Theory of absorption of electromagnetic waves by small particles

É. A. Manykin, P. P. Poluéktov, and Yu. G. Rubezhnyĭ

Moscow Engineering-Physics Institute
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Absorption of optical and infrared radiation by small particles with sizes smaller than the electron mean free path is considered. Emission and absorption of the quanta occurs during collisions with the surface. It is shown that for a transparent dielectric the absorption is nonlinear and proceeds via cumulative ionization. The breakdown intensities are found to be proportional to the square of the frequency. If the electron energy losses due to inelasticity of the collisions with the walls are negligible (at a particle size R greatly exceeding a critical value R^*, the breakdown threshold is proportional to $R\ln(n_a^{1/3}R)$ (n_a is the atom density in the particle) and inversely proportional to the pulse duration τ_p. Under conditions when the energy losses in the collisions play the dominant role ($R\lesssim R^*$), the breakdown intensity is independent of R and τ_p and $I_{\text{thr}}\sim10^9$ W/cm^2 for all transparent dielectrics if optical pulses are employed. For metallic particles under continuous irradiation the absorption cross section σ_{abs} is proportional to R^2, and not to R^3 as when the macroscopic conductivity is used in the theory of absorption by small particles (Landau and Lifshitz, Electrodynamics of Continuous Media, 1957, p. 384 of Russ. ed.). As a consequence, the breakdown threshold is independent of the particle size, whereas in the theory in which macroscopic constants are used the threshold strength increases with decreasing size. It is shown that for metallic particles σ_{abs} is inversely proportional to the square of the frequency; for optical radiation σ_{abs} is of the order of the geometric cross section, whereas for infrared radiation ($\lambda\sim10\ \mu$) σ_{abs} is greater by three orders of magnitude.

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1. INTRODUCTION

At present there are many problems the solutions of which call for knowledge of the character of the absorption of electromagnetic radiation by small particles. These include 1) the study of the destruction of aerosols under the influence of radiation[1]; 2) the influence of impurity particles on the breakdown thresholds in gases[2] and in liquids[3]; 3) the role played by inclusion in solids in electron-heating processes and their influence on the mechanism of damage to material.[4] In the second and third case there are experimental data that indicate that the breakdown thresholds are greatly reduced by the presence of small particles.

In a theoretical analysis of the absorption by small particles, it is customary to use for them macroscopic constants (electric conductivity, thermal conductivity) that are valid for bulky samples of the material. This approach is convenient only for particles with sufficiently large dimensions. For metallic particles these dimensions are determined by the electron mean free path which, say for aluminum at room temperature, is $l\sim0.1\ \mu$. (The estimate was obtained with the aid of the formula for the conductivity of a bulky sample $\sigma\sim e^2nl/mv_F$, where $\sigma\approx10^{18}$ sec^{-1}, [5] e and m are the charge and mass of the electron, while n and v_F are the density and velocity of the electrons on the Fermi surface,

$$n=\frac{2}{(2\pi)^3}\frac{4\pi}{3}k_F^3,\quad v_F=\frac{\hbar k_F}{m},$$

with $k_F=0.93$ at. un. [6]) If the particle dimension $R<l$, then the radiation can be absorbed only as a result of collisions with the surface of the particle, and not collisions with phonons, impurity ions, lattice defects, etc., which determine the absorption in the bulky sample. As a result, the dependence of the absorption cross section on the dimension R is radically altered.

