_e \omega_{pe}^2 k_1 k_2 k_3},$$
$$\omega \gg k v_{Te},\ \omega_1 \gg k_1 v_{Te},\ \omega_2 \gg k_2 v_{Te},\ \omega_{pe} \gg |\mathbf{k}-\mathbf{k}_1| v_{Te}.$$

Substituting the low-frequency field from (3) into (2) and using (5) to (7) we get

$$\varepsilon_k E_k^+ = \frac{8\pi}{ik}\int \tilde{\Sigma}_{k,k_1,k_2,k_3} E_{k_1}^+ E_{k_2}^+ E_{k_3}^- \delta(k-k_1-k_2-k_3)\, dk_1\, dk_2\, dk_3, \tag{8}$$

where

$$\tilde{\Sigma}_{k,k_1,k_2,k_3} = \Sigma_{k,k_1,k_2,k_3} - \frac{8\pi i S_{k,k_1,k-k_1} S_{k-k_1,k_2,k_3}}{\varepsilon_{k-k_1}|\mathbf{k}-\mathbf{k}_1|} \approx \frac{e^2(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)}{8\pi i m_e T_e \omega_{pe}^2 k_1 k_2 k_3}\frac{\varepsilon_{k-k_1}^i}{\varepsilon_{k-k_1}}. \tag{9}$$

We write

$$\delta n_k = -\frac{\varepsilon_k^i}{\varepsilon_k}\frac{1}{4\pi T_e}\int \frac{(\mathbf{k}_2\mathbf{k}_3)}{k_2 k_3} E_{k_2}^+ E_{k_3}^- \delta(k-k_2-k_3)\, dk_2\, dk_3. \tag{10}$$

With the notation introduced here Eq. (8) becomes

$$\varepsilon_k E_k^+ = \int \frac{(\mathbf{kk}_1)}{kk_1}\frac{\delta n_{k-k_1}}{n_0} E_{k_1}^+\, dk_1. \tag{11}$$

If

$$\varepsilon_k \approx 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{3k^2 v_{Te}^2}{\omega_{pe}^2},$$

while the complex amplitude of the Langmuir field in the coordinate representation is

$$\mathbf{E}(\mathbf{r}, t) = (2\pi)^{-4}\int \frac{\mathbf{k}}{k} E_k^+ \exp(i\omega_{pe} t + i\omega t - i\mathbf{kr})\, dk, \tag{11'}$$

we can write Eq. (11) in the form

$$\operatorname{div}\left(2i\omega_{pe}\frac{\partial}{\partial t} + 3v_{Te}\nabla^2\right)\mathbf{E}(\mathbf{r}, t) = \operatorname{div}\omega_{pe}^2\frac{\delta n}{n_0}\mathbf{E}(\mathbf{r}, t). \tag{12}$$

On the other hand, (10) can in the coordinate representation be written as follows:

$$\left(\frac{\partial^2}{\partial t^2} - v_s\nabla^2 - \frac{v_s^2}{\omega_{pi}^2}\nabla^2\frac{\partial^2}{\partial t^2}\right)\delta n(\mathbf{r}, t) = \left(\nabla^2 + \frac{1}{\omega_{pi}^2}\nabla^2\frac{\partial^2}{\partial t^2}\right)\frac{|E(\mathbf{r}, t)|^2}{4\pi m_i} \tag{13}$$

(we used here the fact that $E_{-k}^- = (E_k^+)^*$). In the limit as $\partial/\partial t \ll \omega_{pi}$ Eqs. (12), (13) reduce to the equations used in[3,4] to describe the modulational instability and the collapse.

One should note that the terms in (13) which contain $\omega_{pi}^{-2}\,\partial^2/\partial t^2$ and which are usually dropped are terms with leading derivatives and even when $\omega_{pi}\tau \gg 1$ the correctness of the asymptotic approximation requires a special study. Under conditions when the characteristic time τ becomes of the order of ω_{pi}^{-1} not only the additional terms written down in (13) turn out to be important, but there also appear additional terms in (12) of the same order. They describe electron non-linearities which can be evaluated from the general expression for Σ and S as corrections to their approximate expressions (5) to (7).

3. In the framework of cubic non-linearities there appear several kinds of corrections. First of all we specify that we shall find corrections to the approximate expression $\tilde{\Sigma}$ in which $\varepsilon_k^i \approx -\omega_{pi}^2/\omega^2$ and $\varepsilon_k \approx -\omega_{pi}^2/\omega^2 + \omega_{pi}^2/k^2 v_s^2$, which also appear in the HE, i.e.,

$$\tilde{\Sigma}_{k,k_1,k_2,k_3} = \tilde{\Sigma}^{(0)}_{k,k_1,k_2,k_3} + \delta\tilde{\Sigma}_{k,k_1,k_2,k_3},$$
$$\tilde{\Sigma}^{(0)}_{k,k_1,k_2,k_3} \approx -\frac{e^2(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)}{8\pi i m_e T_e \omega_{pe}^2 k_1 k_2 k_3}\frac{(\mathbf{k}-\mathbf{k}_1)^2 v_s^2}{(\omega-\omega_1)^2 - (\mathbf{k}-\mathbf{k}_1)^2 v_s^2}. \tag{14}$$

We enumerate those effects which must be included in $\delta\tilde{\Sigma}$.

