region of lower frequencies. The influence of the Coulomb field on the collision process (which is not taken into account in the Born approximation) leads to an additional decrease of the height of the peaks. This influence, however, is less significant than for quantizing magnetic fields, owing to the increase of the effective values of the impact parameters at $b \ll 1$. In any case, it cannot lead to a vanishing of the peaks at

$$kT \gg \hbar\omega_B > Z^2 me^4/\hbar^2 \quad (B > Z^2 \cdot 10^9 \text{ G}).$$

At $b \sim 1$, the values of $\Lambda_{\parallel,\perp}$ can be obtained by numerical integration with the aid of formulas (30) and (31). We present the results of the integration for $\Lambda_{\parallel,\perp}$ at $b=10$ and $b=3$ (Fig. 1) and for $\Lambda_\perp$ at $b=0.01$ (Fig. 2). In the calculations we used formula (47), which takes the ion motion into account.


[1]V. P. Silin, Vvedenie v kineticheskuyu teoriyu gazov (Introduction to the Kinetic Theory of Gases), Nauka, 1972, Chap. X.
[2]R. Goldman and L. Oster, Phys. Rev. 136, A602 (1964).
[3]V. N. Sazonov and V. F. Tuganov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. 18, 165 (1975).
[4]G. G. Pavlov and A. D. Kamiker, Pis'ma Astron. Zh. 1, No. 9, 13 (1975) [Sov. Astron. Lett. 1, 181 (1975)].
[5]V. Canuto and H. Y. Chiu, Phys. Rev. A2, 518 (1970).
[6]J. Lodenquai, V. Canuto, M. Ruderman, and S. Tsuruta, Astrophys. J. 190, 141 (1974).
[7]A. El-Gowhari and J. Arponen, Nuovo Cimento 11B, 201 (1972).
[8]V. Canuto, H. Y. Chiu, and L. Fassio-Canuto, Phys. Rev. 185, 1607 (1969).
[9]Yu. A. Gurvich, Zh. Eksp. Teor. Fiz. 61, 1120 (1971) [Sov. Phys. JETP 34, 598 (1972)].
[10]P. S. Zyryanov and V. P. Kalashnikov, Zh. Eksp. Teor. Fiz. 41, 1119 (1961) [Sov. Phys. JETP 14, 799 (1962)].
[11]Yu. N. Gnedin and G. G. Pavlov, Zh. Eksp. Teor. Fiz. 65, 1806 (1973) [Sov. Phys. JETP 38, 903 (1974)].
[12]I. S. Gradshteĭn and I. M. Ryzhik, Tablitsy integralov, summ, ryadov i proizvedeniĭ (Tables of Integrals, Sums, Series, and Products), Nauka, 1971. [Academic, 1966].
[13]G. Bekefi, Radiation Processes in Plasma, Wiley, 1966.
[14]V. Canuto and H. Y. Chiu, Space Sci. Rev. 12, 3 (1971).
[15]E. J. Johnson and E. H. Dickey, Phys. Rev. [B] 1, 2676 (1970).
[16]V. Canuto and D. C. Kelly, Astrophys. and Space Sci. 17, 277 (1972).


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# Interactions and bound states of solitons as classical particles


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The interaction of localized nonlinear waves (solitons) is investigated. A theory is developed of weak interactions whose energy is small compared with the total field energy. In this case, for solitons with close velocities, the motion is described by the classical Newton equations with potential forces determined by the structure of the field far from the maxima. Three basic types of interaction are distinguished; a necessary criterion for the formation of bound states is given. In particular, the bound state of a pair of solitons with tails with an oscillatory structure is investigated. The results are presented of experiments with chains of nonlinear oscillators, in which oscillating solitons and all the types of interaction considered have been observed.



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## 1. INTRODUCTION

The question of the interaction of solitons has already been studied, by analytical and numerical methods, for a number of years. It has been found that, as a result of the interaction of infinitely separated (for $t\to-\infty$) solitons there remain (for $t\to+\infty$) diverging solitons with the same parameters as before the interaction (this property has even been used to define solitons[1]). At the same time, there are now certain exactly soluble equations which permit the existence of bound states of two or more solitons. By means of numerical methods, it has recently been made clear that solutions in the form of unrestrictedly diverging and bound solitons are characteristic not only of exactly integrable types of equations.[2,3] Thus, the numerical calculation carried out in[3] by Kudryavtsev for application to the Ginzburg-Landau equation showed the possibility of the existence of a bound pair of solitons. This question is interesting, in particular, in connection with the possible interpretation of the solitons as field particles. However, up to now there do not exist any general criteria determining the character of the interactions of solitons.

