estimates of the fluctuation[2)] give

$$\Delta\xi_\phi = \frac{\beta}{\pi^4\delta}\left(\frac{T}{T_c}\right)^2.$$

For example, for the garnet $Y_{2.6}Gd_{0.3}Pb_{0.1}Fe_{3.8}Ga_{0.1}O_{12}$ the parameters $\beta \sim 35$, $\delta \sim 3\times10^4$, $\alpha \sim 1.5\times10^{-10}$ cm^2.[5] Then $\Delta\xi_f \sim 10^{-5}\,(T/T_C)^2$ and $\xi_C \sim 10^{-4}/L[\text{cm}]$ (1.27). In this case, with the aid of (3.9) we can find the range of thicknesses L ($10^{-5}\text{cm} \ll L \ll (T_C/T)^2$) for which our analysis can be used.

Because of the presence of fluctuations, the results obtained for the phase transition near the Curie point are valid only for ferromagnets with long-range interaction.

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[1)] In[3] it was concluded that a first-order phase transition that is almost second-order occurs in finite samples. This is connected with the fact that the nonuniformity of the magnetization over the thickness of the plate was not taken into account in that paper.

[2)] The zero-point oscillations give a negligibly small contribution $\Delta\xi_f$.

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Directed collisions between indicator ions containing short-lived nuclei and neighboring atoms in single crystals

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We consider the spectral distribution of γ quanta (or particles) emitted by short-lived excited (or compound) nuclei that move in a crystal and are produced in nuclear reactions induced by parallel beams of monoenergetic particles. By orienting the single crystal with respect to the beam it is possible to produce, with high probability, directed collisions between ions containing short-lived nuclei and neighboring atoms of the crystal. It is found that the velocity change due to scattering can alter the spectrum significantly if the lifetime τ of the nuclei is comparable with the time of flight to the neighboring atoms. Directed collisions permit therefore observation of short-lived compound or γ-excited nuclei with $\tau \sim 10^{-16}$–10^{-14} sec and measurement of their lifetime. Possible applications of the directed-collision technique to the investigation of the local structure of crystals and of vibration dynamics are discussed.

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A method based on controlled atomic collisions in single crystals has been proposed[1,2] to measure the lifetimes of ultra-short-lived nuclei and to analyze the structure of the crystal lattice. It is well known that the crystal lattice exerts an appreciable influence on the motion of fast charged particles, ions, and atoms, as well as on the character of many atomic and nuclear processes that occur in the lattice.[3] The ordered arrangement of the atomic nuclei and of the electrons produces a large anisotropy of the electronic and nuclear stopping losses, producing relatively free "channels" in certain directions of the crystal, and practically blocking the motion in other directions. These orientational singularities of the particle motion manifest them-

selves quite illustratively in the channeling phenomenon[4,5] and in the "shadow" effects, which are widely used for the study of crystals, and the shadow effect has provided a unique possibility of directly measuring the lifetime of compound nuclei.[6]

It is no accident that the most effective methods of measuring are based on the motion and stopping of particles in matter. The characteristic times of these processes form a natural time scale—a microscopic clock for the measurement of the duration of ultra-short-lived nuclear states. Thus, the stopping time of nuclei in matter is used as a microclock to determine the lifetime of excited nuclei by measuring the Doppler shift of γ quanta in the stopping process[7,8] (this process can be analogously used to study compound nuclei). The range times $T_R \sim 10^{-13}-10^{-12}$ sec and correspondingly the times τ that lend themselves to investigation lie in the interval $10^{-14}-10^{-12}$ sec. In the shadow method, the microclock is based on the times of the displacements of the nuclei over distances that change the character of the shadows ($10^{-10}-10^{-9}$ cm); these times are of the order $10^{-9}-10^{-16}$ sec.

Thus, at the present time the least accessible to study is the region of short-lived nuclei with lifetimes $10^{-16}-10^{-4}$ sec. At such short lifetimes, the stopping process no longer can serve as a clock for the measurement of the lifetimes of the nuclei, for within times shorter than 10^{-14} sec the usual stopping mechanisms are incapable in practice to alter the velocity and direction of motion of nuclei having an energy larger than or of the order of 10^3 eV. An appreciable change in the energy and momentum of the nucleus is caused by collisions with scattering through large angles. This process can in principle be used for the measurement of τ,[1] but under ordinary conditions such collisions are quite random and have a very low probability, $\sim 10^{-3}-10^{-5}$.

Directed atomic collisions make possible large-angle scattering with large probability. Compound and excited nuclei accompanied by capture of primary particles form, at low divergence of the incident beam, a directed beam of unstable nuclei (ions); by orienting single crystals relative to the primary beam, it is possible to direct ions with ultrashort-lived nuclei along a line of sites in the lattice and cause them to collide with high probability with the nearest neighbors. If $\tau \sim 10^{-16}-10^{-14}$ sec, then for directed collisions the energy spectrum of the emitted γ quanta (particles) will depend on the time of flight to the collision and on τ, so that the lifetime of the nucleus can be measured. To the contrary, by using indicators with known τ on the order of $10^{-16}-10^{-14}$ sec, it is possible to investigate the local structure of crystals in regions on the order of 10^{-7} cm.

