gation described here can serve as an example of the use of CRNO as a new method for studying the vicinities of singular points of non-singly-connected Fermi surface of polyvalent metals. The results of similar investigations can be used to verify Fermi-surface calculations.

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[1)]The use of the SiO_2 coating in the growth of the indium crystals was proposed by Yu. V. Sharvin.

[1]A. P. Volodin, M. S. Khaĭkin, and V. S. Édel'man, Pis'ma Zh. Eksp. Teor. Fiz. 17, 491 (1973) [JETP Lett. 17, 353 (1973).

[2]M. A. Lur'e and V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. 66, 240 (1974) [Sov. Phys. JETP 39, 114 (1974)]; Fiz. Nizkikh Temp. Sov. J. Low Temp. Phys.

[3]A. P. Volodin, M. S. Khaĭkin, and V. S. Édel'man, Zh. Eksp. Teor. Fiz. 65, 2105 (1973) [Sov. Phys. JETP 38, 1052 (1974)].

[4]M. S. Khaĭkin, S. M. Cheremisin, and V. S. Édel'man, Pribl Tekh. Eksp. No. 4, 225 (1970).

[5]R. G. Chambers, Proc. Phys. Soc. Lond. 86, 305 (1965).

[6]M. S. Khaĭkin, Prib. Tekh. Éksp. No. 3, 95 (1961).

[7]A. P. Volodin, M. S. Khaĭkin, and V. S. Édel'man, Prib. Tekh. Eksp. No. 6, 142 (1974).

[8]R. T. Mina and M. S. Khaĭkin, Zh. Eksp. Teor. Fiz. 51, 62 (1966) [Sov. Phys. JETP 24, 42 (1967)].

[9]M. S. Khaĭkin, Zh. Eksp. Teor. Fiz. 43, 59 (1962) [Sov. Phys. JETP 16, 42 (1963)].

[10]V. F. Gantmakher and I. P. Krylov, Zh. Eksp. Teor. Fiz. 49, 1054 (1965) [Sov. Phys. JETP 22, 734 (1966)].

[11]V. S. Édel'man, E. P. Vol'skiĭ, and M. S. Khaikin, Prib. Tekh. Eksp. No. 3, 179 (1966).

[12]R. T. Mina and M. S. Khaĭkin, Pis'ma Zh. Eksp. Teor. Fiz. 1, No. 2, 34, (1965) [JETP Lett. 1, 60 (1965)].

[13]I. M. Lifshitz, M. Ya. Azbel', and M. I. Kaganov, Elektroniya teoriya metallov (Electron Theory of Metals), Nauka, 1971 [Plenum 1973].

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Permittivity anomaly in metal-dielectric transitions. Theory and simulation

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We formulate some dynamic percolation theory problems whose solution may be of interest in studies of the dynamic properties of disordered systems such as strongly developed doped semiconductors, ferroelectric semiconductors with a diffuse phase transition, island films, and other objects with a nonuniform conductivity and (or) inhomogeneous permittivity. Some of the problems are simulated by means of networks consisting of capacitors and resistors with randomly broken bonds and nodes. The results obtained by such simulation and also the theory of effective media developed in the paper, as well as some considerations based on percolation theory, indicate that the static permittivity of the sample should become infinite for the metal-dielectric transition. This result is used to interpret qualitatively the "polarization catastrophe" observed in the metal-dielectric transition in *n*-silicon.

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1. INTRODUCTION

The methods of percolation theory are widely used to describe the static conductivity of disordered systems, static hopping conductivity, etc. (see, e.g., [1,2]). In this paper we formulate percolation-theory problems whose solutions can be of interest to the study of the dynamic properties of disordered systems, strongly-doped semiconductors, ferroelectric semiconductors, and other physics objects with inhomogeneous conductivity and (or) inhomogeneous dielectric constant. Some of these problems will be investigated experimentally by simulation with a network made up of capacitors and resistors. For an analytic description of the results we use the effective-medium theory, which is a generalization of the theory developed in[3-5].

We use the results, in particular, to interpret experimental data that point to a "polarization catastrophe" (wherein the static dielectric constant becomes infinite) in the metal-dielectric transition in n-type silicon.[6] The results obtained for the flat (two-dimensional) case can be used to interpret the results of experiments with so called island films (see, e.g., [7]).