As shown in the present work, this question can be elucidated in a fairly general formulation applicable to weakly interacting solitons, when at each moment of time the total field differs little from the superposition of the fields of the individual solitons. The most important case of weak interactions is realized when the difference in the velocities of the solitons is small and the distance between their maxima remains large compared with their effective sizes over the duration of the entire process. Such processes yield to a universal approximate description, which enables us not only to solve concrete problems but also to give a classification of the possible types of interactions (one of these is considered below for the first time), from which follow simple criteria for the possibility of formation of bound states, valid not only for two but also for a larger number of interacting solitons.

## 2. THEORY

First of all we shall give a general qualitative description of the process of interaction of solitons. Since, by assumption, the solitons are separated from each other, each given (test) pulse is in the weak field of the tails of the other pulses. The pulse energy $E$ changes on account of the work $A$ performed by these tails.[1)] Because of the weakness of the interaction, we can assume that the field of the tail is fixed and that the work $A$ is related linearly to the magnitudes of the fields of the other solitons at the position of localization of the test soliton.[2)] As a result, the whole process is described by the equations

$$\frac{dE_i}{dt}=\sum_{k\neq i}\alpha_{ik}(v_i;v_k)f(v_k,s_{ik}),\quad i=1,2,\ldots,N, \tag{1}$$

where $N$ is the number of interacting solitons, $v_i$ are their velocities, $f(v_k, s_{ik})$ is the field of the $k$-th soliton at the position of the $i$-th one, and $s_{ik}$ is the distance between the maxima of the solitons. Of course, because the system is conservative, the sum of the right-hand sides of (1) over $i$ should be equal to zero.

It is clear that change of the character of the motion of the solitons when they intereact weakly can be substantial only when the differences in the energies (and velocities) of the solitons are small—only then can the work done by the small tails have an appreciable effect on this difference and, consequently, on the differences in the velocities of the pulses. This means that in the right-hand sides of (1) we can assume that all the $v_i$ are constant and equal, so that only the dependence of $f$ on $s_{ik}$ remains. But since $ds_{ik}/dt=v_i-v_k$, and $E_i$ is related to $v_i$, the system (1) is easily reduced to $N-1$ equations for the quantities $s_{ik}$. Thus, for a pair of solitons we obtain one equation:

$$s_{tt}=2\alpha v_E{}'f(v,s), \tag{2}$$

where $v=\text{const}$.

We note immediately that, in the given approximation, we arrive at the problem of the interaction of two classical particles, the force fields of which correspond to $f(s)$. The function $f(s)$, determined by the exponential tails (which can also oscillate), have, in the general case, the form

$$f(s)\sim\exp(-\lambda_1 s)\begin{Bmatrix}\sin\lambda_2 s\\ \cos\lambda_2 s\end{Bmatrix},$$

and, thus, Eq. (2) can be solved in quadratures. Qualitatively, however, the character of the interaction is determined as follows.

a) For a monotonic $f(s)$ ($\lambda_2=0$) the interaction corresponds either to repulsion only, or to attraction only. For most of the exact $N$-soliton solutions that have been found at present, repulsion is realized. In this case the relative velocity of approach of the solitons (if they were converging in the beginning) decreases, and, for sufficiently small initial values of $\Delta v$, the solitons, having approached to within a certain distance $s_{min}$ of each other, begin to diverge again. If here $\partial E/\partial v>0$, the mutual work has signs such that the soliton that is ahead always increases its energy and the soliton behind decreases its energy. It is easy to see that these conclusions are also valid for an arbitrary number of mutually repelling solitons.

b) If, for monotonic tails ($\lambda_2=0$) the force of the interaction of the solitons corresponds to attraction, then, even if they were diverging in the beginning, later, having moved apart to a maximum separation $s_{max}$ at which their energies become equal, the pulses begin to converge until their fields overlap strongly, after which Eq. (2) is not valid for describing the process.

