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Absorption of sound in helium II at low temperatures and transverse phonon relaxation

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A linear theory of ultrasonic propagation in helium II at low temperatures is developed for the case in which only phonons are responsible for the kinetic phenomena. The pressure range considered is such that the phonons have a decay spectrum. The ultrasonic frequency ω_s is assumed to satisfy the condition $\tau_\parallel^{-1} \gg \omega_s \gg \tau_\perp^{-1}$, where $\tau_\parallel$ and $\tau_\perp$ are the longitudinal and transverse phonon relaxation times, respectively. Expressions are obtained for the absorption coefficient and for the corrections to the sound velocity in this particular frequency range.

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1. INTRODUCTION

The aim of the present work is the construction of a theory of ultrasonic absorption in He II at low temperatures in which only phonons are responsible for the kinetic phenomena, and rotons are not excited. The theory of acoustic phenomena in He II has been developed in detail in the works of Landau and Khalatnikov,[1] Andreev and Khalatnikov,[2] Khalatnikov and Chernikova,[3] Pethick and Ter Haar[4] (see also[5,6]). Two limiting cases have been studied in these researches: 1) $\omega_s\tau \gg 1$ (ω_s is the ultrasonic frequency, τ is the characteristic relaxation time of thermal phonons), when, in particular, the sound absorption coefficient Γ at $\hbar\omega_s \ll T$ is determined by a formula of the Landau–Rumer type[7] and is proportional to ω_s; 2) $\omega_s\tau \ll 1$, when the propagation and absorption of the sound are described by the equations of hydrodynamics and $\Gamma \sim \omega_s^2$.

The slight difference of the phonon spectrum from a linear spectrum leads to the existence of two characteristic phonon relaxation times in He II: the longitudinal time $\tau_\parallel$, which characterizes the fast relaxation of phonons propagating along the given direction, and the transverse relaxation time $\tau_\perp \gg \tau_\parallel$, which characterizes the slow relaxation of phonons propagating at an angle with respect to one another. Landau and Khalatnikov first pointed out the existence of these two relaxation times.[1] They assumed the phonon spectrum to be undamped. The transverse relaxation time is determined here by four-phonon processes and the corresponding collision operator has a complicated integral character.

In the present work, we want to determine how the absorption is affected by the recently discovered experimental fact[8–11] that the phonon spectrum is a decay spectrum, at not too high pressures.

Two widely differing relaxation times define three characteristic frequency intervals:

$$\omega_s \ll \tau_\perp^{-1}; \quad 2)\ \tau_\perp^{-1} \ll \omega_s \ll \tau_\parallel^{-1}; \quad 3)\ \tau_\parallel^{-1} \ll \omega_s. \tag{1.1}$$

With the help of the analytic theory of transverse phonon relaxation, which was developed by us,[12] we have succeeded in considering the first and the second of these intervals. In the first frequency region, our calculation is equivalent to the calculation of the viscosity coefficient with account of transverse relaxation of the phonons, recently performed by Benin.[13] In the second region, the results obtained in the present work are new.

The possible effect of the decay character of the phonon spectrum on acoustic phenomena in He II has been discussed in great detail by Maris,[10,14] and also by Meier and Beck.[15] The kinetic equation for phonons, the distribution of which is perturbed by the sound wave, has been solved numerically in Refs. 10, 14, and 15. In a recent work of Wehner,[16] an analytic approximation of the transverse relaxation operator was proposed in the form of an integral operator with a Gaussian kernel which, in our view, enables one to get only an order-of-magnitude answer.

The results of our work are formulas for the absorption coefficient and the change in the sound velocity in the second frequency interval: $\tau_{\perp}^{-1} \ll \omega_s \ll \tau_{\parallel}^{-1}$. It turns out that the coefficient Γ has the same frequency and temperature dependence in the second interval as in the third. However, the corresponding constant coefficients in the formulas for Γ differ by a factor of six. A corresponding difference in the coefficients has a comparatively simple semi-quantitative explanation. We shall give it at the end of Sec. 3.

The change in the sound velocity δ_c in this frequency interval is proportional to $\ln \omega_s \tau_{\perp}$. This result thus differs from the corresponding expression of Ref. 4 in the third frequency interval, not only in the coefficient in front of the logarithm, but also in the argument of the logarithm.

Unfortunately, our results are difficult to compare with the existing experimental data for the second frequency interval, since the data are scanty in this region. Therefore further experimental work is necessary in this area.

2. DERIVATION OF THE GENERAL EXPRESSIONS FOR THE ABSORPTION COEFFICIENT AND THE SOUND VELOCITY

We shall start out from the following set of equations, which describe the dynamics of nonequilibrium He II (see the work of Landau[17] and the book of Khalatnikov[18]):

$$\partial\rho/\partial t + \operatorname{div}\mathbf{j} = 0, \tag{2.1}$$

$$\partial\mathbf{v}_s/\partial t + \operatorname{grad}(\mu + v_s^2/2) = 0, \tag{2.2}$$

$$\partial E/\partial t + \operatorname{div}\mathbf{Q} = 0, \tag{2.3}$$

$$\partial j_i/\partial t + \partial\Pi_{ik}/\partial x_k = 0. \tag{2.4}$$

Here $\mathbf{v}_s$ is the superfluid velocity, $\rho = \rho_n + \rho_s$ is the density of He II, $\mathbf{j} = \mathbf{j}_0 + \rho\mathbf{v}_s$ is the mass flux density,

$$E = E_0 + \mathbf{v}_s\mathbf{j}_0 + \rho v_s^2/2 \tag{2.5}$$

is the energy density, $\mathbf{Q}$ is the energy flux density,

$$\Pi_{ij} = \rho_s v_{si} v_{sj} + \rho_n v_{ni} v_{nj} + \delta_{ij} p + \tau_{ij} \tag{2.6}$$

is the momentum flux density ($\mathbf{v}_n$ is the normal velocity, p is the pressure and τ_{ij} is the viscous stress tensor).

