Neutron scattering by quantum magnetic excitations in normal and ferromagnetic metals

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The scattering cross section of neutrons by magnetic excitations in the electron liquids of normal and ferromagnetic metals is determined. When the quantization of the orbital motion is inessential, the scattering by Stoner excitations gives rise to the broad line in the energy spectrum of scattered neutrons. In a quantizing magnetic field the broad line splits into a number of narrow lines. Scattering lines corresponding to quantum spin waves are located between scattering lines due to Stoner excitations. The conditions are found under which the scattering cross section by quantum waves is comparable to the scattering cross section by quasiclassical spin waves and is of the order of 1 barn for ferromagnetic metals.

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1. The quantization of the energy levels of the electrons of a metal in a magnetic field, which manifests itself in the discrete values on the Fermi surface, of the electron velocities along the direction of the magnetic field, imposes definite hindrances on the collisionless damping of the waves. The observation of this effect under various conditions has been the subject of a large number of investigations. Gurevich, Skobov, and Firsov[1] have founded the theory of giant quantum oscillations of wave absorption, in which the hindrance on the Landau damping, due to the Cerenkov effect on electrons, manifests itself. Giant oscillations of helicon absorption was first considered by Skobov and Kaner.[2] The corresponding effect for bound magnon-helicon waves was discussed by Glick and Callen.[3]

On the other hand, besides the effect of giant absorption oscillations, it has been theoretically predicted that quantum waves can occur in the regions where the collisionless absorption is forbidden. In an electron gas, such waves were considered independently in a number of papers.[4–8] It was subsequently shown[9] that allowance for the interelectron interaction leads to the possibility of predicting new types of quantum waves, including quantum spin waves (QSW). Finally, it was indicated in[10] that a new type of quantum spin waves can exist in ferromagnetic metals.

We must dwell briefly on the difference between the quantum effect considered in[10] and the effects due to the coupling of spin waves with electromagnetic waves and investigated in[3,11]. Giant oscillations of magnon absorption[3,11] are determined by the hindrance rules for single-particle excitations without spin flips. The role of such excitations, however, is exceedingly small under the conditions of weak coupling between the electromagnetic and spin waves. To the contrary, the pure ferromagnetic quantum effects discussed in[10] give rise to quantum spin waves and to really giant quantum oscillations of magnon absorption, due to the strong interaction of the magnons with the Stoner excitations, i.e., with single-particle excitations with spin flip. As a result of these quantum effects, it turns out[10] that a number of transparency regions or windows appear in the ordinary (quasiclassical) region of the Stoner excitations, and in these regions the existence of such single-particle excitations is forbidden.

When speaking of the experimental possibilities of the discussed effects, we note that experiment has revealed giant helicon-absorption quantum oscillations long ago by Libchaber and Grimes.[12] To the contrary, quantum waves have not yet been observed. The main reason is that in the usually observed integral characteristics of metals, e.g., in the surface impedance, it is difficult to separate the contribution of the quantum waves.

On the other hand there exists a number of experimental methods that make it possible to determine directly the excitation spectrum of a solid.[13,14] One such method, which made it possible recently to investigate in detail the properties of classical spin waves in ferromagnets—magnons, is inelastic scattering of slow neutrons.[15–18] One can hope the investigation of inelastic scattering of neutrons to be useful also in the study of quantum spin waves in metals.

The present paper is devoted to a theoretical analysis of inelastic scattering of slow neutrons by magnetic excitations in normal and ferromagnetic metals. Principal attention will be paid below to revealing the effects of orbital quantization of electrons in the scattering spectrum of neutrons. Such effects manifest themselves most clearly under conditions when magnetic excitations propagate along the direction of the magnetic field. This is precisely why the theory developed below is devoted to inelastic scattering of neutrons, when the vector of the variation of the momentum in the case of scattering is parallel to the magnetic field.

2. The differential cross section for inelastic scattering of neutrons by magnetic excitations is determined by the imaginary part of the magnetic susceptibility $\chi^{\pm}(\omega, k)$[15]:

$$\frac{d^2\sigma}{d\Omega' d\varepsilon_{p'}} \equiv h(\omega, k) = \frac{(\gamma r_0)^2}{4\pi\mu^2}\frac{p'}{p}(N_\omega+1)\,\mathrm{Im}[\chi^+(\omega, k)+\chi^-(\omega, k)]. \quad (2.1)$$

Here $\gamma = -1.913$ is the neutron gyromagnetic ratio; r_0 and μ are the classical radius and the magnetic moment of the electron. $N_\omega = [e^{\hbar\omega/T} - 1]^{-1}$; $\mathbf{p}$, ε_p and $\mathbf{p}'$, $\varepsilon_{p'}$ are the momentum and energy before and after the scattering; $\hbar\omega = \varepsilon_p - \varepsilon_{p'}$; $\hbar\mathbf{k} = \mathbf{p} - \mathbf{p}'$ is assumed to be parallel to the magnetization.

