Interaction between light waves and sound under acoustic nonlinearity conditions

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A new system of equations for studying the interaction between light and nonlinear sound is proposed. The solutions of the inhomogeneous Burgers equation that describes hypersound generation in prescribed light fields are studied analytically and numerically. The dynamics of growth of the harmonics and the total intensity of the wave and the pressure profiles are determined, the latter up to the establishment of stationary conditions. An analysis of the general equation indicates the necessity of taking into account the acoustic nonlinearity if the linear sound absorption is weak. This is due to the large value of the introduced parameter γ.

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1. INTRODUCTION

In the study of stimulated Mandel'shtam-Brillouin scattering (SMBS) and the process of the generation of hypersound by laser radiation,[1,2] a situation can arise in which consideration of acoustic nonlinear effects is necessary. This circumstance was noted by Polyakova[3] and discussed by other authors.[2-6] Their reasoning is based on the known results of the theory of nonlinear sound waves[7] and reduces essentially to the following. In the case of the free propagation of a disturbance, which is specified at the boundary $x=0$ in the form of a harmonic oscillation, distortion of the wave takes place until a sawthooth wave is formed at a distance $x = L_{dis} = \Lambda/2\pi\varepsilon M$ (where Λ is the wavelength, M is the Mach number, ε is the nonlinear parameter). For the appearance of nonlinear effects at distances $x \gtrsim L_{dis}$, it is necessary that the sound absorption be sufficiently small: $L_{dis} \ll L_{abs} = \alpha^{-1}$ (α is the sound absorption coefficient), i.e., the acoustics Reynolds number is large.

According to estimates[1-3] the nonlinearity begins to show at room temperatures $T \sim 300$ °K in the case of hypersonic intensities of the order of 10^2–10^3 W/cm^2, which is achieved in experiments on SMBS. Moreover, when the temperature is lowered, the coefficient of linear attenuation at frequencies $\sim 10^{11}$ Hz falls in crystals from values of $\alpha \sim 10^3$ cm^{-1} to very small values, $\alpha \sim 10^{-3}$ cm^{-1},[8] which contributes to the appearance of the nonlinearity. However, because of the difficulties of experimental observation of the hypersonic wave, the obtained data were inadequate.[2] The fact of generation of the second harmonic of the hypersound is known.[9]

In view of the lack of mathematical tools, the theoretical investigations have been carried out at the estimate level. Moreover, these estimates are based on the results of the free propagation of sound of finite amplitude, since the theory of nonlinear acoustics of waves generated by distributed sources has not yet been developed. A number of results have been obtained recently,[10-12] which enable us to estimate more accurately and to treat the interaction of light with sound with account of the nonlinearity of the latter. Preliminary results have been given by one of the authors.[10]

In the present work, a complete set of equations has been formulated for the calculation of the indicated interactions. With the help of these equations, the simplest problems on the generation of hypersound and SMBS have been solved. Perspectives of further studies in this direction are discussed.

2. FORMULATION OF THE PROBLEM. INITIAL EQUATIONS

The study of SMBS and the process of excitation of hypersound in the field of laser radiation is usually carried out by the method of slowly changing amplitudies, which allows considerable simplification of the initial coupled set of Maxwell equations and the equations of hydrodynamics.[2] Such an approach is specific for the consideration of interactions in media with strong dispersion, in particular for nonlinear optics.[13] In those cases in which acoustic nonlinearity appears, it is necessary to include the amplitude of the higher acoustic harmonics in the description, since even for very high frequencies of laser ultrasound $f \sim 2nc_0/\lambda \sim 10^{11}$ Hz (c_0 is the velocity of sound, n the index of refraction, λ the optical wavelength), dispersion is as a rule unimportant. The appearance of harmonics, which are not taken into account within the framework of the method of slowly changing amplitudes, leads to nonlinear attenuation of the sound wave, which can exceed the usual attenuation and significantly change the dynamics of the process.

The derivation of reduced equations for the complex amplitudes of the pump wave E_p and the Stokes wave E_S in the considered problem does not differ from the standard method and in the case of backscattering leads to equations of well known form[2]:

$$\frac{dE_p}{dx} + k_\omega E_p = -i\frac{\omega_p}{4c} Y\beta\tilde{p}E_S e^{-i\Delta x}, \tag{1}$$

$$-\frac{dE_S}{dx} + k_\omega E_S = -i\frac{\omega_S}{4c} Y\beta\tilde{p}^* E_p e^{i\Delta x}. \tag{2}$$

Here k_ω is the extinction coefficient of the light, β is the compressibility of the medium, Y is the coefficient of nonlinear optical-acoustical coupling, $\tilde{p}$ is the complex amplitude of the sound pressure wave, ω_S and ω_p are the frequencies of the Stokes wave and the pump wave.