2. FORMULATION OF PROBLEM

We recall first the formulation of one of the standard problems of percolation theory. Consider an infinite network made up of identical resistors. We open in random fashion a fraction x of the nodes (sites) of this network (the site problem, see, e.g. [1]). Then at a certain critical value $x = x_c$ the network becomes open ("percolation" of the current over the network ceases). Percolation-theory methods make it possible to estab-

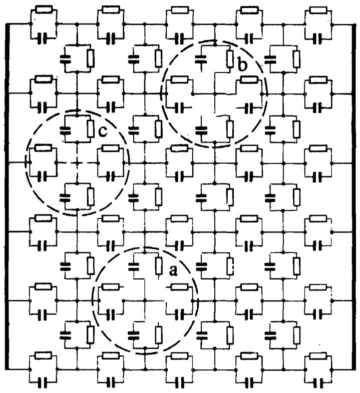

FIG. 1. Small-scale equivalent circuit of the lattice used in the simulation. a) Node in which the resistors are disconnected; b) node in which the capacitors are disconnected; c) node in which both the capacitors and the resistors are disconnected.

lish the topology of the resistance network (the topology of the "infinite cluster"), the behavior of the conductivity, the correlation radius, and other physical quantities near the percolation threshold x_c. By analog simulation and computer calculation it is shown that as $(x-x_c)\to 0$ the conductivity and all other physical quantities depend on $(x-x_c)$ in power-law fashion. Just as in the theory of phase transitions, it is customary to call the exponents of the different physical quantities the critical exponents. The critical exponents seem to depend little on the type of percolation-theory problem (i.e., on whether the bonds or the nodes are missing, or whether some continual models are considered[1,2]). The relations between the critical exponents are analogous to the similarity relations in the theory of phase transitions.[8,9]

To describe the dynamic properties, i.e., the frequency dispersion of the different disordered physical systems in terms of percolation theory, we propose to examine various types of lattice problems of percolation theory, assuming that each element of the lattice (each bond in the lattice) is a network comprising a capacitor, inductor, and active resistor connected in parallel. (A similar generalization can be proposed also for the continual problems of percolation theory.) By specifying different missing fractions x_l, x_c, and x_R of inductors, capacitors, and resistors and also by choosing different relations between the values of L, C, and R, it is possible to simulate various frequency properties of different disordered systems.

We confine ourselves in this paper to a network in which each bond consists of a resistor and capacitor connected in parallel. Such a system, as will be shown below, can be used, for example, for the interpretation of the "polarization catastrophe" in the metal–dielectric transition. Other possible applications of this problem can be the investigation of the phase transitions in ferroelectric semiconductors.

3. EXPERIMENTAL PROCEDURE

We used for the analog situation a square lattice consisting of 512 bonds (256 sites). Each bond comprises a resistor $R=300$ and a capacitor $C=0.5\ \mu F$ connected in parallel. Each of the two opposite sides of this lattice was connected to a metallic busbar (Fig. 1). The capacitance and conductivity of the lattice between these busbars was measured with an alternating current bridge in the range of frequencies from $f_L=150$ Hz to $f_H=8$ kHz. The chosen frequency band satisfies the condition $2\pi f_L \ll 1/RC \ll 2\pi f_H$, satisfaction of which, as we shall show later on, makes it possible to trace the frequency dispersion of the conductivity and of the capacitance of the lattice in all the investigated cases.

Three problems were simulated experimentally.

1. The capacitor connections were not disturbed, so that the capacitors formed a regular rectangular network. The resistances at the nodes were removed in random fashion with a specified removal probability x for each node, as shown in Fig. 1 (node a). The value of x was varied from 0 to 1 in steps of 0.1. For each value we measured the frequency dependences of the capacitance and of the conductivity of the lattice. Such a problem simulates the frequency characteristics of an inhomogeneous system consisting of components having different conductances but identical real parts of the dielectric constant.

2. This problem differs from the first in that the connections between the resistors are left alone, and the capacitors are disconnected from the nodes with probability x in random fashion, as shown in Fig. 1 (node b). This problem simulates the frequency characteristics of an inhomogeneous system consisting of components with different real parts of the dielectric constant and with identical conductivities.

