Dynamic properties of short superconducting filaments

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The dynamic properties of short superconducting junctions are investigated on the basis of the kinetic equations of superconductivity theory. Superconductors with a high concentration of nonmagnetic impurities are considered. The decrease of the effective normal resistance due to the finite relaxation rate of the order parameter is calculated for such superconductors.

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The investigation of the dynamic properties of narrow superconducting channels has been the subject of a number of theoretical[1–5] and experimental[6–9] studies. The theoretical papers[1–3] dealt with filaments of infinite length. Short superconducting bridges were investigated by Aslamazov and Larkin,[4] who have shown that such junctions behave like Josephson junctions, i.e., the current contains a term with $\sin\varphi$, where φ is the phase difference. Experiment[7] has revealed corrections to the current due to the dissipative terms connected with $\cos\varphi$. This effect has been theoretically calculated by Likharev and Yakobson,[5] but their results are rigorously valid only for superconductors with large concentration of paramagnetic impurities, when zero-gap conductivity is realized and the simple nonstationary Ginzburg-Landau equations are valid.

In this paper we consider superconductors with ordinary (nonmagnetic) impurities in the "dirty" limit. We investigate the nonstationary Josephson effect for short filaments ($L<\xi(T)$, L is the filament length and $\xi(T)$ is the coherence length) connected to bulky superconducting electrodes. The results differ from those given in[5].

It is assumed for simplicity that the filaments and the electrodes are of the same material. The boundary conditions for this problem were derived by Zaĭtsev[10] on the basis of a microscopic theory, but we use simpler boundary conditions,[5,11] which are valid for sufficiently thin filaments ($d\ll\xi$, d is the filament diameter). They reduce to the fact that the modulus of the order parameter on the boundary between the bank and the filament is equal to the equilibrium value of Δ_0 in the shores. We also assume that the current J is smaller that the critical pair-breaking current J_c in the filament.

To solve nonstationary problems in superconductivity theory at temperatures close to T_c, it is convenient to start from a time-dependent generalization of the Ginzburg-Landau equations, obtained on the basis of the kinetic equations.[12–15] We present here briefly the main results of the nonstationary theory in a form convenient for the investigation of the dynamic behavior of superconducting filaments.

The simplest method of deriving the kinetic equations is based on the Keldysh method.[16] In this method one introduces the matrix Green's function[14]:

$$\hat{G}=\begin{pmatrix} G^R & G \\ 0 & G^A \end{pmatrix}, \tag{1}$$

the elements of which are matrices in spin space and which satisfies the Gor'kov equation with self-energy part $\hat{\Sigma}$ of the same structure.[14] $\hat{\Sigma}$ consists of two terms that are connected respectively with the interaction with the impurities and with the phonons. From the kinetic equations for G and from the self-consistency condition for the order parameter $\Delta(t)$ follows the continuity condition $\partial\rho/\partial t+\operatorname{div}\mathbf{j}=0$. The Green's function G can next be resolved in the usual manner[13] into parts containing G^A and G^R as well as the anomalous function G^a, the latter satisfying a closed system of equations.

We consider the Green's function $\hat{g}_p(t_1t')$ integrated over the energy, equal to

$$\hat{g}_{\mathbf{p}}(t_1t')=\frac{i}{\pi}\int d\xi\,\hat{G}\left(t_1t'\mathbf{x}+\frac{\mathbf{r}}{2},\ \mathbf{x}-\frac{\mathbf{r}}{2}\right)e^{-i\mathbf{pr}}\,d\mathbf{r}, \tag{2}$$

and change over to the frequency representation in the difference variable t_1-t'. The Fourier transforms $g_p^{R(A)}(\mathbf{x}t\varepsilon)$ and $g_p^a(\mathbf{x}t\varepsilon)$ for dirty alloys then take the form[12,14]

$$\begin{gathered} g_p^{R(A)a}(\mathbf{x}t\varepsilon)=g^{R(A)a}(\mathbf{x}t\varepsilon)+l_{tr}\mathbf{p}\mathbf{g}_1^{R(A)}(\mathbf{x}t\varepsilon),\\ \mathbf{g}_1^{R(A)}(\varepsilon)=-p^{-1}(g^{R(A)}\mathbf{d}g^{R(A)}-i\mathbf{p}_s\tau_z),\\ \mathbf{g}_1^{a}(\varepsilon)=-p^{-1}\left[g^R\mathbf{d}g^a+g^a\mathbf{d}g^A+\frac{\partial f_\varepsilon}{\partial\varepsilon}\frac{d\mathbf{p}_s}{dt}(\tau_z-g^R\tau_zg^A)\right],\\ g^{R(A)}(\varepsilon)=\begin{pmatrix}\gamma^{R(A)}(\varepsilon) & f^{R(A)}(\varepsilon)\\ -f^{R(A)}(\varepsilon) & -\gamma^{R(A)}(\varepsilon)\end{pmatrix}. \end{gathered} \tag{3}$$

