volume. This result may be due to the spatial inhomogeneity of the spin density, due both to the distribution of the intensity of the exciting light in the semiconductor volume and to the possible localization of the oriented electrons.

We note the strong temperature dependence of the observed effect when the temperature T_0 is raised from 77 °K to ~100 °K. In this range, the electron spin-relaxation time is $\tau_S \sim T_0^{-n}$, where $n = 2-3$.[14,15] If $\langle S_z \rangle_0$ is small, so that $\langle S_z \rangle_0 = 0.25 T/\tau \approx 0.25 \tau_S/\tau$, then $\beta \sim \langle S_z \rangle_0^2 \sim T_0^{-4} - T_0^{-6}$. The experimentally observed (Fig. 7) vanishing of the structure of the $\rho(H_x)$ curve when T_0 changes from 77 °K to ~100 °K can be attributed to the abrupt decrease of β. At $T_0 = 4.2$ °K, the plots of $\rho(H_x)$ do not differ qualitatively from those observed at 77 °K.

We have not considered here transient processes or a number of dynamic effects (excitation by intermittent light, application of a pulsed magnetic field, etc.) which indicate that the stationary values of the polarization take a long time to reach the steady state. A quantitative interpretation of these results calls for an additional analysis. In particular, it is necessary to explain the difference between the half-widths of the $\rho(H_x)$ curves when the semiconductor is excited with light with constant and alternating-sign circular polarization.

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Magnetization precession in superfluid phases of He^3

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Solutions of the Leggett equations are obtained which describe the motion of magnetization in both superfluid phases of He^3 located in a strong dc magnetic field. Rotation of magnetization by an ac magnetic field is described. The results are compared with available experimental data.

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1. INTRODUCTION

Progress in the understanding of the nature of the superfluid phases of He^3 is due to a considerable degree to the study of the properties of these phases by the NMR method. The corresponding experiments lend themselves to a quantitative interpretation with the aid of the system of equations for spin dynamics in triplet pairing, obtained by Leggett[1]:

$$\dot{\mathbf{S}} = \gamma[\mathbf{S}\times\mathbf{H}] + \mathbf{R}_D(\mathbf{d}), \tag{1}$$

$$\dot{\mathbf{d}} = [\mathbf{d}(\mathbf{n})\times\gamma(\mathbf{H}-\gamma\mathbf{S}/\chi)]. \tag{2}$$

Here **H** is the external magnetic field, **S** is the total spin of the considered amount of helium, χ is its magnetic susceptibility, γ is the gyromagnetic ratio for the He^3 nuclei, and **d** is a vector in spin space and characterizes the spin structure of the wave function of the condensate. Its exact determination (see[2], p. 367), will not be needed here. When the pairing is in the p state, as is the case in He^3, **d** depends linearly on the unit vector $\mathbf{n}$, which determines the direction in momentum space, i.e., $d_i = A_{ik}n_k$. $\mathbf{R}_D$ is the moment resulting from the interaction of the magnetic dipole moment of the He^3 nuclei:

$$(R_D)_k = -{}^2/_5 g_D e_{klm} [\mathrm{Re}(A_{lm}A_{jj}^* + A_{lj}A_{jm}^*)], \tag{3}$$

g_D is the dipole–dipole interaction constant, and e_{klm} is an absolutely antisymmetrical tensor. The information on the properties of the superfluid phases is contained precisely in this additional term of Eq. (1).

Osheroff and Corruccini[3,4] have recently investigated the behavior of the magnetization in both superfluid phases of He^3 by the pulsed NMR method. In these experiments they obtained, in particular, the dependence of the shift of the frequency of the transverse magnetic resonance on the initial angle β_0 between the magnetization $\gamma\mathbf{S}$ and the external field $\mathbf{H}$. Brinkman and Smith[5,6] found stationary solutions of the system (1), (2), corresponding to the precession of the magnetization with a frequency that depends on the angle in accordance with the laws established experimentally by Osheroff and Corruccini for each of the phases. It should be noted, however, that although the final formula of Brinkman and Smith[6] for the shift of the frequency of the transverse resonance in the B phase is correct, there are objections to the method used for its derivation, and the conclusion they drew that a relaxation term must be introduced into Leggett's equations in order to obtain stationary solutions will be shown by the results of the present paper to be in error.

In this paper we use the system of equations (1) and (2) to consider the behavior of the magnetization in sufficiently strong magnetic fields, when the second term in the right-hand side of (1) is small in comparison with the first. The known Van-der-Pol method of oscillation theory makes it possible in this case to obtain solutions that are asymptotic with respect to a small parameter, which in this case is the square of the ratio of the frequency of the longitudinal oscillations of the magnetization to the Larmor frequency $\Omega^2 = 3g_D/\chi H^2$. In the Osheroff–Corruccini experiments, $\Omega^2 \sim 1/100$ for both phases.