For particles of a transparent dielectric, the character of these changes is even more radical. In a bulky dielectric, the absorption of the radiation is due to the following: 1) the electrons in the conduction band at thermodynamic equilibrium; 2) the transfer of the electrons from the easily-ionized impurities to the conduction band; 3) resonant absorption by the impurities; 4) resonant absorption by color centers; 5) absorption by the lattice. The last mechanism is characteristic of the far-infrared region, which we shall not consider. Inasmuch as at room temperature the vacancy density is of the order of 10^7 cm^{-3}, [7] we can neglect absorption by color centers for particles smaller than 0.01 cm. Absorption by impurities becomes negligible if the particle dimension exceeds $R_1=n_{\text{imp}}^{-1/3}$ ($\sim10^{-5}$ cm for an impurity concentration $n_{\text{imp}}\sim10^{15}$ cm^{-3}). Under thermodynamic equilibrium the number of electrons in the conduction band, for a particle of volume V, is $Vn_0\exp(-\varepsilon_g/k_BT)$, where n_0 is the density of the electrons per unit volume, ε_g is the width of the forbidden band, k_B is the Boltzmann constant, and T is the temperature. There will be no linear absorption if the inequality $Vn_0\exp(-\varepsilon_g/k_BT)<1$ is satisfied, i.e., $R<R_2\sim n_0^{-1/3}\exp(\varepsilon_g/3k_BT)$ ($\sim100\ \mu$ at room temperature at $\varepsilon_g\sim1$ eV). Thus, if the dimension R of a small transparent particle is smaller than R_1 or R_2, then the absorption of electromagnetic radiation has a nonlinear character, in contrast to absorption in the bulky sample, namely: the radiation is not absorbed after a certain intensity I_{mp} at which the probability of multiphoton transition from the valence band to the conduction band during the time of the pulse becomes equal to unity; at $I>I_{\text{mp}}$, the initial electrons produced as a result of the multiphoton ionization become "heated" in the field of the wave and, reaching an energy larger than the width of the forbidden band, initiate an electron avalanche. If the particle dimension is smaller than the electron mean free path in the conduction band, then the electron absorbs quanta when colliding with the surface.

Thus, a study of the singularities of the absorption of electromagnetic radiation by small particles is a pressing problem whenever the collision of the electron with the surface of the particle becomes significant. In this paper we construct a theory for the aforementioned processes in the case of particles of transparent dielectrics and metals.

We shall show that the absorption of radiation by small particles with dimensions smaller than the mean free path in the conduction band has the following features: in the case of small transparent-dielectric particles there are no electrons in the conduction band, so that the absorption of the radiation is due to an electron avalanche that starts with a certain number of initial electrons produced by multiphoton transitions from the valence band to the conduction band. Consequently, the absorption of radiation by such particles and their destruction have a threshold, with the threshold intensity proportional to the particle dimension, to the square of the radiation frequency, and to the reciprocal of the pulse direction. In the case of small metallic particles, the radiation is absorbed in collisions with the surface, the absorption cross section being proportional to R^2, and not to R^3 as in the case when the macroscopic conductivity is used. This is due to the fact that the time between collisions is not constant but is proportional to R. The damage threshold intensity is independent here of the particle dimension (since the amount of the heat removed is proportional to the surface area), whereas in a theory in which macroscopic constants are used the thresholds increase with increasing particle dimension.

2. ABSORPTION OF RADIATION BY AN ELECTRON IN COLLISIONS WITH A WALL

Inasmuch as the electron wave vectors $\mathbf{k}$ satisfy the inequality $kR \gg 1$, we can consider the collision of an electron with a flat wall (we assume also for this purpose that $ka \ll 1$, where a is the distance between atoms). We introduce a potential barrier in the form $U(x) = U_0$ $(x < 0)$, $U(x) = 0$ $(x > 0)$, and assume that the height U_0 of the barrier is much larger than the energies of the incident and scattered electrons (we note that this assumption is perfectly valid for a boundary with a vacuum or a gas; if the particle is an inclusion in some matrix, it is necessary to consider a barrier of finite height). The x axis is chosen perpendicular to the plane surface, the y axis lies in the plane of the x axis and the vector $\mathbf{k}$. By virtue of the inequality $R \ll \lambda$, where λ is the wavelength of the electromagnetic radiation, the field can be regarded as homogeneous: $\mathbf{E} = \mathbf{E}_0 \sin\omega t$; the vector potential is chosen in the form $\mathbf{A} = -c\omega^{-1}\mathbf{E}_0 \cos\omega t$.

