A softening in this region is apparent also from the data of Fürrer and Hälg[5] (single point: $aq/2\pi = 0.347$, $\omega(4.2\ \text{K}) = 3.61 \cdot 10^{12}$ rad/sec, $\omega(78\ \text{K}) = 3.71 \cdot 10^{12}$ rad/sec). The absence of measurements at temperatures directly preceding the superconducting transition, however, made it impossible for the authors of[5] to observe the softening effect described here. Nor can this effect be revealed by such integral characteristics of the frequency spectrum as the specific heat,[15] inasmuch as besides the softening phonons the spectrum also contains phonons whose frequency increases below $T_c$, and in addition, the specific heat is not very sensitive to details of the phonon spectrum.

It is of interest to investigate the anisotropy of the described effects. For a detailed analysis of the results of the present paper we need model calculations that take into account all the contributions made to the damping of the transverse phonons.

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1)Some of the results obtained by us are reported in [10].

2)In particular, these are apparently the causes of the very large (by almost one order of magnitude) discrepancies between the phonon natural width given in[4,5], even at $T = 300\,^\circ$K, when the contribution $2\Gamma_{ph}$ to the total width of the neutron resonance is quite large. (In addition, different procedures were used in these papers to calculate the resolution.) Our estimates of $2\Gamma_{ph}$ at $T = 300\,^\circ$K give values closer to the results of [4].


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# Distinctive features of the phonon spectrum in solid helium


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The effect of the defecton-phonon interaction on the phonon spectrum in crystalline helium is theoretically considered. It is shown that a substantial renormalization of the phonon group velocities occurs in the neighborhood of the threshold frequencies corresponding to the decay of a phonon into a defecton pair. The phonon spectrum breaks off at the threshold points and disappears, when the interaction is sufficiently strong, in the region of frequencies corresponding to the continuum of the free defecton pairs. The coupling between the phonons and the bound defecton states leads to an additional modification of the spectrum. It is also found that the defecton-phonon interaction leads to a nonmonotonic dependence of the intensity of the one-phonon mode in the dynamic form factor on the transferred momentum.


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## 1. INTRODUCTION

The weakness of the interaction, coupled with the smallness of the atomic mass, is the source of quite a number of properties that qualitatively distinguish solid $He^3$ and $He^4$ from normal classical crystals. To these properties pertains, in particular, the fact that the energy of formation of a point defect in crystalline helium turns out to be appreciably lower than the cutoff energy ($\omega_D$) of the phonon excitations and, as was first noted by Andreev and Lifshitz,[1] can, in principle, become zero.

The object of the present paper is to consider the possible distinctive features of the phonon spectrum of solid helium that are caused by the interaction with the defecton excitations under conditions when the minimum energy of creation of the latter is finite. We shall show that, as we approach the creation threshold for one-particle excitation pairs of the "particle-hole" type from the low-frequency side, the phonon group velocity decreases sharply, tending at the threshold point to the value of the one-particle excitation velocity. At this point the phonon-spectrum curve breaks off abruptly and, in the case of a sufficiently intense interaction, vanishes in the frequency range corresponding to the continuum of the one-particle excitations. It reappears only at frequencies corresponding to the upper pair-production threshold, above which pair production is impossible. The shape of the phonon-spectrum curve in the region immediately above the upper threshold is similar to that obtaining near the lower threshold.

Recently, Andreev[2] found that there can exist in solid helium bound defecton states, including bound pairs of the particle-hole type. Since the spectrum of the bound pair lies below the spectrum of the free-pair continuum, the bound states cannot affect the phonon spectrum in the immediate vicinity of the production threshold for the free pairs. As to the region where, in the absence of interaction, the spectral curves (for the phonons and bound pairs) could intersect, the coupling between them leads to a modification of the spectrum in accordance with the principle of (mutual) non-intersectability of terms of the same symmetry.[3]

It is characteristic that the distinctive features of the phonon spectrum that are in question here do not depend on the statistics obeyed by the atoms and, hence, by the one-particle defecton excitations of the crystal. This circumstance is connected with the assumption that the minimum energy of formation of a defecton pair substantially exceeds the temperature of the crystal. As a result, there are virtually no thermal defectons, and their statistics-dependent contribution to the renormalization of the phonon spectrum is exponentially small. For this reason, the analysis of the properties of the phonon spectrum will be carried out for the $T=0$ case.

The distinctive features of the phonon spectrum are closely related with the properties of the crystal's dynamic form factor, which plays an important role in the description of experiments on inelastic neutron scattering. The third section of the paper is devoted to the consideration of some properties of the dynamic form factor that are due to the defecton-phonon interaction.

## 2. DISTINCTIVE FEATURES OF THE PHONON SPECTRUM

The phonon spectrum in a crystal, like the other branches of the Bose excitations of the collective type, is determined by the poles of the two-particle vertex part $\Gamma$[4] as a function of the frequency transfer $\omega$ and the wave vector $\mathbf{k}$. With allowance for this circumstance in the representation of an arbitrary system of functions satisfying the Bloch periodicity conditions, we can write the function $\Gamma$ in the form[5]

$$\hat{\Gamma}(p,p';k)=\hat{\hat{\Gamma}}(p,p';k)+\sum_{\alpha}\hat{g}_{\alpha}(p,k)D_{\alpha}(\omega,\mathbf{k})\hat{g}_{\alpha}^{(+)}(p',k). \qquad (1)$$

The function $D_\alpha(\omega, \mathbf{k})$ explicitly contains phonon poles ($\alpha$ is the number of the phonon branch), and can, naturally, be called the phonon Green function. The quantities $\hat{\hat{\Gamma}}$, $\hat{g}_\alpha$, and $\hat{g}_\alpha^{(+)}$ do not possess such poles. The four-dimensional vectors $p+k$, $p'$ and $p$, $p'+k$ corresponds to the energies and quasimomenta of the particles respectively before and after the interaction: $p=\{\varepsilon, \mathbf{p}\}$, $p'=\{\varepsilon', \mathbf{p}'\}$, $k=\{\omega, \mathbf{k}\}$. The functions $\hat{\Gamma}$, $\hat{\hat{\Gamma}}$, $\hat{g}_\alpha$, and $\hat{g}_\alpha^{(+)}$ are matrices with respect to the band indices numbering the basis functions in whose representation they are written.