Equations (1) and (2) do not describe relaxation processes, and we shall therefore not deal in this paper with questions for which allowance for these processes is essential. Allowance for relaxation should not influence the stationary solution and has little effect (to the extent that the damping is small) on the frequency of the small oscillations. For processes in which the relaxation plays an important role, the analysis of Eqs. (1) and (2) may turn out to be useful as the first step in the subsequent allowance for the damping.

2. TRANSFORMATION OF LEVELS

We shall consider for the time being the process of taking the magnetization out of equilibrium, when the field $\mathbf{H}$ is constant. To separate the effects due to $\mathbf{R}_D$, we change over to a coordinate system that rotates with Larmor frequency $\omega_L = -\gamma H$, and introduce the dimensionless quantities $\mathbf{M} = \gamma\mathbf{S}/\chi H$ and $t' = \omega_L t$, i.e., we measure the time in Larmor periods, while the unit of magnetization measurements will be taken to be its value in the equilibrium state. The system (1) and (2) then takes the form

$$\dot{M}_k = {}^2/_{15}\Omega^2 e_{kln} [\mathrm{Re}(A_{ln}A_{jj}^* + A_{lj}A_{jn}^*)], \tag{4}$$

$$\dot{\mathbf{d}} = -[\mathbf{M} \times \mathbf{d}]. \tag{5}$$

We are interested in magnetization changes that are slow in comparison with the Larmor period. To solve (5) with slowly varying $\mathbf{M}(t')$, we employ the standard adiabatic perturbation theory procedure (see[7], p. 94). We find first the instantaneous eigenvalues and eigenvectors of the right-hand side of (5)

$$[\mathbf{M} \times \mathbf{a}^{(m)}] = \mu^{(m)}\mathbf{a}^{(m)}, \quad m = -1, 0, +1. \tag{6}$$

The eigenvalues will be $\mu^{(m)} = imM$, where $M \equiv |\mathbf{M}|$, and the eigenvectors are $\mathbf{a}^{(0)} = \mathbf{M}/M$ and two additional vectors, $\mathbf{a}^{(1)}$ and $\mathbf{a}^{(-1)}$, which form together with $\mathbf{a}^{(0)}$ a complex-orthonormalized basis

$$\mathbf{a}^{(m)*}\mathbf{a}^{(n)} = \delta^{mn}. \tag{7}$$

We seek the solution of (5) in the form

$$\mathbf{d}(t') = \sum_m \alpha_m(t')\mathbf{a}^{(m)}(t') \exp\{i\sigma_m(t')\}. \tag{8}$$

Substituting this expression in (5) we obtain

$$\dot{\sigma}_m = -mM, \tag{9}$$

$$\dot{\alpha}_m = -\sum_m \alpha_n \mathbf{a}^{(m)*}\dot{\mathbf{a}}^{(n)} \exp\{i(\sigma_n - \sigma_m)\}. \tag{10}$$

We note that the conditions (7) offer a freedom in the choice of the eigenvectors $\mathbf{a}^{(1)}$ and $\mathbf{a}^{(-1)}$, owing to the possibility of rotating them around $\mathbf{a}^{(0)}$ through an arbitrary angle. This freedom is sufficient to require satisfaction of the following conditions:

$$\mathbf{a}^{(m)*}\dot{\mathbf{a}}^{(m)} = 0. \tag{11}$$

When (11) is satisfied we have $\dot{\alpha}_n = 0$ with exponential accuracy in Ω^2. As a result we get

$$\alpha_m(t') = \alpha_m(0) = \mathbf{a}^{(m)*}(0)\mathbf{d}(0),$$

$$\mathbf{d}(t') = \sum_m \mathbf{a}^{(m)}(t') \exp\left\{-im\int_0^{t'} M dt\right\} \mathbf{a}^{(m)*}(0)\mathbf{d}(0). \tag{12}$$

It remains to express $\mathbf{a}^{(m)}$ in terms of $\mathbf{M}$. This can be done if it is noted that

$$\sum_m a_i^{(m)}(t') a_k^{(m)*}(0) = R_{ik} \tag{13}$$

is the rotation matrix that transforms the triad of vectors $\mathbf{a}^{(m)}(0)$ into the triad $\mathbf{a}^{(m)}(t')$. This rotation can be described by Euler angles in accordance with the definition

$$\hat{R}(\alpha, \beta, \gamma) = \hat{R}_z(\alpha)\hat{R}_y(\beta)\hat{R}_z(\gamma).$$

Here $R_z(\alpha)$ is the matrix of rotation around the z axis of the initial system of coordinates through an angle α, etc.; α and β are respectively the azimuthal and polar angles of the vector $\mathbf{M}$ in the system $\mathbf{a}^{(m)}(0)$. The conditions (11) connect γ with α and β:

$$\dot\gamma+\dot\alpha\cos\beta=0. \tag{14}$$

Comparing (12) and (13), we easily see that

$$\mathbf{d}(t')=\hat R\left(\alpha,\beta,\gamma-\int_0^{t'} M\,dt\right)\mathbf{d}(0). \tag{15}$$

Together with (14), this formula expresses $\mathbf{d}$, and hence also A_{ik}, in terms of $\mathbf{M}(t')$. Substituting the thus-obtained A_{ik} in the right-hand side of (4), we obtain a vector equation that contains only $\mathbf{M}$. The subsequent transformation of the obtained equation and an investigation of its solutions differ somewhat for different phases of He^3. It will be more convenient to start with the B phase.