The Schrödinger equation describing the scattering of the electron by the wall in the presence of the field is of the form

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = \left(\frac{1}{2m}\hat{\mathbf{p}}^2 + \hat{V}(\omega t) + U(x)\right)\psi(\mathbf{r},t), \tag{1}$$

$$\hat{V}(\omega t) = \frac{1}{m}\hat{\mathbf{p}}\left(-\frac{e}{c}\mathbf{A}(\omega t)\right) + \frac{1}{2m}\left(\frac{e}{c}\mathbf{A}(\omega t)\right)^2. \tag{2}$$

We carry out a unitary transformation of Eq. (1) with the aid of the operator

$$\hat{S} = \exp\left[\frac{i}{\hbar\omega}\int_0^{\omega t} df\,\hat{V}(f)\right]. \tag{3}$$

The function $\hat{S}\psi(\mathbf{r}, t) = \varphi(\mathbf{r}, t)$ satisfies the equation

$$i\hbar\frac{\partial\varphi}{\partial t} = \frac{\hat{\mathbf{p}}^2}{2m}\varphi + \hat{S}U(x)\hat{S}^{-1}\varphi. \tag{4}$$

Substituting the interaction $\hat{V}(\omega, t)$ of the electron with the field in the operator $\hat{S}$, we find that the latter is a spatial-shift operator. Using the properties of the latter, we write

$$\hat{S}U(x)\hat{S}^{-1} = U(x - a_e f_x \sin\omega t), \quad a_e = eE_0/m\omega^2, \quad f_x = (\mathbf{E}_0)_x/E_0. \tag{5}$$

Thus, the problem has been reduced to consideration of scattering by a potential wall oscillating with frequency ω.[1)] Changing over for the sake of simplicity to an impenetrable wall $(U_0 \to \infty)$, we arrive at the following system of equations:

$$i\hbar\frac{\partial\varphi(\mathbf{r},t)}{\partial t} = \frac{\hat{\mathbf{p}}^2}{2m}\varphi(\mathbf{r},t), \qquad \varphi(a_e f_x \sin\omega t, t) = 0. \tag{6}$$

We represent the wave function $\varphi(\mathbf{r}, t)$ in the form of an incident plane wave and reflected plane waves:

$$\varphi(\mathbf{r},t) = \exp\left[-\frac{i}{\hbar}(\varepsilon t + \mathbf{p}\mathbf{r})\right] + \sum_{n=-\infty} a_n \exp\left[-\frac{i}{\hbar}(\varepsilon + n\hbar\omega)t + \frac{i}{\hbar}\mathbf{p}_n\mathbf{r}\right], \tag{7}$$

where $\mathbf{p}$ and $\varepsilon = \mathbf{p}^2/2m$ are the momentum and energy of the incident electron, $\mathbf{p}_n^2 = 2m(\varepsilon + n\hbar\omega)$, $(\mathbf{p}_n)_y = (\mathbf{p})_y$. The last term in (7) describes processes of multiphoton absorption (emission) of an electron in collisions with a wall. We note that we can substitute the lower limit $-\infty$ in (7), since waves for which $\varepsilon + n\hbar\omega < 0$ attenuate rapidly near the surface.