The magnetic susceptibility in the case when the Fermi-liquid interaction is described by a single constant ψ characterizing the spin part of the interaction takes in the presence of a constant external magnetic field the form[9]

$$\chi_\pm(\omega, k) = \frac{\mu^2 S_\pm(\omega, k)}{(\psi - 4\pi\mu^2) S_\pm(\omega, k) - 1}, \qquad (2.2)$$

$$S_\pm(\omega, k) = \frac{2|e|B}{(2\pi\hbar)^2 c} \sum_n \int dp \cdot \frac{n_F[\varepsilon_n^+(p \mp \hbar k)] - n_F[\varepsilon_n^-(p)]}{\pm\hbar\omega + \varepsilon_n^+(p \mp \hbar k) - \varepsilon_n^-(p) \pm i\hbar\tau^{-1}} \qquad (2.3)$$

where $n_F(\varepsilon)$ is the Fermi distribution function; $\varepsilon_n^\sigma(p) = \varepsilon_n(p) - \sigma\hbar\Omega_0/2$ are the energy levels of the electrons (holes) in the magnetic field, n, p, and σ are the quantum numbers of the Landau representation, Ω_0 is the frequency of the spin splitting of the energy levels, and τ is the quasiparticle momentum relaxation time. In normal metals, the frequency Ω_0 is determined by the magnetic field[19] B:

$$\Omega_0 = 2\mu B / (1 + B_0)\hbar,$$

where $B_0 \equiv \psi\nu(\varepsilon_F)$; $\nu(\varepsilon_F)$ is the electron state density on the Fermi surface. In ferromagnets, Ω_0 is due entirely to the exchange interaction of the quasiparticles.[20,21]

Introducing the real and imaginary parts of the quantity

$$S_\pm(\omega, k) = S_\pm'(\omega, k) + i S_\pm''(\omega, k),$$

we obtain for the scattering cross section

$$h(\omega, k) = h^+(\omega, k) + h^-(\omega, k),$$

$$h^\pm(\omega, k) = -\frac{(\gamma r_0)^2}{4\pi} \frac{p'}{p} (N_\omega + 1) \frac{S_\pm''}{[(\psi - 4\pi\mu^2) S_\pm' - 1]^2 + (\psi S_\pm'')^2}. \qquad (2.4)$$

We are interested in the case of low temperatures, $T < \hbar\Omega$, where Ω is the cyclotron frequency. In this situation we can confine ourselves to scattering processes with production of magnetic excitations.

The electron fluid of metals contains magnetic excitations of two types. First are the single-particle excitations with spin flips. By analogy with ferromagnetic metals, we shall call these Stoner excitations also in the case of normal metals. In the region of the Stoner excitations, the imaginary part of the magnetic susceptibility is due mainly to the poles of the integrand in formula (2.3) for the quantity $S_\pm(\omega, k)$; these poles occur at frequencies $\hbar\omega = \varepsilon_n^\mp(p) - \varepsilon_n^\pm(p - \hbar k)$. In addition to the single-particle excitations, in both ferromagnetic and normal metals there exist collective magnetic excitations—spin waves. The dispersion curves of the spin waves, the spectrum of which is determined by the poles of the magnetic susceptibility (2.2), lie on the (ω, k) plane outside the region of the Stoner excitations. The imaginary part of the quantity $S_\pm(\omega, k)$ is due to the finite relaxation time τ. When the dispersion curves fall in the region of the Stoner excitations, the spin waves are strongly damped because of the decay into Stoner excitations (Landau damping).

In the quasiclassical case, when the distance between Landau levels is much less than their width due to the momentum relaxation time τ, the quantization of the orbital motion is inessential. The spin-wave spectrum contains in this case a quasiclassical branch, the dispersion curve of which terminates on the quasiclassical boundary of the region of the Stoner excitations.[19,22]

In the quantum case there appear in the quasiclassical region of the Stoner excitations a large number of regions called transparency windows, in which the imaginary part of $S_\pm(\omega, k)$ vanishes in the approximation $\tau^{-1} = 0$. Besides the classical spin wave, it now becomes possible for new collective excitations to propagate, namely quantum waves whose dispersion curves lie in the transparency windows.

