Photon absorption and emission in Coulomb collisions in a magnetic field

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The photon absorption coefficients for various photon polarizations (dissipative part of the high frequency conductivity tensor) are found in the case of Coulomb collisions in a magnetized nonrelativistic nondegenerate plasma for arbitrary values of $\hbar\omega_B/kT$ and $\hbar\omega/kT$. (ω is the photon frequency, T the electron temperature and ω_B the electron cyclotron frequency). The absorption coefficients are expressed in terms of the longitudinal and transverse effective collision frequencies $\nu_\parallel$ and $\nu_\perp$, which depend differently on the magnetic field strength. The frequency dependences of $\nu_\parallel$ and $\nu_\perp$ are considered and in particular the shape and height of the peaks (cyclotron resonances) for $\omega = s\omega_B (s=1,2...)$ which are most pronounced for $\nu_\perp$ in a quantizing magnetic field with $\hbar\omega_B \gg kT$. The functions $\nu_{\parallel,\perp}(\omega)$ for $\hbar\omega_B/kT=3$, 10 and 0.01 are found by numerical integration. The inverse bremsstrahlung coefficients in a quantizing and classical ($\hbar\omega_B \ll kT$) fields are compared.

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1. INTRODUCTION

The emission and absorption of photons in electron-ion collisions (bremsstrahlung processes, free–free transitions) are among the main processes whereby radiation interacts with a magnetized plasma. To find the emissivity and the absorption coefficients of electromagnetic waves in such a plasma it is necessary to know the anti-Hermitian part of the dielectric tensor $i\varepsilon''_{\alpha\beta}$. In a nonrelativistic plasma without allowance for the motion of the ions and for spatial dispersion, in a coordinate system with a z axis directed along the magnetic field, the tensor $\varepsilon''_{\alpha\beta}$, cited for example in Silin's monograph,[1] is conveniently represented in the form

$$\varepsilon_{\alpha\beta}'' = \frac{c}{\omega} k_\alpha \delta_{\alpha\beta}, \quad k_\alpha = \frac{\omega_p^2}{c} \frac{\nu^{(\alpha)}}{(\omega - \alpha\omega_B)^2}. \tag{1}$$

Here α, $\beta = 0$, ± 1 are the indices of the cyclic coordinates ($\mathbf{e}_0 = \mathbf{e}_z$, $\mathbf{e}_{\pm 1} = \mp 2^{-1/2}(\mathbf{e}_x \pm i\mathbf{e}_y)$), $\omega_B = eB/mc$ and $\omega_p = (4\pi N_e e^2/m)^{1/2}$ are the electron–cyclotron and Langmuir frequencies, ω is the frequency of the electromagnetic field, $\nu^{(0)} = \nu_\parallel$ and $\nu^{(+1)} = \nu^{(-1)} = \nu_\perp$ are the longitudinal and transverse effective electron collision frequencies. In a tenuous plasma, when the refractive indices $n_\alpha \approx 1 - \omega_p^2/2\omega(\omega - \alpha\omega_B)$ differ little from unity, the quantities k_α have the meaning of the absorption coefficients of radiation with different polarizations ($\alpha = 0$ corresponds to linear polarization along the magnetic field, $\alpha = \pm 1$ corresponds to right-hand and left-hand circular polarizations in a plane perpendicular to the magnetic field). Formulas (1) are valid for frequencies much higher than the ion-cyclotron frequency and not too close to the first cyclotron resonance ($|\omega - \omega_B| \gg \nu^{(\alpha)}$).

Silin[1] has derived in his book formulas for $\nu_\parallel$ and $\nu_\perp$ under the condition $\hbar\omega_B \ll kT$, when the quantization of the transverse motion of the electrons can be disregarded. If the Larmor radius of the electron is much larger than the effective dimension of the region of the electron-ion interaction ($\omega_B \ll \max(\omega_p, \omega)$), then $\nu_\parallel \approx \nu_\perp \approx \nu_0$, where ν_0 is the effective collision frequency in the absence of a magnetic field. In the opposite case the magnetic field influences the collision act and the collision frequencies (especially the transverse frequency) can differ noticeably from ν_0. For many applications, particularly in the study of magneto-optical properties of semiconductors, the region of interest is $\hbar\omega_B \gtrsim kT$. This region is of particular interest in astrophysics in connection with the discovery of pulsars. For example, the characteristics of the x rays from pulsars can be attributed to the presence in the radiating region of a dense ($N_e \sim 10^{22} - 10^{28}$ cm^{-3}), hot ($T \sim 10^7 - 10^9$ K), strongly magnetized ($B \sim 10^{10} - 10^{14}$ G) hydrogen-helium plasma, for which the ratio $\hbar\omega_B/kT$ can be either of the order of or much larger than unity.

We shall show that the coefficients k_α can be expressed in the form (1) for arbitrary $\hbar\omega_B/kT$ and $\hbar\omega/kT$, and we shall calculate the values of $\nu_\parallel$ and $\nu_\perp$ for a nondegenerate plasma. We seek the absorption coefficients by examining the interaction of the radiation with the electron in an individual act of electron-ion collision in the Born approximation. This approach was used in a number of studies.[2–8] Goldman and Oster[2] have obtained the probability of bremsstrahlung by a quasi-classical method, which is valid only for $\hbar\omega_B \ll kT$. Sazonov and Tuganov[3] and Pavlov and Kaminker[4] have calculated the emissivity and the absorption coefficients in a very strong magnetic field $\hbar\omega_B \gg kT$, $\omega_B \gg \omega$. Canuto and Chiu[5] obtained cumbersome formulas for the probability of bremsstrahlung of an extraordinary wave at angles 0 and $\pi/2$ to the magnetic field by an electron of given energy in a transition between definite Landau levels. Canuto and Chiu[5] did not average these quantities with the electron distribution function, and carried out neither an analysis of the obtained formulas nor actual calculations with their aid.

The bremsstrahlung and absorption probabilities in Coulomb collisions in a magnetic field were investigated also in a number of studies,[6–8] the results of which, as shown in [3,4] are in error. Gurvich[9] investigated the contribution of the bremsstrahlung processes to $\varepsilon''_{\alpha\beta}$ for an arbitrary magnetic field by the method of the quantum kinetic equations. Although his formulas are valid for any type of interaction potential of the colliding particles, the actual results were obtained only for short-range δ-like potentials at $\hbar\omega_B \gg kT$.