Since the nonequilibrium state is considered, the quantities ρ_n and $\mathbf{v}_n$ must be so defined as to go over in the equilibrium case into the thermodynamic values. We set1)

$$\rho_n = \frac{4}{3c_0(\rho_0)} \int \hbar k N \frac{d^3k}{(2\pi)^3}, \tag{2.7}$$

$$\mathbf{j}_0 = \rho_n(\mathbf{v}_n - \mathbf{v}_s) = \int \hbar \mathbf{k} N \frac{d^3k}{(2\pi)^3}, \tag{2.8}$$

where N is the phonon distribution function, ω is the phonon frequency, and $\mathbf{k}$ is the wave vector;

$$c_0^2 = \rho\, d\mu/d\rho \tag{2.9}$$

is the sound velocity at $T=0$, $\mu_0 = d\varepsilon_0/d\rho$ (ε_0 and μ_0 are the energy density and the chemical potential, respectively, of helium at $T=0$).

The following quantities are also expressed in terms of the distribution function N: the energy in a system of coordinates moving with the superfluid velocity $\mathbf{v}_s$:

$$E_0 = \mathscr{E}_0(\rho) + \int \hbar\omega N \frac{d^3k}{(2\pi)^3}, \tag{2.10}$$

the "chemical potential"

$$\mu = \mu_0(\rho) + \int \hbar \frac{\partial\omega}{\partial\rho} N \frac{d^3k}{(2\pi)^3}, \tag{2.11}$$

the energy flux density

$$Q_i = j_i\left(\mu + \frac{v_s^2}{2}\right) + \int \hbar\tilde{\omega} \frac{\partial\tilde{\omega}}{\partial k_i} N \frac{d^3k}{(2\pi)^3}, \quad \tilde{\omega} = \omega + \mathbf{k}\mathbf{v}_s, \tag{2.12}$$

the pressure

$$p = \mu\rho - \mathscr{E}_0(\rho) + \frac{1}{3}\int \hbar k_l \frac{\partial\omega_1}{\partial k_l} N \frac{d^3k}{(2\pi)^3}, \tag{2.13}$$

the viscous stress tensor

$$\tau_{ij} = \int \hbar\left(k_i \frac{\partial\omega_1}{\partial k_j} - \frac{1}{3}\delta_{ij} k_l \frac{\partial\omega_1}{\partial k_l}\right) N \frac{d^3k}{(2\pi)^3}, \tag{2.14}$$

where $\omega_1 = \omega + \mathbf{k}\cdot(\mathbf{v}_s - \mathbf{v}_n)$.

We confine ourselves to the case in which the velocities v_s and v_n are small in comparison with c_0, which allows us to neglect nonlinearities of the hydrodynamic type:

$$v_n/c \ll 1, \quad v_s/c \ll 1. \tag{2.15}$$

Moreover, we assume the temperature T to be so small that

$$\rho_n/\rho \ll 1. \tag{2.16}$$

Making use of the linearity of the phonon spectrum,2) we can transform Eqs. (2.2)–(2.4) to

$$\frac{\partial v_{si}}{\partial t} + \frac{c_0^2}{\rho}\left(1 + \frac{3}{4} w \frac{\rho_n}{\rho}\right)\frac{\partial\rho}{\partial x_i} + \frac{3}{4} u \frac{c_0^2}{\rho}\frac{\partial\rho_n}{\partial x_i} = 0, \tag{2.17}$$

$$\frac{\partial j_i}{\partial t}+c_1^2\frac{\partial\rho}{\partial x_i}+\frac{1}{4}c_0^2(1+3u)\frac{\partial\rho_n}{\partial x_i}+\frac{\partial\tau_{ij}}{\partial x_j}=0. \tag{2.18}$$

$$\frac{3}{4}\frac{\partial\rho_n}{\partial t}+\rho_n\operatorname{div}\mathbf{v}_n+\frac{1}{c_0^2}\tau_{ij}\frac{\partial v_{sj}}{\partial x_i}=0 \tag{2.19}$$

Here

$$c_1^2=c_0^2[1+(u+{}^3/_4w)\rho_n/\rho] \tag{2.20}$$

is the adiabatic sound velocity,

$$u=\frac{\rho}{c_0}\frac{dc_0}{d\rho},\quad w=\frac{\rho^2}{c_0}\frac{d^2c_0}{d\rho^2}. \tag{2.21}$$

Equation (2.14) can be rewritten similarly as

$$\tau_{ij}=\int\left(\varkappa_i\varkappa_j-\frac{1}{3}\delta_{ij}\right)\hbar\omega N\frac{d^3k}{(2\pi)^3}, \tag{2.22}$$

where

$$\varkappa=\mathbf{k}/k$$

By virtue of the inequalities (2.15) we can neglect the variable components in the quantities ρ_n, ρ_s, and τ_{ij}, which enter as coefficients in the derivatives of $\mathbf{v}_n$ and $\mathbf{v}_s$ in Eqs. (2.1) and (2.17)–(2.19). However, the constant parts of these quantities are generally not determined by the thermodynamic formulas, since the constant part of the phonon distribution function in the nonlinear regime can deviate significantly from equilibrium.$^{3)}$

The system (2.1), (2.17)–(2.19) must be supplemented with the equation for the phonon distribution function N, which we write out in the next section.