However, if the solitons, slipping through each other, begin to diverge again at this stage, then later Eq. (2) is valid again. In this case the process is repeated—at the distance $s_{max}$ the solitons converge again, and so on. As a result there arises a bound state, in which the solitons perform an oscillating motion over an interval $2s_{max}$. For example, the numerical solution found in[3], in which a similar explanation of the appearance of the bound state is given, corresponds to precisely this case.

c) Finally, $\lambda$ can be complex ($\lambda_2\neq0$); then the soliton can have oscillating tails. In this case the sign of the right-hand side of (2) can vary. Here, in principle, both infinite motions and bound states are possible, the latter existing entirely in the framework of weak interactions of the solitons.

It is clear that this simple classification of the weak interactions of solitons is basically also valid for an arbitrary number of solitons of a single type, inasmuch as the character of the tails that determine such interactions is conserved. We shall give now a more systematic analysis of the interaction of solitons, as applied to the generalized Korteveg–de Vries (KdV) equation

$$\frac{\partial u}{\partial t}+u^p\frac{\partial u}{\partial x}+\frac{\partial^q u}{\partial x^q}=0. \tag{3}$$

For $q=3$ and any $p>0$ a solitary solution can be found in analytic form[5]:

$$u=A[\operatorname{sch}\Delta^{-1}(x-vt)]^{2/p},$$
$$\Delta^{-1}=\frac{pv^{1/2}}{2}=\left[\frac{p^2A^p}{2(p+1)(p+2)}\right]^{1/2}.$$

As already mentioned, we can assume that, for weak interaction, a given soliton lies in the small field of the tails of the other pulses. To describe such interactions we can apply an asymptotic-expansion method, in a form close to that which we gave in[6] for the case of aperiodic almost-stationary waves. Namely, we seek the field in the vicinity of a given soliton in the form

$$u_i(x,t)=U_i(\xi_i,\tau)+\sum_{k\neq i}U_k(\xi_k,\tau)+\sum_{n=1}^{\infty}\varepsilon^n u_i^{(n)}(\xi_i,\tau), \qquad (4)$$

where the $U_i$ correspond to stationary solitons with slowly varying parameters $v_i(\tau)$, $\xi_i=x-\int v_i dt$, $\tau=\varepsilon t$ is the "slow" time, and $\varepsilon\ll 1$ is a small parameter of the order of the relative magnitude of the field of the tails of the $k$-th soliton ($U_k$) at the position of the given (the $i$-th) soliton. Substituting (4) into (3), we obtain the equations of the successive approximations:

$$-v_i\frac{\partial u_i^{(n)}}{\partial\xi_i}+\frac{p}{p+1}\frac{\partial}{\partial\xi_i}(U_i^p u_i^{(n)})+\frac{\partial^2 u_i^{(n)}}{\partial\xi_i^q}=H_i^{(n)}, \qquad (5)$$

where, e.g.,

$$H_i^{(1)}=U_{i\tau}+\frac{p}{p+1}\sum_{k\neq i}\frac{\partial}{\partial x}(U_i^p U_k).$$

The requirements that the corrections $u_i^{(n)}$ be bounded are, as usual, equivalent to the conditions for the orthogonality of $H_i^{(n)}$ to the functions conjugate to the eigenfunctions of the operator in the left-hand side of (5), and this, in the given case, leads to the equations

$$\frac{d}{d\tau}\langle U_i^2\rangle=-\frac{2p}{(p+1)^2}\sum_{k\neq i}\left\langle U_i^p\frac{\partial U_k}{\partial\xi_k}\right\rangle, \qquad (6)$$

where

$$\langle\ldots\rangle_i=\int_{-\infty}^{+\infty}\ldots d\xi_i.$$

These equations are equivalent to (1), and, for close velocities, for two solitons an equation of the type (2) follows from them.

It is natural to clarify, first of all, what this approach gives for the ordinary KdV equation ($p=1, q=3$), since here there is the possibility of comparison with the exact solution of the problem. In this case, Eq. (2) takes the form

$$s_{\tau\tau}=16v^2\exp(-v^{1/2}s). \qquad (7)$$

This equation does not have equilibrium states (apart from $s\to\infty$), and the solitons always diverge. From the first integral of this equation it is easy to determine the quantities

$$s_{min}=\frac{2}{v^{1/2}}\ln\frac{8v}{(\Delta v)_\infty},\quad s(t\to\infty)=(\Delta v)_\infty t+\frac{v^{1/2}}{2}\ln\frac{4v}{(\Delta v)_\infty};$$

thus,

$$\psi=\ln\frac{4v}{(\Delta v)_\infty}$$

is the resulting shift in the phases of the solitons. The value of the field at the point of the minimum between the solitons for $s=s_{min}$ is $u_{min}=3(\Delta v)_\infty$. All this agrees with the exact solution[7] for $\Delta v\ll 1$; it is curious that for $u_{min}$ we even obtain the exact value.[8]