To derive a simplified nonlinear acoustic equation, we begin with the wave equation for the sound field

$$\frac{\partial^2 p'}{\partial t^2} - c_0^2\nabla^2 p' - \frac{b}{\rho_0}\frac{\partial}{\partial t}\nabla^2 p' = -\frac{c_0^2}{8\pi}Y\nabla^2(E^2) + L_2(p'^2). \tag{3}$$

Here b is the dissipation coefficient, $L_2(p'^2)$ denotes symbolically the nonlinear terms that are quadratic in p',[7] which are not written out for reasons of brevity. Setting

$$E^2 = {}^1/_2 E_p E_S^* \exp[i(\Omega t - qx + \Delta x)] + \text{c.c.} \tag{4}$$

in Eq. (3), where $q = \Omega/c_0$ and $\Delta = k_p - k_S - q$, and transforming to a co-moving set of coordinates moving along with the wave with the speed of sound, we obtain[10]

$$\frac{\partial p'}{\partial x} - \frac{\varepsilon}{c_0^3\rho_0}p'\frac{\partial p'}{\partial \tau} - \frac{b}{2c_0^3\rho_0}\frac{\partial^2 p'}{\partial \tau^2} = \frac{Yq}{16\pi}\left(1 - \frac{\Delta}{q}\right)^2 A_p A_S \sin\left(\Omega\tau + \Phi - \frac{\pi}{2}\right). \tag{5}$$

A transformation has been carried out in Eq. (5) from the complex amplitudes of the laser waves to the actual amplitudes and phases: $E_p = A_p\exp(i\varphi_p)$, $E_S = A_S\exp(i\varphi_S)$, while the quantity ε denotes the parameter of acoustic nonlinearity, $\Phi = \Delta x + \varphi_p - \varphi_S + \pi/2$. Similar transformations of Eqs. (1), (2) give the following system:

$$\frac{dA_p}{dx} + k_\alpha A_p = -\frac{\omega_p}{4c}Y\beta p A_S \sin\Phi, \tag{6}$$

$$\frac{dA_S}{dx} - k_\alpha A_S = -\frac{\omega_S}{4c}Y\beta p A_p \sin\Phi, \tag{7}$$

$$\frac{d\Phi}{dx} - \Delta + \frac{Y\beta}{4c}p\left(\omega_S\frac{A_S}{A_p} + \omega_p\frac{A_p}{A_S}\right)\cos\Phi = 0. \tag{8}$$

In order that the set of equations (5)–(8) be complete, it is necessary to add to it the coupling between the parameter of the sound field p' and the actual amplitude p of the acoustic pressure:

$$p = \frac{2}{\pi}\int_0^\pi p'(x,\tau)\sin\left(\Omega\tau + \Phi - \frac{\pi}{2}\right)d(\Omega\tau). \tag{9}$$

The relation (9) expresses the fact that only the amplitude of the fundamental amplitude p of the sound field enters into Eqs. (6)–(8), while the behavior of the field p' is determined by the interaction of an infinite number of harmonics, described by Eq. (5).

It is not difficult to generalize the set (5)–(9) to the case of the interaction of waves at arbitrary angles, to the nonstationary case or to the case of nonplane waves. However, even in the simplest situation (5)–(9), when the light waves propagate counter to one another, the equations are rather complicated, and it is necessary to resort to further simplifications for their analysis.

3. GENERATION OF NONLINEAR HYPERSOUND IN A GIVEN LASER RADIATION FIELD

The basic difference of the system just obtained from that usually employed lies in the nonlinearity of the equation of hypersound generation (5). We consider the problem of the excitation of intense hypersound in the field of two opposing light waves with constant amplitudes A_p and A_S. We shall also assume that the optical dispersion of the medium permits synchronous excitation of sound of the difference frequency $\Omega = \omega_p - \omega_S = 2n\omega_p c_0/c$ and set $\Delta = 0$ and $\Phi = \pi/2$. Transforming to dimensionless variables

$$\theta = \Omega\tau, \quad z = b\Omega^2 x/2c_0^3\rho_0 = \alpha x, \quad \Pi = 2\varepsilon p'/b\Omega, \tag{10}$$

we reduce Eq. (5) to the form of an inhomogeneous Burgers equation:

$$\frac{\partial\Pi}{\partial z} - \Pi\frac{\partial\Pi}{\partial\theta} - \frac{\partial^2\Pi}{\partial\theta^2} = A\sin\theta. \tag{11}$$