For the set of Eqs. (2.1), (2.17)–(2.19), we transform to the set of equations for the functions ρ and ρ_n, which are similar to those which describe the propagation of first and second sound in He II in the weakly nonequilibrium case (see the work of Landau[17] and the book of Khalatnikov,[18] Sec. 20):

$$\frac{\partial^2\rho}{\partial t^2}=c_1^2\nabla^2\rho+\frac{1}{4}c_0^2(1+3u)\nabla^2\rho_n+\frac{\partial^2\tau_{ij}}{\partial x_i\partial x_j}, \tag{2.23}$$

$$\frac{\partial^2\rho_n}{\partial t^2}=\frac{4}{3}(1+u)\frac{\rho_n}{\rho}c_0^2\nabla^2\rho+\frac{c_0^2}{3}\nabla^2\rho_n+\frac{4}{3}\frac{\partial^2\tau_{ij}}{\partial x_i\partial x_j}+\frac{4}{3\rho}\tau_{ij}\frac{\partial^2\rho}{\partial x_i\partial x_j}. \tag{2.24}$$

We shall be interested only in the first sound, whose velocity is close to c_0. Then, with the assumed accuracy, we can set $\partial^2\rho_n/\partial t^2=c_0^2\nabla^2\rho_n$ in the second equation. Then, eliminating ρ_n from the given system, we obtain the following equation for ρ:

$$\frac{\partial^2\rho}{\partial t^2}=c_2^2\nabla^2\rho+\frac{3}{2}(1+u)\frac{\partial^2\tau_{ij}}{\partial x_i\partial x_j}+\frac{1+3u}{2\rho}\tau_{ij}\frac{\partial^2\rho}{\partial x_i\partial x_j}, \tag{2.25}$$

where the square of the low-frequency sound velocity in He II[18] is

$$c_2^2=c_1^2+{}^1/_2c_0^2(1+u)(1+3u)\rho_n/\rho. \tag{2.26}$$

We shall further assume that the sound wave is purely sinusoidal and does not contain higher harmonics. (For the linear problem considered below, such an assumption does not need a special justification. In the nonlinear case, the question as to when the nonlinear effects do not lead to the formation of higher harmonics needs special consideration.) Then the following expressions are obtained from (2.25) for the absorption coefficient (in power) Γ and the change in the sound velocity $\delta c = c - c_2$:

$$\Gamma=-\frac{3}{2c_0^2}q(1+u)\operatorname{Im}\frac{\tau_{xx}'\rho'^*}{|\rho'|^2}, \tag{2.27}$$

$$\frac{\delta c}{c}=\frac{3}{4c_0^2}(1+u)\operatorname{Re}\frac{\tau_{xx}'\rho'^*}{|\rho'|^2}+\frac{1+3u}{2\rho c_0^2}\tau_{xx}, \tag{2.28}$$

where the primes denote the alternating parts of the corresponding quantities, which are described in complex form, q is the sound wave vector, and the x axis is its direction of propagation.

As is shown in our paper,[12] the phonon distribution function N is expressed in terms of the effective phonon temperature $\Theta(\varkappa)$, which depends on the direction of $\varkappa$:

$$N=n_0[\hbar ck/\Theta(\varkappa)], \tag{2.29}$$

where n_0 is the Planck function. Substituting (2.29) in (2.22) and integrating, we have

$$\tau_{xx}=\frac{\pi}{360(\hbar c)^3}\int(3\varkappa_x^2-1)\Theta^4\,d\Omega, \tag{2.30}$$

where the integration is carried out over the entire solid angle.

Thus the problem is reduced to setting up and solving the equation for $\Theta(\varkappa)$.

3. CALCULATION OF THE ABSORPTION COEFFICIENT AND THE SOUND VELOCITY

We consider the case $\omega_s\tau_\parallel \ll 1$, in which ω_s is the frequency of the ultrasound, $\tau_\parallel$ is the time of longitudinal relaxation of the phonons. In the hydrodynamic approximation, the interaction of the phonons with the sound is taken into account by the addition of

$$\Delta\omega=(u+\cos\vartheta)ck\frac{\rho'}{\rho}\equiv k\Delta c \tag{3.1}$$

to the phonon frequency.

Multiplying the kinetic equation

$$\frac{\partial N}{\partial t}+\frac{\partial\omega}{\partial k_x}\frac{\partial N}{\partial x}-\frac{\partial\omega}{\partial x}\frac{\partial N}{\partial k_x}=J\{N\} \tag{3.2}$$

by $\hbar c_0k^3dk/(2\pi)^3$ and integrating over k from 0 to ∞, we obtain an equation for the density of the phonon energy in the given direction $\mathscr{E}(\Theta)=(\pi/120)\Theta^4/(\hbar c)^3$:

$$\frac{\partial\mathscr{E}}{\partial t}+c_0\cos\vartheta\frac{\partial\mathscr{E}}{\partial x}-\frac{\partial\Delta c}{\partial x}\left(-4\mathscr{E}\cos\vartheta+\sin\vartheta\frac{\partial\mathscr{E}}{\partial\vartheta}\right)=\left[\frac{\partial\mathscr{E}}{\partial t}\right]_{\text{coll}} \tag{3.3}$$

where ϑ is the angle between the direction of $\varkappa$ and the x axis. The calculation for the right side of (3.3) was carried out in a work of the authors.[12]

We further restrict ourselves to the study of only the linear approximation, in which

$$\Theta=(1+Z)T, \tag{3.4}$$

where T is the mean temperature of the helium and $|Z|\ll 1$.

In the linear approximation, (3.3) takes the form

$$\frac{\partial Z}{\partial t}+c_0\cos\vartheta\frac{\partial Z}{\partial x}+\frac{\partial\Delta c}{\partial x}\cos\vartheta=-\frac{1}{4\tau_\perp}l^2(l^2+2)Z, \tag{3.5}$$

where l^2 is the angular part of the Laplace operator; the expression for the time $\tau_\perp$ is given in Ref. 12.