3. The connections of the capacitors and of the resistors at the nodes were broken in random fashion. Thus, in this problem the lattice contains nodes of type a, nodes of type b, as well as nodes from which both the resistors and the capacitors are disconnected (nodes of type c in Fig. 1).

In all three problems, the procedure of determining the coordinates of the nodes in which the connections were broken was standard: a random-number generator and a BÉSM-4 computer were used to set in correspondence with each node a random number y having an equal probability density distribution from zero to unity. The node was broken at $y \leqslant x$ and remained intact in the opposite case.

4. FORMULAS OF THE EFFECTIVE-MEDIUM THEORY

To interpret the experimental results we have developed an effective-medium theory, which is a generalization of the theory developed in[3-5]. The results of the calculation are as follows:

$$\varepsilon_m/\varepsilon_1=[(A^2+B^2)^{1/2}+A]^{1/2}+e,$$
$$\sigma_m/\sigma_1=[(A^2+B^2)^{1/2}-A]^{1/2}f+s. \qquad (1)$$

Here ε_m and σ_m are the effective values of the dielectric constant and of the conductance of the medium made up of components with dielectric constants ε_1 and ε_2 and with conductances σ_1 and σ_2,

$$A=\frac{1}{2}\left[e^2-\frac{s^2}{F^2}-\beta k-\beta\frac{g}{F^2}\right],$$

$$B=\frac{1}{F}\left[es+\frac{\beta}{2}(k+g)\right],\quad e=\frac{1}{2}[(1+\beta)p-\beta]-\frac{k}{2}[(1+\beta)p-1],$$

$$s=\frac{1}{2}[(1+\beta)p-\beta]-\frac{g}{2}[(1+\beta)p-1],$$

$k=\varepsilon_2/\varepsilon_1$, $g=\sigma_2/\sigma_1$, and $F=\varepsilon_1\omega/\sigma_1$ is the dimensionless frequency.

The quantities β and p depend on the type of problem. For a problem with a continuous medium having randomly distributed components we have $\beta=\frac{1}{2}$ for the three dimensional case and $\beta=1$ for the two dimentional case, while p is equal to the fraction of the volume occupied by the phase with the parameters ε_1 and σ_1.

For the lattice bond problem (i.e., for a lattice in which bonds with parallel-connected resistor R_1 and capacitor C_1 alternate randomly with bonds having parameters R_2 and C_2) we have

$$\beta=2/(z-2),\tag{2}$$

where z is the number of bonds that are connected to a single node (i.e., the number of nearest neighbors). In this case β is independent of the dimensionality of space. For the bond problem, the quantity p is equal to the probability in which a connection with the parameters R_1 and C_1 is encountered. In these cases it is necessary to assume in the foregoing formulas

$$F=\omega C_1R_1,\quad k=C_2/C_1,\quad g=R_1/R_2,\quad \sigma_m/\sigma_1=s_m,\quad \varepsilon_m/\varepsilon_1=e_m,\tag{3}$$

where s_m and e_m are the ratios of the conductance and capacitance of the lattice to the conductance and capacitance of a lattice made up entirely of the elements C_1 and R_1. (In the planar case (square lattice) the conductance and the capacitance of the lattice made up of the elements R_1 and C_1 are equal respectively to R_1^{-1} and C_1. In the three-dimensional case (cubic lattice), the conductivity and capacitance of such a lattice are equal to nR_1^{-1} and nC_1, where n is the number of elements in the edge of the cube.)

Formulas (3) are valid for the lattice site problem.[1)] For the site problem of interest to us, in a two-dimensional square lattice,[4] we have

$$\beta=(\pi-2)/2.\tag{4}$$

In this case $p=(1-x)^2$, where x is the fraction of the nodes to which the bonds with parameters R_2 and C_2 converge. For other lattice problems, the value of β can be found in[5].

It can be verified that in the case when the medium consists of components of one sort (say resistors only), formulas (1) go over into the usual formulas of the effective-medium theory[1,4,5]:

$$s_m=(1+\beta)(p-p_{cr}),\quad p_{cr}=\beta/(1+\beta).\tag{5}$$

p_{cr} is the percolation threshold calculated within the framework of the effective-medium theory.