3. THE B PHASE

In this phase, at any instant of time, the matrix $A_{ik}\equiv\hat A$ is the matrix of rotation about a certain direction $\boldsymbol\nu$ through an angle θ. To simplify the manipulation we assume that at $t=0$ the system was at equilibrium, i.e., $\boldsymbol\nu\parallel\mathbf{H}$ and $\theta=\theta_0\equiv\arccos(-\frac14)$. Under this condition, no terms with Larmor frequency or its harmonics, in which we are not interested here, will appear in the course of the solution. Changing over to a coordinate frame that rotates with ω_L and with respect to the second index of the matrix A, we obtain with the aid of (15)

$$\hat A(t')=\hat R\left(\alpha,\beta,\gamma+\theta_0+\int_0^{t'}(1-M)\,dt\right). \tag{16}$$

Using the explicit expression for the elements of the matrix for the rotation through the Euler angle, we can obtain $\theta(t')$ and $\boldsymbol\nu(t')$:

$$1+2\cos\theta=\mathrm{Sp}\,\hat A=\cos\beta+\cos\Phi+\cos\beta\cos\Phi;$$
$$\Phi=\alpha+\gamma+\theta_0+\int_0^{t'}(1-M)\,dt,$$
$$2\nu_i\sin\theta=-e_{ikl}A_{kl}. \tag{18}$$

Substituting now (16) in (4) and projecting the obtained equations on the directions of $\mathbf{M}$, $\mathbf{M}\times[\mathbf{H}\times\mathbf{M}]$, and $\mathbf{H}\times\mathbf{M}$, we obtain respectively

$$\dot M={}^2/_{15}\Omega_B^2(1+\cos\beta)(1-2\,\mathrm{Sp}\,\hat A), \tag{19}$$
$$M\dot\beta=-{}^2/_{15}\Omega_B^2\sin\beta\sin\Phi(1-2\,\mathrm{Sp}\,\hat A). \tag{20}$$
$$M\dot\alpha={}^2/_{15}\Omega_B^2(1+\cos\Phi)(1-2\,\mathrm{Sp}\,\hat A). \tag{21}$$

Using (21)–(24) we obtain an equation for Φ:

$$\dot\Phi={}^2/_{15}\Omega_B^2M^{-1}(1-\cos\beta)(1+\cos\Phi)(1-2\,\mathrm{Sp}\,\hat A)+1-M. \tag{22}$$

Dividing (19) by (20), we obtain following elementary integration

$$M(1-\cos\beta)=P=\text{const}. \tag{23}$$

The integral (23) enables us to exclude β from (19) and (22), after which these form a closed system

$$\dot M=\frac{4}{15}\Omega_B^2\left(2-\frac{P}{M}\right)\sin\Phi\left[\frac{P}{M}-\frac12+\left(\frac{P}{M}-2\right)\cos\Phi\right], \tag{24}$$
$$\dot\Phi=\frac{4}{15}\Omega_B^2\frac{P}{M^2}(1+\cos\Phi)\left[\frac{P}{M}-\frac12+\left(\frac{P}{M}-2\right)\cos\Phi\right]+1-M. \tag{25}$$

This system is of the Hamilton type ($\dot M=\partial\mathscr{H}/\partial\Phi$, $\dot\Phi=-\partial\mathscr{H}/\partial M$) with Hamiltonian

$$\mathscr{H}_B(M,\Phi)=\frac{(M-1)^2}{2}+\frac{2}{15}\Omega_B^2\left[\left(\frac{P}{M}-\frac12\right)+\left(\frac{P}{M}-2\right)\cos\Phi\right]^2 \tag{26}$$

and therefore has an energy integral

$$\mathscr{H}_B(M,\Phi)=E=\text{const}. \tag{27}$$

The character of the solutions of the system (24) and (25) can be illustratively represented with aid of the so called phase trajectories, i.e., trajectories describing the representative point in terms of the coordinates M and Φ as the system moves. The equation for the phase trajectories is the energy conservation law (27). Since the Hamiltonian depends on P as a parameter, the picture of the phase transition will also depend on this parameter. We can separate three qualitatively differing regions:

1) $0\leqslant P<{}^5/_4$, 2) ${}^5/_4\leqslant P<2$, 3) $P\geqslant 2$.