We have separated here the phase $\varphi(\mathbf{x}t)$; $p_s=\frac{1}{2}(\partial\varphi/\partial x)$ is the superfluid momentum

$$f_\varepsilon=\operatorname{th}(\varepsilon/2T),\quad \mathbf{d}=\partial/\partial\mathbf{x}+i\mathbf{p}_s\tau_z,$$

l_{tr} is the electron mean free path and p is the Fermi momentum.

With the aid of (3) we write down the necessary kinetic equations:

$$\begin{gathered} -D\frac{d}{dx}\operatorname{Sp}\tau_zg_1^a-2|\Delta|i\operatorname{Sp}\tau_xg^a=A_{st}^T,\\ -D\frac{d}{dx}\operatorname{Sp}g_1^a-\frac{\partial f_\varepsilon}{\partial\varepsilon}\operatorname{Sp}\tau_z(g^R-g^A)\frac{\partial\hat{\Delta}}{\partial t}=A_{st}^L, \end{gathered} \tag{4}$$

where $\hat{\Delta}=i|\Delta|\tau_y$, $A_{st}^{T,L}$ are the integrals of the collisions with the phonons, and $D=\frac{1}{3}vl_{tr}$ is the diffusion coefficient. We note that a similar system of equations was thoroughly investigated in[12–14].

We represent g^a in the form

$$g^a = -2(g^R - g^A) f_L(\varepsilon) - 2(g^R \tau_z - \tau_z g^A) f_T(\varepsilon). \tag{5}$$

where the first term is connected with the flowing current, and the second with the response to the longitudinal electric field.[15] Then the system of kinetic equations (4) takes the form

$$D\frac{d}{dx}\left[K^T(\varepsilon\varepsilon)\left(\frac{df_T(\varepsilon)}{dx} - \frac{1}{2}\frac{\partial f_\varepsilon}{\partial\varepsilon}\frac{dp_s}{dt}\right) + 2ip_s K^{TL}(\varepsilon\varepsilon) f_L(\varepsilon)\right]$$
$$- i|\Delta|(f^R(\varepsilon) + f^A(\varepsilon)) f_T(\varepsilon) = {}^1/_8 A_{st}{}^T(f_T). \tag{6}$$
$$D\frac{d}{dx}\left[K^L(\varepsilon\varepsilon)\frac{df_L(\varepsilon)}{dx} + 2ip_s K^{TL}(\varepsilon\varepsilon) f_T(\varepsilon)\right]$$
$$+ \frac{1}{4}(f^R(\varepsilon) - f^A(\varepsilon))\frac{d|\Delta|}{dt}\frac{\partial f_\varepsilon}{\partial\varepsilon} = \frac{1}{8}A_{st}{}^L(f_L),$$

where

$$K^L(\varepsilon\varepsilon_1) = {}^1/_4[(\gamma^R(\varepsilon) - \gamma^A(\varepsilon))(\gamma^R(\varepsilon_1) - \gamma^A(\varepsilon_1)) - (f^R(\varepsilon) - f^A(\varepsilon))(f^R(\varepsilon_1) - f^A(\varepsilon_1))],$$
$$K^T(\varepsilon\varepsilon_1) = {}^1/_4[(\gamma^R(\varepsilon) - \gamma^A(\varepsilon))(\gamma^R(\varepsilon_1) - \gamma^A(\varepsilon_1)) - (f^R(\varepsilon) + f^A(\varepsilon))(f^R(\varepsilon_1) + f^A(\varepsilon_1))], \tag{7}$$
$$K^{TL}(\varepsilon\varepsilon) = {}^1/_2[(f^R(\varepsilon) - f^A(\varepsilon))(f^R(\varepsilon) + f^A(\varepsilon))];$$