If $q=3$, $p<4$, the character of the interaction is qualitatively the same as in the ordinary KdV equation. However, for $q=3$ and $p>4$, it is not difficult to see that the soliton energy falls with increasing $v$, and we have the second of the cases considered above—interaction of solitons, with an attractive potential. Here we can expect the appearance of bound states, if, after passing through the region of strong overlap, the solitons again diverge.

We now consider the case $p=1$, $q=5$. There is no analytic expression for the stationary solitons in this case; they have been investigated numerically by Kawahara.[9] However, as follows from considerations of scaling, the general structure of the solution is given by the formula $u=v\varphi(v^{1/4}\xi)$, whence the relationship between the amplitude and duration of the soliton is immediately determined. By linearizing the equation for the stationary wave near $u=0$ it is also not difficult to determine the structure of the tails, which is important for the analysis of the weak interaction. It is easy to see that for $v<0$ (a slow soliton),

$$u\sim\exp\left(-\frac{v^{1/4}}{2}\xi\right)\cos\frac{v^{1/4}}{2}\xi,$$

i.e., the tails oscillate.[9] In this case Eq. (2) can be written in the form

$$s_{\tau\tau}=Rv^2\exp\left(-\frac{v^{1/4}}{2}s\right)\cos\frac{v^{1/4}}{2}(s+s_0), \qquad (8)$$

where

$$R=\sqrt{2}MC^{-1}(A^2+B^2)^{1/2},$$
$$s_0=\operatorname{arctg}(AB^{-1}),$$
$$A=\langle\varphi^2(z)e^{-z}\cos z\rangle,$$
$$B=\langle\varphi^2(z)e^{-z}\sin z\rangle,\quad C=\langle\varphi^2(z)\rangle.$$

$M$ is determined from the condition

$$\lim_{|z|\to\infty}\varphi(z)=Me^{-z}\cos z.$$

The principal types of motion in the framework of Eq. (8) are conveniently depicted in the phase plane (see Fig. 1b; for comparison, in Fig. 1a the phase plane of Eq. (7) is also shown). For certain initial conditions the solitons diverge without limit after the interaction (infinite trajectories). However, there exist regions of closed trajectories, corresponding to bound states with mutual oscillations of the solitons. As we have pointed out, these states remain present in the framework of the weak interaction. At the same time, it is natural to assume the existence of strongly bound states, due to the oscillatory character of the field of a soliton at small values of $|\xi|$.

Also of interest is the presence of points of equilibrium in the phase plane. In the given approximation these points correspond to nonoscillating bound states, i.e., stationary two-soliton waves (more precisely, double-

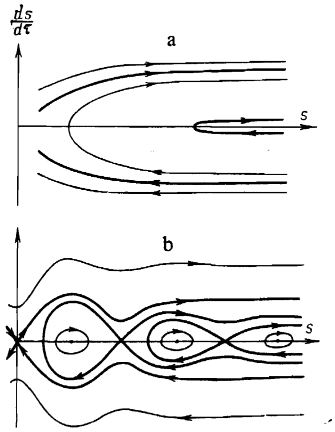


FIG. 1.

humped solitons). The possibility of the existence of such stationary waves with two and more maxima in the principal part of the solution merits further study for immediate application to strongly bound solitons.[3)]

To conclude the theoretical section we note the possibility of a weak interaction of another type, opposite in a certain sense to the type considered, when the energies (velocities) of the solitons differ greatly, i. e., $v_2 \ll v_1$ (cf. also[10]). Usually this also means a large difference in the sizes and durations of the solitons. Then one of the solitons serves as a smooth perturbation (a pedestal) for the other. The solution of such a problem for a short pulse can again be sought in the form of the series (4), but $U_2$ now corresponds not to the tail of the second soliton but to its entire field, assumed fixed. Then, e. g., in the case $p=1$ the change in the amplitude of the fast soliton is described by the equation

$$(B^m)_\tau = B^m U_{2x}, \quad B = v_1 - U_2, \quad m = \frac{2q-1}{q-1}. \tag{9}$$

The values $U_2$ and $U_{2x}$ are here taken locally, with $x = \int v_1 dt$.