The fact that the phonon poles of the vertex part $\Gamma$ have a factorizable form allows us to treat the function $D_\alpha(\omega, \mathbf{k})$ in the same way as any other one-particle Bose Green's function. In particular, for the corresponding retarded Green function $D_\alpha^R(\omega, \mathbf{k})$, we can write down a Dyson type of equation

$$[D_\alpha^R(\omega,\mathbf{k})]^{-1}=[\bar{D}_\alpha^R(\omega,\mathbf{k})]^{-1}-P_\alpha^R(\omega,\mathbf{k}), \qquad (2)$$

in which the contribution, $P_\alpha^R(\omega, \mathbf{k})$, of the processes of decay of a phonon into a pair of defecton excitations has been explicitly separated out. The function $P_\alpha^R(\omega, \mathbf{k})$ is determined by diagrams of the type shown in Fig. 1. A line with the plus sign in the diagram denotes a particle-type defecton (an interstice), while a line with the minus sign denotes a hole-type defecton excitation (a vacancy). Bearing in mind the analytic properties of the function $D_\alpha^R(\omega, \mathbf{k})$, we can represent the quantity $P_\alpha^R(\omega, \mathbf{k})$ in the form of a spectral decomposition:

$$P_\alpha^R(\omega,\mathbf{k})=\int_{-\infty}^{\infty}d\omega'\,\frac{\rho_\alpha(\omega',\mathbf{k})}{\omega-\omega'+i\delta},\quad \delta\to+0,$$
$$\omega\rho_\alpha(\omega,\mathbf{k})\geqslant 0. \qquad (3)$$

The spectral function $\rho_\alpha(\omega, \mathbf{k})$ is completely determined by the defecton-excitation states and the defecton-phonon interaction.

Let us first assume that there are no defecton-defecton bound states. Then the spectral function $\rho_\alpha(\omega, \mathbf{k})$ will be determined by the contribution of only the free pairs. Having an energy gap that is substantially greater than the temperature, the defecton excitations are long-lived. Indeed, the processes of phonon emission by the defectons are forbidden by the laws of conservation of energy and quasi-momentum as a result of the large difference between the phonon and defecton velocities, while the fusion of two defectons into a phonon is excluded because of the absence of thermal defectons. The only possible process that is capable of making a contribution to the defecton decay is the scattering of the defectons by phonons. The probability of these processes are, however, small because of the smallness of the number of thermal phonons $\sim(T/\omega_D)^3$, owing to which we can neglect them.

With allowance for these remarks, the expression for the spectral density $\rho_\alpha(\omega, \mathbf{k})$ for $\omega>0$ has the form

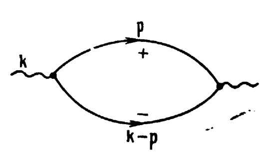


FIG. 1.

$$\rho_\alpha(\omega,\mathbf{k})=\int\limits_{\omega>0}\frac{d\mathbf{p}}{(2\pi)^3}A_\alpha(\mathbf{p},\mathbf{k})\,\delta(\omega-\varepsilon_+(\mathbf{p})-\varepsilon_-(\mathbf{k}-\mathbf{p})),\tag{4}$$

where $\varepsilon_+(\mathbf{p})$ and $\varepsilon_-(\mathbf{k}-\mathbf{p})$ are the energies of the particle- and hole-type defectons with quasi-momenta $\mathbf{p}$ and $\mathbf{k}-\mathbf{p}$ respectively. The function $A_\alpha(\mathbf{p},\mathbf{k})$ is positive. Owing to the band character of the defecton spectrum, the function $\rho_\alpha(\omega,\mathbf{k})$ for $\omega>0$ is different from zero within the frequency range

$$\omega^{(1)}(\mathbf{k})<\omega<\omega^{(2)}(\mathbf{k}),$$

in which the energy sum $\varepsilon_+(\mathbf{p})+\varepsilon_-(\mathbf{k}-\mathbf{p})$ is contained. The frequencies $\omega^{(1)}(\mathbf{k})$ and $\omega^{(2)}(\mathbf{k})$ correspond to the minimum and maximum of the expression $\varepsilon_+(\mathbf{p})+\varepsilon_-(\mathbf{k}-\mathbf{p})$ for a given value of the vector $\mathbf{k}$.

Let us determine the form of the spectral function for $\omega\to\omega^{(1)}(\mathbf{k})$ and $\omega\to\omega^{(2)}(\mathbf{k})$. Let us denote by $\mathbf{p}_1(\mathbf{k})$ and $\mathbf{p}_2(\mathbf{k})$ the values of the quasi-momentum $\mathbf{p}$ (for a fixed $\mathbf{k}$ vector) at which the expression $\varepsilon_+(\mathbf{p})+\varepsilon_-(\mathbf{k}-\mathbf{p})$ has its minimum and maximum respectively. According to the foregoing, these points are not singular points for the defecton spectrum, and near them we can expand the function $\varepsilon_+(\mathbf{p})+\varepsilon_-(\mathbf{k}-\mathbf{p})$ in powers of the difference $\mathbf{q}_1=\mathbf{p}-\mathbf{p}_1(\mathbf{k})$ and $\mathbf{q}_2=\mathbf{p}-\mathbf{p}_2(\mathbf{k})$:

$$\begin{aligned}&\varepsilon_+(\mathbf{p})+\varepsilon_-(\mathbf{k}-\mathbf{p})\cong\omega^{(1)}(\mathbf{k})+\alpha_{ik}(\mathbf{k})q_1^iq_1^k,\quad \mathbf{p}\to\mathbf{p}_1(\mathbf{k});\\&\varepsilon_+(\mathbf{p})+\varepsilon_-(\mathbf{k}-\mathbf{p})\cong\omega^{(2)}(\mathbf{k})-\beta_{ik}(\mathbf{k})q_2^iq_2^k,\quad \mathbf{p}\to\mathbf{p}_2(\mathbf{k}).\end{aligned}\tag{5}$$

Because the points $\mathbf{p}_1(\mathbf{k})$ and $\mathbf{p}_2(\mathbf{k})$ are extremum points, there are no linear terms in the expansion in $q_1$ and $q_2$. The principal values of the matrices $\alpha_{ik}$, $\beta_{ik}$ are nonnegative. We shall also assume that not a single one of them vanishes. Then on the basis of (4) and (5) we obtain for the function $\beta_\alpha(\omega,\mathbf{k})$ for frequencies close to $\omega^{(1)}(\mathbf{k})$ and $\omega^{(2)}(\mathbf{k})$ the following expressions:

$$\begin{aligned}&\rho_\alpha(\omega,\mathbf{k})\cong\frac{A_\alpha(\mathbf{p}_1,\mathbf{k})}{4\pi^2(\det\hat{\alpha})^{1/2}}\sqrt{\omega-\omega^{(1)}(\mathbf{k})}\,\theta(\omega-\omega^{(1)}(\mathbf{k})),\quad\omega\to\omega^{(1)}(\mathbf{k});\\&\rho_\alpha(\omega,\mathbf{k})\cong\frac{A_\alpha(\mathbf{p}_2,\mathbf{k})}{4\pi^2(\det\hat{\beta})^{1/2}}\sqrt{\omega^{(2)}(\mathbf{k})-\omega}\,\theta(\omega^{(2)}(\mathbf{k})-\omega),\quad\omega\to\omega^{(2)}(\mathbf{k}).\end{aligned}\tag{6}$$