Substituting the function (7) in the boundary condition (6) and using for the Bessel functions the generating function

$$\exp(i\alpha\sin\omega t) = \sum_{m=-\infty}^{\infty} J_m(\alpha)\exp(im\omega t), \tag{8}$$

we obtain a system of equations for the amplitudes a_n:

$$(-1)^n J_n(\alpha_0) + \sum_{k=-\infty}^{\infty} a_k J_{n+k}(\alpha_k) = 0, \qquad \alpha_k = \frac{1}{\hbar}(\mathbf{p}_k)_x f_x a_e. \tag{9}$$

For optical fields, the quantity a_e reaches a value 1 Å only at high intensities (exceeding the breakdown threshold) $I \sim 10^{13}$ W/cm^2. For infrared radiation this value is still quite large, $I \sim 10^8$ W/cm^2. We must therefore assume that $\alpha_k \ll 1$, by virtue of which, to determine the amplitude a_k, it is necessary to use one of the equations

(9) in which a_k is multiplied by a zero-order Bessel function, after substituting the obtained values of the amplitudes with indices from zero to $(|k|-1)$ sign k. As a result of this iteration we obtain

$$a_0=-1,\quad a_1=-a_{-1}=-\alpha_0,\quad a_2=-\frac{\alpha_0\alpha_1}{2},\quad a_{-2}=-\frac{\alpha_0\alpha_{-1}}{2}$$
$$a_3=-\frac{\alpha_0}{4}(\alpha_0^2-\alpha_1^2+\alpha_1\alpha_2),\quad a_{-3}=\frac{\alpha_0}{4}(\alpha_0^2-\alpha_{-1}^2+\alpha_{-1}\alpha_{-2}). \tag{10}$$

As seen from (10), with increasing number k the amplitudes decrease like $(\alpha_i)^k$ with a small numerical coefficient. In the collision of an electron with the wall we can therefore confine ourselves to single-photon processes, the probability of which is the ratio of the flux of the reflected electrons to the incident flux:

$$W_1=|a_1|^2\sqrt{\frac{\varepsilon+\hbar\omega}{\varepsilon}},\quad W_{-1}=|a_{-1}|^2\sqrt{\frac{\varepsilon-\hbar\omega}{\varepsilon}}\quad(\varepsilon>\hbar\omega). \tag{11}$$

Averaging (11) over the position of the scattering plane, we get

$$\overline{W}_{\pm1}(\varepsilon)=\lambda_1\sqrt{\varepsilon(\varepsilon\pm\hbar\omega)},\quad \lambda_1=ma_e^2/2\hbar^2. \tag{12}$$

When considering the "heating" of an electron as a result of collisions with the surface of a small particle, we must note the probability $W_\pm(\varepsilon)$ of the absorption and emission of a photon per unit time; this probability is the product of $\overline{W}_\pm(\varepsilon)$ by the number $\beta v/R$ of collisions with the walls (R is the characteristic dimension of the particle, v is the electron velocity, and β is a coefficient on the order of unity and depends on the shape of the particle). We thus obtain ultimately

$$W_\pm(\varepsilon)=\lambda\varepsilon\sqrt{\varepsilon\pm\hbar\omega},\quad \lambda=\lambda_1\sqrt{\frac{2}{m}}\beta\frac{1}{R}. \tag{13}$$

3. DEVELOPMENT OF ELECTRON AVALANCHE IN THE CASE OF A SMALL TRANSPARENT PARTICLE

The absorption of the radiation by a small transparent particle, leading eventually to its destruction, is the result of the development of an electron avalanche in the conduction band. The avalanche begins with a certain number n_0 of initial electrons (e.g., $n_0=1$), produced as a result of a multiphoton transition from the valence band.

We denote by $n(\varepsilon,t)d\varepsilon$ the number of electrons in the conduction band of the particle in the energy interval from ε to $\varepsilon+d\varepsilon$; we can then write down the equation

$$\frac{dn(\varepsilon,t)}{dt}=W_+(\varepsilon-\hbar\omega)n(\varepsilon-\hbar\omega,t)+W_-(\varepsilon+\hbar\omega)n(\varepsilon+\hbar\omega,t)-(W_+(\varepsilon)+W_-(\varepsilon))n(\varepsilon,t). \tag{14}$$