Since $U_{1,2} \sim v_{1,2}$, to within terms of order $v_2/v_1 \ll 1$ we obtain $v_{1\tau} = (1 - m^{-1})U_{2\tau}$. From this it follows directly that the total field $u_{\max}$ at the symmetry point, at which the fast soliton reaches the peak of the slow one, is equal to $[v_{1\infty} + v_{2\infty} - m^{-1} v_{2\infty}\varphi(0)]\varphi(0)$. In the case of the KdV equation ($q=3$) it follows from this that $u_{\max} = 3(\Delta v)_\infty$, as for the case of a small difference in the velocities $v_{1,2}$. This result is again found to be exact. After integration of the expression for $v_{1\tau}$ in the case $q=3$, we obtain that the resulting phase shift of the soliton is equal to $4v_1^{-1}v_2^{1/2}$, which also follows from the exact solution for $v_2 \ll v_1$.[7] Of course, in the given case, in view of the large difference in the velocities the formation of bound states is impossible, and from this point of view it is less qualitatively interesting, although the possibility of a calculation of the resulting phase shift that does not require knowledge of the exact solution can again be seen here.

Of course, the theory expounded above is not fully rigorous and complete, since, in the first place, in certain cases its results lead to the necessity of taking strong interactions into account, and, secondly, there is no rigorous proof of the asymptotic convergence of the given method (the latter, however, is not especially important physically, since the character of the approximation is sufficiently clear). At the same time, the results obtained give an intuitive interpretation of the process of interaction of solitons and find quantitative confirmation in comparisons with exact analytic solutions in those cases where the latter exist. The possibility of the existence of bound states of solitons with oscillating tails is confirmed by experiments in an electromagnetic system, the results of which are given briefly below.

## 3. EXPERIMENT

To investigate the interactions experimentally, artificial wave-guides (lines) in the form of chains of identical links consisting of constant inductances and nonlinear capacitances (semiconductor diodes) have been used (Fig. 2). As has already been noted, such lines are extremely convenient and suitable analog systems for the study of nonlinear waves. Solitons and a number of processes associated with them, including damping and interaction, have already been observed in them.[4]

In the simplest case, the waves in such a system are described by the KdV equation. However, the introduction of inductive coupling between neighboring links, as shown in Fig. 2, changes the character of the dispersion, adding terms with higher derivatives to the corresponding equation. When the coupling coefficient has the value $M = 0.085$ the equation for the traveling waves in such a system coincides with (3) for $q=5$, $p=1$, i. e., the existence of solitons with oscillating tails and the formation of bound states are possible. Experimentally, the processes have been studied principally in such a system (an $MLC$-line), but for comparison we also give certain results pertaining to systems without inductive coupling ($LC$-lines), describable by the KdV equation.

As is well known, in an $LC$-system, from a sufficiently long initial pulse a group of unrestrictedly diverging solitons, arranged in order of decreasing amplitudes, is formed. In an $MLC$-system, under certain initial conditions the process can proceed in an analogous way, but in this case solitons with a field of alternating sign in the tails are formed, as can already be seen from Fig. 3a.

The process of decay of the longer pulse shown in Fig. 3b is interesting. Here the initial duration of the pulse is approximately four times greater than the duration $\tau_0$ of the stationary soliton of the corresponding amplitude. Therefore, as we might have expected, four solitons arise; however, three of them remain bound in a group, and only one splits off and stands apart from this group. This figure visually confirms the possibility of the for-

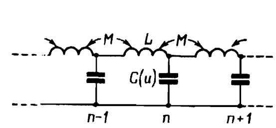


FIG. 2.

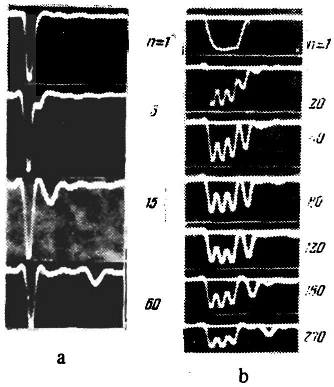


FIG. 3. Decay into solitons of pulses with initial duration $\tau \gtrsim \tau_0$ (a) and $\tau \sim 4\tau_0$ (b).

mation of bound states of solitons in the framework of Eq. (3) with $q=5$, $p=1$.