We consider in this article directed collisions and the possibilities of lifetime measurements and crystal-structure investigations, with (a,γ) nuclear reactions as an example. We note, however, that the method is fully applicable to compound nuclei with corresponding lifetimes, the relative change of the energy of the secondary particles being larger for the compound nuclei than for γ quanta.[2]

1. SCATTERING OF INDICATOR IONS BY VIBRATING CRYSTAL ATOMS

We consider a compound or γ-excited nucleus with energy larger than or of the order of 10 keV, produced near the zero position (0) in a crystal as a result of capture of a fast primary particle a in the nuclear reaction

$$a+A \to B^* \to B+\gamma \quad \text{or} \quad a+A \to B^* \to b+C.$$

If the beam of particles a has low divergence and the particle velocities are almost equal, then the nuclear reactions give rise to a directed beam of excited nuclei, which also have an almost constant velocity $\mathbf{v}$. We shall henceforth consider γ-excited nuclei, but all the results are essentially applicable also to compound nuclei with similar lifetimes.

By orienting the crystal relative to the beam of particles a, and consequently also relative to the beam of indicator ions B^*, we can direct the particles at a small angle α to some chain of the crystal atoms. We confine ourselves to the case when all the atoms of the chain with $n = 1, 2, 3, \ldots$ are the same and are separated from one another by an equal distance d_0, while the zeroth atom, the reaction with which produces the indicator nucleus, can generally speaking be an impurity and its zero equilibrium position can be located at a different distance d from the first atom.

When moving along the chain, the ion interacts with its atom and is scattered. We are interested in the case of sufficiently high velocities, when the quasi-classical approximation is valid and the interaction of the ion with individual atoms of the chain can be regarded independently. As a result of these interactions, the ion is deflected, after passing each atom, through a certain angle, until it leaves the cylindrical region of the effective interaction with the chain atoms.

Depending on the relation between the effective scattering amplitude a and the average thermal (or quantum) displacement of the atom u_T, the ion will have time to move past various numbers of atoms n, prior to leaving the interaction region. We shall consider mainly the case when the cross section $\sigma = \pi a^2$ for scattering through an angle larger than a/d_0 is large enough, so that the condition $u_T \lesssim a$ is satisfied. What occurs mainly here is scattering by the first atom of the chain ($\bar{n} \approx 1$) at once through angles $\theta > \theta_0 = a/d_0$. At $\bar{n} \approx 1$, the probability $w(\theta)$ of scattering the indicator ion (i.e., the ion whose nucleus is in the excited state), through an angle θ is determined in the quasi-classical approximation by the expression

$$w(\theta) = 2\pi p(\theta) \left| \frac{dp(\theta)}{d\theta} \right| W_1[p(\theta)]g; \quad g = e^{-d/u_T}. \tag{1}$$

Here $p(\theta)$ is the impact distance corresponding to scattering through an angle θ, and $W_1(p)$ is the probability of a given value p and is determined by the distribution of the thermal displacements for the pair of atoms 0 and 1.

According to (1), to determine $w(\theta)$ it is necessary to

know the probability distribution of the impact distances $W_1(p)$. Assume that the ion starts its motion at the point $\mathbf{u}_0$ corresponding to the displacement $\mathbf{u}_0$ of the initial atom A from its zero position, along a line making a small angle α with the line joining the positions 0 and 1, and is scattered near the first atom (which is displaced from the site by the vector $\mathbf{u}_1$). We introduce the vector $\mathbf{p}$, drawn from the first atom perpendicular to this line, and equal to

$$\mathbf{p}=\mathbf{u}_{0\perp}-\mathbf{u}_{1\perp}-\mathbf{e}_\perp d\sin\alpha,$$

where $\mathbf{u}_{0\perp}$ and $\mathbf{u}_{1\perp}$ are the projections of $\mathbf{u}_0$ and $\mathbf{u}_1$ on a plane perpendicular to the direction $\mathbf{v}$ of the incident ion, while $\mathbf{e}_\perp$ is a unit vector in the plane containing the line 0–1 and the vector $\mathbf{v}$, and is perpendicular to the latter. Obviously, the absolute value of the vector $\mathbf{p}$ is equal to the impact distance.