When considering electron multiplication with the aid of Eq. (14), we neglect the energy loss due to the inelasticity of the collisions of the electrons with the walls. A justification for this can be found in the next section. We assume that the width ε_g of the forbidden band is much larger than the quantum energy $\hbar\omega$, so that we can expand in powers of $\hbar\omega$ in (14). After substituting (13) in (14) we obtain a diffusion equation of the Fokker–Planck type:

$$\frac{dn(\varepsilon,t)}{dt}=s\frac{d}{d\varepsilon}\varepsilon\frac{d}{d\varepsilon}(\sqrt{\varepsilon}\,n(\varepsilon,t)),\quad s=\lambda(\hbar\omega)^2. \tag{15}$$

We assume also that the doubling of the electrons takes place as soon as their energy reaches values exceeding the width ε_g of the forbidden band, and the electrons are "produced" with zero energy. Then the first boundary condition for Eq. (15), namely the condition for an infinite sink at the point $\varepsilon=\varepsilon_g$, [10,11] takes the form

$$n(\varepsilon_g,t)=0. \tag{16}$$

Since the electron flux along the energy axis, as seen from (15), was given by the formula

$$j(\varepsilon,t)=-s\varepsilon\frac{d}{d\varepsilon}(\sqrt{\varepsilon}\,n(\varepsilon,t)),$$

the second boundary condition, namely that the flux along the energy axis at the point $\varepsilon=0$ be equal to double the flux at $\varepsilon=\varepsilon_g$ (the condition for the doubling of the flux[10–12]) takes the form

$$\varepsilon\frac{d}{d\varepsilon}(\sqrt{\varepsilon}\,n(\varepsilon,t))|_{\varepsilon=0}=2\varepsilon\frac{d}{d\varepsilon}(\sqrt{\varepsilon}\,n(\varepsilon,t))|_{\varepsilon=\varepsilon_g}. \tag{17}$$

We seek the solution of (15) in the conventional form

$$n(\varepsilon,t)=\exp(\gamma_0 t)\,n(\varepsilon). \tag{18}$$

Then the function $n(\varepsilon)$ satisfies the equation

$$\gamma n(\varepsilon)=\frac{d}{d\varepsilon}\varepsilon\frac{d}{d\varepsilon}(\sqrt{\varepsilon}\,n(\varepsilon)),\quad \gamma=\frac{\gamma_0}{s} \tag{19}$$

and the boundary conditions

$$n(\varepsilon_g)=0,\quad \varepsilon\frac{d}{d\varepsilon}\sqrt{\varepsilon}\,n(\varepsilon)|_{\varepsilon=0}=2\varepsilon\frac{d}{d\varepsilon}\sqrt{\varepsilon}\,n(\varepsilon)|_{\varepsilon=\varepsilon_g}. \tag{20}$$

A solution of (19) is the expression[13]:

$$n(\varepsilon)=\varepsilon^{-1/2}Z_0(i4\gamma^{1/2}\varepsilon^{1/4}), \tag{21}$$

where $Z_0(\varepsilon)$ is a Bessel function. Making the assumption $\beta_1=4\gamma^{1/2}\varepsilon_g^{1/4}\gg 1$ (the validity of which will be demonstrated later on), we can use the asymptotic form of the Bessel function, by virtue of which, after substitution in the first boundary condition of (20), the solution (21) is expressed in terms of a Macdonald function:

$$n(\varepsilon)=A\varepsilon^{-1/2}K_0(4\gamma^{1/2}\varepsilon^{1/4}). \tag{22}$$

With the aid of the second condition of (20) we obtain an equation for the quantity $\beta_1=4\gamma^{1/2}\varepsilon_g^{1/4}$:

$$\frac{1}{4}=\sqrt{\frac{2\pi}{\beta_1}}\left(\frac{1}{8}+\beta_1\right)e^{-\beta_1}, \tag{23}$$

from which we get $\beta_1=2.82$. Thus, the growth rate γ_0 of the avalanche is equal to

$$\gamma_0=\frac{\beta_1^2}{16\sqrt{\varepsilon_g}}s=0.497\sqrt{\frac{m}{\varepsilon_g}}\left(\frac{eE_0}{m\omega}\right)^2\frac{\beta}{R}. \qquad (24)$$

Expression (24) shows that the growth rate of the avalanche (the reciprocal electron-multiplication time) increases in direct proportion to the decrease of the particle radius and in proportion to the square of E_0/ω. On the other hand, it has a square-root dependence on the width of the forbidden band. Thus, γ_0 remains essentially unchanged on going from one transparent material to another (other conditions being equal).