In this paper we obtain formulas for the effective frequencies of the Coulomb collisions $\nu_\parallel$ and $\nu_\perp$ at arbitrary relations between $\hbar\omega$, $\hbar\omega_B$, and kT (Sec. 2); these formulas are convenient for analysis and calculations. In Sec. 3 we analyze the frequency dependences of $\nu_\parallel$ and $\nu_\perp$ at $\hbar\omega_B \gg kT$. Near the cyclotron harmonics $\omega = s\omega_B (s = 1, 2, \ldots)$ the quantity $\nu_\perp$ has peaks, the heights and widths of which are determined by different broadening mechanisms, which are considered in the same section. The function $\nu_\parallel(\omega)$ also oscillates with the period ω_B, but the amplitude of the oscillations is much smaller than for $\nu_\perp$. At $\omega \gg \omega_B$, the oscillations decrease rapidly ($\nu_{\parallel,\perp} \to \nu_0$). At $\omega \ll \omega_B$ our results coincide with the known results of[3,4]. In Sec. 4 we consider the case $\hbar\omega_B \lesssim kT$. At $\hbar\omega_B \lesssim kT$ and $\omega \ll \omega_B$ we obtain Silin's results.[1] At $\omega = s\omega_B$ the frequency $\nu_\perp$ can oscillate also at $\hbar\omega_B \ll kT$. At an arbitrary value of $\hbar\omega_B/kT$ the values of $\nu_\parallel$ and $\nu_\perp$ can be easily obtained by numerical integration with the aid of the formulas of Sec. 2. We present the results for $\hbar\omega_B/kT = 10$, 3, and 0.01.

Knowing the absorption coefficients k_α and the corresponding refractive indices n_α, which are independent of temperature if the spatial dispersion is neglected also in a quantizing magnetic field,[10] it is easy to calculate the coefficients $k_{1,2}$ for the absorption of normal waves (ordinary and extraordinary), propagating in an arbitrary direction in a rarefied plasma, by means of the formulas presented by Gnedin and one of us.[11] In a plasma with a definite electron temperature T, the generation power of these waves (the emissivity) is

$$Q_j = k_j B_\omega(T)/2, \quad j = 1, 2,$$

where $B_\omega(T)$ is Planck's function, and the total emissivity is

$$Q = Q_1 + Q_2 = [{}^1/_2(k_{+1} + k_{-1})(1 + \cos^2\vartheta) + k_0 \sin^2\vartheta] B_\omega(T)/2,$$

where ϑ is the angle between the direction of the emission and the magnetic field. Bearing these relations in mind, we confine ourselves henceforth to a determination of the coefficients k_α.

2. DERIVATION OF THE FORMULAS FOR THE COEFFICIENTS k_α IN AN ARBITRARY MAGNETIC FIELD

We consider the absorption of photons by a nonrelativistic electron as it moves in a homogeneous magnetic field $B = B_z$ and in the Coulomb field $U(r) = -Ze^2/r$ of an immobile ion with charge Ze. If we choose as the vector potential of the magnetic field $\mathbf{A} = (-By, 0, 0)$, then the wave functions and the energy levels of the electron in the absence of a field $U(r)$ take the form

$$|np_xp_z\rangle = (L_xL_z)^{-1/2} \exp[i(p_xx + p_zz)/\hbar]\chi_{np_x}(y); \tag{2}$$

$$\chi_{np_x}(y) = (2^n n! \rho_0 \pi^{1/2})^{-1/2} \exp(-\eta^2/2) H_n(\eta), \quad \eta = (y - y_0)/\rho_0; \tag{3}$$

$$E_{np_z} = (n + {}^1/_2)\hbar\omega_B + p_z^2/2m. \tag{4}$$

Here $n = 0, 1, 2, \ldots$ is the number of the Landau level, p_z is the projection of the electron momentum on the direction of the magnetic field, p_x is the quantum number that determines the y coordinate of the center of the Larmor circle: $y_0 = p_x\rho_0^2\hbar^{-1}$, $\rho_0 = (c\hbar/eB)^{1/2} = (\hbar/m\omega_B)^{1/2}$ is the magnetic length, L_x and L_z are the normalization lengths, and $H_n(\eta)$ is a Hermite polynomial. In (2)–(4) we did not write out the spin parts of the wave functions and the energy levels, since we confine ourselves to the dipole approximation, i.e., we do not take into account the spatial dispersion of the plasma.

When a Coulomb field is added to the magnetic field, the wave functions and the energy levels can be obtained only approximately. We use the Born approximation, which is valid if the electron energy is much larger than the electron binding energy with the ion. Then the probability of the bremsstrahlung and absorption of a photon are expressed in terms of the squares of the moduli of the matrix elements

$$\langle f|v_\alpha|i\rangle = \left(\frac{\hbar\omega_B}{m}\right)^{1/2} \frac{M_\alpha{}^{fi}}{\hbar(\omega - \alpha\omega_B)}, \tag{5}$$

$$M_{+1}{}^{fi} = (n')^{1/2}\langle n'-1, p_x'p_z'|U|np_xp_z\rangle - (n+1)^{1/2}\langle n'p_x'p_z'|U|n+1, p_xp_z\rangle, \tag{6}$$

$$M_{-1}{}^{fi} = -(M_{+1}{}^{if})^*, \quad M_0 = (p_z' - p_z)(m\hbar\omega_B)^{-1/2}\langle n'p_x'p_z'|U|np_xp_z\rangle. \tag{7}$$

Here v_α are the cyclic components of the electron velocity in the magnetic field, and $|i\rangle = |np_xp_z\rangle$ and $|f\rangle = |n'p_x'p_z'\rangle$ are the states of the electron before and after the interaction with the photon. In the derivation of (5)–(7) we took into account the relations

$$v_0|np_xp_z\rangle = p_zm^{-1}|np_xp_z\rangle; \tag{8}$$

$$\left.\begin{matrix} v_{+1} \\ v_{-1} \end{matrix}\right\} |np_xp_z\rangle = \left(\frac{\hbar\omega_B}{m}\right)^{1/2} \left\{\begin{matrix} (n+1)^{1/2}|n+1, p_xp_z\rangle \\ n^{1/2}|n-1, p_xp_z\rangle \end{matrix}\right. \tag{9}$$

and the energy conservation law, which made it possible to express the level-energy difference in (5) in terms of the photon frequency.