where $\theta(x)=1$ for $x>0$, $\theta(x)=0$ for $x<0$. Hence with the aid of the relation (3) we have

$$\begin{aligned}&P_\alpha(\omega,\mathbf{k})\cong P_\alpha(\omega^{(1)}(\mathbf{k}),\mathbf{k})+\frac{A_\alpha(\mathbf{p}_1,\mathbf{k})}{4\pi^2(\det\hat{\alpha})^{1/2}}(\omega^{(1)}(\mathbf{k})-\omega)^{1/2},\quad\omega\to\omega^{(1)}(\mathbf{k});\\&P_\alpha(\omega,\mathbf{k})\cong P_\alpha(\omega^{(2)}(\mathbf{k}),\mathbf{k})-\frac{A_\alpha(\mathbf{p}_2,\mathbf{k})}{4\pi^2(\det\beta)^{1/2}}(\omega-\omega^{(2)}(\mathbf{k}))^{1/2},\quad\omega\to\omega^{(2)}(\mathbf{k}).\end{aligned}\tag{7}$$

The square roots here are defined such that for $\omega^{(1)}(\mathbf{k})<\omega<\omega^{(2)}(\mathbf{k})$ the imaginary part of the function $P_\alpha(\omega,\mathbf{k})$ is negative.

Let us denote the solution to the dispersion equation

$$[D_\alpha^R(\omega,\mathbf{k})]^{-1}=0\tag{8}$$

by $\omega=\omega_\alpha(\mathbf{k})$. Outside the interval corresponding to the spectrum of the defecton pairs the function $\omega_\alpha(\mathbf{k})$ is, when the phonon self-damping effects are neglected, real. The lower and upper thresholds for defecton-pair production are determined by the equations

$$\omega_\alpha(\mathbf{k})=\omega^{(1)}(\mathbf{k}),\quad\omega_\alpha(\mathbf{k})=\omega^{(2)}(\mathbf{k}).\tag{9}$$

In order to elucidate the nature of the phonon spectrum near an arbitrary point, $\mathbf{k}_1$, lying on the lower-threshold surface $\omega_\alpha(\mathbf{k})=\omega^{(1)}(\mathbf{k})$, let us determine the form of the phonon Green function for $\omega\to\omega^{(1)}(\mathbf{k}_1)$, $\mathbf{k}\to\mathbf{k}_1$. Let us introduce the notation:

$$\Omega=\omega-\omega_\alpha(\mathbf{k}_1),\quad\boldsymbol{\varkappa}=\mathbf{k}-\mathbf{k}_1.\tag{10}$$

Let us assume that in the vicinity of the point $\mathbf{k}_1$ the phonon spectrum is nondegenerate. Then the expansion of the function $[\bar{D}_\alpha^R(\omega,\mathbf{k})]^{-1}$, which does not contain a contribution of the decay into defecton pairs, in powers of the quantities $\Omega$ and $\boldsymbol{\varkappa}$ has an analytic character. Taking into account the formulas (2) and (7), we obtain

$$[D_\alpha^R(\omega,\mathbf{k})]^{-1}=\frac{1}{a}[\Omega-\mathbf{c}\boldsymbol{\varkappa}-b(\mathbf{v}\boldsymbol{\varkappa}-\Omega)^{1/2}],\qquad\omega\to\omega_\alpha(\mathbf{k}_1),\quad\mathbf{k}\to\mathbf{k}_1.\tag{11}$$

Here $a$ and $\mathbf{c}$ are parameters arising from the expansion of the analytic part of the inverse of the Green function and the quantity

$$\mathbf{v}=\left.\frac{\partial\omega^{(1)}(\mathbf{k})}{\partial\mathbf{k}}\right|_{\mathbf{k}=\mathbf{k}_1}$$

corresponds to the group velocity of the defectons; $\mathbf{c}$ is of the order of the phonon velocity unrenormalized by the interaction with the defectons. The parameters $a$, $b$, and $\mathbf{c}$ depend on $\alpha$ and $\mathbf{k}_1$. For brevity of notation, we shall omit the designation of this dependence. Because $A_\alpha>0$ in the formula (7), the signs of the quantities $a$ and $b$ in (11) coincide. In the following section we shall show that the quantity $a$ is positive; therefore, $a$, $b>0$.

The substitution of the expression (11) into (8) leads to a dispersion equation for the phonon spectrum in the neighborhood corresponding to the lower threshold for the point $\mathbf{k}_1$:

$$\Omega-\mathbf{c}\boldsymbol{\varkappa}-b(\mathbf{v}\boldsymbol{\varkappa}-\Omega)^{1/2}=0.\tag{12}$$

The solution to this equation corresponding to the condition $\Omega=0$ for $\boldsymbol{\varkappa}=0$ has the form

$$\Omega\cong\mathbf{v}\boldsymbol{\varkappa}-(\mathbf{c}-\mathbf{v},\boldsymbol{\varkappa})^2/b^2.\tag{13}$$

the frequency determined by it satisfies the inequality $\Omega<\mathbf{v}\boldsymbol{\varkappa}$, i.e., $\omega<\omega^{(1)}(\mathbf{k})$, and, consequently, the solution (13) corresponds to the prethreshold frequency region. We can convince ourselves by a direct verification that the found root of Eq. (12) applies only for $(\mathbf{c}-\mathbf{v},\boldsymbol{\varkappa})<0$. In the region $(\mathbf{c}-\mathbf{v},\boldsymbol{\varkappa})>0$, however, Eq. (12) does not have solutions—real or complex—in the vincinity of the point $\boldsymbol{\varkappa}=0$. Thus, the phonon spectrum breaks off at the threshold point $\boldsymbol{\varkappa}=0$. As can be seen from (13), the phonon group velocity then tends, as we approach the threshold, to the defecton velocity $\mathbf{v}$. The situation arising here is similar to the situation that arose in the analysis of the properties of the excitations of superfluid helium near the end of the spectrum in Pitaevskiĭ's paper.[6]

Analysis of the phonon spectrum near the upper threshold for decay into a defecton pair leads to the conclusion that the solution to the equation analogous to (12) has, near an arbitrary threshold point, $\mathbf{k}_2$, the form

$$\Omega'\cong\mathbf{v}'\boldsymbol{\varkappa}'+(\mathbf{c}'-\mathbf{v}',\boldsymbol{\varkappa}')^2/b'^2,\tag{14}$$

where $\Omega' = \omega - \omega_\alpha(\mathbf{k}_2)$, $\varkappa' = \mathbf{k} - \mathbf{k}_2$, and the parameters $\mathbf{v}'$, $\mathbf{c}'$, $b'$ have the same origin as the parameters $\mathbf{v}$, $\mathbf{c}$, $b$ in the lower-threshold case. The solution (14) corresponds to $\Omega > \mathbf{v}'\varkappa'$, i.e., $\omega > \omega^{(2)}(\mathbf{k})$, and is valid for $(\mathbf{c}' - \mathbf{v}', \varkappa') > 0$. In the region $(\mathbf{c}' - \mathbf{v}', \varkappa') < 0$, however, the dispersion equation does not possess roots.