Greatest interest attaches to the effective-medium formulas that follow from (1) in the limiting cases of very low ($F\to 0$) and very high ($F\to\infty$) frequencies. For problem 1, in which $g=R_1/R_2=0$ and $k=C_2/C_1=1$, we obtain from (1) as $F\to\infty$:

$$e_m=1,\quad s_m=p.\tag{6}$$

For $F\to 0$ there are three cases. At $p>p_{cr}=\beta/(1+\beta)$, when the percolation over the resistors is not blocked, we obtain

$$s_m=(1+\beta)(p-p_{cr})+\frac{pp_{cr}(1-p)}{(p-p_{cr})^3}F^2\tag{7}$$

(this coincides with (5) at $F=0$), and

$$e_m=\frac{(1-\beta)p+\beta p_{cr}}{p-p_{cr}}.\tag{8}$$

At $p=p_{cr}$ we have

$$s_m=(\beta F/2)^{1/2},\quad e_m=(\beta/2F)^{1/2}.\tag{9}$$

We note that at the percolation point the capacitive susceptance Fe_m has exactly the same absolute value as the active conductance s_m. At $p<p_{cr}$ we have

$$s_m=\frac{pp_{cr}(1-p)}{(p_{cr}-p)^3}F^2,\quad e_m=\frac{p_{cr}}{p_{cr}-p}.\tag{10}$$

Formulas (7), (8), and (10) are valid at $F\ll(p-p_{cr})^2$. Formulas (9) are valid for $(p-p_{cr})^2\ll F\ll 1$.

It is seen from (8) and (10) that the effective-medium theory predicts that the effective capacitance (i.e., the effective dielectric constant of the medium) becomes infinite in the low-frequency limit as the percolation conduction threshold is approached (polarization catastrophe). As seen from the expressions for s_m and e_m (9), at the percolation point ($p=p_{cr}$) there is anomalous dispersion of the conductance and of the lattice capacitance (i.e., of the conductivity and of the dielectric constant of the medium):

$$\left.\frac{ds_m}{dF}\right|_{F\to 0}\to\infty,\quad \left.\frac{de_m}{dF}\right|_{F\to 0}\to\infty.$$

The physical meaning of these results will be discussed in Secs. 5 and 6.

We note that although the effective specific capacitance e_m of the system becomes infinite at the percolation point as $F\to 0$ (see (9)), the capacitive susceptance tends to zero like $\sqrt{F}$ as $F\to 0$.

For problem 2, in which $k=C_2/C_1=0$ and $g=R_1/R_2=1$, we have at $F=0$

$$s_m=1,\quad e_m=p.\tag{11}$$

As $F\to\infty$ and at $p>p_{cr}=\beta/(1+\beta)$, when the percolation is over the capacitors, we have

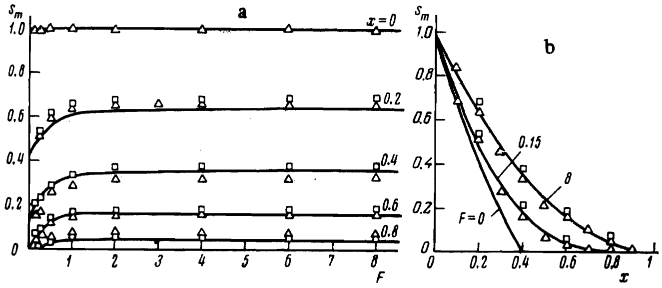

FIG. 2. Dependence of the relative conductance s_m on the dimensionless frequency $F=\omega C_1 R_1$ (a) and on the probability of finding a disconnected node (b) for problem 1. Solid curve—results of calculations within the framework of the effective-medium theory; experimental points: Δ for realization 1, □—for realization 2. In Fig. b, the experimental points are shown for the values $F=0.15$ and $F=8$.

$$s_m=\frac{(1-\beta)p+\beta p_{cr}}{p-p_{cr}},\quad e_m=(1+\beta)(p-p_{cr})+\frac{pp_{cr}(1-p)}{(p-p_{cr})^3F^2}. \qquad (12)$$

At $p=p_{cr}$, just as for problem 1, we get

$$e_m=(\beta/2F)^{1/2},\quad s_m=(\beta F/2)^{1/2}. \qquad (13)$$

At $p<p_{cr}$

$$e_m=\frac{pp_{cr}(1-p)}{(p_{cr}-p)^3F^2},\quad s_m=\frac{p_{cr}}{p_{cr}-p}. \qquad (14)$$

Formulas (12) and (14) are valid at $F(p-p_{cr})^2\gg 1$. Formulas (13) are valid at $1/(p-p_{cr})^2\gg F\gg 1$.