We confine ourselves to the case of relatively high symmetry, when the vibrations of the zeroth and the first atoms in the XY plane perpendicular to the 0–1 direction are isotropic. In the harmonic approximation the probability distribution of the displacements of any linear combination of the coordinates of the crystal atoms, particularly $u_{0\perp}-u_{1\perp}$, is Gaussian, i.e., it can be written in the form

$$w(\mathbf{u}_{0\perp}-\mathbf{u}_{1\perp})=\frac{1}{2\pi\beta}\exp\left[-\frac{1}{2\beta}(\mathbf{u}_{0\perp}-\mathbf{u}_{1\perp})^2\right],\quad \beta=\langle(u_{0x}-u_{1x})^2\rangle. \tag{2}$$

The angle brackets denote here quantum-statistical averaging.

Substituting the presented expression for $\mathbf{p}$ in (2) and averaging over the different directions of the vector $\mathbf{p}$ in the XY plane, we obtain the following formula for the probability $W_1(p)$ of the impact distance corresponding to the scattering of the ion by the first atom:

$$W_1(p)=\frac{1}{2\pi\beta}\exp\left(-\frac{p^2}{2\beta}\right)\exp\left(-\frac{d^2\sin^2\alpha}{2\beta}\right)I_0\left(\frac{dp\sin\alpha}{\beta}\right), \tag{3}$$

where $I_0(x)$ is a Bessel function of imaginary argument.

Formulas (1) and (3) determine the angular distribution of the indicator ions and hence the energy distribution of the Doppler shifts of the emitted γ quanta. With the aid of formulas (1) and (3) we can easily find the integral of $w(\theta)$ with respect to the angles; this integral determines the joint probability w_1 of the emission of γ quanta by the nuclei of the scattered ions, as well as the probability w_0 of the emission of γ quanta by nuclei of ions that have not been deflected:

$$w_1=\int w(\theta)\,d\theta=g,\quad w_0=1-w_1=1-g. \tag{4}$$

2. FREQUENCY DISTRIBUTION OF THE γ QUANTA

We first consider the case discussed above, when the thermal displacements are small in comparison with the amplitude for the scattering through angles $\theta>\theta_0$, or else the quantity g is small, so that only the interaction of the ion with the first atom of the chain is significant, and the scattering probability is determined by formulas (1) and (3). We characterize the spectrum of the emitted photons by specifying their distribution functions $f(\omega)=N(\omega)/N_i$, where $N(\omega)d\omega$ is the number of photons in the frequency interval $d\omega$, and N_i is the number of indicator ions.

Assume first that the incident ions are directly strictly parallel to the line joining the positions 0 and 1, and we are investigating the distribution of the quanta emitted in the same direction. The emission of photons from the nuclei of ions which have not yet undergone collision leads to the appearance in the spectrum $f(\omega)$ of a narrow line $w_0f_0(\omega)$ with a combined natural and apparatus width Γ, shifted by $\omega_D=\omega_0v/c$ relative to the frequency ω_0 of the photon emitted by the immobile nucleus, and with an integral intensity w_0. The emission of phonons by the nuclei of ions that undergo scattering through an angle θ leads to a relative broad distribution in the spectrum, since the Doppler shift of such photons is equal to

$$\Delta\omega_D=v_1(\theta)v^{-1}\omega_D\cos\theta,$$

where $v_1(\theta)<v$ the velocity of the ions scattered through an angle θ (in the laboratory frame).

Thus, the Doppler shift can vary in an interval of width $\sim\omega_D$. Therefore, taking (1) into account, the combined distribution of the emitted photons in frequency is given by

$$f(\omega)=(1-g)f_0(\omega-\omega_D)+gf_1(\omega),\quad f_1(\omega)=\omega_D^{-1}\sigma(\theta)W_1[p(\theta)]\chi(\omega). \tag{5}$$

Here $\sigma(\theta)=2\pi p(\theta)\left|dp(\theta)/d\theta\right|$ is the cross section for scattering through the angle θ, ω is reckoned from the frequency ω_0, the function $f_0(\omega)$ is normalized to unity, while the angle $\theta(\omega)$ corresponding to the given Doppler shift ω, the velocity v_1, and the smooth function $\chi(\omega)$ are determined by the formulas

$$\omega(\theta)=\frac{v_1(\theta)}{v}\omega_D\cos\theta,\quad 2\rho\frac{v_1(\theta)}{v}\cos\theta=\frac{v_1^2(\theta)}{v^2}(1+\rho)-(1-\rho),$$
$$\chi^{-1}(\omega)=\frac{1}{\omega_D}\left|\frac{d\omega(\theta)}{d\theta}\right|=2(1+\rho)\left(\frac{v_1}{v}\right)^3\sin\theta\left[\left(\frac{v_1}{v}\right)^2(1+\rho)+(1-\rho)\right]^{-1}, \tag{6}$$

$\rho=m_0/m$, where m_0 is the mass of the indicator ion and m is the mass of the scattering atom.

