Oscillations of a vortex system in rotating He II

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New data are presented on the oscillations of a vortex system in rotating He II. A vessel with He II magnetically suspended without support is employed for observing the vortex oscillations. The results are compared with the theory. It is shown that the frequency of oscillations is mainly determined by pinning of the vortex to the bottom of the vessel. Beats of the vortex oscillations are observed.

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The investigation of the oscillations of vortices formed in rotating superfluid is of great interest. In the study of elastic waves propagating along the vortices, information was obtained on the elastic constant ν_s by Hall, [1] Andronikashvili and Tsakadze[2] and by Nadirashvili and Tsakadze, [3] and its numerical value was established. It was shown experimentally[3] that the penetration depth of the deeply penetrating wave (the S^- wave), first calculated in the theoretical work of Manaladze and Matinyan, [4] is actually very large and exceeds the penetration depth of the viscous wave in magnitude.

In addition to waves propagating along vortices, there can also exist waves whose wave vector is directed perpendicular to the vortices. These waves can exist only in a regular lattice. Such "sound" oscillations of a triangular vortex lattice were predicted theoretically and studied by Tkachenko. [5] Theoretical works devoted to the study of the properties of the Tkachenko waves were completed by Stauffer, [6] Dyson and Ter Haar, [7] Tkachenko, [8] Beatto[9] and others. The presence of a standing Tkachenko wave in the vortex lattice of a rotating neutron superfluid liquid was used by Ruderman[10] for the elucidation of the oscillation of the velocity of rotating pulsar–neutron stars following the jump of their angular velocity.

In the recent work of Sonin, [11] the oscillations of a system of vortices was studied theoretically for the case in which the wave vector has components both along the vortices and perpendicular to them. Earlier, D. Tsakadze and S. Tsakadze[12] and Tkachenko[13] reported experiments in which the latter case is evidently realized.

In the present work, we report new data on the observation of the oscillation of a system of vortices in rotating He II.

DESCRIPTION OF THE APPARATUS

We used cylindrical vessels, made of plastic. Vessel No. 1 had the following dimensions: diameter 64 mm, height $h=50$ mm, thickness of the wall 0.2 mm. In several cases, the polished end faces of this vessel were coated by a single layer of sand particles with linear dimensions ~50 μ, to reduce the slip of the vortices relative to these surfaces. With the purpose of reducing the distance between the solid surfaces, two coaxial cylindrical surfaces were inserted in vessel No. 1, with diameters 43 and 22 mm. Vessel No. 2 has the following dimensions: diameter 15 mm, $h=70$ mm. In vessel No. 2, diaphragms in the form of disks were inserted in several experiments; these were parallel to the plane of the vessel bottom and were separated by 7 mm from one another. In this case, the vortices were decreased by a factor of about 10.

The vessels, located on the shaft of a small electric induction motor, were suspended in a magnetic field without support and, after an initial pulse, created by short-time operation of the electric motor, could rotate for a period of several hours. The damping of the rotation is determined chiefly by the viscosity of the helium vapor.

The rate of rotation of the vessels filled with liquid helium was measured with the help of a light beam reflected from a mirror attached to the axis of the apparatus. The ray reflected from the mirror was incident on a photoconductor and created an electric pulse, which was recorded by a digital printout unit and the electronic computer M-100. The treatment of the experimental data was carried out with the aid of a high-speed computer. Sometimes, partial compensation of the retarding moment acting on the freely suspended apparatus through the action of the helium vapors was carried out. For this purpose, a weak current was passed through the winding of the electric motor. Before each measurement, the vessel was filled with liquid helium by means of a special apparatus. More detailed information on the apparatus can be found in the works of D. Tsakadze and S. Tsakadze. [14,15]

EXPERIMENTAL RESULTS

It is natural that for the excitation of oscillations of a vortex system it is necessary to perturb it. We have used two methods: a strong acceleration of an already rotating vessel and radial oscillations of the vessel with the vortices. The period of the radial oscillations amounted to ~1.4 sec. In both cases, oscillations of the rate of rotation were observed, which are very quickly damped out for the vessel with smooth end faces in the case of generation of the oscillations by acceleration, while the oscillations in the case of radial vibrations of the same vessel could last as long as desired (see curves a, b in Fig. 1). When the vessel was filled