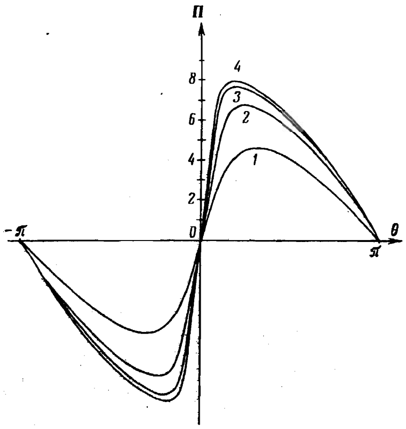

FIG. 1. Profile of the pressure wave at various distances from the boundary of the medium at $A = 20$: curve 1—for $\alpha x = 0.3$; 2—for $\alpha x = 0.5$; 3—for $\alpha x = 0.7$; 4—for $\alpha x = 0.9$.

The variable Π has the meaning of the running Reynolds number, z is the distance in units of the characteristic damping length of the sound of frequency Ω. The number $A = \varepsilon Y\beta_q^2 A_p A_S/16\pi\alpha^2$ can serve as a criterion for the appearance of nonlinearity. The case $A \ll 1$ corresponds to excitation of linear sound. At $A \gg 1$, effective generation of the harmonics should occur.

Equation (11) is linearized by the Hopf-Cole substitution $\Pi = 2\dot{U}/U$ just as in the case of the homogeneous Burgers equation. The equation thus obtained allows separation of the variables, and the temporal part reduces to the Mathieu equation. Specifying the boundary condition in the form $\Pi(z=0, \theta) = 0$, we find the solution

$$\Pi = 2\frac{\partial}{\partial\theta}\ln\left[\sum_{n=0}^{\infty} a_{2n}\exp\left(-\frac{\lambda_{2n}(A)}{4}z\right)\text{ce}_{2n}\left(\frac{\theta}{2}, A\right)\right],$$
$$a_{2n} = \int_0^{2\pi}\text{ce}_0\left(\frac{\theta}{2}, A\right)d\theta \Big/ \int_0^{2\pi}\text{ce}_{2n}^2\left(\frac{\theta}{2}, A\right)d\theta \tag{12}$$

(the notation used is that from the book of Strutt[14]).

Equation (12) enables us to follow the evolution of the wave profile. The results of the calculation from Eq. (12), with account of the first three terms of the series and with representation of the Mathieu functions with accuracy to the tenth harmonic, are shown in Fig. 1 for the value $A = 20$. Inclusion of a large number of terms of the series (12) makes it possible to increase the ac-

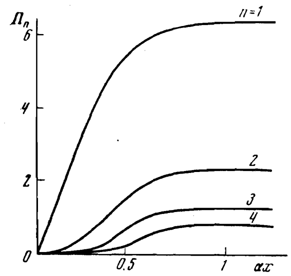

FIG. 2. Ratios of the amplitudes Π_n of the first several harmonics of sound at different cross sections of the medium at $A = 16$.

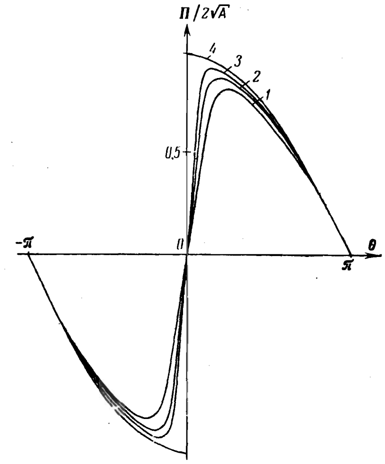

FIG. 4. Form of the stationary profile of the wave at different values of A: curve 1—for $A = 8$, 2—for $A = 16$, 3—for $A = 32$, 4—for $A \gg 1$.

curacy of the calculation at small z. Simultaneously with the numerical treatment of Eq. (12), we have carried out numerical integration of (11) with the help of a specially chosen difference scheme, which allows us to describe correctly the discontinuities that are formed.[15] Similar results were obtained by this method for the temporal shape of the wave. In addition, it has been used for the investigation of the growth dynamics of the harmonics (Fig. 2) and of the total intensity of the wave (Fig. 3). As is seen, the growth of the harmonics follows a different rule than in the case of free propagation.

