choice of the equation for ν_t, can be made only after more complete experimental data on the structure of the mixing zone become available.

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Dependence of the magnetic moment of a layer magnet on the magnetic field

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The dependence of the magnetic moment of a layer magnet on the magnetic field is found in the leading logarithmic approximation.

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The magnetic-moment measurements of[1] show that in layer magnets with ferromagnetic intraplanar interaction and weak anisotropy there is a region of fields in which the moment is a linear function of the logarithm of the external magnetic field. At the same time, the explanation of this fact, based on the spin-wave approximation, cannot be regarded as satisfactory, since this approximation is not valid in the two-dimensional case. It has been shown by Polyakov[2] that in the two-dimensional case the interaction of spin waves turns out to be extremely important and determines the dependence of the physical quantities on the logarithm of the characteristic lengths (in[2] the question of the behavior of the correlation functions was considered). He also proposed a method of summing the leading logarithmic terms of the low-temperature expansion. Here we shall apply this method to determine the dependence of the magnetic moment on the external magnetic field.

In the problem under consideration there are several characteristic lengths: first, the correlation length R_c due to the magnetic field; secondly, the length scale associated with the intraplanar anisotropy; thirdly, the distance between the layers in which the magnetic atoms are situated, and, finally, the distance a between magnetic atoms in the plane. The magnetic moment has different dependences on the external field, depending on the relative sizes of these length scales (cf.[3]). The results of the work of Pokrovskiĭ and Uĭmin[3] pertain to the case of very weak fields, when the correlation length due to the magnetic field is much larger than the length scale associated with the intraplanar anisotropy. In this paper we consider the case of sufficiently strong fields (the criterion will be given below), which are, nevertheless, much smaller than the saturation field.

In the following we shall consider a generalized Heisenberg model with an N-component "spin." We shall call the generators of the group of rotations in N-dimensional space a spin operator (in the Heisenberg model, $N=3$). We shall be interested in the dependence of physical quantities on the logarithm of the magnetic field, and therefore we can neglect the noncommutativity of the spin operators and the difference between lattice sums and integrals. The problem of calculating the thermodynamic averages then becomes classical, and it is necessary to replace the spin operators by N-component unit vectors. The Heisenberg Hamiltonian then acquires the form

$$\mathscr{H}={}^1/_2J\int d^2\mathbf{x}[(\partial_\mu\mathbf{n})^2-2a^{-2}\mathbf{Bn}(\mathbf{x})], \tag{1}$$

where J is the exchange integral, $\mathbf{B}$ is the external magnetic field, $\mathbf{n}$ is a unit N-component vector, and a is the distance between magnetic atoms in the lattice. In (1) we have neglected the interplanar interaction and the anisotropy. It can be seen from the Hamiltonian (1) that the correlation length is related to the magnetic field: $R_c=a(J/B)^{1/2}$.

We define the magnetic moment M by the formula

$$M=\left\langle\frac{\mathbf{Bn}(\mathbf{x})}{B}\right\rangle_{\mathbf{n}}=\Omega^{-1}\int\prod_{\mathbf{y}}d\mathbf{n}(\mathbf{y})e^{-\beta H}\frac{\mathbf{Bn}(\mathbf{x})}{B}, \quad (2)$$

where Ω is the partition function, $\beta=T^{-1}$ (T is the temperature), and the integration over $\mathbf{n}$ runs over the unit sphere in N-dimensional space.

Using (1) and (2), it is easy to find the first term of the low-temperature expansion of M. It is clear from symmetry considerations that the magnetic moment is directed along the field, and therefore it is convenient to parametrize $\mathbf{n}(\mathbf{x})$ in such a way that the deviations of $\mathbf{n}$ from this direction are small:

$$\mathbf{n}(\mathbf{x})=(1-\varphi^2)^{1/2}(\mathbf{B}/B)+\varphi_a\mathbf{e}_a, \quad (3)$$

where $\mathbf{e}_a$ are the set of unit vectors perpendicular to $\mathbf{B}$ and φ_a are the spin components. We rewrite the integral (2) in the parameters φ:

$$M=\Omega^{-1}\int_{-1}^{1}\prod_{\mathbf{x}}d^{N-1}\varphi(\mathbf{x})\exp(-\beta H_{\text{eff}})[1-\varphi^2(0)]^{1/2}, \quad (4)$$

$$H_{\text{eff}}=\frac{J}{2}\int d^2x\left[(\partial_\mu\varphi)^2+(\partial_\mu(1-\varphi^2)^{1/2})^2-2m^2(1-\varphi^2)^{1/2}-\frac{T}{Ja^2}\ln(1-\varphi^2)\right], \quad (5)$$

where $m^2=B/Ja^2$.