4. DAMAGE CRITERION

Let us formulate the condition for the destruction of a small transparent particle in analogy with the "forty-generation criterion" in a gas.[10,11] We assume that the destruction begins when all the particle atoms are ionized, i.e., when the conduction band contains n_aV electrons, where n_a is the density of the atoms and V is the volume of the particles. To obtain this number of electrons it is necessary that $\log_2(n_aV)$ doublings take place, amounting to 26, 17, and 7 generations if R is equal to 10^{-5}, 10^{-6}, and 10^{-7} cm, respectively.

Thus, we have the following condition for the breakdown of a small transparent particle:

$$\gamma_0\tau_p \approx 3\ln(n_a^{1/3}R),$$

or, if we use the expression for the intensity $I=cE_0/8\pi$,

$$\frac{I_{\text{thr}}\ \tau_p}{\omega^2}\approx\varkappa R\ln(n_a^{1/3}R), \qquad \varkappa\approx\frac{3}{4\pi}\frac{\sqrt{m\varepsilon_g}\,mc}{\beta c^2}, \qquad (25)$$

where τ_p is the pulse duration, and $\varkappa$ is a constant that depends on the material and on the shape of the particle (if $\varepsilon_g\approx 10$ eV and $\beta\approx 1$, then $\varkappa\approx 0.35\times10^{-17}$ g/cm).

For a particle with $R\sim10^{-6}$ cm, in the case of optical (λ 0.5 μ) radiation of duration $\tau_p\sim 10$ nsec, we have $I_{\text{thr}}\sim 2\times10^9$ W/cm^2; for infrared pulses ($\lambda\sim 10$ μ) of the same duration we have $I_{\text{thr}}\sim 0.5\times10^7$ W/cm^2.

Let us determine the conditions under which we can neglect the energy lost in the collisions of the electron with the walls, i.e., let us find the region of applicability of the developed theory.

The electron transfers energy to the molecules when it collides with the surface. If the collision time ($\sim\hbar/\varepsilon\sim10^{-16}$ sec) is small in comparison with the characteristic acoustic frequencies ($\sim10^{13}$ sec^{-1}), then the electron loss energy in elastic collisions with approximately $\lambda_e^2 n_{\text{sur}}$ atoms (λ_e is the electron wave length and $n_{\text{sur}}\approx n_a^{2/3}$ is the surface density of the atoms). Thus, in one collision of the electron with the surface the fraction of the electron energy converted into oscillation energy is $\delta\sim 2m/M_a\lambda_e^2 n_{\text{sur}}$ ($\sim10^{-5}$), where M_a is the mass of the atom. This loss can be neglected if the energy $\hbar\omega(\overline{W}_{+1}-\overline{W}_{-1})$ absorbed on the average in each collision$^{2)}$ is much larger than $\varepsilon\delta$:

$$\hbar\omega(\overline{W}_{+1}(\varepsilon)-\overline{W}_{-1}(\varepsilon))\gg\varepsilon\delta, \text{ or } \lambda_1(\hbar\omega)^2/\varepsilon\gg\delta.$$