The presence of collective and single-particle excitations in the electron liquid leads to the presence of scattering lines of various types in the spectrum of the scattered neutrons. Besides the lines due to scattering by spin waves, there appear also lines due to scattering by the Stoner excitations. The scattering cross section (2.4) has a maximum at values of ω and k satisfying the condition

$$(\psi - 4\pi\mu^2) S_\pm'(\omega, k) - 1 = 0. \qquad (2.5)$$

Outside the region of the Stoner excitations, this equation determines the spin-wave spectrum.[9] The width of the maximum of the scattering cross section is determined in this case by the small quantity τ^{-1}. Equation (2.5) has solutions also in the region of the Stoner excitations. The poles of the magnetic susceptibility (2.2) have in this case a large imaginary part and do not correspond to any definite collective mode, but the fluctuations of the spin density near such solutions are large, and it is this which leads to an increase of the scattering cross section.

The different nature of the spin splitting of the energy levels in normal and ferromagnetic metals leads to a substantial difference of both their spin-wave spectra and their Stoner-excitation spectra. Accordingly, the cases of normal and ferromagnetic metal will be considered separately. With an aim at revealing the main regularities of neutron scattering by quantum spin waves, we shall assume the quasiparticle dispersion to be isotropic and quadratic.

3. Being interested in neutron scattering in a normal electron fluid, we consider first the case when the quantization of the energy levels of the electrons in a magnetic field can be neglected. The summation over n in (2.3) can then be replaced by integration and we obtain for the real part of $S_\pm(\omega, k)$

$$S_\pm'(\omega, k) = -\nu(\varepsilon_F)\left\{1 \mp \frac{\omega}{2kv_F} \ln\left|\frac{\pm\omega - \Omega_0 + kv_F}{\pm\omega - \Omega_0 - kv_F}\right|\right\}. \qquad (3.1)$$

The region of the Stoner excitations is determined by the inequalities

$$|\omega \mp \Omega_0| \leqslant kv_F. \qquad (3.2)$$

Inside this region, the imaginary part of $S_\pm(\omega, k)$ is

$$S_{\pm}''(\omega, k) = -\nu(\varepsilon_F)\pi\omega/2kv_F. \quad (3.3)$$

The cross section for scattering by Stoner excitations is described by formulas (2.4), (3.1), and (3.3). The Fermi-liquid interaction manifests itself in the form of the line of the scattering by the Stoner excitations, which has a maximum at energies $\hbar\omega_S(k)$ determined by Eq. (2.5). The magnitude of this maximum depends directly on the Fermi-liquid interaction, and its value at $\psi \gg 4\pi\mu^2$ is

$$\frac{(\gamma r_0)^2}{4\pi}\frac{p'}{p}(N_{\omega_S}+1)\frac{\nu(\varepsilon_F)}{B_0^2}\frac{2kv_F}{\pi\omega_S(k)}, \quad (3.4)$$

where v_F is the Fermi velocity of the electrons.

Outside the region of the Stoner excitations, Eq. (2.5) describes the well known spectrum of the quasiclassical spin waves.[19] In particular, in the long-wave case the spectrum of these waves is given by

$$\omega_S(k) = \Omega_0(1+B_0)\left[1+\frac{k^2v_F^2}{3B_0\Omega_0^2}\right]. \quad (3.5)$$

Here

$$S_{\pm}''(\omega, k) = -\nu(\varepsilon_F)\frac{\Omega_0\tau^{-1}}{(\omega\mp\Omega_0)^2} \quad (3.6)$$

and for the cross section for scattering with excitation of these spin waves we obtain

$$h_{\mathrm{sw}}(\omega, k) = \frac{(\gamma r_0)^2}{4\pi}\frac{p'}{p}(N_\omega+1)\nu(\varepsilon_F)\frac{\Omega_0\tau^{-1}}{[\omega-\omega_S(k)]^2+\tau^{-2}}. \quad (3.7)$$

Thus, at a given scattering vector $\hbar k$, the energy spectrum of the scattered neutrons consists in the quasiclassical case of a broad line of scattering by Stoner excitations and a narrow line of scattering with excitation of quasiclassical spin waves. In the limit of small scattering vectors the distance between these lines is of the order of $\hbar[\Omega_0 - \omega_S(k=0)]$. For realistic values[19] of B_0 in attainable fields ($B \sim 10^5$ G) this quantity is of the order of 10^{-4}–10^{-3} eV. The total cross section for scattering by quasiclassical spin waves is in this case of the order of 10^{-4} b.