It follows from the given data that the wave profile becomes stabilized and at $z \gg 1$ it ceases to depend on the distance traveled by the wave; since $\lambda_0 - \lambda_n < 0$, it follows that at $z \gg 1$ only the first term will be significant in the sum (12) and at large distances from the boundary we have[10,11]:

$$\Pi = 2\frac{\partial}{\partial\theta}\ln\left(\mathrm{ce}_0\left(\frac{\theta}{2}, A\right)\right). \tag{13}$$

This expression is the solution of the stationary equation

$$\Pi\frac{d\Pi}{d\theta} + \frac{d^2\Pi}{d\theta^2} = -A\sin\theta, \tag{14}$$

which is obtained from (11) for Π independent of z.

The stationary profile is shown in Fig. 4 for various values of A. It is not difficult to estimate the distance z_0—the contribution of the solution (12) to the stationary regime (13). At $A \ll 1$, the distance $z_0 \sim 1$ (at these distances, the transient processes are attenuated and the boundary conditions no longer have an effect). At $A \gg 1$, the difference $\lambda_0 - \lambda_2 \approx 8\sqrt{A}$ and, in order that the second term in the sum (12) be small in comparison with the first, it is necessary that $z\sqrt{A} \sim 1$. Consequently, at large A, the distance at which the steady state sets in is estimated as $z_0 \sim A^{-1/2}$. The stationary form of the wave at $A \gg 1$ is determined from the equation $\Pi\, d\Pi/d\theta = -A\sin\theta$:

$$\Pi = 2A^{1/2}\cos(\theta/2)\,\mathrm{sign}\,\theta, \quad -\pi \leq \theta \leq \pi. \tag{15}$$

This corresponds to the upper curve in Fig. 4.

Thus, in the limit of large Reynolds numbers, the profile no longer has a sawtooth shape, as was the case for free propagation of the wave. The more exact asymptotic expression is

$$\Pi = 2A^{1/2}[\cos(\theta/2) - 3\exp\{-2A^{1/2}\theta\}/(1+2\exp\{-2A^{1/2}\theta\})], \quad 0 < \theta \leq \pi. \tag{16}$$

Hence we can estimate the thickness of the shock front at $A^{-1/2}$. Consequently, the quantity $A^{1/2}$ can serve as a measure of the number of harmonics participating effectively in the interaction. This is confirmed by Fig. 2—in the case $A = 16$, we have four interacting harmonics; the others are negligibly small.

We now determine the intensity of the hypersound in the saturation regime. We use Eq. (14). Integrating it over θ, we obtain

$$\frac{d\Pi}{d\theta} + \frac{1}{2}\Pi^2 = A\cos\theta + \frac{1}{2}C, \quad C = \frac{1}{2\pi}\int_0^{2\pi}\Pi^2\, d\theta, \tag{17}$$

where C is proportional to the intensity of the hypersound. After linearization of Eq. (17), we find $C = -\lambda_0$. Then

$$I_{ac} = |\lambda_0|\rho_0 c_0^5 \alpha^2/b^2\Omega^2. \tag{18}$$

At $A \ll 1$, we have $|\lambda_0| \approx A^2/2$ and $I_{ac} = Y^2 I_p I_S \Omega^2/8c^2\rho_0 c_0^3\alpha^2$, which agrees with the usual linear result. In the other limiting case $A \gg 1$, the asymptote yields $|\lambda_0| \approx 2A$ and $I_{ac} = Yc_0(I_p I_S)^{1/2}/n\varepsilon c$, i.e., the growth of intensity of the sound is slowed in the increase in the intensity of the pump due to the nonlinear damping.

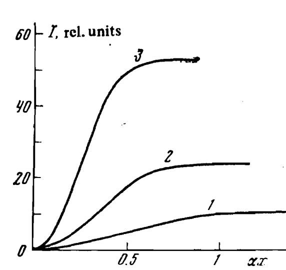

FIG. 3. The dynamics of the total sound intensity at different values of A: curve 1—for $A = 8$, 2—for $A = 16$, 3—for $A = 32$.

The solution of the inhomogeneous Burgers equation (11) with harmonic dependence of the right side on θ is of greatest interest for physical applications; however, this solution and its analysis are rather complicated. For elucidation of the qualitative features of the process of generation at $A \gg 1$, it is useful to consider the two simplest cases, replacing the right side of (11) by the linear functions

$$f_1(\theta)=\begin{cases}-A(1+\theta/\pi), & -\pi\leqslant\theta<0\\ A(1-\theta/\pi), & 0<\theta\leqslant\pi\end{cases}, \qquad (19)$$

$$f_2=A\theta/\pi, \quad -\pi<\theta<\pi.$$

We shall seek a solution in the form $\Pi = a(z)\, f_{1,2}(\theta)$. For $a(z)$ we get the equation

$$a'\pm a^2/\pi=A, \qquad (20)$$

solving which, with the condition $a(z=0)=0$, we obtain

$$a_1=(A\pi)^{1/2}\,\mathrm{th}[z(A/\pi)^{1/2}], \quad a_2=(A\pi)^{1/2}\,\mathrm{tg}[z(A/\pi)^{1/2}]. \qquad (21)$$