In particular, in the hard-sphere model we have

$$f_1(\omega)=\frac{\sigma}{2\omega_D}(1+\rho)W_1[p(\theta)]Y\left(\frac{\omega}{\omega_D}\right),$$
$$Y(x)=1\ \text{at}\quad (\rho-1)/(\rho+1)<x<1,$$

where σ is the total scattering cross section and $Y(x)=0$ outside the indicated interval.

Analogously, if the photons are observed in a direction perpendicular to the ion motion, we have $\Delta\omega_D=0$ for the photons emitted prior to the ion scattering and for the photons emitted after the scattering we have

$$\Delta\omega_D=\frac{v_1(\theta)}{v}\omega_D\sin\theta\cos\varphi$$

(φ is the azimuthal angle) and

$$f(\omega)=(1-g)f_0(\omega)+\frac{gv}{\pi\omega_D}\int_{\theta_1}^{\theta_2}\frac{\sigma(\theta')W_1[p(\theta')]d\theta'}{[v_1^2(\theta')\sin^2\theta'-v_1^2(\theta_1)\sin^2\theta_1]^{1/2}}$$

where

$$v_1(\theta_{1,2})v^{-1}\sin\theta_{1,2}=\omega/\omega_D.$$

3. USE OF THE METHOD OF DIRECTED COLLISIONS

The line width $w_0 f_0(\omega)$ in the spectrum can be much smaller than the width $\sim\omega_D$ of the smooth distribution of the γ quanta emitted by the scattered ions. For example, if $v/c=10^{-2}$ and the γ-quantum energy is $\omega_0=100$ keV, then $\omega_D=10^3$ eV, and the apparatus broadening can be equal to 10^2 eV. It is then possible to separate experimentally the peak from the smooth distribution and to obtain the ratio of their integral intensities:

$$\frac{w_0}{w_1}=\frac{1-g}{g}=e^{d/v\tau}-1.$$

Since d and v are known, knowledge of this quantity makes it possible to determine the lifetime τ of the excited state of the nucleus.

In practice, the intensity ratio can be measured if $0.1<d/v\tau<2$ (at smaller $d/v\tau$ the integral intensity of the peak is smaller than 0.1 of the total intensity and is difficult to separate, while at large $d/v\tau$ the intensity of the smooth distribution becomes exponentially small). Therefore the lifetimes of the indicator nulcei should lie in the interval

$$10d/v>\tau>d/2v$$

or 4×10^{-15} sec $>\tau>3\times10^{-17}$ sec at $d=2$ Å and 3×10^8 cm/sec $>v>5\times10^7$ cm/sec. The upper limit of this interval, as will be shown below, can be raised almost one more order of magnitude by using collisions not with the nearest neighbors but with more remote atoms.

Naturally, at noticeable ratios Γ/ω_D, both the peak and the smooth distribution become smeared out and overlap. Therefore at $\Gamma\sim\omega_D$ the separation of the peak from the smooth distribution on the experimental curve of the γ-quantum spectrum (which is a convolution of the function $f(\omega)$ and of the apparatus-broadening function) becomes difficult. Obviously, preliminary reduction of the experimental data is necessary in this case, to eliminate the instrumental smearing and to separate the "physical" distribution $f(\omega)$. This reduction can be carried out, in particular, with a computer if the values of Γ and the shape of the apparatus-broadening curve are known.

The character of the smooth distribution $f_1(\omega)$ depends essentially on the value of β, i.e., on the thermal and quantum displacements of the crystal atoms. For example, at $\alpha=0$ we have in accordance with (5) and (3)

$$f_1(\omega)=\frac{1}{\omega_D}\sigma[\theta(\omega)]\chi(\omega)\frac{1}{2\pi\beta}\exp\left[-\frac{p^2(\theta(\omega))}{2\beta}\right]. \qquad (7)$$

At low temperatures, much lower than the Debye temperature, for suffuciently heavy atoms, there can be realized the case $a_0\gg u_T$, where a_0 is the scattering amplitude at small $p(\theta)$ (for copper, e.g., $\langle u_{0x}^2\rangle=0.17\cdot10^{-2}$ Å^2 at $T=0$, and a_0 can be larger than 0.1 Å). Then, as seen from (6) and (7), $f_1(\omega)$ is significant only at frequencies greatly differing from ω_D (far from the peak), corresponding to small impact distances $p(\theta)$. The width of the function $f_1(\omega)$ is much smaller in this case than ω_D, and two weakly overlapping peaks will be observed in the spectrum, $f_1(\omega)$ (with maximum at $\omega_D(\rho-1)/(\rho+1)$ and $f_0(\omega)$ (with maximum at ω_D). It is obvious that this should facilitate the separation of the "collisionless" $f_0(\omega)$ and "collision-dominated" $f_1(\omega)$ parts of the spectrum. At large β, however, (for example, at higher temperatures), collisions with appreciable impact distances become possible, i.e., the scattering angles and the frequencies ω lie in large intervals ($\sim\omega_D$ for ω).