Under the conditions of a quantizing magnetic field, relations (3.1) and (3.3), which determine the cross section for the scattering by Stoner excitations, should be replaced by the following:

$$S_{\pm}'(\omega, k) = \frac{\Omega}{2kv_F}\nu(\varepsilon_F)\cdot \sum_\sigma\sum_{n=0}^{N^\sigma}\sigma\ln\left|\frac{\pm\omega-\Omega_0+kv^\sigma(n)-\sigma\hbar k^2/2m}{\pm\omega-\Omega_0-kv^\sigma(n)-\sigma\hbar k^2/2m}\right|, \quad (3.8)$$

$$S_{\pm}''(\omega,k) = \mp\frac{\pi\Omega}{2kv_F}\nu(\varepsilon_F)\sum_\sigma\sum_{n=0}^{N^\sigma}\sigma\int_{-kv^\sigma(n)}^{kv^\sigma(n)}dx\,\delta\left(\pm\omega-\Omega_0-\frac{\sigma\hbar k^2}{2m}-x\right). \quad (3.9)$$

Here N^σ is the number of Landau levels occupied by the quasiparticles with spin σ and $v^\sigma(n)$ is the maximum longitudinal velocity of the particles at the level with energy $\varepsilon_n^\sigma(p)$. As follows from (3.9), $S_\pm''(\omega, k)$ vanishes now not only outside the quasiclassical region of the Stoner excitations (3.2), but also in the transparency windows. Relations (3.8) and (3.9) show that even in the absence of a Fermi-liquid interaction the line shapes for scattering by Stoner excitations are changed on account of the Landau quantization. A quasiclassical line with width on the order of $2\hbar kv_F$ breaks up into a number of narrow lines, the distance between which is of the order of $2\hbar k|v^+(n) - v^-(n)|$. The Fermi-liquid interaction leads to an additional change in the line shape for scattering by Stoner excitations. In particular, in each narrow line there appears a maximum of the scattering cross section at ω and k determined by Eq. (2.5), and the magnitude of the maximum depends on the Fermi-liquid interaction.[1)]

Outside the quasiclassical region of the Stoner excitations, the neutron scattering cross section is determined by the quasiclassical spin waves (see (3.7)). A new effect is produced by the presence of the transparency windows, in which quantum spin waves exist.

In the long-wave limit, the QSW spectrum takes the form $\omega_{\mathrm{QSW}}^i = \Omega_0 + c_i k$, where the phase velocities are given by

$$\sum_n \ln\left|\frac{c_i+v^+(n)}{c_i-v^+(n)}\frac{c_i-v^-(n)}{c_i+v^-(n)}\right| = 0.$$

The cross section for scattering with excitation of such waves is ($\tau^{-1}=0$)

$$h_{\mathrm{QSW}}(\omega, k) \approx \frac{(\gamma r_0)^2}{4\pi}\frac{p'}{p}(N_\omega+1)\frac{\nu(\varepsilon_F)}{B_0^2}\frac{\hbar k^2}{m}\sum_i\frac{v_F}{c_i}\delta(\omega-\omega_{\mathrm{QSW}}) \quad (3.10)$$

In the region of not too small values of k and for quantum waves of low velocity ($c_i \ll v_F$) this quantity turns out to be comparable with the cross section for scattering by quasiclassical spin waves. The distance between the line for the scattering by the QSW and the nearest Stoner-scattering line is $\hbar k|v^+(n) - c_i|$. For quantum waves of low velocity, this quantity can reach values $\sim 10^{-4}$ eV.

The presence in the scattered-neutron spectrum of lines of scattering by quantum spin waves is determined by the region of the existence of the quantum waves, which depends on the ratio of the energy of the spin splitting of the energy levels $\hbar\Omega_0$ and the energy of the cyclotron quantum $\hbar\Omega$.[9] Thus, at $\Omega_0 < \Omega$, transparency windows exist on the (ωk) plane up to $k=0$. The lines of scattering by the Stoner excitations will be separated from each other in this case and in the limit of small momentum transfers. Between these lines, peaks of scattering by quantum spin waves should be observed. In the opposite case $\Omega_0 > \Omega$ the transparency windows appear only at finite values of k, and in the limit of small changes of the momentum the spectrum of the scattered neutrons is determined by the Stoner excitations and by the quasiclassical spin waves. The narrow quantum lines of scattering by Stoner excitations will overlap; the line shape will have a complicated form determined, on the one hand, by the Landau quantization and on the other hand by the Fermi-liquid interaction of the quasiparticles. In going to larger momentum transfers, transparency windows appear, the lines of scattering by the Stoner excitations are then separated and sharp maxima due to scattering by QSW appear between them.