If we introduce the average electron energy $\bar\varepsilon$ (on the order of several electron volts), then the breakdown condition (25) can be rewritten in the form

$$\lambda_{1\,\text{thr}}\ \frac{(\hbar\omega)^2}{\varepsilon}\sim\frac{\sqrt{m\varepsilon_g}}{\varepsilon\beta\tau_p}R\ln(n_a^{1/3}R), \qquad (26)$$

from which we get for $\varepsilon_g\sim 10$ eV at a pulse duration $\tau_p\sim 10$ nsec, in the case of particles with $R\sim10^{-6}$ cm, a value $\lambda_{1\,\text{thr}}(\hbar\omega)^2/\bar\varepsilon\sim10^{-4}$, i.e., the elastic losses can be neglected (up to $R\sim 50$ Å). If we increase the duration of the pulse, then a theory that does not take the losses into account is valid only for those particles whose dimensions exceed the value R^* determined from the equation

$$\frac{\sqrt{m\varepsilon_g}}{\varepsilon\beta\tau_p}R^*\ln(n_a^{1/3}R^*)\approx\delta. \qquad (27)$$

Thus, for particle breakdown by pulses of duration 10 μsec and longer it is necessary to take into account the energy lost by scattering from the walls at all particle dimensions.

If the particle radius is smaller than R^*, the breakdown criterion differs from (26) and takes the form $\lambda_1(\hbar\omega)^2/\bar\varepsilon\gtrsim\delta$, i.e.,

$$\frac{I_{\text{thr}}}{\omega^2}\gtrsim\varepsilon\delta\frac{mc}{4\pi e^2}. \qquad (28)$$

Expression (28) determines the condition under which the absorbed energy exceeds the losses in one collision. Thus, if $R<R^*$, the threshold intensity is independent of the particle dimension and of the pulse duration, and is proportional to the square of the frequency. In the case of infrared radiation ($\lambda\sim 10$ μ), $I_{\text{thr}}\sim 4\times10^6$ W/cm^2; for optical radiation ($\lambda\sim 0.5$ μ) we have $I_{\text{thr}}\sim 1.5\times10^9$ W/cm^2 (it is assumed in the estimates that $\bar\varepsilon\sim 5$ eV).

The following remark is in order. The damage thresholds were determined by considering an electron avalanche, which presupposes the presence of initial electrons in the conduction band. Experiments on the photoconductivity in transparent materials under the influence of optical radiation show that free carriers appear in silicate glasses and in quartz at $I\approx 3.5\times10^8$ W/cm^2, and in ruby at $I\approx 10^{10}$ W/cm^2.[14] Thus, destruction of glass and quartz particles occurs in fields determined by the avalanche development. For ruby particles with $R<0.05$ μ, for which $I_{\text{thr}}<10^{10}$ W/cm^2, the destruction at threshold intensities will have a random character, since multiphonon ionization does not insure delivery of electrons to the conduction band.

5. ABSORPTION OF RADIATION BY SMALL METALLIC PARTICLES

As noted initially, if the dimension of a metallic particle is smaller than the electron mean free path, then the absorption of radiation is due to collisions with the particle surface. Let us find the absorption cross section in this case. By virtue of the Fermi principle, the electrons cannot emit quanta in collisions; on the other hand, radiation can be absorbed by the electrons only in an energy interval on the order of the quantum energy $\hbar\omega$ near the Fermi surface, and the number of these electrons is

$$\Delta N_F = \sqrt{2m}\frac{mV}{\pi^2\hbar^3}\sqrt{\varepsilon_F}\hbar\omega,$$

where V is the volume of the particle, and ε_F is the Fermi energy (we confine ourselves to single-photon processes). The energy absorbed per unit time is

$$\varepsilon = \hbar\omega W_1 \Delta N_F = \frac{64\beta}{411}\left(\frac{\varepsilon_F}{\hbar\omega}\right)^2 R^2 I, \quad I = \frac{cE_0^2}{8\pi}.$$

Thus, the cross section for the absorption of radiation by a small metallic particles takes the form

$$\sigma_{abs} = \frac{64\beta}{411}\left(\frac{\varepsilon_F}{\hbar\omega}\right)^2 R^2. \tag{29}$$

The main feature of formula (29) is the proportionality to R^2, whereas in the theory that makes use of macroscopic constants the cross section for absorption by small particles is proportional to R^3.[15] Let us see how this changes the dependence of the threshold values on the dimension.