The first case corresponds to nonlinear limitation of growth of the sound amplitude. Here the second harmonic generated by the wave of fundamental frequency is in antiphase with the harmonic component of double the pump frequency. The second solution corresponds to the regime of "explosive instability." Here in-phase generation of harmonics occurs, which leads to an unbounded increase in the amplitude of the sound. In other words, the energy brought into the medium by the distributed external force does not have time to reach the front and be dissipated in it; destruction of the sample can serve, for example, as the limitation mechanism.

It is important to note that the first of the formulas (21) gives the qualitatively correct representation of the process of hypersound generation in the case of harmonic external stimulation (11). The characteristic distance at which steady state is reached and the stationary value of the amplitude (cf. with (15)) turn out to be the same here. Therefore, the first model will be used by us in the description of nonlinear damping of sound in SMBS.

4. SOME PROBLEMS OF SMBS WITH ACCOUNT TAKEN OF ACOUSTIC NONLINEARITY

We now consider the stationary SMBS process for a pump that significantly exceeds its threshold value, when it is possible to neglect the terms $k_\omega A_p$, $k_\omega A_S$ in Eqs. (6), (7). We also set $\Delta=0$—the case of complete synchronism, $\Phi=\pi/2$. In the absence of optical losses, as is well known, these equations have the integral $\omega_S A_p^2-\omega_p A_S^2=\mathrm{const}$. If the scale of the optical nonlinearity x_{nl} is much less than the interaction length L of the waves, then this constant can be set equal to zero, which corresponds to scattering in a semi-infinite medium. Assuming the following relation of the scales everywhere below: $x_{nl}\ll L\ll k_\omega^{-1}$, we write down the set (5)–(9) in the form

$$\frac{\partial p'}{\partial x}-\frac{\varepsilon}{c_0^3\rho_0}p'\frac{\partial p'}{\partial\tau}-\frac{b}{2c_0^3\rho_0}\frac{\partial^2 p'}{\partial\tau^2}=\gamma_1 A_l^2\sin\Omega\tau,$$
$$\frac{dA_l}{dx}=-\gamma_2 pA_l, \quad p=\frac{2}{\pi}\int_0 p'(x,\tau)\sin\Omega\tau\, d(\Omega\tau). \qquad (22)$$

Here, for simplicity, we have neglected the difference between ω_p and ω_S, and denote

$$\omega_p=\omega_S=\omega, \quad A_p=A_S=A_l, \quad \gamma_1=Yq/16\pi, \quad \gamma_2=Y\omega\beta/4c. \qquad (23)$$

The behavior of the solution of the system (22) near the boundary can be determined by finding the derivatives $\partial^n p'(0,\tau)\,\partial x^n$ and $d^nA(0)/dx^n$ in terms of the boundary conditions $A_l(x=0)=A_0$, $p'(x=0,\tau)=0$. With accuracy up to three terms, such a calculation gives

$$p'=(\gamma_1A_0^2/\gamma_2)^{1/2}[(z-{}^1/_2az^2+{}^1/_6(a^2-2)z^3)\sin\Omega\tau+{}^1/_3\gamma z^3\sin 2\Omega\tau+\ldots],$$
$$A_l=A_0(1-{}^1/_2z^2+{}^1/_6az^3), \qquad (24)$$

where

$$\gamma=4n\varepsilon/Y, \quad x_{nl}=(\gamma_1\gamma_2A_0^2)^{-1/2}, \quad z=x/x_{nl}, \quad a=\alpha x_{nl}.$$

Thus the dynamics of the process will be determined by the ratio of the scales of α^{-1} and x_{nl}.

In the case of strong linear damping ($\alpha x_{nl}=a\gg 1$), there will be an increase in the pressure from zero to maximum over distances $\sim\alpha^{-1}$ and then a slow decrease. Its character is determined by the solution of the set (22) under the assumptions that the terms $\partial p'/\partial x$ and $(\varepsilon/c_0^3\rho_0)p'\partial p'/\partial\tau$ are small in comparison with $\alpha p'$:

$$p=\frac{\gamma_1A_0^2}{\alpha}\left(1+\frac{z}{a}\right)^{-1}, \quad A_l=A_0\left(1+\frac{z}{a}\right)^{-1/2}. \qquad (25)$$

This argument is supported by the numerical integration of the system (22)[16] given in the review of Starunov and Fabelinskiĭ.[2]