If the scattering cross section $\sigma(\theta)$ (and consequently also $p(\theta)$) is known, then an analysis of the smooth distribution $f_1(\omega)$ by formulas (6) and (7) makes it possible to determine the value of β and its temperature dependence. According to (2), it is determined by the thermal displacements, but is connected not with the mean squared displacements of the individual atoms (like the Debye-Waller factor, which is obtained from x-ray or Mössbauer experiments), but with the relative displacements of the pair of neighboring atoms and contains information not only on the mean squares $\langle u_{0x}^2\rangle$ and $\langle u_{1x}^2\rangle$, but also on the correlation function $\langle u_{0x}u_{1x}\rangle$ of the displacements (according to a known estimate,[9] if the indicator ion coincides with the lattice ions, then β is 30–40% smaller than $2\langle u_{0x}^2\rangle$). Therefore the instrumental determination of β, especially in the case of impurity indicator ions, can yield new useful information on the vibrations of the crystal and impurity atoms.

Conversely, if β is assumed known, then formulas (6) and (7) make it possible to determine the angular dependence of $\sigma(\theta)$ from the experimental distribution $f_1(\omega)$ for $\alpha=0$ at $\beta\sim a_0^2$. In the case of very small $\beta\ll a_0^2$, it is convenient for this purpose to measure $f_1(\omega)$ at different α (different crystal orientations). According to (3) and (5), in this case at $d\sin\alpha\gg\sqrt{\beta}$ the relatively sharp peak $f_1(\omega)$ is located at a certain frequency $\omega_m(\theta)$ and corresponds to $p(\theta)=d\sin\alpha$. Since the position of the peak determines in accordance with (6), the scattering angle θ, while the angle α determines the impact distance $p(\theta)$, it follows that at small β it is possible to obtain $\sigma(\theta)$ without knowing the value of β.

With increasing α, the value of $p(\theta)\approx d\alpha$ increases and the scattering angle θ decreases, so that the maximum α_m of the peak of the γ quanta emitted by the scattered ions approaches the frequency ω_D of the γ quanta emitted by the unscattered ions. In order for the directivity of the collisions to manifest itself and for the effect to be observable, the distance $\omega_D-\omega_m$ between peaks should exceed a quantity on the order of (or somewhat smaller than) the apparatus width Γ. According to (6), at small θ this condition takes the form

$$\frac{\omega_D-\omega(\theta)}{\omega_D}=\frac{1}{2}(\rho+1)\theta^2(p=d\alpha)>\frac{\Gamma}{\omega_D},\quad \beta<a_0^2. \qquad (8)$$

The condition (8) ceases to be satisfied starting with $\alpha = \alpha_c$.

By determining $\theta(p)$ for the well-known Lindhard potential $V_L(r)$ [10]:

$$V_L(r) = \frac{Z_1 Z_2 e^2}{r}\left[1 - \frac{r}{(r^2+3a_L^2)^{1/2}}\right],$$

we obtain at $\theta^* < 1$:

$$\theta^* = 3ba_L^2/p(p^2+3a_L^2), \qquad a_L = 0.885 a_B (Z_1^{2/3}+Z_2^{2/3})^{-1/2}, \quad b = 2Z_1Z_2e^2/m_0v^2, \tag{9}$$

where θ^* is the scattering angle in the c.m.s., a_L is the screening radius, a_B is the Bohr radius, and b is the collision diameter. It is known that

$$\operatorname{tg}\theta = (\rho+\cos\theta^*)^{-1}\sin\theta^*,$$

i.e., at small ρ we have $\theta \approx \theta^*$, and at large ρ we obtain $\theta \approx \rho^{-1}\sin\theta^*$.

We write down the condition (8) in the form

$$\frac{9}{2}(\rho+1)\frac{a_L^4 b^2}{d^2\alpha^2(d^2\alpha_c^2+3a_L^2)^2} > \frac{\Gamma}{\omega_D}.$$

For example, at $Z_1 = 10$, $Z_2 = 50$, $m_0v^2/2 = 100$ keV, when $a_L = 0.11$ Å and $b = 0.072$ Å, this estimate yields $\alpha = \alpha_c = 0.04$ for $\rho = 0.2$, $d = 2.5$ Å, and $\Gamma/\omega_D = 0.2$. At larger angles $\alpha > \alpha_c$, when the last condition ceases to be satisfied, the spectra of the γ quanta emitted by the scattered and unscattered ions merge and the collisions can be regarded as non-directed.

From the foregoing estimates we see that the directivity effect of the collisions manifests itself in a narrow interval of angles and has a strong dependence on the crystal orientation. These effects become most strongly pronounced for atoms that are located near the initial position of the indicator ion. Obviously, the directions to these atoms depend on the initial position. Therefore an investigation of the characteristic dependences of the considered effects on the crystal orientation relative to the beam direction provides a method of determining the initial positions of the ions that serve as indicators. This method can be particularly useful if the indicator ions are impurity ions.