1) The effects of the breakdown of quasi-neutrality which can be found from (9) provided we do not neglect unity compared to $\varepsilon_k - 1$; as a result we get the correction

$$\delta\tilde{\Sigma}^{(1)}_{k,k_1,k_2,k_3} \approx \tilde{\Sigma}^{(0)}_{k,k_1,k_2,k_3}\frac{(\varepsilon_{k-k_1}^e - 1)}{(\varepsilon_{k-k_1}^i - 1)(\varepsilon_{k-k_1} - 1)},$$

for subsonic motions $\omega-\omega_1 \ll |k-k_1| v_s$ we have

$$\delta\tilde{\Sigma}^{(1)}_{k,k_1,k_2,k_3} \approx \frac{e^2(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)}{8\pi i m_e T_e \omega_{pe}{}^2 k_1 k_2 k_3} \frac{(\omega-\omega_1)^2}{(\mathbf{k}-\mathbf{k}_1)^2 v_s{}^2},$$

for supersonic motions $\omega-\omega_1 \gg |k-k_1| v_s$

$$\delta\tilde{\Sigma}^{(1)}_{k,k_1,k_2,k_3} \approx \frac{e^2(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)}{8\pi i m_e T_e \omega_{pe}{}^2 k_1 k_2 k_3} \frac{(\mathbf{k}-\mathbf{k}_1)^2 v_s{}^2}{\omega_{pi}{}^2}.$$

2) Corrections connected with the time dependence of the non-linear processes for field amplitudes given by (11′) when we must expand the currents in $\Delta\omega=\omega-\omega_{pe}$. The quantities $\Delta\omega$ play the role of non-linear corrections proportional to $k^2 v_{Te}^2/\omega_{pe}$ if non-linear terms are neglected. In the given case we must consider the more general problem without approximating $\Delta\omega$ by a linear expression as Eq. (8) takes into account all non-linearities which are cubic in the field. The actual calculation of these corrections leads in the approximations which are linear in $\Delta\omega$ to the general important result:

$$\delta\tilde{\Sigma}^{(2)}_{k,k_1,k_2,k_3}=0. \tag{15}$$

which is valid in the general three-dimensional case. Its consequences will be discussed below.

3) Electronic non-linearities connected with the expansion in $k^2 v_{Te}^2/\omega_{pe}$. In the case of subsonic motions we get

$$\delta\tilde{\Sigma}^{(3)}_{k,k_1,k_2,k_3} \approx \frac{e^2(\mathbf{kk}_1)}{8\pi i m_e{}^2\omega_{pe}{}^4 k_1 k_2 k_3}[-6(\mathbf{kk}_2)(\mathbf{kk}_3)+2k_3{}^2(\mathbf{k}_2\mathbf{k}_3)+11k_2{}^2(\mathbf{k}_2\mathbf{k}_3)$$
$$+2k_3{}^2(\mathbf{kk}_2)-2k_2{}^2(\mathbf{kk}_3)-2(\mathbf{kk}_2)(\mathbf{k}_2\mathbf{k}_3)-3(\mathbf{kk}_3)(\mathbf{k}_2\mathbf{k}_3)+k_2{}^2k_3{}^2+(\mathbf{k}_2\mathbf{k}_3)^2$$
$$+7k^2(\mathbf{k}_2\mathbf{k}_3)-4(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)-6(\mathbf{kk}_2)(\mathbf{k}_2\mathbf{k}_3)-3(\mathbf{k}_1\mathbf{k}_3)(\mathbf{k}_2\mathbf{k}_3)],$$

and correspondingly for supersonic motions

$$\delta\tilde{\Sigma}^{(3)}_{k,k_1,k_2,k_3} \approx \frac{e^2}{8\pi i m_e{}^2\omega_{pe}{}^4 k_1 k_2 k_3}[-6(\mathbf{kk}_1)(\mathbf{kk}_2)(\mathbf{kk}_3)+2(\mathbf{kk}_3)(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)$$
$$+2k_3{}^2(\mathbf{kk}_1)(\mathbf{kk}_2)-2k_2{}^2(\mathbf{kk}_1)(\mathbf{kk}_3)-2k^2(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)+2(\mathbf{kk}_1)^2(\mathbf{k}_2\mathbf{k}_3)$$
$$-k_3{}^2k_2{}^2(\mathbf{kk}_1)+(\mathbf{k}_2\mathbf{k}_3)^2(\mathbf{kk}_1)+k^2k_1{}^2(\mathbf{k}_2\mathbf{k}_3)-k_1{}^2(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)+$$
$$+6(\mathbf{kk}_2)(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)].$$

4) And, finally, an additional term arises due to the possibility of a process which proceeds through a virtual wave at the doubled frequency $(2\omega_{pe})$. If we denote the field at the doubled plasma frequency by $E^{2\pm}$ we can write the non-linear current up to terms cubic in the field in the following form:

$$\rho_k{}^+ = \int S_{k,k_1,k_2} E_{k_1}{}^- E_{k_2}{}^{2+}\delta(k-k_1-k_2)\,dk_1\,dk_2$$
$$+\int \Sigma_{k,k_3,k_1,k_2} E_{k_1}{}^+ E_{k_2}{}^+ E_{k_3}{}^- \delta(k-k_1-k_2-k_3)\,dk_1\,dk_2\,dk_3.$$