In the other limiting case of small linear damping ($a\ll 1$), it follows from (24) that the amplitude of the fundamental of the sound reaches a maximum at distances $\sim x_{nl}$, which is in agreement with the well known solution of the system (22), linearized for sound, in the case $\alpha=0$:

$$p=(\gamma_1A_0^2/\gamma_2)^{1/2}\,\mathrm{th}\, z, \quad A_l=A_0\,\mathrm{sech}\, z. \qquad (26)$$

However, the amplitude of the sound second harmonic here is not small in comparison with the amplitude of the fundamental, since $\gamma\gg 1$. This indicates that in the case of small linear damping of hypersound, and in the appearance of optical saturation ($x_{nl}\ll L$), the acoustic nonlinearity must in principle be taken into account.

For the qualitative description of the process in this case, we replace $\sin\Omega\tau$ in Eqs. (22) by the sawtooth function $f(\Omega\tau)$, in correspondence with the first of Eqs. (19). Setting $p'(x,\tau)=p_0(x)\,f(\Omega\tau)$, we reduce the system (22) to the form

$$\frac{dp_0}{dx}+\frac{\varepsilon\Omega}{\pi c_0^3\rho_0}p_0^2=\gamma_1A_l^2, \quad \frac{dA_l}{dx}=-\gamma_2p_0A_l. \qquad (27)$$

This can be reduced to a single equation

$$A_l\frac{d^2A_l}{dx^2}-\left(1+\frac{2}{\pi}\gamma\right)\left(\frac{dA_l}{dx}\right)^2=-\gamma_1\gamma_2A_l^4. \qquad (28)$$

The limiting approach to linear sound $\gamma\to 0$ enables us to obtain the well-known solution (26) from Eq. (28). However, in condensed media, we usually have $\gamma \gg 1$. For example, for quartz $\gamma = 26$, for water, $\gamma = 25$, for CS_2, $\gamma = 12.5$. Thus allowance for the acoustic nonlinearity in the case of small linear damping of the sound and $x_{nl} \ll L$ is significant. New scales appear here and the regularities are qualitatively different in comparison with the case that is linear in sound $\gamma = 0$ (Eq. (26)).

Equation (28) can be integrated once:

$$\frac{dA_l}{dx} = \pm\left(\frac{\gamma_1\gamma_2}{\gamma-1}\right)^{1/2} A_l^2(1 - CA_l^{2(\gamma-1)})^{1/2}. \tag{29}$$

For completion of the boundary conditions $A_l(x=0) = A_0$ and $p_0(x=0) = 0$, the arbitrary constant C should be equal to $A_0^{-2(\gamma-1)}$. The solution (29) is conveniently analyzed in the phase plane (Fig. 5). By virtue of the second of Eqs. (27), we can judge the behavior of the amplitude of the pressure $p_0(x)$ from the shape of the phase surface for A; its value is proportional to the slope angle φ of the radius vector of the representative point.

Figure 6 shows the curves $A_l(x)$ and $p_0(x)$, obtained by numerical integration of (29). For comparison, the solutions of (25) are given by the dashed lines, without taking into account the acoustic nonlinearity. It is easy to see that the curves of the acoustic pressure differ greatly from one another. The nonlinear absorption leads to the appearance of a pressure maximum and the subsequent decrease of its amplitude is proportional to x_{nl}/x. Since saturation of $p_0(x)$ no longer sets in, the scattering properties of the medium are weakened and the process of reduction of $A_l(x)$ is slowed. At large distances $A_l \sim p_0$, this is a purely nonlinear dependence (in the linear case, we have $p_0 \sim A_l^2$).

Upon decrease in $p_0(x)$, the nonlinear damping of the sound gradually ceases to be the determining effect and account of linear damping will be necessary. Such an account can be carried out within the framework of the method with local dependence of the pressures on the field strength (cf. with (25)). We use the stationary solution (13), in which we shall assume the number A to be slowly dependent on x. The condition of applicability of this approximation is the smallness of the distance at which the solution (12) assumes the stationary value (13) in comparison with the scale of the optical nonlinearity:

$$A^{-1/2}(x) \ll \alpha[\gamma_1\gamma_2 A_l^2(x)]^{-1/2}. \tag{30}$$

The inequality (30) reduces to the condition $\gamma \gg 1$, which is always satisfied. Thus, the running scale of the acoustic nonlinearity is connected with the scale of the optical nonlinearity, but is always smaller than it.