In solid solutions, the values of ρ, a_L, and u_T for atoms of different types are different, and the resultant distribution $f_1(\omega)$ corresponds to a superposition of the distributions that are formed in the scattering of ions by atoms of different sorts. The fraction of the corresponding contributions is proportional to the probability of finding these atoms near the initial position of the indicator ion. Therefore an analysis of $f_1(\omega)$ (which is simpler in the case of strongly differing ρ and a_L) could provide a method of investigating the short-range order in solid solutions, particularly the local short-range order near the impurity indicator ions.

We note that if the lifetime of the indicator ion is $\tau \gg d/v$ but much shorter than the stopping time of the scattering ions (which can be $\sim 10^{-12}$ sec), then the spectral distribution $f(\omega)$ is practically independent of τ, but retains the singularities connected with the directed collisions, since the scattered ions hardly have time to be stopped prior to the radiation (the correction for the stopping within a time τ can be easily taken into account). Then, if the angles α are large ($\alpha \gg \alpha_c$) the distribution $f(\omega)$ reduces to $f_0(\omega - \omega_D)$. On the other hand, if $\omega \ll \omega_c$, then according to (1) and (5) the narrow peak in the spectrum vanishes in our case $d/v\tau \ll 1$, and only the distribution $f_1(\omega)$ remains. By observing orientational effects of this type, it is therefore possible to determine the initial positions of the indicator ions in the lattice, the short-range order in solid solutions, and the differential scattering cross sections with the aid of the more abundant nuclei with noticeably larger $\tau \sim 10^{-13}$ sec. It is obvious that the excited-state lifetime itself drops out in this case and cannot be determined.

To determine the lifetimes of nuclei with relatively large τ, it may be useful to employ collisions not with the nearest neighbors but with more remote atoms. They should be chosen such that the deflections of the ion by atoms lying closer to the initial position do not violate the conditions under which the collisions with the considered atoms can be treated in the impact approximation. This can be done because of the smallness of the ratios a/d and u_T/d. Consider, for example, a geometry in which the atom forming the indicator ion is located initially in the (000) site of a body-centered cubic lattice and the ion beam is directed towards the sites $(022n_0)$. The largest deflection of the ion by the nearest atoms takes place on passing through the site (002). In the case of potential (9) at $d_0/n_0 > \sqrt{3}a_L, u_T$, this deflection displaces the ion from the $(022n_0)$ site by an amount

$$\delta \approx 3ba_L^2 n_0^4/d_0 \qquad (n_0 \gg 1)$$

(d_0 is the length of the edge of the cubic cell), which is of the order of $3\times10^{-4}n_0^4$ Å for the parameter values given above. If $\delta < u_T$, then these deflections have practically no effect on the distribution $f_1(\omega)$, and if $a > \delta > u_T$, then the shape of the smooth distribution changes, but the ratio of its intensity to the intensity of the peak remains unchanged, i.e., the considered method for determining τ can be used. If, for example, $a \approx u_T \approx 0.1$ Å, then it is possible to use in this manner collisions by atoms with $n_0 = 4$, and to increase the upper limit of the interval of possible values of τ by 3–4 times in comparison with the case of collisions at the nearest neighbors. For the "more free" direction to the site $(2n_0+1,\ 2n_0+1,\ 1)$ the ion is deflected most strongly by the (220) atom and δ decreases by a factor of two, while the possible value of n_0 increases by $2^{1/4}$ times. Obviously, to use collisions with larger n_0 we need a particularly small divergence of the initial beam, smaller than $a_L/n_0d_0 \sim 0.2$–$0.3°$.

The condition for the applicability of the results obtained under the assumption that the ion collides effectively only with the first atom of the chain can be written in the form $p^2(\theta_0) > 2\beta$, where $\theta_0 = p(\theta_0)/d$ (a second requirement, $m/m_0 \gg a_L/d$, is practically always satisfied). For the Lindhard potential, according to (9) it takes at $bd > 3a_L^2$ the form

$$\beta < (bd)^{1/2} a_L. \qquad (10)$$

If the indicator ion is produced in the site of a one-atom crystal lattice and the force constants of the initial atom are close to the force constants of the crystal atoms, then at a temperature higher than the Debye temperature, in accordance with the known estimate (see, e.g., [9]), we have

$$\beta \approx {}^2/_3 10^{-2} T d_c^2 / T_{melt}$$

where d_c is the distance between the neighboring atoms of the lattice and T_{melt} is the melting temperature. According to this estimate, β is relatively small, amounting to 0.04 Å^2 at $T = T_{melt}$ and $d_c = 2.5$ Å. For the parameter values given above, $Z_1 = 10$, $Z_2 = 50$, and $m_0 v^2/2 = 100$ keV, the condition (10) is therefore satisfied even at the melting temperature. If $T \ll T_{melt}$, then the inequality (10) is valid also for smaller Z or for larger ion energies. The condition (10) can be violated only in the case of small Z_1 and Z_2 and in the case of very high energies at $T \sim T_{melt}$.