4. An essential difference between the electron liquid of a ferromagnet and the liquid of a normal metal is the presence of an exchange frequency Ω_0, which depends relatively little on the magnetic field.[10] The value of Ω_0 amounts in typical ferromagnets (iron, cobalt, nickel) to approximately 1 eV (see, e.g., [21]) and greatly exceeds the cyclotron frequency in attainable magnetic fields. We are interested in low-frequency excitations $\omega \lesssim \Omega$, and distinguish in this case between the quasiclassical and quantum cases. Bearing in mind that the quasiclassical collective excitations in ferromagnets are the subject of a large number of both theoretical and experimental studies, we present here only some results that seem new to us and pertain to the vicinity of the quasiclassical of the limit of Stoner excitations. Principal attention will be paid below to effects of a quantizing magnetic field. Since these effects manifest themselves most strongly under weak ferromagnetism conditions $\hbar\Omega_0 < 2\varepsilon_F$, which is apparently observed in iron,[21] it is natural to consider precisely this case. We first report the results of an analysis of neutron scattering in the quasiclassical limit, when the orbital quantization can be neglected.

In the region of the Stoner excitations

$$\pm(\omega \mp \Omega_0 - kv^{\pm} - \hbar k^2/2m) \geqslant 0, \tag{4.1}$$

we have for the real part of $S_\pm(\omega, k)$

$$(\psi - 4\pi\mu^2) S_\pm' - 1 \approx -\frac{\omega_M(k) \mp \omega}{2kv_F} \ln\left|\frac{k+k_0}{k-k_0}\right|, \tag{4.2}$$

where $\omega_M(k)$ is the quasiclassical magnon frequency,[10] v^σ is the velocity of particles with spin σ on the Fermi surface, and k_0 corresponds to the intersection of the boundaries of the region (4.1) and the straight line $\omega = 0$. The imaginary part of $S_\pm''(\omega, k)$ is given in this case by (3.3).

It follows from (2.4), (3.3), and (4.2) that the cross section for the scattering of neutrons by Stoner excitations has a maximum if $\omega = \omega_M(k)$ (cf. [22]). The width of this maximum increases with increasing momentum transfer $\hbar k$ (see Fig. 1).

Outside the region of the Stoner excitations, the real part of $S_\pm(\omega, k)$ is given as before by (4.2), while the imaginary part takes the form

$$S_\pm''(\omega, k) = \pm \frac{1}{\psi} \frac{\tau^{-1}}{2kv_F} \ln\left|\frac{k+k_0}{k-k_0}\right|. \tag{4.3}$$

Using formulas (2.4), (4.2), and (4.3) we obtain for the scattering cross section in this case

$$h_{sw}(\omega, k) = \left(\frac{\gamma r_0}{4\pi\mu}\right)^2 \frac{\Omega_M}{\Omega_0} \frac{p'}{p} (N_\omega + 1) \frac{2kv_F\tau^{-1}}{(\omega - \omega_M)^2 + \tau^{-2}} \ln^{-1}\left|\frac{k+k_0}{k-k_0}\right|, \tag{4.4}$$

where $\Omega_M = 8\pi\mu M/\hbar$, and M is the magnetization per unit volume. Formula (4.4) describes neutron scattering by quasiclassical magnons and has a sharp maximum at the frequency $\omega_M(k)$.

Under conditions when the effects of the orbital quan-

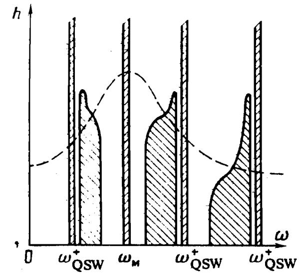

FIG. 1. Dependence of the differential neutron scattering cross section on the energy transfer in the case when the change hk of the neutron momentum by scattering is equal to $\hbar k^{(1)}(0) \equiv m[v^+(0) - v^-(0)]$ and the quasiclassical dispersion curve of the magnons falls in the transparency windows. The broad lines correspond to scattering by Stoner excitations, and the narrow lines correspond to QSW. For comparison, the figure shows the cross section for scattering by Stoner excitations in the quasiclassical case (dashed curve).

tization become significant, transparency windows, in which the imaginary part of $S_\pm''$ given by (3.9) vanishes, appear in the quasiclassical region of the Stoner excitations (4.1). On the (ωk) plane, the transparency windows are determined by the conditions

$$\pm[\Omega_0 - kv^-(n\mp1) - \hbar k^2/2m] < \omega < \pm[\Omega_0 - kv^+(n) + \hbar k^2/2m]$$
$$(n = 0, 1, \ldots, N^-). \tag{4.5}$$

The regions (4.5) begin on the axis $\omega = 0$, where they are bounded by the points $k^{(1)}(n) = m[v^+(n) - v^-(n)]$, and terminate at $\omega = \Omega$ at the points

$$k_+^{(2)}(n) = m[v^+(n) - v^-(n-1)] \text{ and } k_-^{(2)}(n) = m[v^+(n-1) - v^+(n)]$$

respectively for $S_\pm''$.