In (3.5), $\partial Z/\partial t=-i\omega_s Z$, and $\partial Z/\partial x=(iq-\Gamma/2)Z$. The estimates which we give below show that the quantity Γ on the left side of Eq. (3.5) can usually be neglected in comparison with its right side. With the same accuracy we can set the sound velocity equal to c_0 in the first term on the left side. Then (3.5) is rewritten in the form

$$[1-\cos\vartheta+i(4\omega_s\tau_\perp)^{-1}l^2(l^2+2)]Z=(\rho'/\rho)(u+\cos\vartheta)\cos\vartheta. \tag{3.6}$$

By virtue of the symmetry of the problem, Z depends only on ϑ and the given equation must be solved under conditions of the absence of energy and momentum sources at the points $\vartheta=0, T$. These conditions are formulated as follows:

$$\begin{gathered}\sin\vartheta\frac{d}{d\vartheta}\left(\frac{1}{\sin\vartheta}\frac{d}{d\vartheta}\sin\vartheta\frac{d}{d\vartheta}+2\right)Z(\vartheta)\Big|_{\vartheta=0,\pi}=0,\\ \sin^3\vartheta\frac{d}{d\vartheta}\frac{1}{\sin\vartheta}\frac{dZ}{d\vartheta}\Big|_{\vartheta=0,\pi}=0.\end{gathered} \tag{3.7}$$

In the calculation of the viscosity coefficient in Ref. 12, we have assumed that Z does not contain the zeroth and first spherical harmonics, which describe the change in the temperature and the drift velocity of the phonon system as a whole. Here we shall assume that Z represents its complete response to the sound perturbation, i.e., it contains all the spherical harmonics (including the zeroth and first, which, naturally, do not make a contribution to the absorption).

The quantity τ_{xx} (2.30) in the expressions for the absorption coefficient and the corrections to the sound velocity takes the following form in linear approximation:

$$\tau_{xx}=\frac{\pi^2T^4}{45(\hbar c)^3}\int\limits_0(3\cos^2\vartheta-1)Z(\vartheta)\sin\vartheta\,d\vartheta. \tag{3.8}$$

We have succeeded in solving Eq. (3.6) in two limiting cases: 1) $\omega_s\tau_\perp\ll 1$ and 2) $\omega_s\tau_\perp\gg 1$. In the first case, we have the solution $Z(\vartheta)$ in the form of a series in spherical harmonics. It turns out that the coefficients for the zeroth and first spherical harmonics are of zeroth order in the small parameter $\omega_s\tau_\perp$, and the coefficient in the case of the second spherical harmonic, which determines the absorption is first order in $\omega_s\tau_\perp$. In addition, with increase in the numbers of the harmonics, the power to which the parameter $\omega_s\tau_\perp$ is raised also increases. The first three terms of this expansion have the form

$$Z(\vartheta)=\left[\frac{1+u}{2}+\frac{1+3u}{2}\cos\vartheta-i\frac{\omega_s\tau_\perp}{24}(1+u)(1+3\cos 2\vartheta)\right]\frac{\rho'}{\rho}. \tag{3.9}$$

Substituting (3.9) in (3.8), and (3.8) in (2.16), we obtain the well known (see Ref. 18, p. 175, and Ref. 19) expression for the sound absorption coefficient which, as it should in the hydrodynamic limit, is expressed in terms of the viscosity coefficient η:

$$\Gamma=3(1+u)^2\eta\omega_s^2/\rho c_0^3, \tag{3.10}$$

where

$$\eta=\frac{\pi^2}{675}\frac{T^4\tau_\perp}{(\hbar c_0)^3} \tag{3.10a}$$

(see Refs. 12, 13).

At $\omega_s\tau_\perp\gg 1$, the relaxation term in Eq. (3.6) exists only in a small range of angles $\vartheta\ll 1$. Outside this interval,

$$Z(\vartheta)=\frac{\rho'}{\rho}\frac{(u+\cos\vartheta)\cos\vartheta}{1-\cos\vartheta}. \tag{3.11}$$

On the other hand, at $\vartheta\ll 1$, Eq. (3.6) can be simplified by setting $\cos\vartheta=1$ on the right side, putting on the left side

$$\cos\vartheta=1-\frac{\vartheta^2}{2},\quad l^2=\frac{1}{\vartheta}\frac{d}{d\vartheta}\vartheta\frac{d}{d\vartheta}$$

and neglecting the term 2 in comparison with l^2. Introducing the dimensionless variables

$$\xi=2\cdot 6^{-4}\omega_s\tau_\perp\vartheta^6, \tag{3.12}$$

$$Z(\vartheta)=2\cdot 6^{-4/3}(\rho'/\rho)(1+u)(2\omega_s\tau_\perp)^{1/3}\xi^{1/6}z(\xi), \tag{3.13}$$

we get the following equation for $z(\xi)$:

$$[\xi+i(D^2-{}^1/_{36})^2]z(\xi)=\xi^{1/2},\quad D=\xi\frac{d}{d\xi}. \tag{3.14}$$

Let $(\omega_s\tau_\perp)^{-1/6}\ll\vartheta_1\ll 1$. Then $Z(\vartheta)$ at $\vartheta>\vartheta_1$ is determined by the expression (3.11) and at $\vartheta<\vartheta_1$ by the expression (3.13). As $\xi\to 0$, the function $z(\xi)$ should satisfy the boundary conditions

$$\begin{gathered}\xi^{-1/6}(D-{}^1/_6)(D+{}^1/_6)^2z(\xi)=0,\\ \xi^{1/6}(D^2-{}^1/_{36})z(\xi)=0.\end{gathered} \tag{3.15}$$

In the region $\vartheta\sim\vartheta_1$ the functions (3.11) and (3.13) should be identical. For this to happen, it suffices for $z(\xi)$ to have the asymptotic form as

$$z(\xi)\sim\xi^{-1/6}. \tag{3.16}$$

Substituting (3.11) and (3.13) in (3.8), we get

$$\tau_{xx}=\frac{2\pi^2T^4}{135(\hbar c_0)^3}\frac{\rho'}{\rho}(1+u)I, \tag{3.17}$$

where

$$I=\ln\frac{8\omega_s\tau_\perp}{81}-9+\ln\frac{1}{\xi_1}+\int\limits_0^{\xi_1}z(\xi)\frac{d\xi}{\xi^{1/2}}, \tag{3.18}$$

$\xi_1=2\cdot 6^{-4}\omega_s\tau_\perp\vartheta_1^6\gg 1$. For the calculation of I, we need to solve Eq. (3.14). The solution of the equation is given in the Appendix, and it turns out that

$$I=\ln(72\omega_s\tau_\perp)+4C-9-i\frac{\pi}{2}, \tag{3.19}$$

where C is Euler's constant.