Expressing in the usual manner the coefficient k_α of absorption of radiation with polarization $\mathbf{e}_\alpha$ (see the Introduction) in terms of (5), we obtain formula (1), where the effective collision frequencies are

$$\nu^{(\alpha)} = N_i \frac{\omega_B}{4\pi\hbar^4\omega} \sum_{n,n'=0}^{\infty} \iint_{-\infty}^{\infty} dp_z\, dp_z' [f_n(p_z) - f_{n'}(p_z')] A_\alpha^{n'n}(p_z'p_z)$$
$$\times \delta[\hbar\omega_B(n'-n) - \hbar\omega + (p_z'^2 - p_z^2)/2m], \tag{10}$$

$$A_\alpha^{n'n}(p_z'p_z) = \frac{\hbar(L_xL_z)^2}{m\omega_B} \iint_{-\infty}^{\infty} dp_x\, dp_x' |M_\alpha{}^{fi}|^2. \tag{11}$$

Here N_i is the ion concentration, $f_n(p_z)$ is the electron distribution function. The term containing $f_{n'}(p_z')$, takes into account the contribution of the induced transitions.

To calculate $A_\alpha^{n'n}(p_z'p_z)$ we expand the potential $U(r)$ in a Fourier integral

$$U(r) = (2\pi\hbar)^{-3} \int d\mathbf{q}\, U_\mathbf{q} e^{i\mathbf{q}\mathbf{r}/\hbar}, \tag{12}$$

and, using formula (7.374) of[12], transform its matrix element into

$$\langle n'p_x'p_z'|U|np_xp_z\rangle=(2\pi\hbar L_xL_z)^{-1}\int dq_x dq_y$$

$$\times\delta(p_z'-p_z-q_z)S_{n'n}\exp\left(\frac{iq_y(p_x+q_x/2)}{\hbar m\omega_B}\right)U_q;\quad q_z=p_z'-p_z; \qquad (13)$$

$$S_{n'n}=\left(\frac{n!}{n'!}\right)^{1/2}e^{-u/2}u^{(n'-n)/2}L_n^{n'-n}(u),\quad u=\frac{q_x^2+q_y^2}{2m\hbar\omega_B}. \qquad (14)$$

Here $L_n^{n'-n}(u)$ is a Laguerre polynomial. Expression (14) for $S_{n'n}$ is valid at $n'\geq n$; in the opposite case these subscripts must be interchanged, this being equivalent to multiplication by $(-1)^{n'-n}$. Substituting (13) in (6) and (7) and integrating with respect to p_x' and p_x, we obtain

$$A_0^{n'n}(p_z'p_z)=\frac{(p_z'-p_z)^2}{2\pi m\hbar\omega_B}\int dq_x dq_y|U_qS_{n'n}|^2, \qquad (15)$$

$$A_{+1}^{n'n}(p_z'p_z)=(2\pi)^{-1}\int dq_x dq_y|U_q(n'^{1/2}S_{n'-1,n}-(n+1)^{1/2}S_{n',n+1})|^2, \qquad (16)$$

$$A_{-1}^{n'n}(p_z'p_z)=(2\pi)^{-1}\int dq_x dq_y|U_q((n'+1)^{1/2}S_{n'+1,n}-n^{1/2}S_{n',n-1})|^2. \qquad (17)$$

It is seen from (10) that the quantities $A_\alpha^{n'n}(p_z'p_z)$ determine the probabilities of absorption (emission) of a photon if the electron undergoes the transition $np_z\to n'p_z'$. Expressions (16) and (17) were obtained by Canuto and Chiu.[5]

We now use (14) and simplify (16) and (17), taking into account the recurrence relations for the Laguerre polynomials ([12], p. 1051). Then

$$A_0^{n'n}(p_z'p_z)=(p_z'-p_z)^2n!(n'!)^{-1}\int_0^\infty du\,u^{n'-n}e^{-u}[L_n^{n'-n}(u)]^2|U_q|^2, \qquad (18)$$

$$A_{+1}^{n'n}=A_{-1}^{n'n}=\hbar m\omega_Bn!(n'!)^{-1}\int_0^\infty du\,u^{n'-n+1}e^{-u}[L_n^{n'-}(u)]^2|U_q|^2. \qquad (19)$$

It follows from these formulas that in an arbitrary magnetic field there are two effective collision frequencies–longitudinal $\nu_\parallel=\nu^{(0)}$ and transverse $\nu_\perp=\nu^{(+1)}=\nu^{(-1)}$. For an arbitrary electron distribution function and for an arbitrary potential $U(r)$ they can be determined from the formula

$$\left\{\begin{matrix}\nu_\parallel\\ \nu_\perp\end{matrix}\right\}=N_i\frac{\omega_B}{4\pi\hbar^4\omega}\sum_{n,n'=0}^\infty\frac{n!}{n'!}\iint_{-\infty}^\infty dp_z dp_z'[f_n(p_z)-f_{n'}(p_z')]\int_0^\infty du|U_q|^2u^{n'-n}$$

$$\times\left\{\begin{matrix}(p_z'-p_z)^2\\ uhm\omega_B\end{matrix}\right\}e^{-u}[L_n^{n'-n}(u)]^2\delta\left[\hbar\omega_B(n'-n)+\frac{p_z'^2-p_z^2}{2m}-\hbar\omega\right]. \qquad (20)$$

In the present paper we are interested in bremsstrahlung processes in a nondegenerate plasma with definite electron-component temperature T. Then

$$U_q=-\frac{4\pi Ze^2\hbar^2}{q^2},\quad f_n(p_z)=\frac{1-\exp(-\hbar\omega_B/kT)}{(2\pi mkT)^{1/2}}\exp\left(-\frac{n\hbar\omega_B}{kT}-\frac{p_z^2}{2mkT}\right) \qquad (21)$$

and in place of (20) we have

$$\nu_{\parallel,\perp}=\frac{4}{3}\left(\frac{2\pi}{m}\right)^{1/2}\frac{N_iZ^2e^4}{(kT)^{3/2}}\frac{kT}{\hbar\omega}\left[1-\exp\left(-\frac{\hbar\omega}{kT}\right)\right]\Lambda_{\parallel,\perp}, \qquad (22)$$

where

$$\left\{\begin{matrix}\Lambda_\parallel\\ \Lambda_\perp\end{matrix}\right\}=\frac{3}{8}(1-e^{-b})\sum_{n,n'=0}^\infty\iint_{-\infty}^\infty dv\,dv'\int_0^\infty du\,u^{n'-n}\frac{\exp[-(n+v^2/2)b-u]}{[u+(v'-v)^2/2]^2}$$

$$\times\left\{\begin{matrix}(v'-v)^2\\ u\end{matrix}\right\}[L_n^{n'-n}(u)]^2\delta\left(n'-n-\frac{v'^2-v^2}{2}-\frac{x}{b}\right),\quad b=\frac{\hbar\omega_B}{kT},\quad x=\frac{\hbar\omega}{kT}. \qquad (23)$$