A comparison of formulas (6)–(10) and (11)–(14) shows that when e_m is replaced by s_m and F is replaced by $1/F$, the corresponding formulas for problems 1 and 2 coincide. It can be shown that with such a substitution, the initial formulas (1) also coincide for problems 1 and 2. This agreement is the consequence of the electrostatic analogy and should be a property not only of the effective-medium theory but also of the exact solutions of these problems.

5. RESULTS AND DISCUSSION

Problem 1. (The connections between the capacitors are undisturbed, and the resistances are disconnected in random fashion, $C_2=C_1$, $R_2=0$.) The experimental results obtained for this problem are shown in Figs. 2 and 3, in which they are compared with the results of calculations within the framework of the effective-medium theory (see Sec. 4). It is seen from the figures that at all the investigated values of x and F the experimental data for both s_m and e_m are in very good agreement with the results of the calculations, despite the relatively small size of the investigated model system (16×16 nodes).

It is seen from Fig. 2a that, as follows from expression (6), at $F\gg 1$ the dimensionless conductivity is $s_m=p=(1-x)^2$. Thus, measurements of the conductance of the disordered system at frequencies that are larger than Maxwellian can yield information on the concentration of the conducting phase.

As shown in Sec. 4 (see the formulas for e_m (8) and (10), the effective-medium theory predicts the onset of a "polarization catastrophe" at the percolation point. As seen from Figs. 3a and 3b at $x=0.4$ (which coincides, at the accuracy determined by the finite dimensions of the lattice, with the percolation threshold $x_c=0.3972$ of this problem), the effective lattice capacitance indeed increases sharply with decreasing frequency.

It should be noted that the predictions of the effective-medium theory may turn out to be incorrect near the percolation point p_{cr}, inside the so called critical region ($|p-p_{cr}|\lesssim 0.1$ [1,2]). Thus, for example, in the analysis of the Hall effect in the two-dimensional case, the theory of the effective medium predicts correctly the behavior of the Hall constants both inside and outside the critical region.[10] In the three dimensional case, the theory of the effective medium, while describing well the behavior of the Hall constant outside the critical region, leads to a qualitatively incorrect result for the critical region: According to this theory, the Hall constant should change by not more than a factor of two at the percolation point,[11] whereas percolation theory, in qualitative agreement with experiment, points to a divergence of the Hall constant at the percolation point.[10] However, even outside the critical region (at $p-p_{cr}\gtrsim 0.1$), i.e., in the region of applicability of the formulas of the effective-medium theory, they point to a considerable increase (by a factor of four) of the effective dielectric constant of the inhomogeneous medium. In addition, as will be shown in Sec. 6, qualitative considerations based on percolation theory also point to a divergence of the effective dielectric constant of the medium as the percolation point is approached. In contrast to the effective-medium theory, however, within the framework of which the character of the divergence does not depend on the dimensionality of space, percolation theory shows that in the three-dimensional case the divergence should be much weaker than in the two-dimensional case.

Our results can be used for a qualitative interpreta-

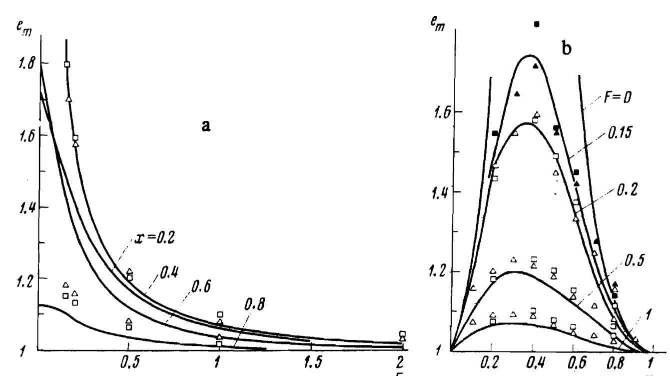

FIG. 3. Dependence of the relative capacitance e_m on the dimensionless frequency f (a) and on x, (b); solid curves——results of calculations within the framework of the effective-medium theory. In Fig. a the experimental points are shown for $x=0.8$ and $x=0.4$, while in Fig. b the points ▲ and ■ pertain to the frequency $F=0.15$.