The following final expressions are obtained from (2.27) and (2.28) (with account of (3.17) and (3.19)) for the absorption coefficient and for the correction to the sound velocity:

$$\Gamma=\frac{\pi^3}{90}\frac{(1+u)^2T^4}{\rho\hbar^3c_0^5}q, \tag{3.20}$$

$$\frac{\delta c}{c_0}=\frac{\pi^2}{90}\frac{(1+u)^2T^4}{\rho\hbar^3c_0^5}[\ln(72\omega_s\tau_\perp)+4C-9]. \tag{3.21}$$

The difference of the coefficient 1/90 in Eqs. (3.20), (3.21) from what would be obtained under the assumption that the collision operator in the kinetic equation is written down with the help of a constant which has the meaning of a relaxation time can be made clear in the following way.

The integral (3.8) is logarithmically large. This enables us to determine both the coefficient before $\ln\omega_s\tau_\perp$ and its imaginary part without solving the Eq. (3.6). It can be shown that for this it suffices to replace the operator l^2 in (3.6) by $1/\vartheta^2$. Then

$$I=3\int\limits_0\frac{\vartheta\,d\vartheta}{\vartheta^2/2+i/\omega_s\tau_\perp\vartheta^4}. \tag{3.22}$$

If the transverse relaxation operator were a constant, then the factor ϑ^{-4} in the second component in the denominator would be absent. We would then have

$$I=3(\ln\omega_s\tau_\perp-i\pi/2). \tag{3.23}$$

In fact,

$$I=\ln\omega_s\tau_\perp-i\pi/2. \tag{3.24}$$

We must now take into consideration the fact that Eq. (3.23) is calculated under the assumption that the dispersion of the thermal phonons is less than their damping. In the opposite limiting case, the imaginary part of the integral is twice as large (hence the sixfold increase discussed above), and the quantity under the logarithm in the real part is proportional to the dispersion of the phonons.

4. CONCLUSION

We now elucidate the limits of applicability of the given theory. To be able to use the transverse phonon relaxation operator, it is necessary that the characteristic range of angles

$$\vartheta\sim(\omega_s\tau_\perp)^{-1/6},$$

in which the phonon distribution function is significantly perturbed be larger than the quantity $\varepsilon_T^{1/2}$—the characteristic value of the angle between the wave vectors of the phonons taking part in a single elementary act:

$$\xi_T(\omega_s\tau_\perp)^{1/3}\ll1. \tag{4.1}$$

Here $\varepsilon(k)>0$ is a function which characterizes the departure of the phonon dispersion law

$$\omega(k)=c_0k[1+\xi(k)]$$

from linearity, and $\varepsilon_T=\xi(T/\hbar c)$; the fundamental approximation, which is at the basis of the theory of transverse phonon relaxation,[12] has the form

$$\xi_T\ll1. \tag{4.2}$$

We rewrite the inequality (4.1) in the following way:

$$(\omega_s\tau_\parallel)^{1/3}(\tau_\perp/\tau_\parallel)^{1/3}\xi_T\ll1. \tag{4.3}$$

However, we now consider a special interval, $\omega_s\tau_\parallel\ll1$. Further, as has been pointed out in Ref. 12, $\tau_\perp/\tau_\parallel\sim\xi_T^{-2}$. It is then clear that the inequality (4.1) is always satisfied.$^{4)}$

Another inequality on which the theory is based calls for the possibility of neglecting the quantity Γ in comparison with the sum of the remaining components in the left side of (3.5), which is equal to $q\vartheta^2$ in order of magnitude. It has the form

$$\Gamma/q\ll\vartheta^2\sim(\omega_s\tau_\perp)^{-1/3}, \tag{4.4}$$

and can be rewritten as

$$(\omega_s\tau_\parallel)^{1/3}(\tau_\perp/\tau_\parallel)^{1/3}\Gamma/q\ll1. \tag{4.5}$$

The condition that the inequality (4.5) be satisfied for all frequencies ω_0 of the allowed frequency interval is

$$\frac{1}{\xi_T^{2/3}}\frac{(1+u)^2T^4}{\rho\hbar^3c_0^5}\ll1. \tag{4.6}$$

This inequality thus imposes an upper bound on the allowed temperature interval.

If the quadratic approximation were valid for the function ξ_T, $\xi_T=\gamma(T/\hbar c)^2$, then the inequality (4.6) would be rewritten in the form

$$T\ll[\gamma^2\rho c/(1+u)^6\hbar]^{1/8}T_0, \tag{4.7}$$

where $T_0=(\rho\hbar^3c_0^5)^{1/4}$ is the characteristic temperature, equal approximately to 7 °K. In the case $\gamma=(4-10)\cdot10^{-17}$ cm^2, the dimensionless coefficient on the right side of (4.7) is of the order of unity. In this case, the inequality (4.7) would impose no limitation on the allowed temperature interval.

The fact is, however, that the function ξ_T increases with increase in T more slowly than according to a quadratic law at temperatures of the order of 0.5 °K; particularly at sufficiently high temperatures. Therefore, if the frequency of the ultrasound is sufficiently high, cases are possible in which the inequality (4.5) is violated.