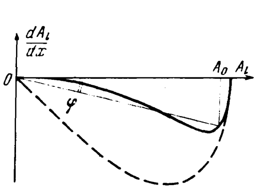

FIG. 5. Form of the phase plane of Eq. (29) with account and without account of the nonlinearity of the sound; the dashed curve is for $\gamma = 0$, the solid curve for $\gamma = 14$.

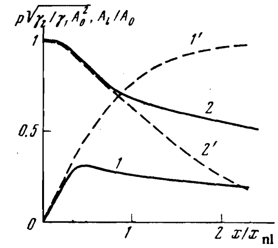

FIG. 6. Dynamics of the pressure wave p (curves 1, 1′) and the light waves A_l (curves 2, 2′) in the case of small linear sound absorption: 1, 2—with account of the acoustic nonlinearity ($\gamma = 14$); 1′, 2′—without account of the acoustic nonlinearity ($\gamma = 0$).

This allows us to neglect the derivative $\partial/\partial x$ in Eq. (11), and the system (22) reduces to the single equation

$$\frac{dA_l}{dx} = -\gamma_2 p A_l,$$
$$p = \frac{2}{\pi}\int_0^\pi \frac{b\Omega}{\varepsilon}\frac{\partial}{\partial\theta}\ln\left[\mathrm{ce}_0\left(\frac{\theta}{2}, \frac{\varepsilon Y A_l^2}{4\pi b^2\Omega^2\beta}\right)\right]\sin\theta\, d\theta. \tag{31}$$

For simplification of its analysis, we approximate the dependence $p(A_l)$ by the more convenient functions. We denote by $N^2 = \varepsilon Y A_0^2/4\pi b^2\Omega^2\beta$ the value of the number A on the boundary. At $N^2A_l^2/A_0^2 \ll 1$, the quantity $p = (b\Omega/\varepsilon)N^2A_l^2/A_0^2$; in the opposite case, $N^2A_l^2/A_0^2 \gg 1$, we have $p = 16(b\Omega/\varepsilon)NA_l/3\pi A_0$ (the latter follows from Eq. (15)). We shall assume that

$$p = \frac{b\Omega}{\varepsilon}\frac{N^2A_l^2/A_0^2}{1 + (3\pi/16)NA_l/A_0}. \tag{32}$$

Equation (31) with the approximations (32) is easily integrated, and its solution is represented in the form

$$A_l = \frac{A_0}{N}\left\{\left[\left(\frac{3\pi}{16}\right)^2 + \frac{2\alpha}{\gamma}(x + x_0)\right]^{1/2} - \frac{3\pi}{16}\right\}^{-1}. \tag{33}$$

Equation (33) describes the transition from the regime with strong nonlinear acoustic effects to the purely dissipative stationary regime. At large distances, the pressure amplitude is proportional to A_l^2 and decreases according to a hyperbolic law. At $x \gg x_{nl}$, we have $A_l = A_0(\alpha x_{nl}^2/2x)^{1/2}$ (cf. (25)).

Since the found solution (29) closely corresponds to the problem of sound generation in the field of light waves incident on the crystal from the outside (see, for example, Ref. 17), we estimate the effect of the nonlinear damping for just this situation. Let the pump wave with intensity $I_p(0)$ be incident on the quartz crystal of length L from the left, and the wave at the Stokes frequency with intensity $I_S(L)$ from the right. In order that the Stokes wave emerging from the crystal have the intensity $I_S(0) = I_p(0)$, it is necessary that $I_S(L) = I_p(L)$. However, the values of $I_p(L)$, calculated from Eqs. (26) and (29) (without account and with account of the nonlinearity of the sound), will diverge. Thus, for example, at $I_p(0) = 40$ MW/cm^2, $L = 1.5$ cm, the value $I_p(L)$ calculated from Eq. (26) amounts to 55 kW/cm^2. The formulas (29) used give the value $I_p(L) = 22$ MW/cm^2.

If, at room temperatures, when the linear damping of the hypersound is large, practically all the light scattering takes place in a thin boundary layer of the medium with dimension of the order of α^{-1}, then at low temperatures, the situation is different. The nonlinear damping of the sound increases the role of the volume in the process of light scattering (see Fig. 6); the small duration of the giant laser pulse leads to non-stationarity of the SMBS, which limits the length of interaction to a quantity of the order of 10^{-1}–10^{-2} cm. Whereas this suffices for the development of SMBS at room temperatures, at low temperatures such a length does not assure the effective transformation of the light.