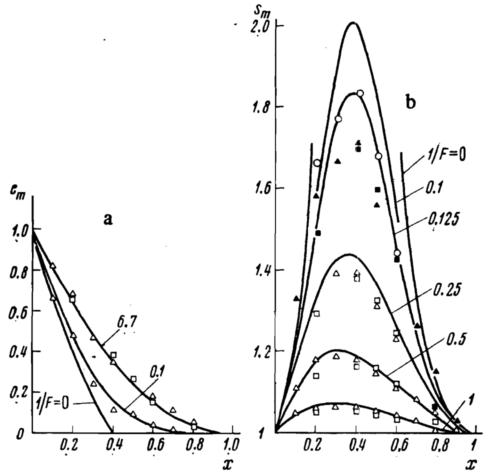

FIG. 4. Dependence of the relative capacitance e_m of the lattice (a) and of the relative conductance of the lattice s_m (b) on x. Solid lines—results of calculations within the framework of the effective-medium theory. In Fig. b, the experimental points ▲ and ■ pertain to $1/F = 0.25$, while ○ pertains to $1/F = 0.1$.

tion of the experimental data obtained in[6], where a "polarization catastrophe" was observed in the metal-dielectric transition in silicon. The abrupt increase of the dielectric constant at low frequency when the transition point was approached was interpreted as a result of polarization of the donors and was described quantitatively with the aid of the Clausius-Mosotti relation. However, on approaching the transition point, the dielectric constant increases more rapidly than predicted by the Clausius-Mosotti formula. As noted in[6], this may be due to the inhomogeneity of the system. Our results illustrate the mechanism with the aid of which the inhomogeneity of the system can cause or enhance the divergence of the dielectric constant near the metal-transition point. An example of physical objects of practical value to which our results pertain in the two-dimensional case is provided by island films.[7]

Problem 2. (The connection between the resistors is not broken, the capacitors are disconnected randomly, $R_1 = R_2$, $C_2 = 0$.) The experimental data compared with the calculations within the framework of the effective-medium theory in Fig. 4, confirm the arguments advanced in Sec. 4 concerning the analogy of problems 1 and 2. We note, however, that in this problem, at the percolation point, the active conductance of the system increases without limit ($\propto \sqrt{F}$, see (13)). From the mathematical point of view, the reason for this increase of σ is that at the percolation point the capacitive susceptance Fe_m, which increases in proportion to F, is equal to the active conductance (see, (13)). A physical explanation of the increase of the dielectric constant in problem 1 will be presented in Sec. 6 with the aid of percolation theory. The explanation of the increase of the conductance in problem 2 can be obtained in the same manner, by using the electrostatic analogy mentioned in Sec. 4.

Thus, the results obtained for problem 2 indicate a sharp increase of the losses at the percolation point at high frequencies. Among the real physical systems that can be set in correspondence with our model system, we can mention ferroelectrics with diffuse phase transitions and with composition fluctuations.

Problem 3. (Independent opening of the connections of the capacitors ($C_2 = 0$) and of the resistors ($R_2 = 0$) at the nodes). This problem is a particular case of the more general problem in which the capacitances and resistances are independently disconnected with equal probabilities $(1 - p_e)$ and $(1 - p_s)$. This problem can be used to simulate the behavior of complicated mixtures made up of components with different values of the conductivity and of the dielectric constant. For this problem it is also possible to construct an effective-medium theory, but since the corresponding formulas are too cumbersome, we confine ourselves to a qualitative examination of the main features of the behavior of systems of this type. Such a system has three percolation thresholds, for percolation over the resistances, for percolation over the capacitances, and an overall percolation threshold when neither active nor capacitive current can flow through the system. The size of this overall threshold can obviously be obtained by solving the percolation problem in a lattice in which each bond is a parallel junction of two conducting bonds of different "color," say "blue" and "green." Assume that the probability of breaking the "blue" bond $(1 - p_s)$ and the probability of the breaking of the "green" bond $(1 - p_e)$ are given. Then the probability of breaking the double bond is $(1 - p_s)(1 - p_e)$. The overall percolation threshold occurs when the following condition is satisfied:

$$(1-p_s)(1-p_e) = 1 - p_{cr}. \tag{15}$$

The foregoing arguments are illustrated in Fig. 5. The curves separating the region 4 from region 5 in this figure is described by expression (15).