It is worth noting that in the case when the component of the defecton velocity along the direction of the wave vector $\mathbf{k}$ is negative (i.e., when $\mathbf{nv} < 0$, $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$), the component of the phonon group velocity $\mathbf{n}\,\partial\omega_\alpha(\mathbf{k})/\partial\mathbf{k}$ along the same direction, being positive in the region of small values of the vector $\mathbf{k}$, goes, according to (13), through zero as we approach the lower threshold. In other words, in this case the phonon-spectrum curve goes through a maximum. It is evident that at points sufficiently far from the lower threshold on the low-frequency side and from the upper threshold on the high-frequency side the role of the effects of the interaction with the defectons diminishes, and the phonon group velocities tend to the corresponding—to them—unrenormalized values. At the threshold points, however, they are comparable to the defecton group velocities, which, according to the available experimental data, is significantly lower than the phonon group velocities.

Let us recall that the conclusion about the breaking off of the phonon spectrum at the lower threshold and its regeneration at the upper threshold was arrived at on the basis of an expansion of the function $[D_\alpha^R(\omega, \mathbf{k})]^{-1}$ in the immediate vicinities of the threshold points. In order to understand the physical meaning of the disappearance of the poles of the $D$ function and elucidate the possibility of the regeneration of the phonon spectrum in the frequency range $\omega^{(1)}(\mathbf{k}) < \omega < \omega^{(2)}(\mathbf{k})$ far from the threshold values of $\omega$, let us consider in greater detail the case of weak coupling, namely, the case when the parameters $b^2$ and $b'^2$, which have the dimension of frequency, are small compared to the width of the defecton spectrum:

$$b^2,\ b'^2 \ll \omega^{(2)}(\mathbf{k}) - \omega^{(1)}(\mathbf{k}). \tag{15}$$

In this case the expressions (7) and (11) and Eq. (12) have a region of applicability wider than that of the solutions (13) and (14), which correspond to a power series expansion in the quantities $(\mathbf{c} - \mathbf{v}, \varkappa)/b^2$ and $(\mathbf{c}' - \mathbf{v}', \varkappa')/b'^2$.

As follows from the formulas, (7), for the polarization operator, the two-valued function $D_\alpha^{(*)}(\omega, \mathbf{k})$ of the complex frequency $\omega$ can be set in correspondence with the function $D_\alpha^R(\omega, \mathbf{k})$. In the upper and lower half-planes of the first sheet of the Riemann surface the function $D_\alpha^{(*)}(\omega, \mathbf{k})$ coincides with the analytic continuations of the retarded, $D_\alpha^R(\omega, \mathbf{k})$, and advanced, $D_\alpha^A(\omega, \mathbf{k})$, phonon Green functions. The first and second sheets are connected by a branch cut along the segment $\omega^{(1)}(\mathbf{k})$, $\omega^{(2)}(\mathbf{k})$ of the real axis. In analyzing the mechanism underlying the disappearance of the poles of the $D^R$ function, it is useful to consider the poles of the function $D_\alpha^{(*)}(\omega, \mathbf{k})$ in the near-threshold frequency region. At values of $\omega$ close to $\omega^{(1)}(\mathbf{k})$, they are determined by the equation

$$\Omega - \mathbf{c}\varkappa \mp b(\mathbf{v}\varkappa - \Omega)^{1/2} = 0,$$

which follows from (12), and which has the solution

$$\Omega_{1,2} = \mathbf{c}\varkappa - \frac{b^2}{2} \pm \frac{b^2}{2}\left[1 - 4\frac{(\mathbf{c}-\mathbf{v}, \varkappa)}{b^2}\right]^{1/2}. \tag{16}$$

For $(\mathbf{c} - \mathbf{v}, \varkappa) < 0$, the first root, $\Omega_1$, corresponds to the first sheet of the function $D_\alpha^{(*)}(\omega, \mathbf{k})$. For $(\mathbf{c} - \mathbf{v}, \varkappa)^2/b^4 \ll 1$, it goes over into the expression (13). The second root, $\Omega_2$, corresponds to a pole of $D_\alpha^{(*)}(\omega, \mathbf{k})$ on the second sheet of the Riemann surface. For $(\mathbf{c} - \mathbf{v}, \varkappa) = 0$, the first pole $\Omega_1$ crosses to the second sheet, where the pole $\Omega_2$ also continues to remain. For $(\mathbf{c} - \mathbf{v}, \varkappa) < b^2/4$, both of them are real. At the points $(\mathbf{c} - \mathbf{v}, \varkappa) = b^2/4$ the poles $\Omega_1$ and $\Omega_2$ merge and become complex when $(\mathbf{c} - \mathbf{v}, \varkappa) > b^2/4$, the two poles being then complex conjugates. If $(\mathbf{c} - \mathbf{v}, \varkappa) < b^2/4$, then the real parts of the two roots satisfy the condition $\mathrm{Re}(\mathbf{v}\varkappa - \Omega) > 0$, and the analytic continuation of the function $D_\alpha^R(\omega, \mathbf{k})$ from the real axis into the lower half-plane along the shortest path does not contain them as its poles. The situation changes abruptly when $(\mathbf{c} - \mathbf{v}, \varkappa) = b^2/2$. If $(\mathbf{c} - \mathbf{v}, \varkappa) > b^2/2$, then the pole

$$\Omega_2 = \mathbf{c}\varkappa - \frac{b^2}{2} - i\frac{b^2}{2}\left[4\frac{(\mathbf{c}-\mathbf{v}, \varkappa)}{b^2} - 1\right]^{1/2} \tag{17}$$

turns out to be in a region of the second sheet which can be reached by an analytic continuation of the function $D_\alpha^R(\omega, \mathbf{k})$ from the real axis into the lower-plane along the shortest path. The poles of the function $D_\alpha^{(*)}(\omega, \mathbf{k})$ have, in the case of $\omega$ close to $\omega^{(2)}(\mathbf{k})$, properties similar to those just considered.