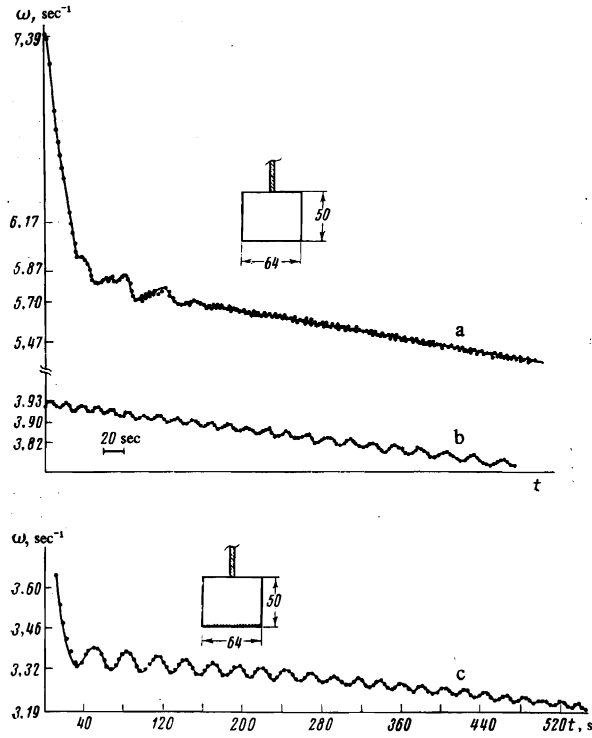

FIG. 1. Oscillations of the angular velocity of vessel No. 1 with He II at 1.46 °K. Curve a—vessel with smooth surfaces after rapid acceleration; b—the same vessel in the case of radial oscillations; c—the vessel with polished surface after rapid acceleration (the scale is logarithmic on the ordinate axis).

with He I, no oscillations of the rate of rotation were observed either in the case of acceleration of the vessel or in the case of radial oscillations.

The amplitude of the oscillations varied in the experiments in the limits $\sim(0.2-0.005)\ \mathrm{sec}^{-1}$. It was impossible in our case to distinguish smaller oscillations from the noise. In the case of use of a vessel with polished faces, the damping of the oscillations of the rate of rotation decreased very strongly after acceleration of the vessel (see Fig. 1, curve c).

The frequency of oscillations of vessels No. 1 and No. 2 in the case of identical angular velocities is the same within limits of error. Moreover, the frequency does not change upon insertion of coaxial surfaces in vessel No. 1. These facts indicate that the frequency of the oscillations does not depend on the radius of the vessel. However, upon decrease in the length of the vortices by means of diaphragms placed in vessel No. 2, the frequency of the oscillations increases by a factor of two. In many cases, beats of the oscillations of the rate of rotation were observed (Fig. 2).

As mentioned earlier, Ruderman,[10] attempting to explain the oscillations of the rate of rotation of pulsars after their acceleration, assumed that they were produced by the appearance of standing Tkachenko waves in the vortex lattice of the superfluid neutron liquid. The spectrum of the Tkachenko waves has the form[13]

$$\Omega^2=c_T^2q^2. \quad (1)$$

Here Ω is the frequency of the oscillations, q the wave vector, and c_T the velocity of propagation of the wave:

$$c_T^2=\varkappa\omega/8\pi, \quad (2)$$

ω is the angular velocity of rotation, $\varkappa$ is the quantum of circulation.

For Tkachenko waves in an infinitely long cylinder, Ruderman found that the period of the fundamental mode of the oscillations is equal to

$$\theta=\frac{4\pi}{5}\left(\frac{m}{\hbar\omega}\right)^{1/2}R, \quad (3)$$

m is the mass of the boson, R the radius of the cylinder. The result obtained from this formula agrees well with the observed periods of oscillation of the pulsars[10]; however, in comparison with the data of our experiments, it gives a period that is too large by a factor of 8. This means that the Ruderman model is not valid for our case.

In the work of Sonin,[11] a possible explanation of the smallness of the period of oscillations in our experiments was given. Sonin considered the oscillations of

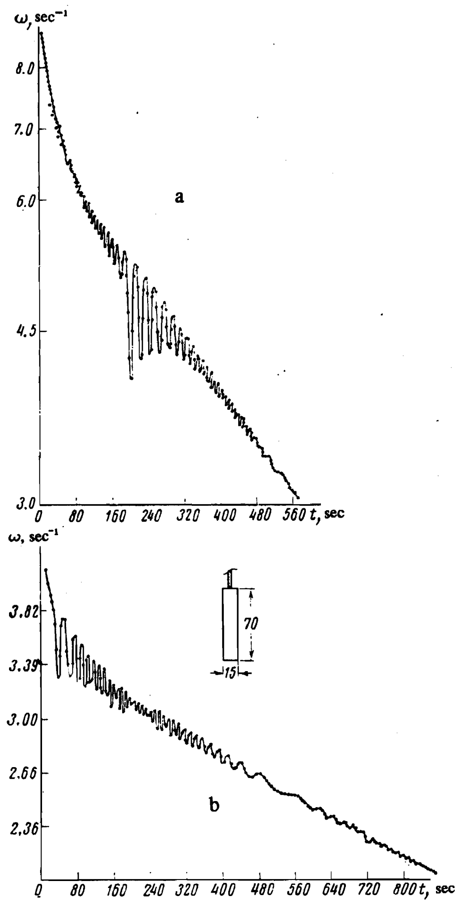

FIG. 2. Beats arising after rapid acceleration of vessel No. 2 (T = 1.46 °K) in the case of different angular velocities of rotation (the scale is logarithmic on the ordinate axis).