At low temperatures $T\ll J$, small φ are found to be important in the integral (4), and therefore, to first order in T/J, we have

$$M\approx1-\frac{1}{2}\langle\varphi^2\rangle=1+\frac{N-1}{8\pi J}T\ln\frac{B}{J}. \quad (6)$$

Calculating the next terms in the expansion, it is easy to see that terms appear of the form $[(T/J)\ln(B/J)]^n$, which must be summed in order to find the dependence of the moment on the field in the leading logarithmic approximation. The circumstance that turns out to be essential in the solution of the problem is that, in fields $B\ll T$, the correlation functions of spins at distances $a\ll R\ll R_c$ do not depend on R_c (while the logarithmic contributions to the magnetic moment arise from precisely these characteristic lengths), and allowance for the magnetic field reduces to the introduction of an infrared cutoff.

The summation of the leading logarithmic terms of the low-temperature expansion in the case of the two-dimensional Heisenberg model is a highly nontrivial problem. This is connected with the fact that the fields $\varphi(\mathbf{x})$ for $\mathbf{B}=0$ have zero scaling dimensions, and all the Feynman graphs give logarithmic contributions. However, this difficulty can be circumvented using the renormalization-group method developed for application to our problem by Polyakov[2] and Berezinskiĭ and Blank.[4]

We shall calculate the integral (2) by integrating first over $\mathbf{n}(\mathbf{x})$ with short wavelengths such that $a\leqslant q^{-1}\leqslant R^{(1)}$ (q is the wave number), then over fluctuations with wave numbers $R^{(1)}\leqslant q^{-1}\leqslant R^{(2)}$, and so on. At each step we shall choose the range of wavelengths so that the condition $(T/J)\ln(R^{(n+1)}/R^{(n)})\ll1$ is fulfilled. In the integration over the short-wave distributions $\mathbf{n}(\mathbf{x})$, for which $q^2\gg B/Ja^2$, the effect of the magnetic field can be neglected. As was shown by Polyakov,[2] the form of the Hamiltonian is then reproduced, and the temperature, which, in the low-temperature expansion, plays the role of the interaction constant, is renormalized:

$$T_R=T\left[1-\frac{N-2}{2\pi J}T\xi\right]^{-1}, \quad (7)$$

where T_R is the effective temperature of the fluctuations with wavelengths greater than R, and $\xi=\ln(R/a)$.

The integrations described above can be performed in the following way. We write $\mathbf{n}(\mathbf{x})$ in the form

$$\mathbf{n}(\mathbf{x})=Z[\mathbf{n}_0,\boldsymbol{\psi}]\mathbf{n}_0(\mathbf{x})+\boldsymbol{\psi}(\mathbf{x}), \quad (8)$$

where $\boldsymbol{\psi}(\mathbf{x})$ are the short-wavelength fluctuations, $\mathbf{n}_0(\mathbf{x})$ are the spin distributions with wave numbers $q\leqslant R^{-1}$, and Z is determined from the condition $\mathbf{n}^2=\mathbf{n}_0^2=1$:

$$Z=-\mathbf{n}_0\boldsymbol{\psi}+[1-\boldsymbol{\psi}^2+(\mathbf{n}_0\boldsymbol{\psi})^2]^{1/2} \quad (9)$$

(the sign in front of the square root is chosen using the condition that $\mathbf{n}=\mathbf{n}_0$ when $\boldsymbol{\psi}=0$).

We substitute (8) and (9) into (2) and integrate over $\boldsymbol{\psi}(\mathbf{x})$. Then,

$$M=\langle\mathbf{Bn}(\mathbf{x})/B\rangle_{\mathbf{n}}=\langle Z\rangle_{\boldsymbol{\psi}}\langle\mathbf{Bn}_0(\mathbf{x})/B\rangle_{\mathbf{n}_0}, \quad (10)$$

$$M_R=\langle\mathbf{Bn}_0(\mathbf{x})/B\rangle_{\mathbf{n}_0}=\Omega_R^{-1}\int\prod_{\mathbf{y}}d\mathbf{n}_0(\mathbf{y})e^{-F[\mathbf{n}_0]}(\mathbf{Bn}_0(\mathbf{x})/B), \quad (11)$$

$$F[\mathbf{n}_0]=\beta\langle H\rangle_{\boldsymbol{\psi}}=\frac{J}{2T_R}\int d^2x(\partial_\mu\mathbf{n}_0)^2-\beta R^{-2}\langle Z\rangle_{\boldsymbol{\psi}}\int d^2x\,\mathbf{Bn}_0(\mathbf{x}), \quad (12)$$

$$Z_M=\langle Z[\mathbf{n}_0,\boldsymbol{\psi}]\rangle_{\boldsymbol{\psi}}=1-\frac{N-1}{4\pi J}T\ln\frac{R}{a}, \quad (13)$$

where Ω_R is the partition function:

$$\Omega_R=\int\prod_{\mathbf{x}}d\mathbf{n}_0(\mathbf{x})e^{-F[\mathbf{n}_0]}.$$

Only the first logarithmic terms have been taken into account in formulas (10)–(13).