In order to elucidate the physical meaning of the poles of the function $D_\alpha^{(*)}(\omega, \mathbf{k})$, let us use the method proposed in[7] (see also[4]). If at the initial moment of time the crystal was in the one-phonon excited state, then the amplitude of the probability for finding it in the same state at the moment of time $t$ is, in accordance with[7,4], determined by the integral

$$F_\alpha(\mathbf{k}, t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} D_\alpha(\omega, \mathbf{k}), \tag{18}$$

where $D_\alpha(\omega, \mathbf{k})$ is the causal Green function figuring in the relation (1). Using the properties of the functions $D_\alpha$, $D_\alpha^R$, $D_\alpha^A$, and $D_\alpha^{(*)}$, let us move the integration contour in (18) into the lower half-plane of the complex frequency, as shown in Fig. 2. The sections of the contour marked by the continuous and dashed lines correspond respectively to the first and second sheet of the function $D_\alpha^{(*)}(\omega, \mathbf{k})$. If we neglect the self-damping of the phonons, then the integral of the function $(2\pi)^{-1} \times (D_\alpha^R(\omega, \mathbf{k}) - D_\alpha^A(\omega, \mathbf{k}))e^{-i\omega t}$ along the vertical section $\mathrm{Re}\,\omega = 0$ vanishes (see[7,4]). Therefore, this section of the contour has been omitted in the figure. The points $A$, $B$, and $C$ denote the poles of the function $D_\alpha^{(*)}(\omega, \mathbf{k})$

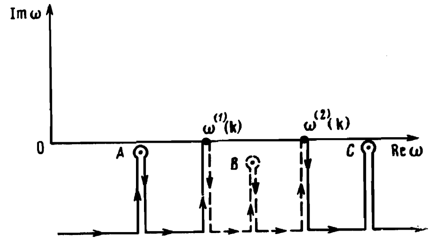

FIG. 2.

that are encountered in the path of the integration contour when it is shifted into the lower half-plane. Indeed, according to the analysis carried out above, for a given value of the wave vector the integration contour can encompass not more than one of the poles $A$, $B$, and $C$. For $(\mathbf{c}-\mathbf{v}, \varkappa)<0$ this pole is of the type $A$; for $(\mathbf{c}-\mathbf{v}, \varkappa)>b^2/2$, $(\mathbf{c}'-\mathbf{v}', \varkappa')<-b'^2/2$, it is a pole of the type $B$, while for $(\mathbf{c}'-\mathbf{v}', \varkappa')>0$ it is of the type $C$. In the intervals $0<(\mathbf{c}-\mathbf{v}, \varkappa)<b^2/2$ and $-b'^2/2<(\mathbf{c}'-\mathbf{v}', \varkappa')<0$, not one of the poles of the function $D_\alpha^{(*)}(\omega, \mathbf{k})$ falls on the integration contour.

Clearly, the poles under consideration can correspond to weakly damped elementary excitations of the phonon type only in the case when the contribution from their circumvention to the integral (18) exceeds the contributions from the integration along the vertical sections $\mathrm{Re}\,\omega=\omega^{(1)}(\mathbf{k})$, $\omega^{(2)}(\mathbf{k})$. The latter contributions decrease in time according to a power law, in contrast to the exponential decrease of the pole contributions. For $\omega_\alpha(\mathbf{k})<\omega^{(1)}(\mathbf{k})$, or $\omega_\alpha(\mathbf{k})>\omega^{(2)}(\mathbf{k})$, their contribution turns out to be negligible when the self-damping of the phonons is neglected. If, on the other hand, the wave vector $\mathbf{k}$ lies in the space between the surfaces $\omega_\alpha(\mathbf{k})=\omega^{(1)}(\mathbf{k})$ and $\omega_\alpha(\mathbf{k})=\omega^{(2)}(\mathbf{k})$, then the pole contribution to the integral (18) will, when the imaginary part of a type-$B$ pole, (17), is taken into account, predominate only upon the fulfilment of the inequalities $|(\mathbf{c}-\mathbf{v}, \varkappa)|\gg b^2$, $|(\mathbf{c}'-\mathbf{v}', \varkappa')|\gg b'^2$, which, with allowance for the formula (17), implies

$$\omega-\omega^{(1)}(\mathbf{k})\gg b^2,\quad \omega^{(2)}(\mathbf{k})-\omega\gg b'^2. \tag{19}$$

The satisfaction of these inequalities is possible only when the conditions (15) are fulfilled. Thus, only in the case of a sufficiently weak interaction, satisfying the inequalities (15), can phonons in the continuum region for the free defecton pairs exist as weakly-damped excitations at frequencies determined by the conditions (19). If, on the other hand, the interaction is not weak, then the pole contribution to the integral (18) is comparable to, or less than, the contributions from the integration along the segments $\mathrm{Re}\,\omega=\omega^{(1)}(\mathbf{k})$, $\omega^{(2)}(\mathbf{k})$, and phonons as elementary excitations are absent at wave vectors lying in the space between the surfaces $\omega_\alpha(\mathbf{k})=\omega^{(1)}(\mathbf{k})$ and $\omega_\alpha(\mathbf{k})=\omega^{(2)}(\mathbf{k})$. The integrals along the vertical segments $\mathrm{Re}\,\omega=\omega^{(1)}(\mathbf{k})$ and $\mathrm{Re}\,\omega=\omega^{(2)}(\mathbf{k})$ clearly correspond to the excitation of the defecton pairs.

Thus far, we have assumed that the spectrum of the defecton pairs reduces to the free-pair continuum. In reality, however, as was indicated above, bound defecton pairs can, according to[2], exist in solid helium. Let us denote by $\varepsilon_l(\mathbf{k})$ the unrenormalized—by the dynamical interaction with the phonons—energy of the $l$-th branch of the bound state of the pairs. The correction, due to this branch, to the spectral density $\rho_\alpha(\omega, \mathbf{k})$, of the phonons for $\omega>0$ is equal to

$$\rho_\alpha{}^l(\omega, \mathbf{k})=N_\alpha{}^l(\mathbf{k})\,\delta(\omega-\varepsilon_l(\mathbf{k})),\quad N_\alpha{}^l(\mathbf{k})>0.$$

The corresponding contribution to the polarization operator for $\omega\sim\varepsilon_l(\mathbf{k})$ has, according to (3), the form

$$P_\alpha{}^l(\omega, \mathbf{k})=\frac{N_\alpha{}^l(\mathbf{k})}{\omega-\varepsilon_l(\mathbf{k})+i\delta},\quad \delta\to+0.$$

The dispersion equation (8) with allowance for the correction to the polarization operator $P_\alpha^l(\omega, \mathbf{k})$ in the expression, (2), for the Green function gives the spectrum of two interacting branches of the phonon and bound-defecton-pair excitations. The general properties of the solution to (8) in the wave-vector region where the branches under consideration could, in the absence of interaction, intersect are, as usual, determined by the principle of nonintersectability of terms of the same symmetry.[3] Since by the very nature of the bound states $\varepsilon_l(\mathbf{k})<\omega^{(1)}(\mathbf{k})$, their presence can in no way affect the above-considered properties of the phonon spectrum in the imediate vicinity of the lower threshold.