The evaluation of this expression gives the same result for subsonic as for supersonic motions:

$$\delta\tilde{\Sigma}^{(4)}_{k,k_3,k_1,k_2} \approx \frac{e^2}{8\pi i m_e{}^2\omega_{pe}{}^4 k_1 k_2 k_3}[6(\mathbf{kk}_1)(\mathbf{kk}_2)(\mathbf{kk}_3)+(\mathbf{kk}_1)(\mathbf{kk}_3)(\mathbf{k}_2,\mathbf{k}-\mathbf{k}_3)$$
$$+(\mathbf{kk}_2)(\mathbf{kk}_3)(\mathbf{k}_1,\mathbf{k}-\mathbf{k}_3)+{}^1/_2(\mathbf{kk}_3)(\mathbf{k}_1,\mathbf{k}-\mathbf{k}_3)(\mathbf{k}_2,\mathbf{k}-\mathbf{k}_3)+(\mathbf{kk}_3)(\mathbf{kk}_2)k_1{}^2$$
$$+(\mathbf{kk}_3)(\mathbf{kk}_1)k_2{}^2+{}^1/_4(\mathbf{kk}_3)k_1{}^2(\mathbf{k}_2,\mathbf{k}-\mathbf{k}_3)+{}^1/_4(\mathbf{kk}_3)k_2{}^2(\mathbf{k}_1,\mathbf{k}-\mathbf{k}_3)+$$
$$+{}^4/_3|\mathbf{k}-\mathbf{k}_3|^{-2}({}^1/_4k_1{}^2(\mathbf{k}_2,\mathbf{k}-\mathbf{k}_3)+{}^1/_4k_2{}^2(\mathbf{k}_1,\mathbf{k}-\mathbf{k}_3)+{}^1/_2(\mathbf{k}_1,\mathbf{k}-\mathbf{k}_3)(\mathbf{k}_2,\mathbf{k}-\mathbf{k}_3))$$
$$\times({}^1/_4(\mathbf{kk}_3)(\mathbf{k}-\mathbf{k}_3)^2-(\mathbf{kk}_3)(\mathbf{k},\mathbf{k}-\mathbf{k}_3)+k_3{}^2(\mathbf{k},\mathbf{k}-\mathbf{k}_3))].$$

Summing all the expressions obtained we get the following corrections for subsonic motions:

$$\delta\tilde{\Sigma}_{k,k_1,k_2,k_3} \approx \frac{e^2}{8\pi i m_e{}^2\omega_{pe}{}^4 k_1 k_2 k_3}\Big[-\frac{7}{4}(\mathbf{kk}_2)(\mathbf{kk}_1)(\mathbf{kk}_3)-{}^{15}/_4(\mathbf{kk}_1)(\mathbf{kk}_3)(\mathbf{k}_2\mathbf{k}_3)$$
$$-{}^3/_4(\mathbf{kk}_2)(\mathbf{kk}_3)(\mathbf{k}_1\mathbf{k}_3)+{}^5/_8k_1{}^2(\mathbf{kk}_2)(\mathbf{kk}_3)-{}^{11}/_8k_2{}^2(\mathbf{kk}_1)(\mathbf{kk}_3)$$
$$-{}^1/_8k_1{}^2(\mathbf{kk}_3)(\mathbf{k}_2\mathbf{k}_3)-{}^1/_8k_2{}^2(\mathbf{kk}_3)(\mathbf{k}_1\mathbf{k}_3)+{}^1/_4(\mathbf{kk}_3)(\mathbf{k}_1\mathbf{k}_3)(\mathbf{k}_2\mathbf{k}_3)$$
$$-{}^5/_8k^2(\mathbf{kk}_3)(\mathbf{k}_1\mathbf{k}_2)+{}^3/_4(\mathbf{kk}_3)^2(\mathbf{k}_1\mathbf{k}_2)-{}^3/_8k_3{}^2(\mathbf{kk}_3)(\mathbf{k}_1\mathbf{k}_2)+2k_3{}^2(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)$$
$$-2(\mathbf{kk}_1)(\mathbf{kk}_2)(\mathbf{k}_2\mathbf{k}_3)+2k_3{}^2(\mathbf{kk}_1)(\mathbf{kk}_2)+k_2{}^2k_3{}^2(\mathbf{kk}_1)+(\mathbf{k}_2\mathbf{k}_3)^2(\mathbf{kk}_1)$$
$$-7k^2(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)-4(\mathbf{kk}_1)^2(\mathbf{k}_2\mathbf{k}_3)+11k_2{}^2(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)-6(\mathbf{kk}_1)(\mathbf{kk}_2)(\mathbf{k}_2\mathbf{k}_3)$$
$$-3(\mathbf{kk}_1)(\mathbf{k}_1\mathbf{k}_3)(\mathbf{k}_2\mathbf{k}_3)-(\mathbf{kk}_3)[\mathbf{k}_1\times\mathbf{k}_2]^2+(\mathbf{k}_1\mathbf{k}_2)[\mathbf{k}\times\mathbf{k}_3]^2$$
$$+\frac{4}{3}\frac{[\mathbf{k}_1\times\mathbf{k}_2]^2[\mathbf{k}\times\mathbf{k}_3]^2}{|\mathbf{k}_1+\mathbf{k}_2|^2}\Big],$$