The character of the phase trajectories is determined by the locations of the stationary points of the system, i.e., the point at which $\dot M=0$ and $\dot\Phi=0$, and by the location of the separatrices, i.e., the phase trajectories passing through the stationary points. We will recall that motion along the separatrix takes place with an infinite period. Since the Hamiltonian depends on $\cos\Phi$, it suffices to consider any interval of variation of Φ with length 2π. We shall assume that $-\pi\leqslant\Phi\leqslant\pi$, with the straight lines $\Phi=\pi$ and $\Phi=-\pi$ coinciding.

At $P<\frac54$ there are four stationary points: two centers (see Fig. 1)

$$M_C=1,\quad \cos\Phi_C=\frac{P-{}^1/_2}{2-P}. \tag{C_1, C_2}$$

and two saddles

$$\Phi_{S1}=0,\quad M_{S1}=1+\frac{4}{15}\Omega_B^2\frac{P}{M^2}\left(5-4\frac{P}{M}\right), \tag{S_1}$$
$$\Phi_{S2}=\pm\pi,\quad M_{S2}=1. \tag{S_2}$$

Substituting the stationary solutions corresponding to the centers C_1 and C_2 in (21), we see that for these we have $\dot\alpha=0$, i.e., in an immobile coordinate system these solutions correspond to precession of the magnetization at the Larmor frequency. At small deviations of M and Φ from the stationary values C_1 and C_2, they will execute small oscillations about these values, with frequency

$$\omega_{\mathrm{tr}}^2={}^4/_5\Omega_B^2({}^5/_4-P).$$

It may be more convenient to express this frequency in terms of the value of β_c for the stationary point

$$\omega_{\mathrm{tr}}^2={}^1/_5(1+4\cos\beta_c)\Omega_B^2. \tag{28}$$

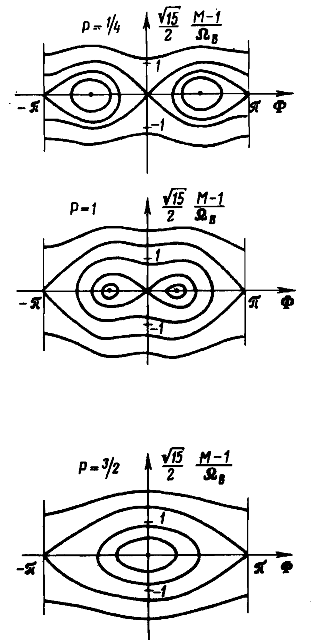

FIG. 1.

At $\beta_c = 0$, these oscillations go over into longitudinal magnetization oscillations (cf. [1]). The energies corresponding to the saddle points S_1 and S_2 become comparable at a certain $P = P_1 \approx \frac{1}{2}$. At this value of P a change takes place in the character of the arrangement of the separatrices.

With increasing P, the points C_1, C_2, and S_1 come closer together and coalesce into one at $P = \frac{5}{4}$. At $\frac{5}{4} \leq P < 2$ there remain only two stationary points: S_1, which has now become a center, and S_2, which has remained a saddle. The solution S_1 corresponds to precession of the magnetization at a frequency different from the Larmor frequency. The frequency shift is obtained by substituting S_1 in (21):

$$\delta\omega = \dot{\alpha} = \frac{4}{3}\frac{\Omega_B^2}{M_{s1}}\left(\frac{4}{5}\frac{P}{M_{s1}} - 1\right) = -\frac{4}{15}\frac{\Omega_B^2}{M_{s1}}(1 + 4\cos\beta_{s1}). \tag{29}$$

This formula coincides with that previously obtained by Brinkman and Smith.[6] The small oscillations about S_1 occur at a frequency

$$\omega_{tr}^2 = -\frac{2}{15}\Omega_B^2(1+4\cos\beta_{s1})(1+\cos\beta_{s1}). \tag{30}$$

We have left out from the last formula the terms $\sim\Omega_B^4$.

At $P > 2M_{S_1}$ the point S_1 becomes a saddle, while S_2 becomes a center. This case will not be considered here, since the point S_2 corresponds to $\cos\beta_{S_2} > 1$.

4. ROTATION OF THE MAGNETIZATION AND COMPARISON WITH EXPERIMENTS FOR THE *B* PHASE

The experimental results for the frequency shift of the transverse resonance are well described by formula (29) at angles β_{S_1} that are not too close to π. However, starting with $\beta_{S_1} \approx 140-150°$, an appreciable discrepancy between the experimental points and the theoretical curve is observed (see the figure in[4]), and when the magnetization is rotated through an angle larger than π, a strong scatter of the experimental data is observed. To ascertain the cause of this apparent discrepancy between theory and experiment, it is necessary to consider the process of rotation of the magnetization, as a result of which the magnetization is inclined a certain angle β_0 to the direction of $\mathbf{H}$. This rotation is effected by turning on for a certain time an alternating magnetic field directed perpendicular to the main field H and having the Larmor frequency, i.e., $\frac{1}{2}H_1 e^{it'}$. The deflecting field is constant in the coordinate frame that rotates at the Larmor frequency.