Figure 3 shows a schematic representation of the spectrum of the phonons of the $\alpha$-th branch and of the defecton pairs along some given direction, $\mathbf{n}$, of the wave vector $\mathbf{k}=|\mathbf{k}|\mathbf{n}$, $\mathbf{n}^2=1$. For simplicity, we show only one branch of the bound defecton pairs. The hatched part of the figure corresponds to the continuum for the free defecton pairs; the phonon spectrum is absent from the region of this continuum if the interaction is not weak. The dashed lines represent the spectrum of the phonons and the bound defecton pairs in the absence of any interaction between them.

## 3. THE DYNAMIC FORM FACTOR

One of the most important quantities characterizing a crystal is the dynamic form factor $S(\omega, \mathbf{k})$:

$$\begin{aligned}S(\omega,\mathbf{k})&=\int dt\, e^{i\omega t}S(\mathbf{k},t),\\ S(\mathbf{k},t)&=\langle\hat{n}_\mathbf{k}(t)\hat{n}_{-\mathbf{k}}(0)\rangle.\end{aligned}\tag{20}$$

Here $\hat{n}_\mathbf{k}(t)=\int d\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}}\hat{n}(\mathbf{r}, t)$ is the Fourier transform of the Heisenberg particle-number-density operator and the symbol $\langle\circ\circ\circ\rangle$ denotes averaging over the statistical ensemble. The relation[8]

$$S(\omega,\mathbf{k})=-\frac{1}{\pi}\left[1-\exp\left(-\frac{\omega}{T}\right)\right]\mathrm{Im}\,R(\omega,\mathbf{k}) \tag{21}$$

relates the dynamic form factor to the density-density correlation function defined by the equalities

$$\begin{aligned}R(\omega,\mathbf{k})&=\int dt\, e^{i\omega t}R(\mathbf{k},t),\\ R(\mathbf{k},t)&=-i\langle[\hat{n}_\mathbf{k}(t),\hat{n}_{-\mathbf{k}}(0)]\rangle\theta(t),\end{aligned}\tag{22}$$

where $[,]$ is a commutator and $\theta(t)$ is the step function. Since the quantity $R(\omega, \mathbf{k})$ can be expressed in terms of the two-particle, equal-argument (pairwise) Green function $G^{II}$,[4] while the latter is linearly related to the vertex part $\Gamma$, the quantity $R(\omega, \mathbf{k})$ with allowance for the equality (1) can be written in the form

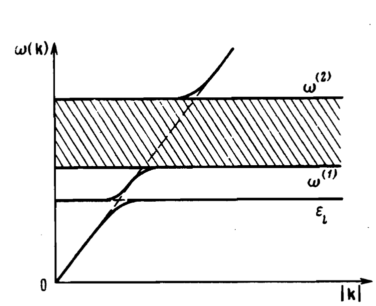


FIG. 3.

$$R(\omega, \mathbf{k}) = \tilde{R}(\omega, \mathbf{k}) + \sum_\alpha B_\alpha(\omega, \mathbf{k}) D_\alpha^R(\omega, \mathbf{k}) B_\alpha^{(+)}(\omega, \mathbf{k}). \quad (23)$$

In contrast to the retarded phonon Green function $D_\alpha^R(\omega, \mathbf{k})$, the functions $B_\alpha(\omega, \mathbf{k})$, $B_\alpha^{(+)}(\omega, \mathbf{k})$, and $\tilde{R}(\omega, \mathbf{k})$ do not contain one-phonon poles, on account of which their frequency dependence is determined by the contribution of only the many-phonon and defecton excitations. Notice that the argument $\mathbf{k}$ of the functions $S(\omega, \mathbf{k})$ and $R(\omega, \mathbf{k})$ can assume arbitrary values, including those that extend beyond the limits of the first Brillouin zone. In the latter case the function $D_\alpha^R(\omega, \mathbf{k})$, like the excitations determined by it, should be understood in the scheme of the extended zones of the vector $\mathbf{k}$.

Assuming the state of the crystal to be invariant under time reversal, we can show that

$$B_\alpha^{(+)}(\omega, \mathbf{k}) = B_\alpha(\omega, \mathbf{k}).$$

Taking this equality into account, we can, on the basis of (23), write the imaginary part of the function $R(\omega, \mathbf{k})$ in the form

$$\operatorname{Im} R(\omega, \mathbf{k}) = \operatorname{Im} \tilde{R}(\omega, \mathbf{k}) + \sum_\alpha \operatorname{Im} D_\alpha^R(\omega, \mathbf{k}) \operatorname{Re} B_\alpha^2(\omega, \mathbf{k}) + \sum_\alpha \operatorname{Re} D_\alpha^R(\omega, \mathbf{k}) \operatorname{Im} B_\alpha^2(\omega, \mathbf{k}). \quad (24)$$

In separating out the one-phonon contribution from this expression, we should bear in mind that, while the frequency widths of the peaks corresponding to the contributions of the many-phonon and defecton excitations correspond to the energy widths of the spectrum of these excitations, the frequency width of the one-phonon contribution is determined by the phonon damping. It is natural that the separation of the phonon peak can be carried out only in the case when the phonon damping is small compared to the indicated spectral widths. When this condition is not fulfilled, it should be assumed that there is no one-phonon contribution to the dynamic form factor. This means, in particular, that if the phonon-defecton interaction is not weak, then a one-phonon contribution to the dynamic form factor does not, in accordance with the results of the preceding section, exist in the frequency interval corresponding to the free-defecton spectrum.

Let us consider the influence of the phonon-defecton interaction on the intensity of the phonon peak in the wave-vector region where the phonon damping is weak. In this case the separation of the one-phonon contribution $S_\alpha^{\text{ph}}(\omega, \mathbf{k})$ from the dynamic form factor does not meet with any difficulties. It is clearly determined by the sum of the second and third terms on the right-hand side of the formula (24) with the phonon Green function replaced by the corresponding pole expression. Let us define the integrated intensity of the one-phonon contribution to the dynamic form factor of a crystal by the equality

$$I_\alpha(\mathbf{k}) = \int_{\omega_<^\alpha(\mathbf{k})}^{\omega_>^\alpha(\mathbf{k})} d\omega\, S_\alpha^{\text{ph}}(\omega, \mathbf{k}). \quad (25)$$

The limits of the integration here have been chosen such that their half-sum $\frac{1}{2}(\omega_>^\alpha(\mathbf{k}) + \omega_<^\alpha(\mathbf{k}))$ corresponds to the center of the phonon peak, while their difference satisfies the inequalities

$$\gamma_\alpha(\mathbf{k}) \ll \omega_>^\alpha(\mathbf{k}) - \omega_<^\alpha(\mathbf{k}) \ll \Delta\omega(\mathbf{k}) \quad (26)$$