indices with arbitrary direction of the wave vector and took into account the presence of boundaries. He showed that even an insignificant bending of the vortices (which is very difficult to avoid in the case of vortices with bounded length because of the presence of contact with the bottom of the vessel) leads to an appreciable change in the spectrum of oscillations, which takes the form

$$\Omega^2=(2\omega)^2\frac{p^2}{p^2+q^2}+c_T^2q^2. \quad (4)$$

Here p is the component of the wave vector along the vortices and q is the same in the direction perpendicular to the vortex direction. The case $p=0$ corresponds to pure Tkachenko oscillations; however, even a small bending of the vortices ($p\neq 0$) gives a significant correction, since $2\omega \gg c_T q$. For the characteristic frequencies of oscillations in the cylinder, we obtain the condition

$$\frac{1+\beta}{4}-\frac{q^2}{q^2-u^2}\left(\frac{1}{4}-\frac{J_2[R(q^2-u^2)^{1/2}]}{R(q^2-u^2)^{1/2}J_1[R(q^2-u^2)^{1/2}]}\right)=0, \quad (5)$$

where β is the ratio of the moment of inertia of the vessel to the moment of inertia of the liquid, $u=2\omega/c_T\sqrt{AL}$, L is the length of the vessel. The quantity A is connected with the slip coefficient:

$$b=\frac{K_t^2}{A-K_t},\quad K_t=\left(\frac{2\omega}{\varkappa\ln(r_n/a)}\right)^{1/2}.$$

In the limit $u\gg 1/R$ (which corresponds into the situation in our experiments), we have for the fundamental eigenfrequency

$$\Omega=2\omega/\sqrt{AL}. \quad (6)$$

As can be seen, the frequency of the oscillations in this case is generally not dependent on the "rigidity" of the vortex lattice (c_T).

Unfortunately, the formula (6) cannot be applied directly in a test of the experimental data, since the slip coefficient b, which determines A, depends strongly on Ω. However, the real and imaginary parts of b can be determined from an analysis of the damped oscillations. As shown in Ref. 11, the damped oscillations in Fig. 1a correspond to $b\approx(25-6i)\ \mathrm{cm}^{-1}$, which is in agreement with the order of magnitude from the data of Gamtsemlidze *et al.*,[16] who determined the imaginary part of b and obtained $b\sim -2i\ \mathrm{cm}^{-1}$.

For the damped oscillations in Fig. 1c, we have $\Omega=(18-0.003i)\ \mathrm{sec}^{-1}$. The corresponding slip coefficient $b\approx(37-1.8i)\ \mathrm{cm}^{-1}$. The imaginary part of the slip coefficient, obtained by us, agrees with the coefficient of Gamtsemlidze *et al.*[16]

It should be noted that in Ref. 16, apparatus was used with roughnesses of $\sim 50\ \mu$. The introduction of the roughnesses contributes to good pinning of the vortices, decreasing the slip and the viscous interaction of the vortices with the bottom of the vessel, which is the reason for the decrease in the damping of the oscillations.

As is seen from Eq. (6), the frequency of the fundamental mode of the oscillations does not depend on the radius of the vessel, which is confirmed by experiment. In Ref. 11 it was shown that the difference between the fundamental and the next frequency is equal to

$$\Delta\Omega=3.7\ c_T^2/\Omega R^2 \quad (7)$$

and the generation of beats between these two frequencies is possible. However, the period of the beats, observed experimentally (Fig. 2) is smaller by a factor of 6 than that predicted by Eq. (7). The following can be said concerning the disagreement of the calculated and measured periods of the beats: Eq. (7) gives the greatest period, corresponding to neighboring eigenfrequencies. Since, because of the closeness, not only the very close frequencies can interact, but also those separated by a relatively large interval of frequency, then observation of beats with periods smaller than that predicted by Eq. (7) is possible experimentally. Evidently, such a possibility is realized in our experiments.

So far as the agreement of the observed periods of oscillation of pulsars with the Ruderman formula (3) is concerned, possibly the vortices in the pulsars are deformed less and the role of oscillations with $p\neq 0$ is small, in view of the large length of the vortices in the pulsars, of the small amplitude of the oscillations and the degree of smoothness of the internal surface of their envelop. In this connection, it is appropriate to recall that, according to Ref. 14, the behavior of the pulsar is better modeled in the case of a smooth surface of the vessel with He II.

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