In the absence of a shift of the transverse-resonance frequency, the magnetization precesses under the influence of the deflecting field in a plane perpendicular to this field. Owing to the influence of the aforementioned shift, the motion of the magnetization will be more complicated and it must be determined by solving the equations of motion of the magnetization in the presence of a deflecting field. Allowance for this field leads to the following changes of the system (19)–(22): in Eqs. (20)–(22) there will appear increments proportional to $h \equiv H_1/2H$,

$$M\dot{\beta} = hM\cos\alpha - {}^2/_{15}\Omega_B^2 \sin\beta\sin\Phi[1 - 2\,\mathrm{Sp}\,\hat{A}], \tag{31}$$

$$M\sin\beta\dot{\alpha} = -hM\sin\alpha\cos\beta + {}^2/_{15}\Omega_B^2\sin\beta(1+\cos\Phi)[1-2\,\mathrm{Sp}\,\hat{A}], \tag{32}$$

$$M\dot{\Phi} = -hM\sin\alpha\,\mathrm{ctg}\,\beta(1-\cos\beta) + {}^2/_{15}\Omega_B^2(1-\cos\beta) \times(1+\cos\Phi)[1-2\,\mathrm{Sp}\,\hat{A}] + M(1 - |\mathbf{M} - \mathbf{h}|), \tag{33}$$

and Eq. (19) remains unchanged.

In the experiments of Osheroff and Corruccini, h and Ω_B^2 are of the same order of magnitude. Thus, the system written out here has two time scales. According to (31) and (32) we have $\dot{\alpha} \sim \dot{\beta} \sim h, \Omega_B^2$, and Eqs. (19) and (32) describe oscillations of frequency $\sim\Omega_B \gg h$, i.e., the magnetization motions described by the system (19) and (31)–(33) constitute oscillations of frequency $\sim\Omega_B$ about a slowly varying equilibrium position. The oscillations are excited at the instant when the deflecting field is turned on or off, and their energy is of the order of h^2, with the coefficient of h^2 depending on the actual law governing the variation of h with time. The oscillations will be more intense if the time during which the field is turned on is of the order of $1/\Omega_B$. If this time is of the order of unity (in Larmor periods), then oscillations can be excited at the Larmor frequency and its multiples, so that it is necessary to avoid in the experiment the two indicated regions.

In the process of the rotation of the magnetization, as will be shown, a change takes place in the energy of the system by an amount on the order of h, i.e., much larger than the energy of the oscillations excited at the instant when the field is turned on. By virtue of the conservation of the adiabatic invariant, which in this case, as is well known,[8] is the ratio of the oscillation energy to their frequency, the energy of the oscillations will remain small so long as the adiabaticity condition $\omega_{tr} \gg h$ is satisfied. This means that the oscillations will not influence substantially the motion of the mean value of the magnetization. In order to find this motion it is necessary to substitute the Φ and M corresponding to this stationary point about which the oscillation takes place in (31) and (32). As a result we obtain a system of equations describing the variation of α and β under the influence of the deflecting field. We note that the small parameter in the description of the rotation process is $h/\omega_{tr} \sim h/\Omega_B$, and not Ω_B^2 as in the preceding section, in connection with which the results obtained here have an accuracy on the order of h/Ω_B, i.e., ~10%.

At $\beta < \theta_0$, the stable stationary point corresponds to vanishing of the square bracket in the right-sides of (31)–(33) and (19), and as a result the system for α and β takes the form

$$\dot{\beta} = h\cos\alpha, \tag{34}$$

$$\sin\beta\dot{\alpha} = -h\sin\alpha\cos\beta. \tag{35}$$

The solution of this system, satisfying the initial condition $\mathbf{M} \parallel \mathbf{H}$ at $t'=0$, is

$$\beta = ht', \quad \sin\alpha = 0, \tag{36}$$

i.e., at a pulse duration corresponding to angles smaller than θ_0, the angle of rotation of the magnetization is proportional to the pulse duration. After turning off the deflecting field, the magnetization executes small oscillations about one of the stationary points, C_1 or C_2, and its mean value precesses at the Larmor frequency about the field H.

When β approaches θ_0, as seen from (28), the adiabaticity condition is violated in a small region of angles $|\beta-\theta_0| \sim (h/\Omega_B)^2 \sim 10^{-2}$–$10^{-3}$. Experiment revealed damped irregularities of the precession in a much wider region $|\beta-\theta_0| \sim 10°$. The cause of these irregularities may be that at $\beta \approx \theta_0$ the difference between the energies corresponding to the stationary points C_1, C_2, and S_1 becomes small and comparable with the energy of the oscillations of the magnetization, excited when the field h is turned off. This means that after h is turned off, motions along the phase transitions can occur and subtend over all three indicated points. Irregular precession at the Larmor frequency sets in only after these motions are damped. The width of the region in which one should expect the irregularities to appear is determined, by virtue of the foregoing, by the condition $E_{S_1} - E_C \sim h^2$, i.e., according to (26) we have $\theta_0 - \beta \sim h/\Omega_B$. For H_1 from the paper of Osheroff and Corruccini[4] this yields 2–5°.