($\gamma_\alpha(\mathbf{k})$ and $\Delta\omega(\mathbf{k})$ are respectively the frequency widths of the phonon peak and the multiphonon- and defecton-excitation background). The possibility of the fulfilment of the condition (26) is ensured by the requirement that the phonon damping be weak. With allowance for what has been said above about the determination of the quantity $S_\alpha^{\text{ph}}(\omega, \mathbf{k})$ we shall have on the basis of the relations (21), (24), and (25) the following expression for the integrated intensity of the one-phonon peak:

$$I_\alpha(\mathbf{k}) = \left[1 - \exp\left(-\frac{\omega_\alpha(\mathbf{k})}{T}\right)\right]^{-1} \operatorname{Re} B_\alpha^2(\omega_\alpha(\mathbf{k}), \mathbf{k}) Q_\alpha(\mathbf{k}), \quad (27)$$

$$Q_\alpha(\mathbf{k}) = \left[\frac{\partial}{\partial\omega} \operatorname{Re}(D_\alpha^R(\omega, \mathbf{k}))^{-1}\right]^{-1} \Big|_{\omega=\omega_\alpha(\mathbf{k})}. \quad (28)$$

In computing $I_\alpha(\mathbf{k})$, we neglected the quantities of relative order of magnitude $\gamma_\alpha(\mathbf{k})(\omega_>^\alpha(\mathbf{k}) - \omega_<^\alpha(\mathbf{k}))^{-1}$, $(\omega_>^\alpha(\mathbf{k}) - \omega_<^\alpha(\mathbf{k}))/\Delta\omega(\mathbf{k})$. Within this degree of accuracy, the intensity $I_\alpha(\mathbf{k})$ does not, in particular, contain any contribution from the third term of the expression (24), a term which affects the shape of the one-phonon peak.

According to (11), the residue at the pole of the phonon Green function (28) in the immediate vicinity of the lower threshold has the form

$$Q_\alpha(\mathbf{k}) = 2a\,|(\mathbf{c} - \mathbf{v}, \boldsymbol{\varkappa})|/b^2. \quad (29)$$

Notice that the weakness of the phonon damping simultaneously implies also the smallness of $\operatorname{Im} B_\alpha(\omega, \mathbf{k})$ in comparison with $\operatorname{Re} B_\alpha(\omega, \mathbf{k})$. Therefore, $\operatorname{Re} B_\alpha^2(\omega, \mathbf{k}) > 0$. Since, by virtue of its definition, the dynamic form factor is positive, the formulas (27) and (29) lead to the conclusion that $a > 0$. It follows from the equalities (27) and (29) that the integrated intensity of the one-phonon peak, $I_\alpha(\mathbf{k})$, vanishes at the threshold points.

In order to understand its behavior as the lower threshold is approached from the side of wave vectors corresponding to low frequencies, let us note that, like that of the polarization operator $P_\alpha^R(\omega, \mathbf{k})$, the diagram expansion of the quantity $B_\alpha(\omega, \mathbf{k})$, defined by the equality (23), will contain a diagram of the type shown in Fig. 1. As can be seen from the expansion (3), the contribution of this diagram increases in absolute value as the threshold is approached. This circumstance creates the prerequisite that, before vanishing at the threshold itself, the function $I_\alpha(\mathbf{k})$ should go through a maximum as the threshold is approached. Since, on the other hand, the total contribution of the diagrams for $B_\alpha(\omega, \mathbf{k})$ that do not have defecton pairs as intermediate states decreases with increasing $|\mathbf{k}|$ (this is, in particular, corroborated by the expression for the function $I_\alpha(\mathbf{k})$ in the harmonic approximation, an expression which is proportional to the Debye–Waller factor), the indicated maximum should, if it occurs, be preceded by a minimum of the function $I_\alpha(\mathbf{k})$. In such a situation, instead of the classical monotonically decreasing dependence on $|\mathbf{k}|$, the integrated intensity $I_\alpha(\mathbf{k})$ has an oscillatory character in the prethreshold region. Since the characteristic frequency range in which a substantial change occurs in the threshold part of the function $B_\alpha(\omega, \mathbf{k})$ is the frequency width, $\Delta\omega_d = \omega^{(2)}(\mathbf{k}) - \omega^{(1)}(\mathbf{k})$, of the continuum spectrum of the defecton pairs, the "period" of oscillation of the function $I_\alpha(\mathbf{k})$, i.e., the distance between its minimum and its zero at the threshold, is qualitatively determined by a quantity of the order of $\Delta\omega_d$.

A similar analysis leads to the conclusion that the function $I_\alpha(\mathbf{k})$ vanishes also at the points of the upper threshold and that it will certainly go through a maximum as we move away from this threshold on the higher-frequency side. As to the minimum, there is, in contrast to the region near the lower threshold, no basis for it, since, as we move away from the upper thresholds on the side of higher frequencies and larger wave vectors, both the threshold and the nonthreshold parts of the quantity $B_\alpha(\omega, \mathbf{k})$ decrease.

A necessary condition for the nonmonotonic dependence of the integrated intensity, $I_\alpha(\mathbf{k})$, of the one-phonon peak on the absolute value of the wave vector is, naturally, that the threshold diagram shown in Fig. 1 make a sufficiently large contribution to the function $B_\alpha(\omega, \mathbf{k})$. The magnitude of this contribution is determined by both the amplitude of the defecton-phonon interaction and the overlap of the defecton wave functions, the dependence on this overlap being expressed by the integral

$$J(\mathbf{p}, \mathbf{k}) = \int_{v_c} d\mathbf{r}\, e^{-i\mathbf{k}\mathbf{r}} \psi_{+,\mathbf{p}}(\mathbf{r}) \psi_{-,\mathbf{k}-\mathbf{p}}(\mathbf{r}). \tag{30}$$

Here $\psi_{+,\mathbf{p}}(\mathbf{r})$ and $\psi_{-,\mathbf{k}-\mathbf{p}}(\mathbf{r})$ are the wave functions of the particle- and hole-type defectons with quasi-momenta $\mathbf{p}$ and $\mathbf{k}-\mathbf{p}$ respectively. The integration is restricted to the volume, $v_c$, of the unit cell of the crystal. Let us emphasize that the possible nonmonotonic dependence of the integral $J(\mathbf{p}, \mathbf{k})$ on $\mathbf{k}$ can serve as the factor that promotes the oscillations in the intensity, $I_\alpha(\mathbf{k})$, of the one-phonon peak. For example, the possibility of the integral $J(\mathbf{p}, \mathbf{k})$ having a maximum in the second or third Brillouin zone is not to be excluded. In this case the oscillations of the function $I_\alpha(\mathbf{k})$ are more probable at $\mathbf{k}$ values lying outside the limits of the first Brillouin zone. Finally, among the conditions necessary for the oscillatory behavior of the function $I_\alpha(\mathbf{k})$ must be included the requirement that the lower threshold for decay into a defecton pair for a given branch of the phonon spectrum be accessible. Such a condition is not realized in classical crystals.