From a comparison of (22) with the known[13] formulas for the effective collision frequency at $B=0$, it follows that as $B\to 0$ we have

$$\Lambda_{\parallel,\perp}\to\Lambda_0=\exp(\hbar\omega/2kT)K_0(\hbar\omega/2kT), \qquad (24)$$

where K_n is a Macdonald function. At $\hbar\omega\ll kT$ the quantity Λ_0 goes over into the Coulomb logarithm $\Lambda_0\approx\ln(4kT/\hbar\omega)-C$, where $C\approx 0.58$ is the Euler constant. We note that the formulas presented here are valid for a sufficiently tenuous plasma and for high frequencies ($\omega\gg\omega_p$) when the maximum impact parameter in the Coulomb logarithm is determined not by the Debye radius ρ_D, but by the distance $\sim(kT/m\omega^2)^{1/2}$, that the electron traverses during one period of oscillations of the electromagnetic wave. To take into account the Debye screening it would be necessary to add the term $\hbar/2m\omega_B\rho_D^2$ in the square brackets of the denominator in (23).

We proceed to the calculation of the quantities $\Lambda_\parallel$ and $\Lambda_\perp$. We first use the fact that the δ-function in (23) depends only on $n'-n\equiv s$ and, assuming s to be fixed, we carry out the summation in (23) over n if $s>0$ or over n' if $s<0$, with the aid of formula (8.976) of[12]. Then

$$\left\{\begin{matrix}\Lambda_\parallel\\ \Lambda_\perp\end{matrix}\right\}=\frac{3}{8}\sum_{s=-\infty}^\infty\iint_{-\infty}^\infty dv\,dv'\int_0^\infty du\exp\left[\frac{sb}{2}-\frac{bv^2}{2}-u\,\mathrm{cth}\,\frac{b}{2}\right]$$

$$\times\left\{\begin{matrix}(v'-v)^2\\ u\end{matrix}\right\}\frac{I_s[u/\mathrm{sh}(b/2)]}{[u+(v'-v)^2/2]^2}\delta\left(s+\frac{v'^2-v^2}{2}-\frac{x}{b}\right), \qquad (25)$$

where $I_s(u)$ is a modified Bessel function. Integrating in (25) with respect to v', we obtain with the aid of the δ-function

$$\left\{\begin{matrix}\Lambda_\parallel\\ \Lambda_\perp\end{matrix}\right\}=\frac{3}{8}\sum_\pm\sum_{s=-\infty}^\infty\int_{-\infty}^\infty\frac{dv}{|v_\pm|}\int_0^\infty du\,I_s\left[\frac{u}{\mathrm{sh}(b/2)}\right]$$

$$\times\left\{\begin{matrix}2a_\pm\\ u\end{matrix}\right\}\exp\left[-\frac{bv^2}{2}-u\,\mathrm{cth}\,\frac{b}{2}+\frac{sb}{2}\right](u+a_\pm)^{-2}\Theta(v_\pm^2), \qquad (26)$$

where

$$v_\pm=\pm[v^2-2(s-x/b)]^{1/2},\quad a_\pm=(v_\pm-v)^2/2,$$

and Θ is the Heaviside function. The two signs ± in this formula correspond to two scattering channels, when the z component of the electron velocity is directed after the scattering either along the magnetic field (+ sign) or in the opposite direction (− sign). The second method of transforming (25) consists of introducing new variables $(v+v')/2$ and $v'-v$ and integrating with respect to $(v'+v)/2$ with the aid of a δ-function. The resultant double integral reduces to a single integral

$$\left\{\begin{matrix}\Lambda_\parallel\\ \Lambda_\perp\end{matrix}\right\}=\frac{3}{4}e^{x/2}\sum_{s=-\infty}^\infty\left(\mathrm{sh}\frac{b}{2}\right)^{-|s|}\int_0^\infty dy\,yg^{-1}\left(y+\mathrm{cth}\frac{b}{2}+g\right)^{-|s|}$$

$$\times\left\{\begin{matrix}4hb^{-1}(4y/b+1)^{-1}K_1(h)\\ [|s|+(y+\mathrm{cth}(b/2))g^{-1}]K_0(h)\end{matrix}\right\};$$

$$g=(y^2+2y\,\mathrm{cth}(b/2)+1)^{1/2},\quad h=\tfrac{1}{2}|x-sb|(4y/b+1)^{1/2}. \qquad (27)$$

The third method consists of using the expansion of the δ-function in a Fourier integral with respect to the variable t; this enables us to sum in (25) over s with the aid of formula (8.511) of[12]. Integrating then with respect to $(v'+v)/2$, introducing $w=(v'-v)^2/2$, and making the substitution $t\to t+i/2$, we obtain

$$\left\{\begin{matrix}\Lambda_\parallel\\ \Lambda_\perp\end{matrix}\right\}=\frac{3}{8}\left(\frac{b}{\pi}\right)^{1/2}e^{x/2}\int_{-\infty}^{\infty}dt\,e^{ixt}\int_0^\infty\frac{dw}{w^{1/2}}\int_0^\infty du\,\frac{\exp(-Aw-Du)}{(u+w)^2}\left\{\begin{matrix}2w\\ u\end{matrix}\right\},\qquad(28)$$

$$A=b\left(t^2+\frac{1}{4}\right),\qquad D=\operatorname{cth}\frac{b}{2}-\cos(bt)/\operatorname{sh}\frac{b}{2}.\qquad(29)$$

We integrate (28) with respect to u and w. As a result we get

$$\Lambda_\parallel=\frac{3}{2}b^{1/2}e^{x/2}\int_0^\infty\cos xt\left[\frac{\sqrt{A}}{A-D}-\frac{D}{(A-D)^{3/2}}\ln\frac{A^{1/2}+(A-D)^{1/2}}{D^{1/2}}\right]dt,\qquad(30)$$

$$\Lambda_\perp=\frac{3}{4}b^{1/2}e^{x/2}\int_0^\infty\cos xt\left[-\frac{\sqrt{A}}{A-D}+\frac{2A-D}{(A-D)^{3/2}}\ln\frac{A^{1/2}+(A-D)^{1/2}}{D^{1/2}}\right]dt.\qquad(31)$$

Formulas (26), (27), (30), and (31) give three representations for the quantities $\Lambda_\parallel$ and $\Lambda_\perp$, and one of these expressions turns out to be the most suitable in different regions of the parameters x and b.