At $\beta > \theta_0$ it is necessary to use the stationary point S_1: $\Phi_{S_1}=0$, $M_{S_1} \approx 1$, in which case

$$\dot{\beta} = h\cos\alpha, \tag{37}$$

$$\dot{\alpha}\sin\beta = -h\sin\alpha\cos\beta - {}^4/_{15}\Omega_B^2\sin\beta(1+4\cos\beta). \tag{38}$$

The system has an integral

$$U_D^{(B)}(\beta) - \mathbf{hM} = \text{const}, \tag{39}$$

which has the meaning of conservation of the dipole energy $U_D^{(B)}(\beta)$ and the Zeeman energy in the field $\mathbf{h}$, with

$$U_D^{(B)}(\beta) = \mathscr{H}_B(M_{S1}, \Phi_{S1}) = {}^1/_{30}\Omega_B^2 h^{-1}(1+4\cos\beta)^2. \tag{40}$$

The constant in the right-hand side of (39) is determined from the condition $\mathbf{h}\cdot\mathbf{M}=0$ at $\beta=\theta_0$, and then

$$\sin\alpha\sin\beta = {}^1/_{30}\Omega_B^2 h^{-1}(1+4\cos\beta)^2. \tag{41}$$

This equation together with (37) describes the variation of $\mathbf{M}$ in the course of rotation at angles $\beta > \theta_0$. In particular, it is possible to obtain t' as a function of β by integrating $dt' = d\beta/\cos\alpha$, where α is expressed in terms of β with the aid of (41). In this region, the angle of rotation is no longer proportional to the pulse duration. The difference will be larger the farther away we are from the point θ_0. It is seen also from (37) that there exists a maximum value β_{max}, which is reached at $\alpha=\pi/2$. Substituting $\sin\alpha=1$ in (41), we obtain an equation for β_{max}:

$$\sin\beta_{max} = {}^1/_{30}\Omega_B^2 h^{-1}(1+4\cos\beta_{max})^2. \tag{42}$$

When comparing this result with experiment, we must see to it that the obtained values of β_{max} be not too close to π, for in the vicinity of π, as follows from (30), the adiabaticity condition is violated. In this case the non-adiabaticity region is $\pi-\beta \sim h/\Omega_B$ and the time of passage through this region coincides in order of magnitude with the period of the longitudinal oscillations, as a result of which the energy can become redistributed here among the oscillations and the motion of the equilibrium position. The energy transfer depends on the phase of the magnetization oscillations, which should be regarded as random. Such an energy transfer leads to a scatter of the experimental point after passage through the danger region.

Table I lists the values of $\pi - \beta_{max}$ for different Ω_B and h, calculated from formula (42) and extracted from the experimental point.[4] The experimental values of $\pi - \beta_{max}$ were obtained from the frequency shift of the transverse resonance assuming that the formula (29) is correct.

The difference between experiment and theory is less than 15%, and is in agreement with the estimate given above for the accuracy of the described approach. The next to the last column of the table lists the values of $\pi - \beta \equiv \chi_{ad}$ at which $h = \omega_{tr}(\beta)$, i.e., the frequency of the rotation of the magnetization in the field h coincides with the frequency of the longitudinal oscillations, while in the last column the maximum values of the adiabaticity parameter is $\lambda \equiv h/\omega_{tr}(\beta_{max})$. For those h for which

TABLE I.

T/T_c	ν, kHz	H_1, Oe	$180° - \beta_{max}$, expt.	$180° - \beta_{max}$, theory	χ_{ad}	λ
0.743	2009	7.26	25°	25°	7°	0.31
0.743	2009	2.30	37°	44°	2°	0.07
0.743	2009	5.0	29°	32°	5°	0.18
0.743	2440	5.8	22°	26°	6°	0.28
0.743	2440	2.1	35°	42°	2°	0.07
0.743	2440	3.9	29°	33°	4°	0.14
0.503	2009	5.0	37°	41°	4°	0.11
0.503	2440	5.8	32°	36°	4°	0.14
0.503	2440	3.9	37°	42°	3°	0.09

$\lambda \leqslant 0.1$, no scatter of the experimental points is observed at large angles of rotation of the magnetization, thus offering evidence in favor of the proposed explanation of the scatter for the other h.

5. THE A PHASE

In the A phase the vector $\mathbf{d}(\mathbf{n})$, the motion of which is determined by Eq. (2), can be represented in the form of a product of a real vector $\mathbf{d}$, which does not depend on $\mathbf{n}$, by a complex scalar function $f(\mathbf{n})$. Equations (4) and (5) for the A phase in a rotating coordinate system can be written in the form

$$\dot{\mathbf{M}} = -\Omega_A^2 [\mathbf{d}\times\mathbf{l}](\mathbf{d}\mathbf{l}), \tag{43}$$

$$\dot{\mathbf{d}} = -[\mathbf{M}\times\mathbf{d}], \tag{44}$$

$$\dot{\mathbf{l}} = -[\mathbf{H}\times\mathbf{l}]. \tag{45}$$

Here $\mathbf{l}$ is a unit vector that indicates the direction in which the gap in the spectrum of the A phase of He^3 vanishes. This vector, when describing the precession of the magnetization, can be assumed to be immobile because of the large inertia corresponding to the change of the spatial orientation of the condensate.