In the transparency windows, the dispersion equation of the spin waves (2.5) has solutions corresponding to the quantum spin waves. The spectrum of the QSW in ferromagnets turns out to be different for waves of different polarization. It was shown earlier[10] that the character of the spectrum of left-polarized QSW is determined by the possibility of the magnon quasiclassical dispersion curve falling in the transparency window. Namely, if the magnon dispersion curve crosses the transparency windows, then magnon-absorption giant quantum oscillations occur, similar to those predicted for ultrasound.[1] Far from the limits of the transparency window, the frequency of left-polarized quantum waves ω^+_{QSW} is close to $\omega_M(k)$. As the limits of the window are approached, the QSW dispersion curve turns out to be quite close to the corresponding limiting curve. To the contrary, if the quasi-classical curve does not fall in the transparency windows, then the QSW dispersion curve lies near the right-hand boundary of the windows.

Since there are no right-polarized quasiclassical spin waves in ferromagnets, the dispersion curve of right-polarized QSW, as follows from (2.5) and (3.8), is always adjacent to the left boundary of the transparency windows:

$$\omega_{QSW}=\omega(n)-\delta\omega, \quad \omega(n)=-\Omega_0+kv^+(n)-\hbar k^2/2m, \tag{4.6}$$

$$\delta\omega=\frac{\hbar k}{m}\frac{[k-k^{(1)}(n)][k_-^{(2)}(n+1)-k]}{k_-^{(2)}(n+1)-k^{(1)}(n+1)}$$
$$\times\left[\frac{v^+(n)}{v^-(n)}\right]^2\exp\left\{-\frac{2kv_F}{B_0\Omega}[(\psi-4\pi\mu^2)S_-'(\omega(n),k)-1]\right\},$$

where $S'_\pm$ is given by Eq. (4.2).

The contribution of the quantum waves whose frequencies are close to the quasiclassical magnon frequency $\omega_M(k)$ to the neutron scattering cross section is described by formula (4.4) under the condition

$$(\Omega\tau)^{-1}<\hbar\Omega_0/\varepsilon_F, \tag{4.7}$$

which coincides with the condition for the existence of QSW with frequency $\omega_M(k)$. The separation of the contribution made to the scattering cross section by QSW whose frequencies differ from the corresponding limiting frequencies determined by (4.5) by a small amount $\delta\omega$ is subject to the more stringent conditions

$$\delta\omega\tau\gg 1. \tag{4.8}$$

For the cross section for scattering by such quantum waves we have

$$h^{\pm}_{QSW}(\omega,k)=\left(\frac{\gamma r_0}{4\pi\mu}\right)^2\frac{\Omega_M}{\Omega_0}\frac{p'}{p}(N_\omega+1)\frac{2kv_F}{|B_0|\Omega_0}\frac{|\delta\omega|\tau^{-1}}{(\omega-\omega^{\pm}_{QSW})^2+\tau^{-2}}. \tag{4.9}$$

We note that the scattering lines (4.9) can in principle be resolved in windows with small $n\ll N^-$ in magnetic fields as low as $B\sim 10^4-10^5$ G, where $\hbar\delta\omega$ amounts to $10^{-4}-10^{-5}$ eV.

Let us discuss finally the scattering of neutrons by Stoner excitations under conditions when quantization manifests itself. From (2.4), (3.8), and (3.9) it follows that quantization of the orbital motion of the electrons, just as in the case of normal metals, leads to the appearance of a fine structure of the spectrum of the neutrons scattered by Stoner excitations in a ferromagnet. The broad quasiclassical scattering line described by Eqs. (2.4), (3.3), and (4.2) splits into a number of narrow lines located in the region and arising when the change in the neutron momentum $\hbar k$ due to scattering exceeds the minimum possible value of the wave number in the transparency windows. This condition takes the form $k>k_+^{(2)}(0)$ and $k>k^{(1)}(0)$ for $h^\pm(\omega,k)$.