3. INVERSE BREMSSTRAHLUNG IN A QUANTIZING MAGNETIC FIELD ($\hbar\omega_B\gg kT$)

If $e^{-b/2}\ll 1$, then the plasma electrons are at the Landau ground level $n=0$. In this case formulas (26) and (27) for $\Lambda_\parallel$ and $\Lambda_\perp$ are simpler. Instead of (26) we obtain

$$\left\{\begin{matrix}\Lambda_\parallel\\ \Lambda_\perp\end{matrix}\right\}=\frac{3}{8}\sum_{\pm}\sum_{s=0}^{\infty}\frac{1}{s!}\int_{-\infty}^{\infty}\frac{dv}{|v_\pm|}\int_0^\infty du\,\frac{u^s\exp(-u-bv^2/2)}{(u+a_\pm)^2}\left\{\begin{matrix}2a_\pm\\ u\end{matrix}\right\}\Theta(v_\pm^2),\qquad(32)$$

and instead of (27)

$$\left\{\begin{matrix}\Lambda_\parallel\\ \Lambda_\perp\end{matrix}\right\}=\frac{3}{4}e^{x/2}\sum_{s=0}^{\infty}e^{-sb/2}\int_0^\infty\frac{y\,dy}{(y+1)^{s+2}}\left\{\begin{matrix}\dfrac{2(y+1)|x-sb|}{b(1+4y/b)^{1/2}}K_1(h)\\ (s+1)K_0(h)\end{matrix}\right\}.\qquad(33)$$

In (32) and (33), the number s of the cyclotron harmonic has the meaning of the number of the Landau level to which the electron is excited. The harmonics with $s<0$ make an exponentially small contribution at $b\gg 1$.

Let us investigate the dependence $\Lambda_{\parallel,\perp}$ on the frequency $x=\hbar\omega/kT$. According to (32) and (33)

$$\Lambda_{\parallel,\perp}=\sum_{s=0}^{\infty}\Lambda_{\parallel,\perp}^{(s)},\quad e^{sb-x}\Lambda_{\parallel,\perp}^{(s)}(x<sb)=\Lambda_{\parallel,\perp}^{(s)}(x>sb).\qquad(34)$$

Recognizing that $a_\pm\ll 1$ at $x-sb\ll b$ and that $a_+\approx a_-\approx(x-sb)/b$ at $x-sb\gg 1$, we obtain from (32) the following at $x>sb$: if $x-sb\ll b$

$$\Lambda_\parallel^{(0)}=\frac{3}{2}e^{x/2}K_0\left(\frac{x}{2}\right),\quad\Lambda_\perp^{(0)}=\frac{3}{4}\left(\ln\frac{b}{x}-1-C\right)e^{x/2}K_0\left(\frac{x}{2}\right),\qquad(35)$$

$$\Lambda_\parallel^{(1)}=\frac{3}{2}\left[\frac{x-b}{b}\left(\ln\frac{b}{x-b}-1-C\right)K_1\left(\frac{x-b}{2}\right)-\frac{2}{b}K_0\left(\frac{x-b}{2}\right)\right]e^{(x-b)/2},\qquad(36)$$

$$\Lambda_\parallel^{(s>1)}=\frac{3(x-sb)}{2s(s-1)b}e^{(x-sb)/2}K_1\left(\frac{x-sb}{2}\right),\quad\Lambda_\perp^{(s\geq 1)}=\frac{3}{4s}e^{(x-sb)/2}K_0\left(\frac{x-sb}{2}\right);\qquad(37)$$

if $x-sb\ll 1$

$$\Lambda_\parallel^{(0)}=\frac{3}{2}\left(\ln\frac{4}{x}-C\right),\quad\Lambda_\perp^{(0)}=\frac{3}{4}\left(\ln\frac{4}{x}-C\right)\left(\ln\frac{b}{x}-1-C\right),\qquad(38)$$

$$\Lambda_\parallel^{(1)}=\frac{2}{b}\left(\ln\frac{b}{4}-1\right),\qquad(39)$$

$$\Lambda_\parallel^{(s>1)}=\frac{3}{s(s-1)b},\quad\Lambda_\perp^{(s\geq 1)}=\frac{3}{4s}\left(\ln\frac{4}{x-sb}-C\right);\qquad(40)$$

if $1\ll x-sb\ll(s+1)b$

$$\Lambda_\parallel^{(0)}=\frac{3}{2}\left(\frac{\pi}{x}\right)^{1/2},\quad\Lambda_\perp^{(0)}=\frac{3}{4}\left(\frac{\pi}{x}\right)^{1/2}\left(\ln\frac{b}{x}-1-C\right),\qquad(41)$$

$$\Lambda_\parallel^{(1)}=\frac{3}{2}\left(\frac{\pi}{x-b}\right)^{1/2}\left[\frac{x-b}{b}\left(\ln\frac{b}{x-b}-1-C\right)-\frac{2}{b}\right],\qquad(42)$$

$$\Lambda_\parallel^{(s>1)}=\frac{3(\pi(x-sb))^{1/2}}{2s(s-1)b},\quad\Lambda_\perp^{(s\geq 1)}=\frac{3}{4s}\left(\frac{\pi}{x-sb}\right)^{1/2};\qquad(43)$$

if $x-sb\gg(s+1)b$

$$\Lambda_\parallel^{(s)}={}^3/_2\pi^{1/2}b(x-sb)^{-3/2},\quad\Lambda_\perp^{(s)}={}^3/_4\pi^{1/2}b^2(s+1)(x-sb)^{-5/2}.\qquad(44)$$

The values of $\Lambda_{\parallel,\perp}^{(s)}$ at $x<sb$ are obtained from (35)–(44) with the aid of (34).