We shall assume that at the initial instant the system was at equilibrium, i.e., $\mathbf{l}\parallel\mathbf{d}$ and $\mathbf{l}\perp\mathbf{H}$. This limitation is not essential. Transforming now the system (43)–(45), as was done for the B phase, we obtain an equation for M, α, β, and Φ, in the right-hand sides of which there are contained, besides slowly varying terms, also terms that change with the Larmor frequency and its multiples. The existence of such terms was noted by Brinkman and Smith.[5] Since we are not interested here in motions with frequencies on the order of Ω_A and Ω_A^2, we average the obtained system over the time interval τ satisfying the inequality $1 \ll \tau \ll 1/\Omega_A$. As a result we obtain a system of equations describing slow variations of the magnetization

$$\dot{M} = {}^1/_8\Omega_A^2(1+\cos\beta)^2\sin 2\Phi, \tag{46}$$

$$M\dot{\alpha} = {}^1/_8\Omega_A^2[2\cos\beta+(1+\cos\beta)\cos 2\Phi], \tag{47}$$

$$M\dot{\beta} = -{}^1/_8\Omega_A^2\sin\beta(1+\cos\beta)\sin 2\Phi, \tag{48}$$

$$M\dot{\Phi} = {}^1/_8\Omega_A^2(1-\cos\beta)[2\cos\beta+(1+\cos\beta)\cos 2\Phi]+M(1-M). \tag{49}$$

This system has an integral $P \equiv M(1-\cos\beta) = \mathrm{const}$, which enables us to exclude β from (46) and (49), after which these form a Hamiltonian system with a Hamiltonian

$$\mathscr{H}_A(M,\Phi) = \frac{(M-1)^2}{2} - \frac{\Omega_A^2}{8}\left[\left(1-\frac{P}{M}\right)^2 + \frac{1}{2}\cos 2\Phi\left(2-\frac{P}{M}\right)^2\right]. \tag{50}$$

The stationary solutions are obtained by equating to zero the right-hand sides of (46) and (49). There exist two such solutions:

$$\Phi_c = 0, \quad M_c = 1 + \frac{\Omega_A^2}{8}\frac{P}{M_c^2}\left(4-3\frac{P}{M_c}\right);$$

$$\Phi_s = \pm\frac{\pi}{2}, \quad M_s = 1 - \frac{\Omega_A^2}{8}\frac{P^2}{M_s^3}.$$

The principal interval of the variation of Φ is in this case $-\pi/2 \leqslant \Phi \leqslant \pi/2$. The first of the written solutions is a center, the second a saddle (see Fig. 2). The frequency of the small oscillations about the center in principle or of Ω_A^2 is determined by the formula

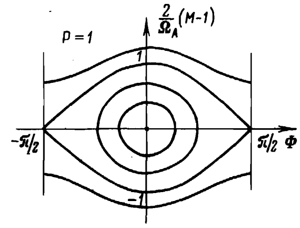

FIG. 2.

$$\omega_{\mathrm{tr}}^2 = \frac{1}{4}\Omega_A^2\left(2-\frac{P}{M}\right)^2 = \frac{1}{4}\Omega_A^2(1+\cos\beta_c)^2. \tag{51}$$

At $P=0$ it goes over into the frequency of the longitudinal oscillations of the magnetization. Substitution of $\Phi=0$ in the right-hand side of (47) yields the frequency shift of the transverse resonance

$$\delta\omega = \dot{\alpha} = \frac{1}{2}\frac{\Omega_A^2}{M_c}\left(1-\frac{3}{4}\frac{P}{M_c}\right) \approx \frac{\Omega_A^2}{8}(1+3\cos\beta_c). \tag{52}$$

The last expression coincides with that proposed by Osheroff and Corruccini[4] and obtained by Brinkman and Smith.[5] The formula describes well the experimental data at all angles between $\mathbf{M}$ and $\mathbf{H}$.