and for supersonic motions

$$\delta\tilde{\Sigma}_{k,k_1,k_2,k_3} \approx \frac{e^2}{8\pi i m_e{}^2\omega_{pe}{}^4 k_1 k_2 k_3}\Big[-\frac{7}{4}(\mathbf{kk}_1)(\mathbf{kk}_2)(\mathbf{kk}_3)+5k^2(\mathbf{kk}_1)(\mathbf{k}_2\mathbf{k}_3)$$
$$-6(\mathbf{kk}_1)^2(\mathbf{k}_2\mathbf{k}_3)-{}^{19}/_4(\mathbf{kk}_1)(\mathbf{kk}_3)(\mathbf{k}_2\mathbf{k}_3)+2k_3{}^2(\mathbf{kk}_1)(\mathbf{kk}_2)-{}^{11}/_8k_2{}^2(\mathbf{kk}_1)(\mathbf{kk}_3)$$
$$+k^2k_1{}^2(\mathbf{k}_2\mathbf{k}_3)-{}^3/_4(\mathbf{kk}_3)(\mathbf{kk}_2)(\mathbf{k}_1\mathbf{k}_3)+{}^1/_4(\mathbf{kk}_3)(\mathbf{k}_1\mathbf{k}_3)(\mathbf{k}_2\mathbf{k}_3)+{}^5/_8k_1{}^2(\mathbf{kk}_2)(\mathbf{kk}_3)$$
$$-{}^1/_8k_2{}^2(\mathbf{kk}_3)(\mathbf{k}_1\mathbf{k}_3)-{}^1/_8k_1{}^2(\mathbf{kk}_3)(\mathbf{k}_2\mathbf{k}_3)-{}^3/_8k^2(\mathbf{kk}_3)(\mathbf{k}_1\mathbf{k}_2)+{}^3/_4(\mathbf{kk}_3)^2(\mathbf{k}_1\mathbf{k}_2)$$
$$-{}^3/_8k_3{}^2(\mathbf{kk}_3)(\mathbf{k}_1\mathbf{k}_2)-{}^1/_2(\mathbf{kk}_3)[\mathbf{k}_1\times\mathbf{k}_2]^2+{}^1/_2(\mathbf{k}_1\mathbf{k}_2)[\mathbf{k}\times\mathbf{k}_3]^2$$
$$+\frac{2}{3}\frac{[\mathbf{k}_1\times\mathbf{k}_2]^2[\mathbf{k}\times\mathbf{k}_3]^2}{|\mathbf{k}_1+\mathbf{k}_2|^2}\Big]. \tag{16}$$

The one-dimensional case is of special interest as in the approximation (16) the result vanishes exactly.

4. Before discussing the consequences of these results we find the contribution from the non-linearities of higher (fourth and fifth) order in the field which are connected with S' and Σ' in Eq. (1). They give, respectively, corrections to the right-hand sides of Eqs. (2) and (3):

$$(4\pi)^{-1}ik\varepsilon_k\delta E_k{}^+=2\int S_{k,k_1,k_2}E_{k_1}{}^+\delta E_{k_2}\delta(k-k_1-k_2)\,dk_1\,dk_2$$
$$+\int S'_{k,k_1,k_2,k_3,k_4}E_{k_1}{}^+E_{k_2}E_{k_3}{}^+E_{k_4}{}^-\delta(k-k_1-k_2-k_3-k_4)\,dk_1\,dk_2\,dk_3\,dk_4$$
$$+\int \Sigma'_{k,k_1,k_2,k_3,k_4,k_5}E_{k_1}{}^+E_{k_2}{}^+E_{k_3}{}^-E_{k_4}{}^+E_{k_5}{}^-\delta(k-k_1-k_2-k_3-k_4-k_5)\,dk_1\,dk_2\,dk_3\,dk_4\,dk_5, \tag{17}$$

$$(4\pi)^{-1}ik\varepsilon_k\delta E_k=\int S_{k,k_1,k_2}E_{k_1}E_{k_2}\delta(k-k_1-k_2)\,dk_1\,dk_2$$
$$+\int \Sigma_{k,k_1,k_2,k_3}E_{k_1}E_{k_2}{}^+E_{k_3}{}^-\delta(k-k_1-k_2-k_3)\,dk_1\,dk_2\,dk_3$$
$$+\quad S_{k,k_1,k_2,k_3,k_4}E_{k_1}{}^+E_{k_2}{}^-E_{k_3}{}^+E_{k_4}{}^-\delta(k-k_1-k_2-k_3-k_4)\,dk_1\,dk_2\,dk_3\,dk_4. \tag{18}$$

The coefficients Σ, S', and Σ' are here already symmetrized with respect all indexes bar the first one.

It is sufficient in Eq. (18) to use (3) for the low-frequency field and to substitute the result obtained in the first term of (17), in the second term of (17) also we can use Eq. (3) for the low-frequency field. As a result we get

$$(4\pi)^{-1}ik\varepsilon_k\delta E_k{}^+=2\int \delta\tilde{\Sigma}'_{k,k_1,k_2,k_3,k_4,k_5}E_{k_1}{}^+E_{k_2}{}^+E_{k_3}{}^-E_{k_4}{}^+E_{k_5}{}^-\cdot$$
$$\cdot\delta(k-k_1-k_2-k_3-k_4-k_5)\,dk_1\,dk_2\,dk_3\,dk_4\,dk_5,$$