Relations (35), (38), and (41) were obtained by Kaminker and one of us[4] by another method. It follows from them that at $\omega\ll\omega_B$ and $\hbar\omega_B\gg kT$ we have

$$\nu_\parallel\approx\frac{3}{2}\nu_0,\quad\nu_\perp\approx\frac{3}{4}\nu_0\left(\ln\frac{\omega_B}{\omega}-1-C\right).\qquad(45)$$

The appearance of the additional logarithmically large factor in $\nu_\perp$, which is typical of the case of strong magnetic fields,[1] is due to the increase of the characteristic time of the electron–electron interaction.

When values $x\sim sb-1$ are reached with increasing frequency, transitions to the Landau levels $n=s$ begin to contribute. The quantities $\Lambda_\perp^{(s)}$ take the form of asymmetrical peaks with logarithmically infinite maxima at the points $x=sb$. The quantity $\Lambda_\perp^{(s)}$ decreases exponentially when the frequency decreases from this point, and decreases in power-law fashion when the frequency increases. The quantities $\Lambda_\parallel^{(s)}$ also take the form of peaks, but with a finite maximum ($\sim 3\sqrt{\pi}/2s^{3/2}b^{1/2}$) that is shifted by a distance $\sim sb$ to the right away from the resonance. The logarithmic divergences of $\Lambda_\perp$ at $\omega=s\omega_B$ are due not to the actual form of the potential $U(r)$, but by the root singularities in the density in the number of the final states, which influence in one way or another all the processes in quantizing magnetic fields.[14]

For inverse bremsstrahlung in the scattering of electrons by a δ-function potential, divergences of this type were investigated in[9]. This case differs significantly from Coulomb scattering. Thus, the absolute intensities of the oscillations of the coefficient $k_{\pm 1}$ in Coulomb collisions increase much faster (by approximately s^2 times) with increasing s than in scattering by a short-range potential. According to Gurvich,[9] k increases in proportion to ω_B with increasing magnetic field at $\omega_B\gg\omega$, while $k_{\pm 1}$ tends to a constant value, whereas in our case k_0 tends to a constant value and $k_{\pm 1}$ decreases in proportion to ω_B^{-2}. The increase of the coefficients k_α in scattering by a short-range potential is due to the increase, by a factor $b^{1/2}$, of the maximum value of the transverse momentum transfer in a strong field $b \gg 1$. In scattering by a Coulomb potential, this increase is negligible, since the main contribution is made by small momentum transfers. This difference in the behavior of the high-frequency conductivity at $\omega \ll \omega_B$ makes it possible to distinguish between scattering by short-range potentials (acoustic phonons, point defects) and scattering by ionized impurities in experiments of the type described in[15].

We note that if we use formula (31) for $\Lambda_\perp$, then the logarithmic singularities at $x = sb$ are due to the divergence of the integral with respect to t at the upper limit. Since the variable t has the meaning of the dimensionless interaction time, it follows that the divergences can be regarded as a consequence of an unlimited increase in the time of interaction upon absorption of resonant radiation. In fact, the time of interaction is restricted to the value t_* (see below). If $t_* \gg 1$, then it follows from (31) that the maximum height of the peaks of the function $\Lambda_\perp(s)$ at $x \approx sb$ is equal to

$$(\Lambda_\perp^{(s)})_{max} \approx 3(\ln t_*)/4s. \tag{46}$$

The same effect can be otherwise described as a broadening of the cyclotron resonance, which is equivalent to replacing the δ-function in (25) by a function with a finite width $\delta x \sim t_*^{-1}$. If $\delta x \gg b$, then the resonances smear out to such an extent that they become unobservable.

We present some of the most important broadening mechanisms.

A. Broadening due to the motion and recoil of the ions. Owing to the change of the ion momentum as a result of collision with the electron, its energy changes by an amount $(-2\mathbf{Pq}+q^2)/2M$ (**P** and M are the momentum prior to the collision and the mass of the ion, q is the momentum transferred in the electron). Under cyclotron-resonance conditions we have

$$q \sim q_\perp \sim (sm\hbar\omega_B)^{1/2}, \quad q/P \sim (msb/M)^{1/2} \sim (mx/M)^{1/2} \ll 1,$$

and the recoil of the moving ion leads to smearing of the frequency by an amount $\delta x_i \sim Pq/MkT \sim (msb/M)^{1/2}$, i.e., in (46) we have $t_* \sim (M/msb)^{1/2}$. At $b = 10$ and $s = 2$ we have $t_* \sim 10$.

Formally, the motion and the recoil of the ion can be easily taken into account if $b \ll M/m$ and the ion moves in the interaction region along a straight-line trajectory. This reduces to making in (30) and (31) the substitution

$$A, D \to A, D + mb(t^2 + 1/4)/M. \tag{47}$$

We note that the effect of the Doppler broadening, which is the result of the change of the ion energy upon absorption of a photon with momentum $\sim \hbar\omega/c$, can be taken into account in similar fashion. At $\hbar\omega \ll mc^2$, however, the contribution of this effect is small in comparison with the broadening due to the ion recoil ($\sim \hbar\omega/cq \sim (\hbar\omega/mc^2)^{1/2}$).

B. Broadening due to the action of the Coulomb field on the electron. The influence of the Coulomb field on the electron was taken into account by us only in first order of perturbation theory (the Born approximation). We note that the logarithmic divergences appear at $x = sb$ in the form of a divergence of the integral (26) with respect to v (with respect to p_z) at the point $p_z = 0$, near which the condition for the applicability of the Born approximation $p_z^2/2m > \varepsilon$ (the electron-ion binding energy ε depends, generally speaking, on the magnetic field) may be violated. The criterion for the applicability of the Born approximation in this case is (besides $kT > \varepsilon$) smallness of the integral with respect to p_z from zero to $(2m\varepsilon)^{1/2}$ in comparison with the integral from $(2m\varepsilon)^{1/2}$ to infinity. The sufficient condition for this smallness is $|x - sb| > \varepsilon/kT$.