Observation of directed atom-nucleus collisions does not impose any particularly stringent requirements on the experiment from the point of view of the nuclear technology. With the aid of modern detectors one can measure reliably Doppler shifts at nuclear velocities $v \lesssim 0.001c$. [8] It is possible to use also other methods of recording the directed collisions (see below).

At the same time, the use of directed collision imposes more stringent requirements on the perfection of the crystals and their surface quality, and makes it necessary to take into account the danger of radiation damage to the crystal by the charged particles. These requirements are analogous to those imposed on crystals in experiments aimed at observing channeling and the shadow effect (the preparation of crystals for the observation of the shadow effect is discussed, for example, in [11]). The crystal surface quality is significant, particularly, because directed collisions occur principally near the crystal surface.

Indeed, owing to multiple scattering through small angles, the primary-particle beam will lose its directivity, and its divergence angle will increase. When the beam angular divergence due to multiple scattering exceeds the value α_c, the primary particles begin to produce compound nuclei that do not satisfy the condition for the directed collisions. In order for the number of these nuclei to be small, it is necessary to restrict the divergence of the initial beam, and consequently of the beam of the compound nuclei, to the value

$$\overline{\theta_d^2}(t) \leqslant \alpha_c^2, \qquad (11)$$

where $\overline{\theta_d^2}$ is the mean-squared divergence of the initial beam, due to the multiple scattering, t is the depth of the crystal, up to which the compound nuclei satisfy the condition of directed collisions.

We use the theory of multiple scattering to estimate $\overline{\theta_d^2}$ and t (see [12]). Although the theory pertains to random positions of the scattering centers, it provides good estimates in our case, since the natural beam divergence α and the small-angle scattering lead to randomization of the collisions with the nuclei. The theory yields [12]

$$\overline{\theta_d^2} \approx 0.04 \frac{Z^2 Z_2^2 t}{A\varepsilon} \ln \frac{7800 (Z_2+1) Z_2^{1/3} Z^2 c^2 t}{v^2 A (1+3{,}35\alpha_1^2)}, \qquad (12)$$

where Z_1 and Z_2 are respectively the charges of the primary nuclei and the target nuclei, A is the mass number of the target nuclei, ε is the energy of the primary particles, v is the particle velocity, and $\alpha_1 = ZZ_2^2/\hbar v$. Here ε is in MeV and t is in g/cm^2.

It follows from (11) and (12) that for $\alpha_c = 4 \times 10^{-2}$, $Z_2 \lesssim 30$, $Z = 1$, and $\varepsilon = 1-3$ MeV the condition for directed collisions is satisfied at a depth $t \sim 5 \times 10^{-4}$ g/cm^2 or $t \sim 1$ μ. This, however, does not mean that it is necessary to use only thin crystals. It may be convenient to use thicker crystals but to choose the energy of the incident particles near resonance or near the reaction threshold such that the particles no longer cause the reaction after being scattered through angles $\theta_d \approx \alpha_c$ and experiencing the corresponding energy losses.

Let us explain this by using the resonance reactions (p,γ) as an example. The crystal thickness t corresponds to stopping losses $\delta\varepsilon \approx (\partial\varepsilon/\partial x)t$, where $\partial\varepsilon/\partial x$ is a function of the specific stopping losses. If the initial proton energy ε_0 satisfies the condition $\delta\varepsilon = \varepsilon_0 - \varepsilon_r \lesssim (\partial\varepsilon/\partial x)t$, where ε_r is the proton resonant energy and t is defined by condition (11), then all the nuclei produced in the reaction will correspond to the condition of directed collisions. Actually $\delta\varepsilon \sim 10^4 - 10^5$ eV, i.e., $\delta\varepsilon/\varepsilon_0 \gtrsim 1$ % (the beam in electrostatic accelerators is monoenergetic to a much higher degree). Thus, experiments aimed at observing directed atom-nucleus collisions impose no new exceptional requirements on the experiment, i.e., on the particle beams, radiation detectors, and the crystals. The same accelerator and measurement techniques used to study the lifetimes of nuclei stopped in amorphous targets and to study channeling in simple crystals are sufficient for the observation of directed collisions.

The choice of the (p,γ) reactions that can be investigated by the method of directed atomic collisions can be based on theoretical considerations as well as on data from other experiments. According to theoretical estimates, excited levels with lifetimes $\tau < 10^{-14}$ sec should be expected for the $E1$ and $M1$ transitions and for quantum energies $\varepsilon_\gamma \gtrsim 1$ MeV (see, e.g., [13]). The lifetimes of the excited states, which have turned out to be too short in measurements by the method of attenuation of the Doppler shift, should be verified by the method of directed collisions.