We now make clear the conditions under which it is possible to neglect the nonlinear corrections to the linear theory constructed above. First we must estimate at what sound intensities S_{ac} the inequality

$$|Z|\ll1 \tag{4.8}$$

is satisfied. Making use of the expression (3.13) for

$Z(\vartheta)$ and assuming the function $z(\xi)$ in this expression to be of the order of unity, we obtain

$$S_{ac}\ll\rho c_0{}^3(1+u)^{-2}(\omega_s\tau_\perp)^{-2/3}. \tag{4.9}$$

However, as we can easily establish, this condition is not a very strong limitation on the sound intensity. The inequality (4.8) is a condition on the alternating part (with frequency ω_s) of the phonon distribution function. At the same time, the nonequilibrium contribution to the constant part of the phonon distribution function must satisfy a similar condition (the disequilibrium is due to the sound absorption).

The range of angles in which the sound perturbs the constant part of the distribution function is a quantity of order $(\omega_s\tau_\perp)^{-1/2}$. However, in the process of sound propagation, a broadening of this angular interval occurs due to processes of transverse relaxation. If the "length" of the sound absorption

$$\frac{1}{\Gamma}\sim\frac{\rho\hbar^3c_0{}^5}{(1+u)^2T^4}\frac{1}{q} \tag{4.10}$$

is much greater than the distance $c_0\tau_\perp$ over which the transverse relaxation takes place, then the nonlinear effects become important when the sound intensity is of the order of $\rho_nc_0^3$—the maximum energy density which can be transferred by the phonon component at the given temperature, i.e., the condition of applicability of the linear theory is5)

$$S_{ac}\ll\rho_nc_0{}^3\quad\text{at}\quad c_0\Gamma\tau_\perp\ll1. \tag{4.11}$$

If the sound absorption length is small in comparison with the length of transverse relaxation, then the "increase" of the normal component of the ultrasound takes place in a small interval of angles $\Delta\vartheta$, which is determined from the condition $c_0\Gamma\tau_\perp(\Delta\vartheta)^4\sim1$. Consequently, the criterion of applicability of the linear theory is

$$S_{ac}\ll\rho_nc_0{}^3(\Delta\vartheta)^2\sim\frac{\rho_nc_0{}^3}{(c_0\Gamma\tau_\perp)^{1/2}}\quad\text{at}\quad c_0\Gamma\tau_\perp\gg1. \tag{4.12}$$

In conclusion, we shall now point out the range of temperatures and frequencies in which the effects considered here can be observed. Using a very rough estimate for $1/\tau_\parallel$,

$$\frac{1}{\tau_\parallel}\approx10^2\frac{(1+u)^2T^5}{\rho\hbar^4c^5}, \tag{4.13}$$

we find that at $T=0.6$ °K (i.e., at the upper boundary of the allowable temperature range) $\tau_\parallel\approx10^{-10}$. Thus, at $T=0.6$ °K, the inequality $\omega_s\ll10^{10}$ sec^{-1} should be satisfied. The range of allowed frequencies decreases rapidly upon decrease in the temperature. Thus, at $T=0.3$ °K, we have $\omega_s\ll10^8$ sec^{-1}.

The lower boundary of the considered frequency interval is determined by the condition $\omega_s\gg1/\tau_\perp$. To estimate the time at high temperatures, for example, at $T=0.6°$, it is not convenient to use the expression (4.3) of Ref. 12, since the values of k of importance in the corresponding integral are those for which the quadratic approximation of the function $\xi(k)$ is unsuitable. For this purpose, however, we can use the experimental value of the viscosity coefficient η, which is connected with $\tau_\perp$ by the relation (3.10a). Using the data of the experiments of Whitworth,[20] we have at $T=0.6$ °K and at saturated vapor pressure of helium, $\tau_\perp^{-1}=1.2\times10^5$ sec^{-1}. Thus the lower frequency boundary of applicability of the theory at $T=0.6$ °K is given by the inequality $\omega_s\ll10^5$ sec^{-1}. Upon a decrease in the temperature (or an increase in the pressure) $\tau_\perp^{-1}$ falls off and the corresponding inequalities do not impose real limitations on the allowed frequency interval.

From what has been said above follows the curious conclusion that in the region of temperatures where the kinetics in He II are determined only by individual phonons, the region of applicability of the hydrodynamic theory of sound absorption (in which $\Gamma\sim\omega_s^2$) is almost absent. This means that, of the three frequency intervals enumerated in the Appendix, only the second and third can actually be realized. In both intervals, the absorption coefficient is a linear function of frequency, but the absorption is six times larger in the third interval. The two linear parts are joined by a transition region (the numerical calculations of which have been carried out in the work of Maris for several temperatures[14]).

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APPENDIX: DETERMINATION OF THE FUNCTION $z(\xi)$

An equation of the type (3.14) has been studied by Barnes[21] and in greater detail by Meyer.[22] However, they have considered only the nondegenerate case, in which the differential operator on the left side has the form $\Gamma_i(D-\rho_i)$, where all the parameters ρ_i are different. For our purposes, a more detailed asymptotic form is necessary than in Refs. 21, 22. Here we shall study the degenerate equation and the asymptotic behavior of its solutions.

The calculations have a very simple and symmetric form, if we consider the more general equation

$$[-x+(D^2-\rho^2)^2]z(\rho,x)=e^{-3\pi i/4}x^{1/2} \tag{A.1}$$

in place of (3.14). This equation has the boundary conditions

$$x^\rho(D^2-\rho^2)z(\rho,x)\to0\quad\text{at}\quad x\to0, \tag{A.2}$$
$$x^{-\rho}(D-\rho)(D+\rho)^2z(\rho,x)\to0\quad\text{at}\quad x\to0. \tag{A.3}$$

In our case, $\rho=1/6$ and $x=\xi e^{i\pi/2}$.