where

$$\delta\tilde{\Sigma}'_{k,k_1,k_2,k_3,k_4,k_5}=\Sigma'_{k,k_1,k_2,k_3,k_4,k_5}-\frac{8\pi i S_{k_2+k_3,k_2,k_3}}{|\mathbf{k}_2+\mathbf{k}_3|\varepsilon_{k_2+k_3}}(S'_{k,k_1,k_2+k_3,k_4,k_5}+S'_{k,k_1,k_4,k_5,k_2+k_3})$$
$$-\frac{8\pi i S_{k,k_1,k-k_1}}{|\mathbf{k}-\mathbf{k}_1|\varepsilon_{k-k_1}}S'_{k-k_1,k_2,k_3,k_4,k_5}-\frac{32\pi^2 S_{k,k_1,k-k_1}S_{k_2+k_3,k_2,k_3}}{|\mathbf{k}-\mathbf{k}_1|\varepsilon_{k-k_1}|\mathbf{k}_2+\mathbf{k}_3|\varepsilon_{k_2+k_3}}\cdot$$
$$\cdot(\Sigma_{k-k_1,k_2+k_3,k_4,k_5}+\Sigma_{k-k_1,k_4,k_5,k_2+k_3})+\frac{32\pi^2 S_{k_2+k_3,k_2,k_3}}{|\mathbf{k}_2+\mathbf{k}_3|\varepsilon_{k_2+k_3}}\frac{S_{k_4+k_5,k_4,k_5}\Sigma_{k,k_1,k_2+k_3,k_4+k_5}}{|\mathbf{k}_4+\mathbf{k}_5|\varepsilon_{k_4+k_5}}$$
$$+\frac{4\pi i(8\pi)^2 S_{k,k_1,k-k_1}}{|\mathbf{k}-\mathbf{k}_1|\varepsilon_{k-k_1}}\frac{S_{k-k_1,k_2+k_3,k_4+k_5}S_{k_2+k_3,k_2,k_3}S_{k_4+k_5,k_4,k_5}}{|\mathbf{k}_2+\mathbf{k}_3|\varepsilon_{k_2+k_3}|\mathbf{k}_4+\mathbf{k}_5|\varepsilon_{k_4+k_5}}, \tag{19}$$

and

$$(4\pi)^{-1} ik\varepsilon_k \delta E_k = 2\int \delta S'_{k,k_1,k_2,k_3,k_4} E_{k_1}{}^{+} E_{k_2}{}^{-} E_{k_3}{}^{+} E_{k_4}{}^{-} \times \delta(k-k_1-k_2-k_3-k_4)\, dk_1\, dk_2\, dk_3\, dk_4,$$

where

$$\delta S'_{k,k_1,k_2,k_3,k_4} = S'_{k,k_1,k_2,k_3,k_4} - \frac{8\pi i S_{k_1+k_2,k_1,k_2} \Sigma_{k,k_1+k_2,k_3,k_4}}{|\mathbf{k}_1+\mathbf{k}_2| \varepsilon_{k_1+k_2}} - \frac{(8\pi)^2 S_{k,k_1+k_2,k_3+k_4} S_{k_1+k_2,k_1,k_2} S_{k_3+k_4,k_3,k_4}}{|\mathbf{k}_1+\mathbf{k}_2| \varepsilon_{k_1+k_2} |\mathbf{k}_3+\mathbf{k}_4| \varepsilon_{k_3+k_4}}.$$

Using the approximate expression for the matrix elements occurring in (19), we have

$$\delta\Sigma'_{k,k_1,k_2,k_3,k_4,k_5} = -\frac{e^4 (\mathbf{k}\mathbf{k}_1)(\mathbf{k}_4\mathbf{k}_5)}{8\pi i m_e{}^2 T_e{}^2 \omega_{pe}{}^4 k_1 k_2 k_3 k_4 k_5 |\mathbf{k}-\mathbf{k}_1|^2} \times [(\mathbf{k}_3, \mathbf{k}-\mathbf{k}_1)(\mathbf{k}_2, \mathbf{k}_4+\mathbf{k}_5) + (\mathbf{k}_2, \mathbf{k}-\mathbf{k}_1)(\mathbf{k}_3, \mathbf{k}_4+\mathbf{k}_5)] \frac{\varepsilon'_{k-k_1}}{\varepsilon_{k-k_1}} \frac{\varepsilon^i_{k_4+k_5}}{\varepsilon_{k_4+k_5}}.$$

This expression is analogous to (9) and obtained under the same approximations, i.e., neglecting corrections in $k^2 v_{Te}^2/\omega_{pe}$ and all other kinds of corrections which were discussed in the preceding section.

For subsonic motions the correction terms are thus of order $|E|^2/4\pi nT \ll 1$. This means that we are in fact dealing with weak non-linearities and the theory using the HE can not describe correctly the limit $|E|^2/4\pi nT \approx 1$. On the other hand, for supersonic motions the higher non-linearities turn out to be not too important and the present analysis shows that one can use for supersonic motions the HE up to $|E|^2/4\pi nT \approx 1$. However, for supersonic motions a limit such as $|E|^2/4\pi nT \ll 1$ appears from the conditions that we may neglect the electron non-linearities.

5. We now discuss the role of the corrections described in Sec. 4 which we shall all call electron non-linearities as they are of the same order as the electron non-linearities.