The description of the process of the rotation of the magnetization for the A phase can be carried out in analogy with the procedure used for the B phase, except that the frequency shift of the transverse resonance sets in here from the very beginning, and the dipole energy is of the form

$$U_D^{(A)}(\beta) = \mathscr{H}_A(M_c,\Phi_c) \approx -{}^1/_{16}\Omega_A^2(3\cos^2\beta+2\cos\beta). \tag{53}$$

Instead of (41) we now obtain

$$\sin\alpha\sin\beta = {}^1/_{16}\Omega_A^2 h^{-1}(1-\cos\beta)(5+3\cos\beta), \tag{54}$$

and for $\beta_{\max}$ we have the equation

$$\sin\beta_{max} = {}^1/_{16}\Omega_A^2 h^{-1}(1-\cos\beta_{max})(5+3\cos\beta_{max}). \tag{55}$$

The applicability of Eq. (54), and with it also (55) is limited, just as in the case of the B phase, by the adiabaticity condition $\omega_{\mathrm{pr}}(\beta) \gg h$. This condition, according to (51), is violated in the angle region $\pi-\beta \sim (h/\Omega_A)^{1/2} \sim 30°$ for h and Ω_A from[3]. All the values of $\beta_{\max}$ which are obtained with the aid of (55) for the h and Ω_A used by Osheroff and Corruccini[3] fall in this region, and therefore are generally speaking incorrect. It is possible, however, to describe the rotation of the magnetization also in the non-adiabaticity regions, by using the fact that in this region the moment produced by the dipole–dipole interaction for h and Ω_A from the experiment of[3] is small in comparison with the moment produced by the deflecting field in a ratio $\Omega_A^2/4h$. This parameter assumed in the experiments of Osheroff and Corruccini values from 0.06 to 0.4. By virtue of the indicated smallness we can use the method of successive approximations and omit in the zeroth approximation the terms proportional to Ω_A^2 in those equations which describe the rotation of the magnetization. As a result we have $\beta = ht'$, $\dot{\alpha} = \dot{\Phi} = \dot{M} = 0$.

To obtain the next approximation, it is necessary to substitute the values $M = 1$ and $\Phi = 0$ into the terms proportional to Ω_A^2 of the right-sides of the aforementioned equations, after which it is easily seen that these equations have an integral that is obtained from (54) by expansion in terms of $\Omega_A^2/4h$, and for β_{max} we get

$$\pi - \beta_{max} = \Omega_A^2/4h.$$

For the experimental curves of[3], plotted at $T/T_c = 0.754$ and $T/T_c = 0.854$ we have in accordance with (56) $180° - \beta_{max} \approx 20°$. It must be borne in mind that for these curves $\Omega_A^2/4h \approx 0.4$, so that the employed approximation is not very accurate and the result should be regarded as an estimate. For $T/T_c = 0.930$ we get $180° - \beta_{max} \approx 10°$, and for $T/T_c = 0.952$ we have $180° - \beta_{max} \approx 4°$. In this case $\Omega_A^2/4h$ is equal to 0.2 and 0.06, respectively, and the approximation should give good results.

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Critical magnetic fields in layered thin-film vanadium–carbon structures

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The critical magnetic fields $H_{c\perp}$ and $H_{c\parallel}$ of superconducting layered thin-film C-(V-C-V)-C samples, in which a nonmonotonic dependence of the critical temperature T_c and resistivity ρ on the thickness of the intermediate carbon layer was previously observed, are investigated. It is found that $|dH_{c\perp}/dT|$ and $|dH_{c\parallel}^2/dT|$ also depend near T nonmonotonically on the thickness of the layer. Estimates of the electron density of the states derived on the basis of data on $|dH_{c\perp}/dT|$ and data on the residual resistivity agree qualitatively with the assumption that the nonmonotonic dependence of T_c and ρ on the thickness of the carbon layer is due to variation of the electron density of the states.

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An oscillatory dependence of the critical temperature of superconducting films of thickness $d < 300$ Å on the thickness d_c of the dielectric coating (in the range of d_c from 0 to ~30 Å) was observed in[1–4]. The objects of these investigations were superconducting films of Mo–C,[1,2] Tc,[2] V,[3] and Al,[4] the dielectric being carbon[1–3] and silicon oxide.[4] Similar oscillations were observed in a film system with two vanadium layers, $C-(V-C-V)-C$,[3] as functions of the thickness of the intermediate dielectric C layer.

Golyanov *et al.*[2,3] have observed also correlations between T_c and the resistivity in films having different dielectric-coating thicknesses and in sandwiches with different intermediate dielectric-filler layers, viz., the positions of the maxima of T_c corresponded to the minima of the resistivity. It was indicated in this connection[3] that this correlation can be explained by assuming that the oscillations are the result of a change in the electron density of states.

The easiest way to estimate experimentally the electron density of states N in film samples is to calculate N from data on the perpendicular magnetic fields $H_{c\perp}$ and on the residual resistivity ρ_n.[5–8] This is the method that will be used here to determine N. The investigations were carried out on two-layer vanadium-carbon C–(V–C–V)–C sandwiches. Samples of this type were chosen because they exhibited most distinctly the correlation of T_c and ρ_n.[3] Besides $H_{c\perp}$ and ρ_n we measured also the parallel critical magnetic fields $H_{c\parallel}$.

SAMPLES AND MEASUREMENT PROCEDURE

The C–(V–C–V)–C sandwiches were prepared by ion sputtering in an ultrahigh-vacuum installation.[3] The