All that has been said above enables us to suggest the following scheme for the generation of intense hypersound. Two laser beams of equal intensity I_0 and a frequency difference corresponding to the backward SMBS are directed against one another on a crystal of thickness $L \approx x_{n1}/\sqrt{\gamma}$ placed in a liquid helium cryostat. Here the maximum intensity of the hypersound amounts to $c_0 Y I_0/nc\,\varepsilon$. For example, for quartz and at an intensity $I_0 = 100$ MW/cm^2 ($\tau_L \sim 10^{-7}$ sec), we get $L = 5 \times 10^{-2}$ cm, $I_{ac} = 300$ W/cm^2 (a similar calculation from linear theory yields an estimate that is larger by a factor of 20). We note that even for crystal lengths of the order of a centimeter, intense hypersound cannot be extracted from the crystal, because of its nonlinear absorption.

Retuning the frequency of one of the laser beams and directing the light beams on the crystal at the corresponding angle, we can change the frequency of the generated hypersound.

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[1]I. L. Fabelinskiĭ, Molekulyarnoe rasseyanie sveta (Molecular Scattering of Light) Nauka, 1965. English Translation, Plenum, 1968.
[2]V. S. Starunov and I. L. Fabelinskiĭ, Usp. Fiz. Nauk 98, 441 (1969) [Sov. Phys. Usp. 12, 463 (1970)].
[3]A. L. Polyakova, Pisma Zh. Eksp. Teor. Fiz. 4, 132 (1966); 7, 76 (1968) [JETP Lett. 4, 90 (1966); 7, 57 (1968)].
[4]I. L. Fabelinskiĭ, Izv. Akad. Nauk SSSR Ser. Fiz. 35, 874 (1971).
[5]N. N. Lavrinovich, Zh. Eksp. Teor. Fiz. 60, 69 (1971) [Sov. Phys. JETP 33, 39 (1971)].
[6]S. B. Krivokhizha, Trudy (Works) of the Phys. Inst., Acad. Sci. USSR 72, 3 (1974).
[7]O. V. Rudenko and S. L. Soluyan, Teoreticheskie osnovy nelineĭnoi akustiki (Theoretical Bases of Nonlinear Acoustics) Nauka, 1975.
[8]P. Klemens, in: Physical Acoustics, v. IIIB (Warren Mason, Ed.) New York.
[9]R. G. Brever, Appl. Phys. Lett. 6, 165 (1965).
[10]O. V. Rudenko, Pisma. Zh. Eksp. Teor. Fiz. 20, 445 (1974) [JETP Lett. 20, 203 (1974)].
[11]A. A. Karabutov, E. A. Lapishin and O. V. Rudenko, Prod. Sixth Internat. Symp on Nonlinear Acoustics, Moscow State Univ. Press, 1975, p. 28.
[12]A. A. Karabutov and O. V. Rudenko, Zh. Tekh. Fiz. 45, 1457 (1975) [Sov. Phys. Tech. Phys. 20, 920 (1975)].
[13]S. A. Akhmanov and R. V. Khokhlov, Problemy nelineinoi optiki (Problems of Nonlinear Optics) (VINITI, 1964).
[14]M. J. O. Strutt, Lame, Mathieu and Related Functions in Physics and Technology, (Russ. transl. ONTI, 1935). [German ed. reprinted by Edwards Brothers, 1944].
[15]N. S. Bakhvalov, Chislennye metody (Numerical Methods) Nauka, 1974.
[16]K. Grob, Z. Physik 201, 59 (1965).
[17]M. J. Brienza and A. S. De Maria, Appl. Phys. Lett. 11, 44 (1967).

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Quantized motion of atoms and molecules in electromagnetic fields

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Atoms and molecules may exist in coupled states in the strong field of a standing electromagnetic wave. The absorption spectrum peaks (against the background of the Doppler contour) acquire a fine structure when the distance between levels begins to exceed the line width. This occurs for atoms in fields ~ 10 W/cm^2 and for molecules in fields ~ 0.1 W/cm^2. The peak width is investigated as a function of the frequency detuning of the strong field for broad and narrow molecular resonances. Discontinuities arise in the atomic spectra when the condition $\epsilon(\hbar k) > \hbar\gamma$ is satisfied and they may produce dips of the absorption coefficient. The case of a strong field ($\gtrsim 1$kw/cm^2) is considered when the general shape of the absorption coutour changes, viz., the Doppler contour is replaced by a band whose width is proportional to the field amplitude.

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1. INTRODUCTION

A strong inhomogeneous electromagnetic field acts on atoms and molecules in two ways: the Stark shift alters the energy levels and the particle velocities. The effective modulation of the levels (the dynamic Stark effect) has been investigated in detail, principally in connection with the theory of gas lasers.[1–3] In this theory it is very important that the atoms move with constant unperturbed velocities. We consider in this paper