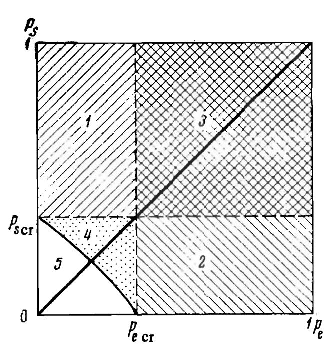

FIG. 5. Diagram characterizing percolation in the problem with capacitors and resistors that are independently disconnected with different probabilities $(1 - p_e)$ and $(1 - p_0)$; 1—region with percolation over the resistors but not over the capacitors, 2—region with percolation over the capacitors but not over the resistors, 3—region with percolation over both the resistors and capacitors, 4—region in which there is current percolation but not over the resistors and capacitors separately, 5—region in which there is no percolation. The dashed lines correspond to the separate thresholds of percolation over the resistors and capacitors. The solid diagonal lines correspond to the case $p_s = p_e$, which was experimentally investigated in problem 3.

Starting from qualitative considerations based on percolation theory, we can expect to observe in this system, near the capacitance and resistance percolation thresholds, effects that are analogous to those described above in the discussion of problems 1 and 2. Let for the sake of argument $p_s < p_e$, i.e., the resistances are disconnected with large probability. Let the quantity p_s be close to the resistance percolation threshold $p_{s\,cr}$. The capacitor network is in this case farther from the percolation threshold $p_{e\,cr}$. Then, in accordance with the concepts concerning the structure of the "infinite cluster," developed in[12,2], we can qualitatively visualize the system in the form of two networks—a resistance grid with characteristic mesh dimension L_s and a capacitor network with characteristic dimension L_e, where L_e is the correlation radius for the capacitors and L_s is the correlation radius for the resistors. Since the resistor network is closer to the percolation threshold, it is of much greater mesh ($L_s \gg L_e$). This gives rise to a situation qualitatively similar to problem 1, and one should expect at low frequencies divergences of the effective specific capacitance (the effective dielectric constant) of the system (see also Sec. 6). Analogously, if $p_s > p_e \sim p_{e\,cr}$, then at high frequencies one should expect the effective conductivity of the system to increase with frequency.

The experimental results obtained by us for problem 3 confirm these considerations. We have observed that if the percolation over the resistors ceases before the percolation over the capacitors, then at low frequencies the function $e_m(x_s)$ has a sharp maximum at the point corresponding to the cessation of the percolation over the resistors (here x_s is the fraction of the disconnected resistors). Such a maximum is observed even when the fraction of the disconnected capacitors is only slightly smaller than the fraction of the disconnected resistors.

6. QUALITATIVE ANALYSIS OF THE EFFECTS OF DIVERGENCES NEAR CRITICAL POINTS IN TERMS OF PERCOLATION THEORY[2)]

For the sake of argument we consider only problem 1, in which only the resistors are disconnected and the effective capacitance of the system diverges at low frequencies. (The analysis for problem 2 at high frequencies can be carried out in perfect analogy.) Let $p < p_{cr}$, i.e., assume no percolation over the resistors, and $(p_{cr} - p)/p_{cr} \ll 1$. In this case the system contains individual finite clusters of interconnected resistors. The characteristic dimension of these clusters is equal to the correlation radius L,[12,2] which increases like $(p_{cr} - p)^{-\nu}$ as $p \to p_{cr}$. The critical exponent ν of the correlation radius was calculated in[9]. In accordance with the universality hypothesis, it does not depend on the type of problem or on the character of the system, and is determined only by the dimensionality of the space.[8,9]

At low frequencies ($F \to 0$), which are the ones at which the effective capacitance diverges, the susceptance ωc_1 of one bond is small in comparison with the conductance R_1^{-1} of the bond. The system in question can therefore be regarded qualitatively (at low frequencies) as "metallic" finite clusters (resisters) separated by a dielectric (capacitors). In order of magnitude, the effective specific capacitance of such a system e_m is equal to the specific capacitance of a capacitor grid C_f (C_f is the mutual capacitance of two neighboring finite clusters) with cell dimension L. Thus, in the two-dimensional case $e_m = C_f/C_1$, and in the three-dimensional case $e_m = C_f/LC_1$.