$$A_{st}^{T,L}(f_T, f_L) = -\frac{N(0)g^2\pi}{4(ps)^2}\int_{-\infty}^{\infty} d\varepsilon_1(\varepsilon_1 - \varepsilon)^2 \operatorname{sign}(\varepsilon_1 - \varepsilon)\operatorname{ch}^{-1}\frac{\varepsilon}{2T}\operatorname{ch}^{-1}\frac{\varepsilon_1}{2T}$$
$$\times \operatorname{sh}^{-1}\frac{\varepsilon - \varepsilon_1}{2T}\left[\operatorname{ch}^2\frac{\varepsilon}{2T} f_{T,L}(\varepsilon) - \operatorname{ch}^2\frac{\varepsilon_1}{2T} f_{T,L}(\varepsilon_1)\right](\gamma^R(\varepsilon) - \gamma^A(\varepsilon)\ (\gamma^R(\varepsilon_1) - \gamma^A(\varepsilon_1) \tag{8}$$

g is the electron-phonon interaction constant, and s is the speed of sound. We have left out of the collision integral the terms with the anomalous Green's functions $f^{R,A}$, which make a small contribution in our case at T values close to T_c.

It is necessary to add to the system (6) expressions for the current density and for the change of the electron density $\rho(t)$. The anomalous part of the current j^a is equal to

$$j^a = -\sigma\int_{-\infty}^{\infty} d\varepsilon\left\{K^T(\varepsilon\varepsilon)\left[\frac{1}{e}\frac{\partial f_T(\varepsilon)}{\partial x} - \frac{1}{2}\frac{\partial f_\varepsilon}{\partial\varepsilon}\frac{dp_s}{dt}\right] + 2ip_s K^{TL}(\varepsilon\varepsilon) f_L(\varepsilon)\right\}, \tag{9}$$

$$\rho(t) = 2eN(0)\left(\int_{-\infty}^{\infty} d\varepsilon\, N_1(\varepsilon) f_T(\varepsilon) - \tilde{\varphi}(xt)\right). \tag{10}$$

Here $\tilde{\varphi}(xt) = \frac{1}{2}(\partial\varphi/\partial t) + e\Phi(xt)$ is the gauge-invariant potential, $N_1(\varepsilon) = \frac{1}{2}(\gamma^R(\varepsilon) - \gamma^A(\varepsilon))$, $\Phi(xt)$ is the scalar electric potential, σ is the conductivity of the normal metal, and $N(0)$ is the density of states on the Fermi surface.

For short bridges $L < \xi(T)$ we can neglect all the terms except the gradient terms, in the static part of the Ginzburg-Landau equation. As a result we obtain for this case the system of equations

$$-\frac{1}{2i}\int_{-\infty}^{\infty} d\varepsilon(f^R(\varepsilon) + f^A(\varepsilon)) f_T(\varepsilon) = \frac{\pi}{4T_c}D\left[2p_s\frac{d|\Delta|}{dx} + |\Delta|\frac{dp_s}{dx}\right],$$
$$\frac{\pi}{8T_c}\frac{d|\Delta|}{dt} + \frac{1}{2}\int_{-\infty}^{\infty}(f^R(\varepsilon) - f^A(\varepsilon)) f_L(\varepsilon) = \frac{\pi D}{8T_c}\left(\frac{d^2}{dx^2} - 4p_s^2\right)|\Delta|. \tag{11}$$

Returning to the first of the equations (6), we note that by virtue of the electroneutrality condition ($\rho(t) = 0$) $f_T(\varepsilon)$ is proportional to the gauge-invariant potential $\tilde{\varphi}(xt)$, the proportionality coefficient being connected with the excitation distribution function f_ε. In first-order approximation with $N_1 \approx 1$ it follows from (10) that

$$f_{0T}(\varepsilon) = \frac{1}{2}\frac{\partial f_\varepsilon}{\partial\varepsilon}\tilde{\varphi}(x). \tag{12}$$

Substituting (12) in the first equation of (6), in which we have retained only the gradient terms (the term with $K^{TL}(\varepsilon\varepsilon)$ is neglected, since it differs from zero in a very narrow energy region $\sim 1/\tau_\varepsilon$, where τ_ε is the energy relaxation time), and assuming that $K^T(\varepsilon\varepsilon)$ depends only on the mean value $|\overline{\Delta}|$, we arrive at the Laplace equation $\nabla^2\Phi = 0$ for the electric potential. This equation was considered in[4]. Its solution gives an approximate value of Φ, which is now used to solve Eqs. (11).