First of all we discuss the problem of the collapse. To do this we must give an exact definition what we understand by the collapse phenomenon. It is widely known and expounded in textbooks[7] that the development of the linear stage of the modulational instability leads to the occurrence of a lowered density. However, one usually is dealing with the non-linear stage of the modulational instability and the collapse corresponds to the fact that the non-linear stage of the modulational instability is, according to Zakharov's assumption,[4] non-linearly not stabilized, i.e., it develops until $|E|^2/4\pi nT \approx 1$, $\delta n/n \approx 1$, $r \sim r_d$. Subsonic density rarefactions are usually small: $\delta n/n \gtrsim m_e/m_i$; to reach $\delta n/n \sim 1$ the rarefactions must go through a supersonic stage. Zakharov[4] obtained self-similar solutions

$$E \sim \frac{1}{t} f\left(\frac{r}{t^{2/3}}\right), \tag{20}$$

which were disputed by Litvak, Fraĭman, and Yunakovskiĭ[8] as they do not conserve the energy flux. Without discussing the validity of the self-similar solutions, we note that the electron non-linearities which were discussed in Sec. 4 come into play when $t \lesssim \omega_{pi}^{-1}$, i.e., the HE can be used when $t \gg \omega_{pi}^{-1}$; up to the time $t \approx \omega_{pi}^{-1}$ the self-similar solution (20) gives

$$|E|^2/4\pi nT \approx 1, \qquad \delta n/n \approx (m_e/m_i)^{1/3}.$$

As the criterion $t \gg \omega_{pi}^{-1}$ must be satisfied, it follows from this that $|E|^2 4\pi nT \ll 1$ and $\delta n/n \ll (m_e/m_i)^{1/3}$. The Landau damping for such perturbations is, even if we assume that $\delta n/n \approx (m_e/m_i)^{1/3}$, a quantity of the order of $\omega_{pe} \times 10^{-6}$, i.e., it is very small and cannot be used as a mechanism for the dissipation of the Langmuir turbulence. The main problem of the possibility of the collapse consists thus, even if we assume the validity of the self-similar solution (20), in whether the non-linear equations which take only the electron non-linearities into account can lead to self-compression effects. One can easily solve this problem. It is important that in the three-dimensional case the corrections due to the time-dependence of the non-linear interactions are absent in the linear approximation in $\Delta\omega$ because of (15). On the right-hand side of the equations we obtained there are no $\Delta\omega$ and, hence, no modulational interaction occurs (the modulational instability arises formally as a consequence of the frequency dependence of the non-linear response). However, one can also understand physically that self-compression must be absent. As we have already stated the equations are applicable when $|E|^2/4\pi nT \ll 1$; in that case the electron non-linearities in the three-dimensional case contain second derivatives with respect to the coordinates and have the same structure as the dispersion term, but smaller by a factor $|E|^2/4\pi nT$. Hence, the linear dispersion dominates and the non-linear compression processes, even if they occur, are small. Langmuir wave packets will thus basically spread out due to the linear dispersion.

The equations which describe such non-linear interactions do in no way differ from the equations with a weak non-linearity or weak turbulence,[7] i.e., they are described by standard methods where it is necessary to use the linear dispersion to describe waves in the non-linearities. This shows the impossibility to realize collapse, even if the solutions (20) were valid.

6. We now consider in detail one-dimensional motions, taking the electron non-linearities into account, and in this connection we discuss the possibility of the occurrence of fast Langmuir solitons with $u \gg v_s$. In contrast to the three-dimensional case not only the terms proportional to $\Delta\omega$, but also the terms of order $k^2 v_{Te}^2/\omega_{pe}^2$ vanish (see (16)). It is therefore necessary to take into account the effects of the next order among which will be effects of the order of products of small parameters. Effects in $(\varepsilon^i_{k-k_1}/\varepsilon_{k-k_2})(k^2 v_{Te}^2/\omega_{pe}^2)$ turn out to be small compared to $\Sigma^{(0)}_{k,k_1}$; taking effects of order $k^4 v_{Te}^4/\omega_{pe}^4$ into account leads to the result:

$$\delta\Sigma^{\text{one-dim}}_{k,k_1,k_2,k_3} = \frac{e^2 k v_{Te}{}^2}{8\pi i m_e{}^2 \omega_{pe}{}^6} \Big[-\frac{335}{2} k_1{}^4 + \frac{121}{2} k_1{}^3 (k_1+k_3) - 17 k_1{}^2 (k_1+k_3)^2 - 40 k_1 (k_1+k_3)^3 + 4 (k_1+k_3)^4 \Big].$$

Taking these terms into account the non-linear equations will be

$$\frac{2i}{\omega_{pe}}\frac{\partial E}{\partial t}+\frac{3v_{Te}^2}{\omega_{pe}^2}\frac{\partial^2 E}{\partial x^2}=\frac{\delta n}{n_0}E-\frac{v_{Te}^4/\omega_{pe}^4}{4\pi nT}\Big[-\frac{335}{2}|E|^2\frac{\partial^4 E}{\partial x^4}+\frac{121}{2}\frac{\partial^3 E}{\partial x^3}\frac{\partial}{\partial x}|E|^2-17\frac{\partial^2 E}{\partial x^2}\frac{\partial^2}{\partial x^2}|E|^2-40\frac{\partial E}{\partial x}\frac{\partial^3}{\partial x^3}|E|^2+4E\frac{\partial^4}{\partial x^4}|E|^2\Big]. \quad (21)$$