At the boundaries of the transparency windows (4.5), in the approximation $\tau^{-1}=0$, we have $|S'_\pm(\omega,k)|=\infty$; the cross section for scattering by Stoner excitations vanishes in this case. The maximum of the scattering cross section in each line is determined by Eqs. (2.5) and (3.8). Just as in the transparency windows, in the region of the Stoner excitations these equations have, besides the quasiclassical solution $\omega=\omega_M(k)$, also quantum solutions adjacent to those window boundaries near which the QSW dispersion curves are located. The frequency of the quantum solution differs from the boundary frequency (see (4.5)) by the same amount $\delta\omega$ as the corresponding frequency of the quantum wave. The width of the maximum of the cross section then turns out to be of the order of $\delta\omega$. The condition for observing this maximum is analogous to the condition (4.8) for the existence of QSW whose dispersion curves are adjacent to the boundaries of the transparency windows. Far from these boundaries, the neutron scattering cross section is determined by formulas (2.4), (3.9), and (4.2). The width of the lines for scattering by Stoner excitations is of the order of $k[v^+(n-1)-v^+(n)]$, which amounts to $\sim\hbar\Omega\Omega_0/\varepsilon_F$ in lines with $n\ll N^-$ and turns out to be approximately equal to Ω if $n\sim N^-$. The distance between the scattering lines is of the same order. The number of narrow lines for scattering by Stoner excitations changes from a value $\sim\varepsilon_F/\hbar\Omega_0$ at $k=\Omega_0/v_F$ to a value on the order of unity if $k\approx(\hbar\Omega_0/\varepsilon_F)^{1/2}mv_F/\hbar$.

The lines of scattering by magnetic excitations with different polarizations may overlap. However, if the conditions (4.7) and (4.8) are satisfied, it becomes possible to separate the contribution made to the scattering cross section by the quantum waves. Indeed, in sufficiently strong fields and pure samples ($B\gtrsim 10^4$ G, $\tau\sim 10^{-9}-10^{-10}$ sec) the ratio of the differential cross section for the scattering of neutrons (4.4) by QSW with frequency $\omega\approx\omega_M(k)$ to the cross section for scattering by Stoner excitations is of the order of $\Omega\tau\gg 1$. For QSW whose dispersion curves are adjacent to the boundaries of the transparency windows, this ratio amounts to $\delta\omega\tau\gg 1$ (see (4.9)).

The most favorable conditions for the observation of QSW occur in transparency windows near the quasiclassical dispersion curve. An estimate of the total cross section (pure atom) for neutron scattering by a QSW of frequency $\omega\approx\omega_M(k)$ yields, say for iron[21,23] ($\hbar\Omega_0\approx 2$ eV, $4\pi M\approx 2\times 10^4$ G) a value on the order of 1 b. The figure shows schematically the dependence of the neutron cross section on the value of ω.

[1)]A similar situation will be considered in detail later on for ferromagnetic metals.

[1]V. L. Gurevich, B. G. Skobov, and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. 40, 786 (1961) [Sov. Phys. JETP 13, 552 (1961)].
[2]B. G. Skobov and É. A. Kaner, Zh. Eksp. Teor, Fiz. 46, 1809 (1964) [Sov. Phys. JETP 19, 1219 (1964)].
[3]A. J. Glick and E. Callen, Phys. Rev. 169, 530 (1968).
[4]M. J. Stephen, Phys. Rev. 129, 997 (1963).
[5]A. L. McWhorter and W. G. May, IBM J. Res. Dev. 8, 285 (1964).
[6]O. V. Konstantinov, V. I. Perel', and S. L. Ginzburg, Fiz. Tverd. Tela (Leningrad) 9, 2139 (1967) [Sov. Phys. Solid State 9, 1684 (1968)].
[7]S. B. Anokhin and A. S. Kondrat'ev, Uch. Zap. Leningr. Gos. Univ. Ser. Fiz. 15, 24 (1968).
[8]E. A. Kaner and V. G. Skobov, Phys. Status Solidi 22, 333 (1967).
[9]P. S. Zyryanov, V. I. Okulov, and V. P. Silin, Pis'ma Zh. Eksp. Teor. Fiz. 8, 489 (1968) [JETP Lett. 8, 300 (1968)]; Zh. Eksp. Teor. Fiz. 58, 1295 (1970) [Sov. Phys. JETP 31, 696 (1970)].
[10]V. P. Silin and A. Z. Solontsov, Zh. Eksp. Teor. Fiz. 69, 2141 (1975) [Sov. Phys. JETP 42, 1089 (1975)].
[11]V. Ya. Demikhovskiĭ and V. M. Sokolov, Izv. Akad. Nauk SSSR Ser. Fiz. 36, 1518 (1972).
[12]A. Libchaber and C. C. Grimes, Phys. Rev. 178, 1145 (1969).
[13]S. V. Vonsovskiĭ, Magnetizm (Magnetism), Nauka, 1971.
[14]A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskiĭ, Spinovye volny (Spin Waves), Nauka, 1967 [Pergamon, 1971].
[15]Yu. A. Izyumov and R. P. Ozerov, Magnitnaya neĭtronogra-