The influence of the Coulomb field at frequencies closer to cyclotron resonances leads to a decrease in the height and broadening of the peaks. At $\varepsilon > (msb/M)^{1/2}kT$ this broadening may turn out to be larger than that considered in subsection A. Then allowance for the latter in the region of applicability of our formulas is immaterial. The values of $\Lambda_\perp$, calculated with allowance for the broadening due to the motion of the ions at $|x - sb| < \varepsilon/kT$, yield the upper limit of the height of the peaks. The frequency interval between resonances $\delta x_c \sim \varepsilon/kT$, in which our results are not accurate, is small in comparison with the distance b between the peaks at $\hbar\omega_B \gg \varepsilon$ ($B \gg Z^2 \cdot 10^9$ G). If $B = 10^{12}$ G, then $\varepsilon \sim 0.1$ keV[16] for $Z = 1$, and at $T = 10^7$ °K ($b \approx 10$) we have $\delta x_c \sim \delta x_i \sim 0.1$ and $\delta x_c/b \sim 0.01$. At $kT \ll \hbar\omega_B \ll \varepsilon$ the broadening can become so large that the function $\Lambda_\perp(x)$ becomes almost smooth.

C. Natural widths of the Landau levels. In very strong magnetic fields, the broadening of the resonances due to the electron losses to synchroton radiation becomes important. In this case

$$t_* \sim kTm^3c^5/e^4B^2\hbar \sim b^{-2}(mc^2/kT)(\hbar c/e^2).$$

At $B = 10^{13}$ G and $T = 10^7$ °K we have $t_* \sim 10$.

D. Collision broadening. The finite lifetime of the electron at a given level due to collisions with particles of the surrounding medium also leads to a broadening of the resonances. In the plasma, the main contribution is made by the the electron-ion collisions and

$$t_* \sim kT/\hbar\nu_{eff} \sim (kT)^{5/2}m^{1/2}/N_iZ^2e^4\Lambda.$$

At $N_i = 10^{23}$ cm^{-3}, $T = 10^7$ °K, and $\Lambda \sim 10$ we have $t_* \sim 10^3$, i.e., the collision broadening can be neglected under these conditions.

We note that the considered broadening mechanisms can alter the value of $\Lambda_\perp$ also at $x \ll b$ (the zeroth cyclotron harmonic). Thus, at $x \ll (mb/M)^{1/2} \ll 1$ we have in place of (36) (with logarithmic accuracy)

$$\Lambda_\perp \approx \frac{3}{8}\left(\frac{1}{2}\ln\frac{M}{mb}\ln\frac{Mb}{m} + \ln\frac{mb}{Mx^2}\ln\frac{M}{m}\right). \tag{48}$$

We consider now the behavior of $\Lambda_{\parallel,\perp}$ at $x \gg b$, when a large number of harmonics with large s contribute to the

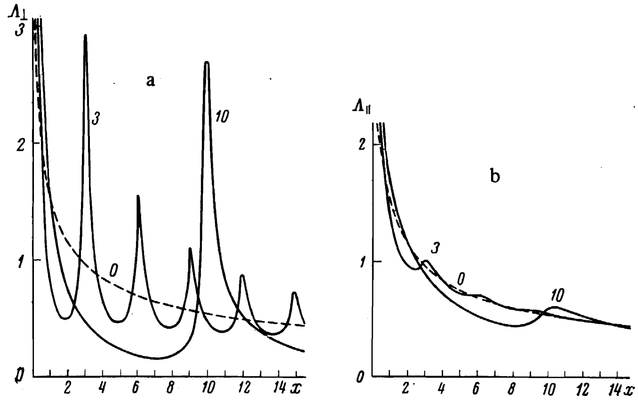

FIG. 1. Plots of $\Lambda_\perp$ (a) and $\Lambda_\parallel$ (b) against the dimensionless frequency $x=\hbar\omega/kT$ at $b=\hbar\omega_B/kT=3$ and 10. The values of b are marked near the curves. The dashed curve is a plot of $\Lambda_0(x)$.

absorption coefficient. Recognizing that at $s\gg 1$ and $x-sb\gg 1$ we have

$$\frac{1}{s!}\int_0^\infty du\frac{u^s e^{-u}}{(u+a_\pm)^2}\approx\frac{1}{(s+a_\pm)^2}\approx\left(\frac{b}{x}\right)^2$$

and replacing in (32) the summation over s by integration, we obtain

$$\Lambda_{\parallel,\perp}(x\gg b)=\Lambda_0(x\gg 1)\,[1+O(b/x)]+\Lambda_{\parallel,\perp}^{(s_*)}, \qquad (49)$$

where $s_*=E(\omega/\omega_B)$ is the number of the last harmonic that contributes to the absorption of a photon of frequency ω. The quantity $\Lambda_\parallel^{(s_*)}$ is negligibly small at $s_*\gg 1$, while $\Lambda_\perp^{(s_*)}$ can reach a value $\sim(\omega_B/\omega)\ln t_*$, i.e., it is also small at sufficiently high frequencies. Thus, even at $\hbar\omega_B\gg kT$ at frequencies $\omega\gg\omega_B$ the values of $\Lambda_{\parallel,\perp}$ tend to Λ_0 and the influence of the magnetic field on the absorption coefficients can be neglected.

4. INVERSE BREMSSTRAHLUNG IN A CLASSICAL MAGNETIC FIELD ($\hbar\omega_B\lesssim kT$)

At $b\ll 1$ it is convenient to represent the integrals with respect to t in (30) and (31) in the form of a sum

$$\Lambda_{\parallel,\perp}=\Lambda_{\parallel,\perp}^{(1)}+\Lambda_{\parallel,\perp}^{(2)}=\int_0^{b^{-1}}\lambda_{\parallel,\perp}\,dt+\int_{b^{-1}}^\infty\lambda_{\parallel,\perp}\,dt,$$

where $\lambda_{\parallel,\perp}$ are the corresponding integrands. If the magnetic field is so small that $b\ll x$, then

$$\Lambda_{\parallel,\perp}^{(2)}\ll\Lambda_{\parallel,\perp}^{(1)}\approx b^{1/2}e^{x/2}\int_0^\infty A^{-1/2}\cos xt\,dt=e^{x/2}K_0(x/2)=\Lambda_0. \qquad (50)$$

We have used here the fact that at $bt\ll 1$ in $\lambda_{\parallel,\perp}$ we have $A-D\ll A,\ D$.