Apart from this there appear non-linear effects proportional to $(\Delta\omega/\omega_{pe})(k^2 v_{Te}^2/\omega_{pe}^2)$ and $(\Delta\omega/\omega_{pe})^2$. Evaluation of terms of order $(\Delta\omega)^2$ leads to the result:

$$\delta\Sigma_{k,k_1,k_2,k_3}\approx\frac{e^2k}{8\pi i m_e^2\omega_{pe}^2 v_{Te}^2}\left(\frac{4k_2}{(k-k_1)}\frac{\Delta\omega\,\Delta\omega_2}{\omega_{pe}^2}-\frac{4k_3}{(k-k_1)}\frac{\Delta\omega\,\Delta\omega_3}{\omega_{pe}^2}\right). \quad (22)$$

Introducing dimensionless variables

$$\xi=xk_d,\quad \tau=\omega_{pe}t,\quad u=v_0/v_{Te},\quad \mathscr{E}^2\to E^2/\pi nT,$$

and using (21) we can write the non-linear equation in the following form:

$$2i\frac{\partial\mathscr{E}}{\partial\tau}+3\frac{\partial^2\mathscr{E}}{\partial\xi^2}-|\mathscr{E}|^2\frac{\partial^2\mathscr{E}}{\partial\tau^2}+u|\mathscr{E}|^2\frac{\partial^2\mathscr{E}}{\partial\xi\,\partial\tau}+u^2\left(\mathscr{E}\frac{\partial^2|\mathscr{E}|^2}{\partial\xi^2}-\frac{\partial\mathscr{E}}{\partial\xi}\frac{\partial|\mathscr{E}|^2}{\partial\xi}\right)=0. \quad (23)$$

We discuss two limiting cases, when

$$(\Delta\omega/\omega_{pe})^2\gg k^4v_{Te}^4/\omega_{pe}^4, \quad (24)$$

and the opposite case.

When (24) is satisfied we can use Eq. (23) to solve the problem of the possibility of the existence of fast Langmuir solitons (spikons). Introducing $\mathscr{E}=|\mathscr{E}|e^{i\Phi}$ and separating the real and imaginary parts we get two equations for $|\mathscr{E}|$ and Φ. Assuming that $|\mathscr{E}|$ is only a function of $\xi-u\tau$ we get as a necessary consequence of the equation that Φ has the form

$$\Phi=\Phi_0-\Omega\tau-\left(\frac{\Omega}{2u}+\frac{u}{6}\right)\xi+\psi,$$

where

$$\frac{\partial\psi}{\partial\tau}=A|\mathscr{E}|^2\left(1-\frac{u^2}{3}|\mathscr{E}|^2\right)^2,\quad A=\text{const}.$$

We consider here the simplest case $A=0$. The solution for the amplitude $|\mathscr{E}|$ then has the form

$$\mathscr{E}=\mathscr{E}_0\,\mathrm{ch}^{-1}(\xi-u\tau)/\xi_0, \quad (25)$$

where

$$\xi_0=6/u\mathscr{E}_0(\Omega+u^2/3)^{1/2},$$

$$\Omega=\frac{u^2}{3}\left\{\frac{5}{3}+\frac{u^2}{3}\mathscr{E}_0^2\pm2\left[\left(1+\frac{u^2}{6}\mathscr{E}_0^2\right)\left(3+\frac{u^2}{6}\mathscr{E}_0^2\right)\right]^{1/2}\right\}. \quad (26)$$

It is clear from solution (25) that spikons may exist. To see this we determine first of all the limits of applicability of the results. On the one hand, it is necessary to satisfy (24) which reduces to the condition $u\gg\xi_0^{-1}$ and the inequality $u\ll1$. As a result of this we have $\xi_0\gg1$. On the other hand, the inequality $u\gg\xi_0^{-1}$ leads to the condition $u\mathscr{E}_0\ll1$, and hence the frequency (26) has the approximate form:

$$\Omega=\frac{u^2}{3}\left(\frac{5}{3}+2\sqrt{3}\right)\approx5.13\frac{u^2}{3}. \quad (27)$$

As $\Omega+u^2/3>0$, the solution (26) with the minus sign must be discarded, and this has been used in (27), i.e., $\xi_0\approx4.2/u^2\mathscr{E}_0$. However, it is necessary to bear in mind that the term with $\tilde{\Sigma}^{(0)}$ is small compared to the terms taken into account, i.e.,

$$\frac{m_e}{m_i}\frac{1}{u^2}\ll\frac{u^2}{\xi_0^2}\sim u^6\mathscr{E}_0^2.$$

The following inequalities must thus be satisfied

$$17.6\gg u^2\mathscr{E}_0^2\gg\left(\frac{m_e}{m_i}\right)u^{-6}\quad\text{or}\quad1\gg u\gg\left(\frac{1}{17.6}\right)^{1/6}\left(\frac{m_e}{m_i}\right)^{1/6}.$$

These inequalities show that fast Langmuir solitons are possible for sufficiently large values of u and $\mathscr{E}_0$. In the other limiting case when the opposite of (24) is satisfied, Eqs. (21) could apparently lead to fast solitons while from dimensional considerations we must have $\xi_0\sim\mathscr{E}_0$, but $\mathscr{E}_0\ll1$ and, hence, $\xi_0\ll1$ which contradicts the condition under which Eq. (24) was obtained. Spikons are thus impossible when (24) is not satisfied.

7. In conclusion we note that there does not occur such a strong compensation of the non-linear processes for three-dimensional motions and the main contribution for fast processes comes from terms proportional to $k^2v_{Te}^2/\omega_{pe}^2$ which, on the one hand, prevents the possibility for the realization of a collapse, and on the other hand, makes the existence of fast three-dimensional solitons difficult in practice.