The interaction of two solitons, which is illustrated by Fig. 4, has been studied in more detail. Here, on successive oscillograms of the process, we also show the position of one of the initial pulses, moving without interaction and serving as a reference pulse that demonstrates the phase shift arising on account of the interaction.

In the KdV equation (Fig. 4a), solitons that are not too greatly different in amplitude approach until they form a symmetric two-humped state and then diverge again owing to the forward transfer of energy. This process, which is described by Lax, [8] has already been observed in an $LC$-system, [4] but Fig. 4a makes it possible to exhibit certain details of the interaction. In particular, it can be seen that, practically up to the point of closest approach, the maximum of the leading soliton propagates with unchanging velocity; but the larger pulse slows down while overtaking it. After the symmetry point has been passed, the solitons exchange roles: the leading soliton begins to accelerate and the trailing one moves with an almost constant velocity. Consequently, the resulting phase shift is acquired extremely sharply near the symmetry point, in a time much shorter than the total time for the exchange of energy between the solitons.

Figure 4b shows the interaction of solitons in the framework of Eq. (3) with $q=5$, $p=1$ (an $MLC$-system). Here the process of formation of a bound state can be clearly seen. After the approach, the solitons do not diverge at all, but (moving, as a whole, somewhat faster than the smaller of the initial pulses, which serves as the "reference" pulse) begin to oscillate about the overall "center of gravity" in such a way that the energy circulates between the solitons, and their amplitudes and velocities vary periodically. One complete period of these oscillations is shown in Fig. 4b.

We note that, here, at no time during the period of the oscillations do the solitons overlap each other completely (the profile of the wave remains two-humped throughout) and, roughly, we can always distinguish time intervals during which the solitons are either only attracting each other or only repelling each other. In this sense their binding is relatively weak.[4)] The pattern changes, however, when the oscillations of the field of the tails are deeper. This can be achieved by increasing the coupling coefficient to $M=0.13$, which corresponds to the appearance of a term with a third derivative in Eq. (3). In this case the work performed over the first oscillations of the solitons turns out to be so large that, when they approach, the distance between their maxima is found to be shorter than their characteristic sizes and the wave profile becomes single-humped (the absorption of one soliton by another). After this a pair of solitons is formed again, and the smaller of them begins to grow monotonically and the larger to diminish monotonically (see Fig. 4c). It is characteristic that, right up to the point of complete overlap, the small soliton only decreased during the approach, and the large one only grew. As a whole, this process proceeds as in the case of interaction of solitons with an attractive potential.

The results obtained confirm the principal conclusions of the theory—in particular, the conclusion that the formation of bound states of solitons is possible as a result of oscillatory structure of their tails.

In conclusion, we note that the approach described here is, in principle, also valid for describing the interaction of two- and three-dimensional solitons. In this case, inasmuch as the motion is not one-dimensional, the pattern of the interaction is qualitatively changed. Thus, in the case of attraction the bound state can have the form of solitons rotating about each other. On the other hand, even in the case of solitons diverging without limit after a collision, the result of the interaction is not as trivial as for one-dimensional motion: the energies (velocities) of the individual solitons before and after the interaction are, in the general case, different. In turn, this should lead to motion of mixed character in an ensemble of a large number of solitons, and to the establishment of stationary distributions, as in an ordinary gas.

Finally, we indicate other physical situations in which the bound states described can be realized. Thus, the propagation of magnetosonic waves at a certain critical angle to the magnetic-field direction is described by Eq.

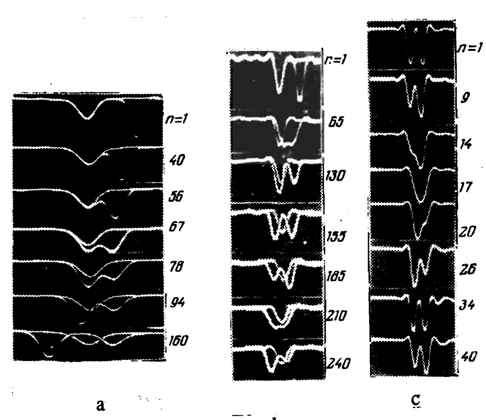


FIG. 4. Interaction of two solitons in an $LC$ line (a) and an $MLC$ line (b, c). In (a) and (b) the reference pulse is also shown.