To proceed further, we must obtain a more exact expression for the function $f_T(\varepsilon)$. The difference between $f_T(\varepsilon)$ and the value that follows from (12) is determined by the change of the distribution function of the excitations. For currents smaller than the critical current, the change of the distribution function of the normal excitations is connected with the destruction of the Cooper pairs due to inelastic scattering of electrons by phonons, and can be obtained from the first equation of (6) without the first term in the left-hand side. The result is analogous to that given in[15] and takes the form

$$f_T(\varepsilon) = \frac{1}{2}N_1(\varepsilon)\frac{\partial f_\varepsilon}{\partial\varepsilon}(N_1(\varepsilon) + 2\tau_\varepsilon\overline{|\Delta|}N_2(\varepsilon))^{-1}\tilde{\varphi}(xt),$$
$$N_2(\varepsilon) = \frac{i}{2}(f^R(\varepsilon) + f^A(\varepsilon)), \tag{13}$$
$$\frac{1}{\tau_\varepsilon} = \frac{N(0)g^2\pi}{32(ps)^2}\int\frac{d\varepsilon_1(\varepsilon_1 - \varepsilon)^2\operatorname{sign}(\varepsilon - \varepsilon_1)\operatorname{ch}(\varepsilon/2T)}{\operatorname{sh}((\varepsilon - \varepsilon_1)/2T)\operatorname{ch}(\varepsilon_1/2T)}N_1(\varepsilon_1).$$

The function $f_L(\varepsilon)$ is determined, as already noted, by the flowing current. For short bridges at a current $J < J_c$ it is small quantity. Indeed, solving the second equation of (6), where we choose as the modulus $|\Delta|$ the solution of the Ginzburg-Landau equation (11) without the left-hand sides and with boundary conditions $f_L(L) = f_L(0) = 0$, we obtain

$$f_L(\varepsilon) \approx \frac{\delta L^2}{48D}\frac{\partial f_\varepsilon}{\partial\varepsilon}\frac{d|\Delta|}{dt}(f^R(\varepsilon) - f^A(\varepsilon))(K^L(\varepsilon\varepsilon))^{-1}, \quad \delta \sim 1. \tag{14}$$

Thus, substituting (13) and (14) in Eqs. (11), we obtain ultimately

$$\beta u|\Delta|\left(\frac{\partial\varphi}{\partial t} + 2e\Phi\right) = 2\left(2p_s\frac{d|\Delta|}{dx} + |\Delta|\frac{dp_s}{dx}\right).$$
$$u\frac{d|\Delta|}{dt}(1 + \gamma) = \left(\frac{d^2}{dx^2} - 4p_s^2\right)|\Delta|. \tag{15}$$

We have changed over here to the dimensionless variables

$$x \to \frac{x}{\xi(T)}, \quad t \to \frac{t}{t_0}, \quad t_0^{-1} = \frac{4T\pi^3}{7\zeta(3)}\left(\frac{T_c - T}{T}\right), \quad u = \frac{\pi^4}{14\zeta(3)}.$$

The voltage between the superconducting banks is equal to

$$V(t) = -\pi\Delta_0^2\mu(l)/4Te, \quad \mu(x) = 2e\Phi, \quad \mu(0) = 0, \quad \mu(l) = -\frac{\partial\varphi_0}{\partial t}, \quad l = \frac{L}{\xi(T)};$$
$$\beta = \frac{T_c i}{\pi\overline{|\Delta|}}\int_{-\infty} d\varepsilon(f^R(\varepsilon) + f^A(\varepsilon))\frac{N_1(\varepsilon)}{N_1(\varepsilon) + 2\overline{|\Delta|}\tau_\varepsilon N_2(\varepsilon)}\frac{\partial f_\varepsilon}{\partial\varepsilon} \approx [1 + (2\tau_\varepsilon\overline{|\Delta|})^2]^{-1/2}. \tag{16}$$

In the calculation of the integral in the last expression we used the following form of the Green's function[15]:

$$g^{R(A)}(\varepsilon)=\frac{-i}{[|\Delta|^2-(\tilde\omega^{R(A)})^2]^{1/2}}\begin{pmatrix}\tilde\omega^{R(A)} & |\Delta| \\ -|\Delta| & -\tilde\omega^{R(A)}\end{pmatrix}$$
$$\omega^{R(A)}=\varepsilon\pm i/2\tau_\varepsilon, \tag{17}$$

and in addition we had assumed that $N_1(\varepsilon)\approx 1$, $N_2 \approx 2\tau_\varepsilon|\overline{\Delta}|/[1+(2\tau_\varepsilon|\overline{\Delta}|)^2]$,

$$\gamma=\frac{2T^2\delta}{3(|\overline{\Delta}|)^2}\frac{ul^2}{\pi^2}\int_{-\infty}^{\infty}d\varepsilon\frac{(f^R(\varepsilon)-f^A(\varepsilon))^2}{K^L(\varepsilon\varepsilon)}\frac{\partial f_\varepsilon}{\partial\varepsilon}. \tag{18}$$