fiya (Magnetic Neutron Diffraction), Nauka, 1966.
[16]H. A. Mook, R. M. Nicklow, E. D. Thompson, and M. K. Wilkinson, J. Appl. Phys. 50, 1450 (1969).
[17]E. D. Thompson and H. A. Mook, J. Appl. Phys. 41, 1227 (1970).
[18]H. A. Mook, J. W. Lynn, and R. M. Nicklow, Phys. Rev. Lett. 30, 556 (1973).
[19]V. P. Silin, Fiz. Metal. Metalloved. 29, 681 (1970).
[20]A. A. Abrikosov and I. E. Dzyaloshinskiĭ, Zh. Eksp. Teor. Fiz. 35, 771 (1958) [Sov. Phys. JETP 8, 535 (1959)].
[21]A. V. Gold, J. Low Temp. Phys. 16, 3 (1974).
[22]E. D. Thompson, Phys. Rev. Lett. 19, 635 (1967).
[23]P. W. Shumate, R. V. Coleman, and K. C. Fivag, Phys. Rev. B1, 395 (1970).

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Investigation of the zero-gap state induced by a magnetic field in bismuth-antimony alloys

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The character of the band motion induced by a magnetic field H is investigated in $Bi_{1-x}Sb_x$ alloys in which the spectrum structure has been changed from a direct to inverted one by a hydrostatic pressure p. The measurements are carried out for alloys with a broad range of concentrations $6.6 \leq x \leq 13$ at.%; in fields H up to 65 kOe and at p up to 15 kbar. It is found that the transition to the zero-gap state induced by the magnetic field occurs only in the inverted region of the alloy spectrum for $p > p_i$. The surface of the zero-gap state in the physical parameter space (composition–pressure–magnetic field) is plotted for $Bi_{1-x}Sb_x$ alloys. By extrapolation it is found that the surface is bounded by parameter values such that $x \lesssim 40$ at.%, $p \lesssim 35$ kbar, and $H \lesssim 1500$ kOe. The directions and velocities of the mutual motion of the L and T bands for the direct and inverted alloy spectra are determined. It is found that transition to the zero-gap state induced by a magnetic field results in the isotropization of the transverse relaxation time of the L carriers.

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INTRODUCTION

A study of a new state of matter, intermediate between that of a metal and an insulator, named the zero-gap (gapless) state (ZGS), has attracted much interest in recent years. A characteristic feature of the ZGS is the absence of a direct gap ε_g in the energy spectrum. This gives rise to a number of unusual properties that cause the matter in the ZGS to differ qualitatively from a metal or an insulator.

The ZGS is the result of high symmetry of the crystal lattice. This situation is realized in gray tin (α-Sn) and also in HgTe, HgSe, HgS, Cd_3As_2, and some other compounds, which have been named natural zero-gap semiconductors. A theory of the ZGS, with a detailed analysis of the crystal symmetry at which this state can arise, was developed by Abrikosov and Beneslavskiĭ.[1]

In addition to the natural zero-gap semiconductors, a rather large class of substances is known in which vanishing of the direct gap ε_g and the transition to the ZGS take place as a result of changes in different physical parameters such as the alloy composition, temperature, pressure, and magnetic field.[2] It is particularly interesting to investigate this case inasmuch as by gradually varying the external action it is possible to observe in succession the restructuring of the energy spectrum of the initial matter as it goes over into the ZGS. At the present time, continuous transitions into the ZGS, induced by external action, have been frequently observed and investigated in solid-solution systems such as $Cd_xHg_{1-x}Te$, $Cd_xHg_{1-x}Se$, $Pb_{1-x}Sn_xTe$, $Pb_{1-x}Sn_xSe$, $Bi_{1-x}Sb_x$ etc. Transitions of these alloys in ZGS as a result of the changes in the composition or pressure have revealed an abrupt decrease in the effective masses and an increase in the carrier mobilities,[3,4] as well as anomalies connected with impurity states.[5]

Among the transitions to the ZGS induced by changes of various external parameters, the least investigated at present are transitions due to the action of a strong magnetic field. These transitions have definite features that distinguish them qualitatively from the transitions to the ZGS caused by changes of other physical parameters. The point is that in a strong magnetic field the electron system in the initial material becomes quasi-one-dimensional and polarized. The end points of the conduction band and of the valence band are determined in this case by the Landau levels corresponding to the quantum numbers $n = 0$ (the levels 0^- and 0^+). The electrons and holes at these levels have oppositely directed spins.

The carriers retain with three degrees of freedom and remain unpolarized following other types of action on the energy spectrum of the material (for example, changes in the alloy composition or hydrostatic compression). Therefore the transitions to the ZGS under the influence of a magnetic field can be regarded as a different class that calls for a special theoretical and