In the averaging over the small-scale fluctuations $\boldsymbol{\psi}(\mathbf{x})$ there arises a difficulty associated with the degeneracy of the quadratic form in the Hamiltonian of the fields $\boldsymbol{\psi}$. However, this difficulty is easily overcome by imposing on the fields $\boldsymbol{\psi}$ the condition $\boldsymbol{\psi}(\mathbf{q})\cdot\mathbf{n}_0(\mathbf{x})=0$ ($\boldsymbol{\psi}(\mathbf{q})$ is the Fourier transform of $\boldsymbol{\psi}(\mathbf{x})$).

From the formulas given it can be seen that after the averaging over the short-wavelength fluctuations we arrive at the initial problem, but with renormalized parameters and a new cutoff $q_{\max}=R^{-1}$. Step-by-step integration over the short-wavelength fluctuations can be continued for so long as $q\gg R_c^{-1}$. When the wavelengths of the distributions $\mathbf{n}(\mathbf{x})$ become of the order of R_c, the logarithmic renormalizations cease.

The above property (the renormalizability) of the Heisenberg model has been well studied in field theory and in the theory of second-order phase transitions.[5,6] The differential equations of the renormalization group have in our case the form

$$\frac{\partial}{\partial \xi} T_R = \frac{N-2}{2\pi J} T_R^2, \tag{14}$$

$$\frac{\partial}{\partial \xi} \ln Z_M = -\frac{N-1}{4\pi J} T_R. \tag{15}$$

These equations are easily solved, and we find

$$Z_M = \left[1 - \frac{N-2}{2\pi J} T \ln \frac{R}{a}\right]^{(N-1)/2(N-2)}, \tag{16}$$

where Z_M is the renormalization factor for the magnetic moment.

The solution of Eq. (14) is the renormalized temperature (7). With logarithmic accuracy, we can substitute R_c in place of R in (7) and (16). It can be seen from (10) that in the renormalizations the magnetic moment is renormalized multiplicatively, i.e., $M_R = Z_M^{-1} M$. The renormalized moment $M_{R_c} = 1$, and therefore

$$M = Z_M(R_c/a).$$

We shall explain the equality $M_{R_c} = 1$. The integrations carried out above can be imagined as amalgamating groups of spins into effective spins. It is clear that the effective spin of a region with characteristic size of the order of R_c is directed parallel to the magnetic moment, i.e., $M_{R_c} \approx 1$. Therefore, with logarithmic accuracy, we have

$$M = \left[1 + \frac{N-2}{4\pi J} T \ln \frac{B}{J}\right]^{(N-1)/2(N-2)}. \tag{17}$$

We note that in the case $N=3$ (the Heisenberg model) the magnetic moment becomes a linear function of the logarithm of the external magnetic field. In the limit $N=2$ (the XY-model), (17) goes over into the result obtained by Berezinskiĭ.[7] It is clear from the derivation of formula (17) that it is valid so long as $T_{R_c} \ll J$. Therefore, we cannot consider very weak fields, and the formula given in (17) is true in the region

$$\exp\{-4\pi J/(N-2)T\} \ll B \ll T. \tag{18}$$

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Parametric excitation of antiferromagnetic modes in strong magnetic fields

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One of the magnetic oscillation modes of an antiferromagnet with "collapsed" sublattices (the so-called antiferromagnetic, or spin-flip, mode) is not connected linearly with the alternating magnetic field but can be excited by parallel pumping with a threshold amplitude inversely proportional to the anisotropy field.

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In an antiferromagnet (AF) in a strong magnetic field equal to double the value of the exchange field between the sublattices ($H_0 = 2H_E \equiv H_c$), the sublattice magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$ collapse (a spin-flip transition takes place).[1–5] At $H_0 \geqslant 2H_E$ the equilibrium value of the antiferromagnetic (AF) vector vanishes: $L_s \equiv (M_1 - M_2)_s = 0$, and for the ferromagnetic (F) vector saturation is reached: $M_s \equiv (M_1 + M_2) = 2M_0$. The sublattice structure thus vanishes, but the system nevertheless remembers its AF origin: besides the ordinary (ferromagnetic) resonance mode there exists a pure antiferromagnetic (spin-flip) sf-mode with frequency

$$\omega_{20} = \gamma[(H_0 - 2H_E)(H_0 - 2H_E + H_A)]^{1/2},$$

where H_A is the anisotropy field, which retains the AF vector in the "easy plane" of the crystal (H_0 is parallel to the easy plane). In the first of these modes, the sublattice moments precess about the magnetic fields, remaining parallel to each other. In the second, the moments $\mathbf{M}_1$ and $\mathbf{M}_2$ precess about the fields over an ellip-