It is not difficult to obtain a particular solution of (A.1) in the form of the following series:

$$z_0(\rho,x)=e^{-3\pi i/4}\frac{\pi^2}{\cos^2\pi\rho}\sum_{n=0}^{\infty}\frac{x^{n+1/2}}{\Gamma^2(n+3/2-\rho)\Gamma^2(n+3/2+\rho)}. \tag{A.4}$$

The linearly independent solutions of the corresponding homogeneous equation have the form $z_1(\pm\rho,x)$ and $z_2(\pm\rho,x)$, where

$$z_1(\rho, x)=\sum_{n=0}^{\infty}\frac{x^{n+\rho}}{\Gamma^2(n+1+2\rho)\,(n!)^2}, \qquad (A.5)$$

$$z_2(\rho, x)=-z_1(\rho, x)\ln x+2\sum_{n=0}^{\infty}\frac{\psi(n+1)+\psi(n+1+2\rho)}{\Gamma^2(n+1+2\rho)\,(n!)^2}x^{n+\rho}, \qquad (A.6)$$

and where $\psi(x)$ is the logarithmic derivative of the Γ function.

It is not difficult to establish the fact that the functions $z_2(\pm\rho, x)$ do not satisfy the boundary conditions at $x=0$. Thus the problem is to construct such a linear combination of the solutions $z_0(\rho, x)$, $z_1(\rho, x)$ and $z_1(-\rho, x)$ which would have the asymptotic behavior (3.16) as $x\to\infty$.

To obtain the asymptotic representations, it is convenient to write out these series in the form of Barnes contour integrals (we shall not require the integral for the function z_2 here, but we write it out, keeping in mind its possible future application):

$$z_0(\rho, x)=e^{-3\pi i/4}\frac{\pi^3}{\cos^2\pi\rho}\frac{1}{2\pi i}\int_L\frac{x^s\,\mathrm{tg}\,\pi s\,ds}{\Gamma^2(s+1-\rho)\Gamma^2(s+1+\rho)}, \qquad (A.7)$$

$$z_1(\rho, x)=\frac{1}{2\pi i}\int_L\frac{\Gamma(-s+\rho)\cos\pi(s-\rho)}{\Gamma^2(s+\rho+1)\Gamma(s-\rho+1)}x^s\,ds, \qquad (A.8)$$

$$z_2(\rho, x)=\frac{1}{2\pi i}\int_L\frac{\Gamma^2(-s+\rho)}{\Gamma^2(s+\rho+1)}x^s\,ds \qquad (A.9)$$

where the contour L begins at the point $+\infty$, goes around the origin in the negative direction and returns to the initial point. It is also convenient to introduce the function

$$\Phi(\rho, x)=\Phi(-\rho, x)=\frac{1}{2\pi i}\int_L\Gamma^2(-s-\rho)\Gamma^2(-s+\rho)x^s\,ds \qquad (A.10)$$

which has a simple asymptotic behavior. That is, by the method of stationary phase we can obtain

$$\Phi(\rho,x)\sim(2\pi^3)^{1/2}x^{-3/8}\exp(-4x^{1/4}) \quad \text{at} \quad |\arg x|<4\pi. \qquad (A.11)$$

With the help of the relation $\Gamma(s)\Gamma(1-s)=\pi/\sin\pi s$, we can write the integrals (A.7)–(A.9) in the form

$$z_0(\rho, x)=\frac{e^{-3\pi i/4}}{\pi\cos^2\pi\rho}\left\{\frac{\cos^4\pi\rho}{2\pi i}\int_L\Gamma^2(-s-\rho)\Gamma^2(-s+\rho)\,\mathrm{tg}\,\pi s\cdot x^s\,ds\right.$$
$$-i\frac{1-4\cos^2\pi\rho}{8}[\Phi(\rho, xe^{2\pi i})-\Phi(\rho, xe^{-2\pi i})]$$
$$\left.-\frac{i}{16}[\Phi(\rho, xe^{4\pi i})-\Phi(\rho, xe^{-4\pi i})]\right\}, \qquad (A.12)$$

$$z_1(\rho, x)=-\frac{i}{16\pi^3}[\Phi(\rho, xe^{4\pi i})-\Phi(\rho, xe^{-4\pi i})-2e^{-2\pi i\rho}\Phi(\rho, xe^{2\pi i})$$
$$+2e^{2\pi i\rho}\Phi(\rho, xe^{-2\pi i})-2i\sin 4\pi\rho\,\Phi(\rho, x)], \qquad (A.13)$$

$$z_2(\rho, x)=-\frac{1}{4\pi^2}[e^{2\pi i\rho}\Phi(\rho, xe^{2\pi i})+e^{-2\pi i\rho}\Phi(\rho, xe^{-2\pi i})-2\Phi(\rho, x)]. \qquad (A.14)$$

Not all the functions Φ in Eqs. (A.12) and (A.13) satisfy the condition $|\arg x|<4\pi$. In order to obtain the asymptotic representation of the function $\Phi(\rho, xe^4\pi^1)$ we can use the relation

$$z_2(\rho, xe^{2\pi in})=e^{2\pi in\rho}[z_2(\rho, x)-2\pi inz_1(\rho, x)], \qquad (A.15)$$

which is a consequence of (A.5) and (A.6). We note that the formulas (A.13)–(A.15) enable us to obtain an asymptotic series for $z_1(\pm\rho, x)$ and $z_2(\pm\rho, x)$ at all values of $\arg x$ and thus to determine all the Stokes coefficients (cf. Ref. 23).