The anomalous current $J^{(a)}$ in dimensionless variable and normalized to the value

$$\frac{Se^2N(0)\pi vl_{tr}}{6T_c}\frac{\Delta_0^2}{\xi(T)}$$

(S is the cross section are of the filament) is equal to

$$J^{(a)}=-a\nabla\mu,\quad a=\frac{1}{2}\int_{-\infty}^{\infty}d\varepsilon\,K^T(\varepsilon\varepsilon)\frac{\partial f_\varepsilon}{\partial\varepsilon}, \tag{19}$$

while the total current takes the form

$$J=\operatorname{Im}\psi^*\nabla\psi-a\nabla\mu,\quad \psi=\Delta/\Delta_0. \tag{20}$$

The Ginzburg-Landau equations (15) are now conveniently written in the form of one equation in the complex function ψ:

$$\left[u_0\left(\frac{d}{dt}+i\mu\right)+u_1\frac{1}{|\psi|}\frac{d|\psi|}{dt}\right]\psi=\frac{d^2\psi}{dx^2}, \tag{21}$$

where $u_0=\beta u$, $u_1=u(1+\gamma-\beta)$. The boundary conditions for the last equation take the form $\psi(0)=1$, $\psi(l)=e^{i\varphi_0}$.

Formulas (16) and (21) lead to simple Ginzburg-Landau equations only in the limit when $\tau_\varepsilon|\Delta|\ll 1$, which corresponds to temperatures very close to critical. The more realistic limit is $\tau_\varepsilon|\Delta|\gtrsim 1$. We can, however, retain the condition $\overline{|\Delta|}^2\tau_\varepsilon/T_c\ll 1$; then $a\approx 1$, $\gamma=0$, and the results are quantiatively valid. Inasmuch as $\beta\neq 1$ (u_1 is not small), all the main features of the problem are preserved.

As the zeroth approximation for the potential in (21) we choose $\mu_0=-(x/l)\partial\varphi_0/\partial t$, [5] and for $|\psi|$ in the left-hand side of this equation we choose the expression given in[4]

$$|\psi_0|^2=1-\frac{4x}{l}\left(1-\frac{x}{l}\right)\sin^2\frac{\varphi_0}{2},\quad \psi_0=1-\frac{x}{l}+\frac{x}{l}e^{i\varphi_0}.$$

We seek the solution of (21) in the form $\psi=\overline{\psi}+\psi_1$, $|\psi_1|\ll|\overline{\psi}|$; $\overline{\psi}$ satisfies the equation (21) with $u_1=0$. The effect of the relaxation of the order parameter is assumed here to be small, i.e.,

$$l^2u_1\overline{\frac{\partial\varphi_0}{\partial t}}\ll 1,\quad l^2u_0\overline{\frac{\partial\varphi_0}{\partial t}}\ll 1.$$

For short filaments, we assume the validity of the following inequalities:

$$\left|u_1\frac{1}{|\psi|}\frac{d|\psi|}{dt}\psi_0\right|\gg\left|u_0\left(\frac{\partial}{\partial t}-\frac{ix}{l}\frac{\partial\varphi_0}{\partial t}\right)\psi_1\right|,\left|u_1\frac{1}{|\psi|}\frac{d|\psi|}{dt}\psi_1\right|, \tag{22}$$

which are then proved by direct verification. We then obtain for ψ_1 the equation

$$\frac{d^2\psi_1}{dx^2}=-u_1\frac{x}{l}\left(1-\frac{x}{l}\right)\frac{\psi_0}{|\psi_0|^2}\frac{\partial\varphi_0}{\partial t}\sin\varphi_0 \tag{23}$$

with ψ_1 subject to the boundary conditions $\psi_1(l)=\psi_1(0)=0$.