The destruction of a particle under stationary irradiation takes place whenever the energy absorbed by the particle exceeds the energy removed to the external medium. To estimate the latter, we confine ourselves to aerosol particles. We assume that the air molecules, which have in collisions with the particle surface an energy $(\frac{3}{2})k_B T_0$, where T_0 is the temperature of the external medium, escapes with an energy $(\frac{3}{2})k_B T_{part}$ (T_{part} is the particle temperature). It is assumed here that the dimension of the particles is much smaller than the mean free path of the air molecules ($\sim 1\ \mu$ under normal conditions), so that heating of the particle does not change the thermal field of the air. For the energy removed per unit time we then obtain Newton's law

$$Q_{rem} = {}^3/_2 k_B n_{air} v_T s_{sur}(T_{part} - T_0), \tag{30}$$

where n_{air} is the density of the gas, v_T is the thermal velocity of the gas molecules, and s_{sur} is the surface area of the particle. Solving the equation $\sigma_{abs} I_{thr} = Q_{rem}$, we find that for metallic particles suspended in air the threshold intensity does not depend on the dimension (for all metals we have the order-of-magnitude value $I_{thr} \sim 10^5$ W/cm^2 in the case of optical radiation and $I_{thr} \sim 10^3$ W/cm^2 if $\lambda \sim 10\ \mu$); on the other hand, when the absorption is determined by using the macroscopic conductivity, the threshold is inversely proportional to the dimension.

It is seen from (29) that in the case of optical irradiation $\sigma_{abs} \sim \sigma_{geom} = \pi R^2$, since $\hbar\omega_{opt} \sim 1-2$ eV and $\varepsilon_F \sim 10$ eV. For infrared radiation ($\lambda \sim 10\ \mu$, $\hbar\omega \sim 0.05$ eV), σ_{abs} exceeds σ_{geom} by approximately three orders of magnitude.

We note in conclusion that these singularities of the radiation absorption may turn out to be significant when considering, for example, the destruction of a transparent solid matrix with absorbing inclusions, or else polycrystalline samples, by electromagnetic irradiation and also when ascertaining the nature of the damage to bulky dielectrics under the assumption that their internal structure has a certain "graininess." The inhomogeneities then serve as point sources of heat or injectors of the electrons into the conduction band of the matrix. If we use short optical pulses ($\tau_p \sim 10$ nsec), then the dimension of the particle can be determined from the particle breakdown threshold, and on this basis one can design a dimension spectrometer for dielectric aerosols.

[1)]It is possible to treat in similar fashion the problem of stimulated emission and absorption of optical photons in the scattering of a slow electron by an atom,[8] as well as the problem of ionization of a hydrogen-like atom in a strong electromagnetic field.[9]

[2)]The energy loss in collisions of an electron with volume phonons can be neglected in comparison with the loss in collisions with the walls. In fact, the wave vector of typical phonons is $k \sim R^{-1} \sim 10^5-10^7$ cm^{-1}, i.e., the phonon energy is $\varepsilon_{ph} = \hbar ck \sim 10^{-5}-10^{-3}$ eV ($c \sim 10^5$ cm/sec is the speed of sound). Assuming by way of estimate that the change of the phonon energy in the collision is equal to ε_{ph}, we find that in one collision with the volume phonon the electron loses a fraction $\varepsilon_{ph}/\varepsilon \sim 10^{-6}-10^{-4}$ of its energy. Since $R < l$, the fraction of energy lost by the electron as a result of collisions with the volume phonons during the time from one collision with the wall to the other is $\delta_{ph} \sim \varepsilon_{ph}R/el \sim 10^{-6}$.

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