Osgood *et al.*,[9] as well as Minkiewicz *et al.*,[10] have carried out, using the method of inelastic neutron scattering, an experimental investigation of the properties of the dynamic form factor of solid $He^4$ in the bcc and hcp phases. According to their results, the dependence of the intensity of the one-phonon peak on the momentum transfer is nonmonotonic. Thus, in[9] anomalous intensities were observed for scattering by phonons that involves momentum transfers larger than the reciprocal-lattice vector, and in[10] oscillations in the intensity of the one-phonon peak were observed at large momentum transfers. The indicated anomalies were connected in[9] with energy transfers of the order of $\omega \sim 1.4$ MeV. This assertion does not contradict the results of the theroretical analysis carried out above, according to which the oscillations of the quantity $I_\alpha(\mathbf{k})$ are caused by threshold phenomena. In contrast to[9], the conclusion is drawn in[10] that the anomalous behavior of the intensity of the one-phonon scattering of neutrons depends not on the energy, but on the momentum transfer. This conclusion is, however, at variance with the fact that, according to the data of the same paper, there are no anomalies in the case of the low-frequency transverse phonon mode $T_1$ in the [011] direction.

In both[9] and[10] the experimental data were analyzed on the basis of the Ambegaokar–Convway–Baym sum rule[11]:

$$\int_{-\infty}^{\infty} d\omega\, \omega S_\alpha^{\mathrm{ph}}(\omega, \mathbf{k}) = \frac{(\mathbf{k}\xi_\alpha)^2 |d(\mathbf{k})|^2}{2m}, \tag{31}$$

which establishes a connection between the one-phonon part of the dynamic form factor $S_\alpha^{\mathrm{ph}}(\omega, \mathbf{k})$ and the Debye–Waller factor $|d(\mathbf{k})|^2$, which determines the equilibrium distribution of the number density of the atoms in the unit cell of a crystal ($m$ is the mass of the atom and $\varepsilon_\alpha$ is the polarization vector of the $\alpha$-th phonon branch). Notice first of all that the relation (31) does not settle the question of how to extract the one-phonon contribution from the dynamic form factor. Furthermore, in deriving the formula (31),[11] it was assumed that any excited state of a crystal can be completely characterized by a displacement of its atoms relative to the lattice sites. Such a description can in no way take into account the possibility of real and virtual processes of defecton-pair production and, in fact, assumes the energies corresponding to these processes to be infinite. Therefore, the sum rule (31) is applicable only to classical crystals, in which the minimum energy necessary for the production of a defecton pair is significantly higher than the maximum phonon energy.

Notice, finally, that the resolving power in the measurements of the one-phonon scattering of neutrons for large momentum transfers is not high.[10] Thus, the comparison of the theory with experiment requires a more detailed experimental investigation of the dynamic form factor and more adequate processing of the data.

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# Dopplerons and the Gantmakher–Kaner effect in tungsten plates with atomically pure surfaces

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The surface resistance of thin single crystal tungsten plates in the radio-frequency range is investigated as a function of the intensity of a magnetic field directed along the normal to the surface. The measurements were made on samples whose surfaces had been purified in a high vacuum ($10^{-11}$ mm Hg) or covered with a monomolecular impurity film. The smooth variation of the real $R(H)$ and imaginary $X(H)$ components of the impedance due to the anomalous skin effect, as well as the component $R^{\rm osc}(H)$ that oscillates with respect to magnetic field and is due to the Doppler-shifted cyclotron resonance, are studied. When the magnetic field is oriented along a fourfold symmetry axis (⟨100⟩), resonance sets in for carriers lying on the inflection of the hole octahedron of the tungsten Fermi surface. Resonance-induced dispersion of the imaginary part of the nonlocal conductivity produces weakly damped circulary polarized transverse waves dopplerons {L. M. Fisher, O. V. Konstantinov, *et al.*, Zh. Eksp. Teor. Fiz. **60**, 759 (1971) and **63**, 224 (1972) [Sov. Phys. JETP **33**, 410 (1971) and **36**, 118 (1973)]; R. G. Chambers and V. G. Skobov, J. Phys. **1**, 202 (1971); D. S. Falk *et al.*, Phys. Rev. B1, 406 (1970)}. It is found that the amplitude of the doppleron signal depends on the state of the sample and increases with increasing crystal purity. The observed changes are attributed to the increase of the specular-reflection coefficient of the resonant electrons. If the magnetic field is not normal to the plate surface, the doppleron wave undergoes collisionless magnetic Landau damping and the signal is reduced to a value comparable with the amplitude of the Gantmakher–Kaner "wave." Purification of the surface (and the ensuing increase specularity) decreases the doppleron amplitude further and produces interference peaks in the Gantmakher–Kaner "waves." The effect of surface currents due to the static skin effect, on the smoothly varying components $R(H)$ and $X(H)$ and the oscillating component $R^{\rm osc}(H)$ of the surface resistance of the plate is discussed.

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## INTRODUCTION

Cyclotron damping that is shifted by the Doppler effect exerts an appreciable influence on the high-frequency properties of degenerate metals. Direct and well-investigated consequences of Doppler-shifted cyclotron resonance (DSCR) are the limited helicon-existence regions in weak magnetic fields $H_0$ and the onset of the Gantmakher–Kaner ratio-frequency size effect in a normal magnetic field. A recent object of investigation has been one more manifestation of DSCR, namely, low-frequency electromagnetic waves found to exist in anisotropic specially compensated conductors and to be produced if the magnetic field is near the cyclotron-damping threshold. These waves were named dopplerons.[1–4] The doppleron modes are transverse circularly polarized waves with length quite close to the extremal displacement $u_{\rm max}$ of some selected group of carriers outside their cyclotron period. For this reason the doppleron wave is outwardly similar to the Gantmakher–Kaner "wave,"[5] which is produced in the same interval of magnetic fields or close to it.

In typical experiments, doppleron waves were excited in tin metallic plates. The magnetic field was oriented along the normal to the sample surface and coincided with a high symmetry axis of the crystal. With changing field $H_0$, the wave length in the metal changed and this led to the appearance of a series of resonant absorption maxima corresponding to satisfaction of the standing-wave conditions in the plate. The impedance oscillations were observed against the background of smooth but much larger changes of the impedance, due to the anomalous skin effect. In compensated metals, these changes turn out to be quite appreciable, since the conductivity of the metal decreases without limit with increasing magnetic field, in proportion to $(\omega_c\tau)^2$, where $\omega_c$ is the cyclotron frequency and $\tau$ is the momentum relaxation time of the electrons.

In the radio-frequency band there exist thus several mechanisms responsible for energy dissipation in the plate: the anomalous skin effect, doppleron waves, and

  