It should be noted that a similar approach to the solution of (21) with $u_1=0$ is incorrect, since inequalities of the type (22) are not satisfied for all x in the limit of short filaments. In this case it is necessary to obtain the exact solution.[5]

The determination of ψ_1 from (23) is trivial, but the result is very cumbersome and is therefore not presented here. We note only that ψ_1 is periodic in φ_0 with a period 2π.

The equation for the total current follows from (20):

$$Jl=\int_0^l\operatorname{Im}\psi^*\nabla\psi\,dx-\mu(l).$$

Substituting here the obtained value for ψ_1, we get the expression

$$\frac{J}{J_{cr}}=\frac{d\varphi_0}{dt}\left\{1+\frac{u_0l^2}{15}(1-\cos\varphi_0)+\frac{u_1l^2}{12}(1+\cos\varphi_0)\times\left[\frac{3(\bar\varphi_0-\sin\varphi_0)}{\sin\varphi_0(1-\cos\varphi_0)}-1\right]\right\}+\sin\varphi_0, \tag{24}$$

where J_{cr} is the critical value of the Josephson current, and $\bar\varphi_0/2$ is the principal value of $\tan^{-1}\tan(\varphi_0/2)$, i.e., the current is periodic function of φ_0 with period 2π.

The influence of the relaxation of the order parameter thus decreases the effective normal resistance by an amount

$$\frac{\Delta R}{R}=\left(\frac{u_0}{15}+\frac{u_1}{24}\right)l^2.$$

A few remarks are in order concerning the investigation of stimulated superconductivity in short bridges by microwave radiation. In this case the function $f_L(\varepsilon)$ is a correction, linear in the field intensity, to the distribution function obtained by Éliashberg.[11,17] For shorter bridges, however, this correction makes a contribution to the current that is small in comparison with the remaining terms of (24).

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Coherent effects in superconducting bridges of variable thickness

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Coherent microwave phenomena in thin-film superconducting tin junctions of "variable" thickness are investigated experimentally. It is shown that the dependence of the superconducting current on phase difference, which is approximately harmonic, is preserved up to the limiting junction dimensions, which exceed the coherence length by several times. Some features of the coherent phenomena, which manifest themselves when the relative dimensions of the junctions and the conditions for the transition from the resistive model to ordered motion of Abrikosov vortice in the film are varied, are elucidated. The experimental results are compared with current theoretical concepts.

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The possible existence of the Josephson effect in superconductor—constriction— superconductor junctions with small cross sections ($a \ll \xi$) was first demonstrated theoretically by Aslamazov and Larkin. Starting from the Ginzburg–Landau equations, they have shown[1] that the Josephson effect can be due in this case to a redistribution of the current I among the normal and superconducting components without breaking the Cooper pairs. According to Aslamazov and Larkin, near the critical temperature T_c, at currents not greatly exceeding the critical value I_c, the current in the junction region is

$$I=I_n+I_s=V/R+I_c\sin\varphi,\qquad d\varphi/dt=2eV/\hbar. \tag{1}$$

I_n and I_s in (1) are the currents of the normal and superconducting electrons, V is the voltage across the junction, R is the junction resistance in the normal state, and φ is the phase difference of the wave functions of the superconducting electrodes. A relation of the type (1) in superconducting point contacts and in film bridges of small size was confirmed many times in experiment.[1)]

At the same time, phenomena typical of Josephson junctions, for example the appearance of current steps on the current-voltage characteristics following application of microwave radiation, are observed in superconducting point junctions and film bridges having transverse dimensions much larger than ξ.[2,3] These phenomena are attributed to the formation and coherent motion of magnetic-flux quanta (vortices) in a direction perpendicular to the transport current. A detailed analysis of the vortical motion in such junctions was carried out in a number of studies.[4,5] It is impossible to obtain in this case a simple analytic relation similar to (1), although the very concept of coherent vortex motion turned out to be quite productive when it came to explain the properties of large-size junctions.

A theoretical analysis of the conditions for the change of the model (1) and the transition to the concept of coherent vortex motion when following a change in the weak-coupling geometry has been carried out in the last few years (see, e.g.,[6]). The variable parameters are in this case usually the geometric dimensions of the junction relative to characteristic parameters such as the coherence length ξ or the penetration depth $\delta_\perp$ of the perpendicular magnetic field into the film. One of the most interesting conclusions[6,7] is the existence of "limiting" dimensions; when these are exceeded, a qualitative change takes place in the character of the dynamic processes in the weak-coupling region.

In the experimental studies known to us,[8,9] the con-