The investigation methods used in the present paper differ in essential ways from the methods using averages over the high frequency which are used in other papers.[3,4] This is just the reason why we were able to determine the limits of the applicability of the HE and to evaluate all kinetic effects. We emphasize also that the electron non-linearities which make the occurrence of collapse impossible are completely determined by kinetic effects and could not be found by a hydrodynamical approach.

Note added in proof (March 26, 1976). Our analysis indicates that terms proportional to $\Delta\omega(k^2v_{Te}^2/\omega_{pe}^2)$ must be taken into account when one determines the structure of fast solitons.

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Nuclear magnetic resonance of V^{51} in a V_3Si single crystal at room temperature

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NMR of V^{51} in a V_3Si single crystal is investigated at room temperature. The anisotropic component K_{an} of the Knight shift is found to be zero. It is noted that calculations based on the band-structure scheme proposed by Labbe and Friedel are not in accord with the obtained temperature dependence of K_{an}.

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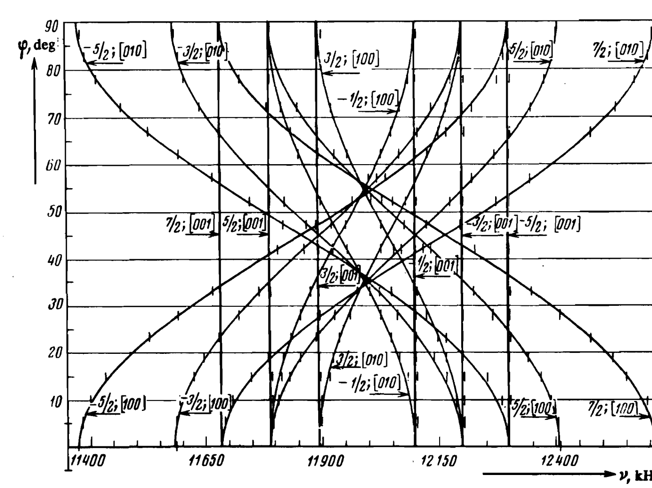
Tret'yakov, Kodes, and Kuritsin, [1] in a report of the results of an investigation of nuclear magnetic resonance (NMR) of V^{51} in polycrystalline V_3Si samples, have shown that in addition to the temperature-dependent isotropic Knights shift (K_{is}) there exists an anisotropic component of the Knight shift (K_{an}), which also changes with changing temperature. Since there was no sufficiently correct method of determining K_{an} from the NMR spectra obtained with polycrystals until recently, it was impossible in[1] to determine K_{an}. However, since K_{an} is directly connected with the anisotropy of the electron density in crystals, and in some cases with the values of the magnetic moments of the atoms contained in the crystal lattice, attempts were made to investigate NMR in single-crystal samples of V_3Si. Tret'yakov *et al.* [2] investigated NMR of V^{51} of single-crystal V_3Si in a polarizing magnetic field H_0 = 4 kOe at 78 °K and have established that $K_{an} = 0.04\%$ for this resonance. The use of the acoustic nuclear resonance method, [3] likewise with a single-crystal sample, has confirmed the value $K_{an} = 0.04\%$ at 78 °K, and yielded $K_{an} = 0.07\%$ at 17 °K. At room temperature, however, the value of K_{an} for NMR of V^{51} has not been determined to date.

The present study is a continuation of[2] and is devoted to an investigation of NMR of V^{51} in the same V_3Si single crystal, for the purpose of obtaining the value of K_{an} at room temperature. The experiment itself as well as the reduction of the experimental data were the same in the present study as in[2], except for the value of the polarizing magnetic field H_0, which could be raised to 10 kOe, owing to the increase in the resistivity of the sample at the higher temperature; this led to an increase in the accuracy of K_{an}. The accuracy with which the crystallographic directions of the single crystal were set relative to the external magnetic field was 2°.

The figure shows a plot of the resonance frequencies of the satellite NMR transitions of V^{51} on the angle φ between the direction of H_0 and the crystallographic direction [100] of the single crystal. The [001] direction remained perpendicular to H_0 for all the spectra. The vertical strokes in the figure correspond to the experimental values of the resonant frequencies, while the solid lines were calculated theoretically at $K_{an} = 0$, $K_{is} = 0.54\%$, and a quadrupole splitting constant $\nu_q = 206$ kHz. The method of determining these parameters from the experimental values of the satellite transition frequencies is given in[2]. Thus, the reduction of experimental data leads to the following NMR parameters of V^{51} in V_3Si at room temperature: $\nu_Q = 206 \pm 1$ kHz, $K_{is} = (0.54 \pm 0.01)\%$, and $K_{an} = (0.00 \pm 0.01)\%$.

The parameters of NMR of V^{51} using the spectrum from a powder obtained by pulverizing a part of the investigated single crystal has yielded the parameters $\nu_Q = 206 \pm 1$ kHz, $K_{is} = (0.57 \pm 0.01)\%$, and $K_{an} = (0 \pm 0.01)\%$. These parameters are identical with the parameters of



Dependence of the resonant frequencies of the NMR satellite transitions of V^{51} of single-crystal V_3Si on the crystal orientation relative to the external magnetic field at room temperature. The fractions in the figure correspond to the quantum magnetic number of the Zeeman level of the V^{51} nucleus.