Since $\arg x=\pi/2$ in our case, the functions $\Phi(\rho, xe^{2\pi i})$ and $\Phi(\rho, xe^{-4\pi i})$ increase exponentially as $|x|\to\infty$, while the functions $\Phi(\rho, x)$ and $\Phi(\rho, xe^{-2\pi i})$ fall off exponentially. In the first term in (A.12), the integral can be carried out over a contour parallel to the imaginary axis. Shifting the contour into the left halfplane, we obtain the following asymptotic representation for the integral which enters into this term:

$$\frac{1}{2\pi i}\int_L\Gamma^2(-s-\rho)\Gamma^2(-s+\rho)\,\mathrm{tg}\,\pi s\cdot x^s\,ds$$
$$=-\frac{1}{\pi}\sum_{n=1}^{N}\frac{\Gamma^2(n-1/2+\rho)\Gamma^2(n-1/2-\rho)}{x^{n-1/2}}+O(x^{-N-1/2}). \qquad (A.16)$$

Using these results, we find that the solution of Eq. (A.1) having no terms that increase exponentially as $|x|\to\infty$ is given by

$$z(\rho, x)=z_0(\rho, x)-e^{-3\pi i/4}\frac{\pi^2}{2\cos^2\pi\rho}[z_1(\rho, x)+z_1(-\rho, x)]$$
$$-e^{-3\pi i/4}\frac{i\pi^2}{\sin 2\pi\rho}[z_1(\rho, x)-z_1(-\rho, x)], \qquad (A.17)$$

or

$$z(\rho, x)=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}g(s)\xi^s\,ds, \qquad (A.18)$$

where

$$g(s)=\frac{1}{\pi}\Gamma^2(-s+\rho)\Gamma^2(-s-\rho)\left[\cos^2\pi\rho\,\mathrm{tg}\,\pi s e^{i(\pi s/2-3\pi/4)}\right.$$
$$\left.+\frac{i}{2}\cos 2\pi\rho e^{i(\pi s/2-3\pi/4)}-\frac{i}{2}e^{-i(3\pi s/2+3\pi/4)}\right]. \qquad (A.19)$$

It is not difficult to demonstrate that the principal term of the asymptotic expansion of $z(\rho, x)$ has the form (3.16) as $|x|\to\infty$.

To calculate L it is convenient to introduce the function

$$\tilde{g}(s)=g(s)+\frac{\Gamma(-s+\rho)}{\Gamma(1/2+\rho)}\frac{1}{s+1/2}. \qquad (A.20)$$

Then

$$z(\rho, \xi)=\tilde{z}(\rho, \xi)+\frac{\xi^{-1/2}}{\Gamma(1/2+\rho)}\int_0^{\xi}e^{-\eta}\eta^{\rho-1/2}\,d\eta, \qquad (A.21)$$

where

$$\tilde{z}(\rho, \xi)=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\tilde{g}(s)\xi^s\,ds. \qquad (A.22)$$

At $\xi_1\gg 1$, we have

$$\int_0^{\xi_1}z(\xi)\xi^{-1/2}\,d\xi\approx\int_0^{\infty}\tilde{z}(\rho, \xi)\xi^{-1/2}\,d\xi+\ln\xi_1-\psi(\rho+1/2). \qquad (A.23)$$

The first integral in (A.23) is calculated with the help of the Mellin inversion formula and is equal to $\tilde{g}(-\frac{1}{2})$.

As a result,

$$I=\ln\frac{8\omega_{\cdot}\tau_{\perp}}{81}-9-2\psi\left(\rho+\frac{1}{2}\right)-2\psi\left(\rho-\frac{1}{2}\right)-i\frac{\pi}{2}, \qquad (A.24)$$

which transforms into (3.19) at $\rho=\frac{1}{6}$.

[1] The velocity c_0 is a variable quantity: it depends on the density ρ. The definition (2.7), however, contains the constant quantity $c_0(\rho_0)$, where ρ_0 is the mean density of HeII. With such a definition, small departures $\delta\rho_n$ from the equilibrium value of ρ_n are proportional to the small departure δs from the equilibrium value of the entropy density S, and $\delta\rho_n/\rho_n = (4/3)\delta S/S$.

[2] The deviation of the phonon spectrum from linear is taken into account only in the calculation of the transverse relaxation operator for the phonons.

[3] In particular, the last term in (2.19) can differ from zero for this reason. In this case, we mean by τ_{ij} a constant quantity (in contrast, for example, to the nonlinear term in Eq. (18.7), of Ref. 18, where τ_{ij} denotes the complete viscous stress tensor).

[4] To make estimates of such a type more precise, we must keep it in mind that the characteristic value ε_T of the quantity $\varepsilon(k)$, which determines the rate of transverse relaxation in the system of phonons, must, as noted in Ref. 12, be of the order of $\xi(AT/\hbar c)$, where A is a number of the order of 10.

[5] If this condition is violated, then the constant part v_n can be of the order of c, which violates the limits of applicability of the initial equations (2.17)–(2.19) themselves.

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The effect of a strong ac field on the dynamical nuclear resonance frequency shift in magnetically ordered crystals

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We use the example of strong stationary NMR saturation in magnetically ordered crystals to show that due to the change in the interaction between spins the magnitude of the dynamical frequency shift (DFS) depends not only on the average value of the z-component of the nuclear spin, but also on the relation between the detuning and the amplitude of the strong ac field. We show also that for an appropriate choice of the parameters of that field there may be no DFS.

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It is well known[1,2] that the hyperfine interaction between nuclear and electron spins in magnetically ordered materials leads to the occurrence of a strong indirect interaction between the nuclear spins (Suhl–Nakamura interaction[3]). This interaction determines the first and second moments of the NMR line. We have $M_1 \gg \sqrt{M_2}$ even at high temperatures ($\hbar\omega/kT \ll 1$).[4] The first moment of the line or, alternatively, the dynamical frequency shift (DFS) depends linearly on the average value of the z-component of the nuclear spin (in temperature terms: on the reciprocal temperature of the nuclear spin system). This fact leads to a change in the resonance frequency for NMR saturation.

So far[5,6] the effect of a strong ac field on the magnitude of the DFS has been considered solely in terms of the change in the z-component of the magnetization. However, if the ac field is sufficiently large, it changes