We stop now to estimate the effective capacitance C_f. The resistors belonging to the neighboring finite clusters are separated by different distances in different places. At some points, the average number of which we shall designate by N, they are separated by only one bond with disconnected resistors, i.e., they are connected only by one capacitor C_1. These points obviously make a contribution C_f^0 on the order of NC_1 to the capacitance C_f. There are also points in which the neighboring clusters are separated by 2, 3, 4, etc. periods of the initial lattice, and all these places also contribute to C_f. We estimate here, however, only the contribution C_f^0 from those points at which the neighboring finite clusters are closest to one another (separated by only one bond), and thus obtain a lower bound for the capacitance C_f.

Let the resistors be disconnected with a probability $1 - p > 1 - p_{cr}$. We connect mentally each of the disconnected resistors in such a system, with probability $(p_{cr} - p)/(1 - p)$. Then the total fraction of the connected resistors becomes $p + (1 - p)(p_{cr} - p)/(1 - p) = p_{cr}$, i.e., a critical situation arises, wherein the finite clusters merge into one infinite cluster. Each two neighboring finite clusters will have at the point $p = p_{cr}$ on the average one common bond. This means that out of the N points, at which the ends of the finite clusters were separated from one another by one bond, one will be connected on the average with probability unity, i.e., $N(p_{cr} - p)/(1 - p) \sim 1$, whence $N \sim 1/(p_{cr} - p)$, and consequently $C_f^0 \sim C_1/(p_{cr} - p)$. Thus, $e_m \gtrsim 1/(p_{cr} - p)$ for the two-dimensional case and $e_m \gtrsim (p_{cr} - p)^{-(1-\nu)}$ for the three-dimensional case.

Thus, qualitative considerations based on percolation theory predict a power-law divergence of the effective dielectric constant of disordered systems on approaching the percolation point, at frequencies much lower than Maxwellian: $e_m \sim 1/(p_{cr} - p)^q$, where q is a new critical exponent, the exact determination of which we consider to be an independent important and interesting problem.

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1)In this case the lattice site problem can be formulated in the following manner. Consider a lattice made up of bonds comprising the elements C_1 and R_1 in parallel. The bonds emerging from each site are replaced, with probability x, by bonds made up of elements C_2 and R_2.

2)The content of this section is fully based on ideas advanced by B. I. Shklovskiĭ in a discussion of the experimental results reported above.

[1]S. Kirkpatrick, Rev. Mod. Phys. **45**, 574 (1973).
[2]B. I. Shklovskiĭ and A. L. Éfros, Usp. Fiz. Nauk 117, 401 (1975) [Sov. Phys. Usp. 18, 845 (1976)].
[3]B. E. Springett, Phys. Rev. Lett. **31**, 1463 (1973).
[4]B. P. Watson and P. L. Leath, Phys. Rev. [B] **9**, 4893 (1974).
[5]N. P. Butcher, J. Phys. C 8, 1324 (1975).
[6]T. G. Kastner, N. K. Klee, G. S. Cieloszyk, and G. L. Salinger, Phys. Rev. Lett. **34**, 1627 (1975).
[7]R. H. Ritchie, Surf. Sci. **34**, 1 (1973).
[8]A. G. Dunn, J. W. Essam, and J. M. Loveluck, J. Phys. C 8, 743 (1975).
[9]M. E. Levinshteĭn, B. I. Shklovskiĭ, M. S. Shur, and A. L. Efros, Zh. Eksp. Teor. Fiz. **69**, 386 (1975) [Sov. Phys. JETP **42**, 197 (1976)].
[10]M. E. Levinshteĭn, M. S. Shur, and A. L. Efros, Zh. Eksp. Teor. Fiz. **69**, 2203 (1975) [Sov. Phys. JETP **42**, 1120 (1976).
[11]M. H. Cohen and J. Jortner, Phys. Rev. Lett. **30**, 696 (1973).
[12]A. S. Skal and B. I. Shklovskiĭ, Fiz. Tekh. Poluprovodn. **8**, 1586 (1974) [Sov. Phys. Semicond. **8**, 1029 (1975)].

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