We present examples of (p,γ) reactions that can be used for such measurements.

1. $p + {}^{12}C \rightarrow {}^{13}N + \gamma$ [14]

In the excited $^{13}N^*$ nucleus there is a level with a lifetime $\tau_\gamma \approx 2 \times 10^{-15}$ sec.

2. $p + {}^{16}O \rightarrow {}^{17}F + \gamma$ [15]

The resonant energies of the protons correspond to a number of excited states with radiative "widths" $\Gamma_\gamma > 1$ eV, i.e., with lifetimes $\tau_\gamma < 10^{-15}$ sec.

3. $p + {}^{23}\mathrm{Na} \to {}^{24}\mathrm{Mg} + \gamma$ [16]

The excited nucleus ^{24}Mg* has three levels with lifetimes $\tau_\gamma < 5 \times 10^{-15}$ sec.

4. $p + {}^{42}\mathrm{Ca} \to {}^{43}\mathrm{Sc} + \gamma$ [17]

In the ^{43}Sc nucleus there are three levels with $\tau_\gamma < 5 \times 10^{-15}$ sec.

The effectiveness of directed collisions depends on the total probability of scattering of the excited nuclei through large angles, larger than θ_0 ($\theta_0 \leq \theta_x \leq \pi$). This probability

$$\bar{w}(\theta_0) = \int_{\theta_0}^{\pi} W_1[p(\theta)]\, d\theta$$

can be made large enough by a suitable choice of the collisions with definite scattering nuclei.

For the reactions $^{12}\mathrm{C}(p,\gamma)^{13}\mathrm{N}$, $^{23}\mathrm{Na}(p,\gamma)^{24}\mathrm{Mg}$, $^{42}\mathrm{Ca}(p,\gamma)^{43}\mathrm{Sc}$ the probability is $w(\theta_0) \geq 0.3$ if $Z \gtrsim 30$. In the reaction $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$, owing to the large value of the kinetic energy of the ^{17}F* nucleus (~ 1 MeV), it is necessary to produce collisions between the ^{17}F* nuclei and atoms having a large charge, for example, in crystals containing lead atoms. If the directed collisions are produced with the aid of (n,γ) reactions by neutrons of energy $\varepsilon_n \sim 10^5$–10^6 eV, then the limitations on the crystal thickness are inessential, since $t \sim 0.1$ cm. As shown by estimates, in the study of (n,γ) reactions it is necessary to have neutron fluxes $J_n \sim 10^5$–10^7 neut/sec with divergence angles $\alpha \sim 2°$. It is less important to have the neutrons monoenergetic. The neutron energy spread $\Delta\varepsilon$ introduces an error $\Delta\varepsilon/2\varepsilon$ into the measurements of τ, i.e., if we allow an error of $\sim 10\%$ in the determination of τ, we can use a sufficiently large part of the neutron spectrum.

So far we have considered reactions of the type $A(a,\gamma)B$, in which the excited nuclei have the same direction as the primary-particle beam. In reactions of the type $a + A \to B^* + b \to C^* + \gamma \to C$ it is also possible to separate excited nuclei C^* having a definite direction of motion, and to cause these nuclei to collide with neighbors. This is done in standard fashion with coincidence systems. The direction of the primary beam is in this case not the same as that of the atomic collisions, i.e., in this case the primary beam can be introduced into the crystal at different angles to the direction of the collisions of the nuclei C^* with the neighbors. It is important that the probability of the distribution of the impact parameters for the nuclei C^* depends in such a collision scheme not only on β but also on the displacement of the nuclei B^*, which is due to capture of the particle a, i.e., on the lifetime τ_B* of the nuclei B^*.[18]

We have considered above a scheme for measuring τ, in which the microclock is the collision direction and the indicator of the decay instant is the change of the energy distribution (Doppler shift) of the γ quanta (particles). The Doppler shift, however, is not the only possible indicator of the instant of decay. In principle it is possible also to record directly the scattering of the ions or the measurement of the angular distributions of the quanta or particles. It is also possible to use the dependence of the spatial distribution of the daughter radioactive nuclei (with large half-life) instant of decay (before and after the collision) and consequently on τ, i.e., it is possible in principle to measure with the aid of directed collisions compound-nucleus lifetimes on the order of 10^{-6}–10^{-14} sec by determining the spatial distribution of the long-lived nuclei. The method of directed atomic collisions makes it thus possible to construct different systems for the measurement of the lifetimes of ultra-short-lived nuclear states (10^{-17} sec $\lesssim \tau \leq 10^{-14}$ sec). It is expected to be useful also for the analysis of crystal structure.[19]

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