With increasing fields, the quantity $\Lambda_\parallel$ remains close to Λ_0, while $\Lambda_\perp$ can change significantly. At $b\gg x$ we have with logarithmic accuracy

$$\Lambda_\perp^{(1)}=\ln b^{-1}=\ln(\rho_e/r_{min}), \qquad (51)$$

where $\rho_e=(kT/m\omega_B^2)^{1/2}$ is the Larmor radius of the electron and $r_{min}=\hbar(mkT)^{-1/2}$ is the minimum impact parameter at $kT>me^4/\hbar^2$ (the Born approximation). The values of $\Lambda_\perp^{(2)}$ depend on the emission frequency

$$\Lambda_\perp^{(2)}=\frac{3}{4}\left(\ln\frac{\omega_B}{\omega}\right)^2 \quad \text{if} \quad \omega\gg\omega_B\left(\frac{m}{M}\right)^{1/2}, \qquad (52)$$

$$\Lambda_\perp^{(2)}=\frac{3}{16}\ln\frac{M}{m}\ln\frac{\omega_B^4 m}{\omega^4 M} \quad \text{if} \quad \omega\ll\omega_B\left(\frac{m}{M}\right)^{1/2}. \qquad (53)$$

In (53) the maximum interaction time is bounded by the thermal motion of the ion, which is taken into account by formula (47). Formulas (52) and (53) are equivalent to formulas (64.37) and (64.39) in Silin's book,[1] where the influence of the Debye screening and the limitation of the time of interaction due to the Coulomb acceleration are also analyzed. We note that at $b\ll 1$ and $x\ll 1$ formula (28) goes over into (64.10) of[1] if the integration limits are suitably cut off in (28) and the substitution $A\to bt^2$ is made in (29), i.e., if it is assumed that the classical approximation of the minimal impact parameter is larger than the quantum approximation ($e^2/kT>\hbar(mkT)^{1/2}$).

The quantity $\Lambda_\perp$ can oscillate also at $b\ll 1$, having maxima at $x\approx sb$. Without allowance for the limitations on the interaction of the time, the heights of the peaks $\Lambda_\perp$ turn out to be logarithmically infinite, as also in the case $b\gg 1$. If the time of interaction is limited to t_*, then a rough estimate of the height of the peak can be obtained by using the expansion[12]

$$\ln\left(\operatorname{ch}\frac{b}{2}-\cos bt\right)=\frac{b}{2}-\ln 2-2\sum_{s=1}^\infty\frac{e^{-sb/2}}{s}\cos sbt. \qquad (54)$$

Substituting this expansion in the formula for $\Lambda_\perp^{(2)}$ and retaining only the resonant term (the sum of the nonresonant terms in $\Lambda_\perp^{(1)}$ is close to Λ_0), we obtain an estimate of the maximum height of the s-th peak:

$$\delta\Lambda_\perp^{(max)}\approx\frac{3}{4s}\ln bt_*. \qquad (55)$$

If, in particular, the interaction time is limited by the departure of the ion from the interaction region, then (see (47)) $t_*\sim b^{-1}(M/m)^{1/2}$ and

$$\delta\Lambda_\perp^{(max)}\sim\frac{3}{8s}\ln\frac{M}{m}\approx\frac{3}{s}. \qquad (56)$$

With decreasing magnetic field, the relative value of the s-th peak decreases because of the increase of Λ_0, due to the shift of the given cyclotron resonance to the

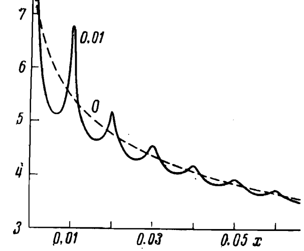

FIG. 2. Plot of $\Lambda_\perp(x)$ at $b=0.01$. The notation is the same as in Fig. 1.

region of lower frequencies. The influence of the Coulomb field on the collision process (which is not taken into account in the Born approximation) leads to an additional decrease of the height of the peaks. This influence, however, is less significant than for quantizing magnetic fields, owing to the increase of the effective values of the impact parameters at $b \ll 1$. In any case, it cannot lead to a vanishing of the peaks at

$$kT \gg \hbar\omega_B > Z^2 me^4/\hbar^2 \quad (B > Z^2 \cdot 10^9 \text{ G}).$$

At $b \sim 1$, the values of $\Lambda_{\parallel,\perp}$ can be obtained by numerical integration with the aid of formulas (30) and (31). We present the results of the integration for $\Lambda_{\parallel,\perp}$ at $b=10$ and $b=3$ (Fig. 1) and for $\Lambda_\perp$ at $b=0.01$ (Fig. 2). In the calculations we used formula (47), which takes the ion motion into account.

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Interactions and bound states of solitons as classical particles

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The interaction of localized nonlinear waves (solitons) is investigated. A theory is developed of weak interactions whose energy is small compared with the total field energy. In this case, for solitons with close velocities, the motion is described by the classical Newton equations with potential forces determined by the structure of the field far from the maxima. Three basic types of interaction are distinguished; a necessary criterion for the formation of bound states is given. In particular, the bound state of a pair of solitons with tails with an oscillatory structure is investigated. The results are presented of experiments with chains of nonlinear oscillators, in which oscillating solitons and all the types of interaction considered have been observed.

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1. INTRODUCTION

The question of the interaction of solitons has already been studied, by analytical and numerical methods, for a number of years. It has been found that, as a result of the interaction of infinitely separated (for $t \to -\infty$) solitons there remain (for $t \to +\infty$) diverging solitons with the same parameters as before the interaction (this property has even been used to define solitons[1]). At the same time, there are now certain exactly soluble equations which permit the existence of bound states of two or more solitons. By means of numerical methods, it has recently been made clear that solutions in the form of unrestrictedly diverging and bound solitons are characteristic not only of exactly integrable types of equations.[2,3] Thus, the numerical calculation carried out in[3] by Kudryavtsev for application to the Ginzburg-Landau equation showed the possibility of the existence of a bound pair of solitons. This question is interesting, in particular, in connection with the possible interpretation of the solitons as field particles. However, up to now there do not exist any general criteria determining the character of the interactions of solitons.

As shown in the present work, this question can be elucidated in a fairly general formulation applicable to weakly interacting solitons, when at each moment of time the total field differs little from the superposition of the fields of the individual solitons. The most important case of weak interactions is realized when the