(3) with $q=5$, $p=1$, inasmuch as the term with the third derivative vanishes for this direction.[11] Another example of interest for applications is the propagation of capillary-gravitational waves in shallow water, for which the realization of solitons with oscillations and the formation of bound states are also possible (cf.[9]).

The authors are grateful to E. N. Pelinovskiĭ for useful comments.

[1)]The possibility of such an interpretation of the interaction of solitons has already been noted previously.[4]

[2)]In the following we consider solitons each of which is determined by only one phase, of the type $x-vt$; this excludes "envelope solitons" (e.g., Langmuir solutions) from consideration, although, in principle, the given approach appears to be perfectly possible for these also.

[3)]This possiblity has now been confirmed by means of a numerical investigation of Eq. (3).

[4)]Weakly bound states in which the solitons are coupled by oscillations further from their maxima have also been observed experimentally. In this case the characteristic period of the oscillations is substantially increased, the changes in the amplitudes and velocities turn out to be considerably smaller, and the oscillograms are less revealing.

[1]A. C. Scott, F. Y. F. Chu, and D. W. McLaughlin, Proc. IEEE 61, 1443 (1973).
[2]R. K. Dodd, R. K. Bullough, and S. Duckworth, J. Phys. A 8, L64 (1975).
[3]A. E. Kudryavtsev, Pis'ma Zh. Eksp. Teor. Fiz. 22, 178 (1975) [JETP Lett. 22, 82 (1975)].
[4]L. A. Ostrovskiĭ, V. V. Papko, and E. N. Pelinovskiĭ, Izv. Vuzov, Radiofizika 15, 580 (1972).
[5]N. J. Zabusky, Nonlinear Partial Differential Equations, Academic Press, N.Y., 1967.
[6]K. A. Gorshkov, L. A. Ostrovsky, and E. N. Pelinovsky, Proc. IEEE 62, 1B, 11 (1974).
[7]V. E. Zakharov, Zh. Eksp. Teor. Fiz. 60, 993 (1971) [Sov. Phys. JETP 33, 538 (1971)].
[8]P. D. Lax, SIAM Rev. 11, 7 (1969) (Russ. transl. in "Matematika" ("Mir", M.) 13, No. 5, 128 (1969)).
[9]T. Kawahara, J. Phys. Soc. Japan 33, 260 (1972).
[10]G. M. Zaslavskiĭ, Zh. Eksp. Teor. Fiz. 62, 2129 (1972) [Sov. Phys. JETP 35, 1113 (1972)].
[11]T. Kakutani and H. Ono, J. Phys. Soc. Japan 26, 1305 (1969).

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# Stationary model of the "corona" of spherical laser targets

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An analytic stationary model of the "corona" of spherical laser targets is considered with account taken of the major physical processes, viz. hydrodynamic processes, laser radiation absorption in the vicinity of the critical point, and electron thermal conductivity. Exressions for the "corona" parameters are derived as functions of the laser pulse parameters (radiation flux and frequency) and of the target (radius and thermophysical properties).

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1. As follows from a number of studies,[1,2] all the known schemes for laser initiation of thermonuclear reactions consist of three physical stages: evaporation, compression, and thermonuclear combustion. These stages are governed by different physical processes and exert different influences on the set of final parameters characterizing the laser-induced thermonuclear fusion process as a whole.

The initial stage of the interaction of the laser radiation with the target material consists of evaporation and heating of a definite fraction of the medium, i.e., formation of a "corona," which is a hot plasma of relatively low density that expands in a direction opposite to the incident radiation. During this stage, a pressure pulse is generated at the boundary between the corona and the dense cold material and accelerates the unevaporated part of the target towards the center. The principal parameter characterizing the evaporation stage is the hydrodynamic efficiency,[3] i.e., the ratio of the energy of the unevaporated part of the target to the total laser-emission energy. The magnitude of this ratio determines both the energy balance in the system, i.e., the temperature of the central region of the target, and the maximum value of the thermonuclear-fuel mass that can be compressed by the radiation to a high density at a given energy.[1] Moreover, the degree of the compression of the target material also depends on the shape and amplitude of the pressure pulse.

The indicated quantities—the hydrodynamic efficiency and the pressure amplitude—depend essentially on the physical state of the corona, a state determined by the parameters of the laser pulse (flux density, duration, radiation frequency) and of the target (radius and thermophysical constants of the evaporated layer).

It should be noted that the process of formation and expansion of the corona as well as the compression pro-

  