$$\frac{\partial}{\partial \xi} T_R = \frac{N-2}{2\pi J} T_R^2, \quad (14)$$

$$\frac{\partial}{\partial \xi} \ln Z_M = -\frac{N-1}{4\pi J} T_R. \quad (15)$$

These equations are easily solved, and we find

$$Z_M = \left[1 - \frac{N-2}{2\pi J} T \ln \frac{R}{a}\right]^{(N-1)/2(N-2)}, \quad (16)$$

where Z_M is the renormalization factor for the magnetic moment.

The solution of Eq. (14) is the renormalized temperature (7). With logarithmic accuracy, we can substitute R_c in place of R in (7) and (16). It can be seen from (10) that in the renormalizations the magnetic moment is renormalized multiplicatively, i.e., $M_R = Z_M^{-1} M$. The renormalized moment $M_{R_c} = 1$, and therefore

$$M = Z_M(R_c/a).$$

We shall explain the equality $M_{R_c} = 1$. The integrations carried out above can be imagined as amalgamating groups of spins into effective spins. It is clear that the effective spin of a region with characteristic size of the order of R_c is directed parallel to the magnetic moment, i.e., $M_{R_c} \approx 1$. Therefore, with logarithmic accuracy, we have

$$M = \left[1 + \frac{N-2}{4\pi J} T \ln \frac{B}{J}\right]^{(N-1)/2(N-2)}. \quad (17)$$

We note that in the case $N=3$ (the Heisenberg model) the magnetic moment becomes a linear function of the logarithm of the external magnetic field. In the limit $N=2$ (the XY-model), (17) goes over into the result obtained by Berezinskiĭ.[7] It is clear from the derivation of formula (17) that it is valid so long as $T_{R_c} \ll J$. Therefore, we cannot consider very weak fields, and the formula given in (17) is true in the region

$$\exp\{-4\pi J/(N-2)T\} \ll B \ll T. \quad (18)$$

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Parametric excitation of antiferromagnetic modes in strong magnetic fields

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One of the magnetic oscillation modes of an antiferromagnet with "collapsed" sublattices (the so-called antiferromagnetic, or spin-flip, mode) is not connected linearly with the alternating magnetic field but can be excited by parallel pumping with a threshold amplitude inversely proportional to the anisotropy field.

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In an antiferromagnet (AF) in a strong magnetic field equal to double the value of the exchange field between the sublattices ($H_0 = 2H_E \equiv H_c$), the sublattice magnetizations $\mathbf{M}_1$ and $\mathbf{M}_2$ collapse (a spin-flip transition takes place).[1–5] At $H_0 \geq 2H_E$ the equilibrium value of the antiferromagnetic (AF) vector vanishes: $L_s \equiv (M_1 - M_2)_s = 0$, and for the ferromagnetic (F) vector saturation is reached: $M_s \equiv (M_1 + M_2) = 2M_0$. The sublattice structure thus vanishes, but the system nevertheless remembers its AF origin: besides the ordinary (ferromagnetic) resonance mode there exists a pure antiferromagnetic (spin-flip) sf-mode with frequency

$$\omega_{20} = \gamma[(H_0 - 2H_E)(H_0 - 2H_E + H_A)]^{1/2},$$

where H_A is the anisotropy field, which retains the AF vector in the "easy plane" of the crystal (H_0 is parallel to the easy plane). In the first of these modes, the sublattice moments precess about the magnetic fields, remaining parallel to each other. In the second, the moments $\mathbf{M}_1$ and $\mathbf{M}_2$ precess about the fields over an elliptic cone, lagging each other 180° in phase.

It follows from the Landau-Lifshitz equations that in the linear approximation the sf mode is not excited by an alternating magnetic field h of any polarization, so that its existence has sometimes been neglected.[8] It is shown below that the corresponding sf mode ω_{2k} of the spin-wave spectrum can be excited parametrically. It lends itself therefore to an experimental investigation, so that other processes in which it takes part become of interest.

The calculations were performed for the simplest model of a two-sublattice AF with anisotropy of the easy plane type. The magnetic energy of the crystal, with allowance for the Dzyaloshinskiĭ field H_D (the z axis is along the crystal axis and $\mathbf{x} \parallel \mathbf{H}_0$) is written in the form

$$\mathscr{H}=2M_0\int dV\left\{H_E m^2+{}^1/{}_2H_A l_z^2+{}^1/{}_2H_A' m_z^2+H_D(m_x l_y-m_y l_x)-mH+\frac{\alpha}{2}\left(\frac{\partial \mathbf{l}}{\partial x_i}\right)^2+\frac{\alpha'}{2}\left(\frac{\partial \mathbf{m}}{\partial x_i}\right)^2\right\}, \tag{1}$$

$$\mathbf{m}=(\mathbf{M}_1+\mathbf{M}_2)/2M_0=\mathbf{m}_0+\boldsymbol{\mu}(t),\quad \mathbf{l}=(\mathbf{M}_1-\mathbf{M}_2)/2M_0=\mathbf{l}_0+\boldsymbol{\lambda}(t),$$

$$|\mathbf{M}_1|=|\mathbf{M}_2|=M_0,\quad \mathbf{H}=\mathbf{H}_0+\mathbf{h}(t),\quad H_E,\ H_A,\ H_D,\ \alpha>0.$$

Denoting the equilibrium angle between $\mathbf{M}_1$ and $\mathbf{H}_0$ by $\pi/2-\psi$, we obtain for ψ the equation

$$H_E\sin 2\psi=H_D\cos 2\psi+H_0\cos\psi, \tag{2}$$

and for the resonance frequencies

$$(\omega_{10}/\gamma)^2=H_0\sin\psi(2H_E\cos^2\psi+H_D\sin 2\psi+H_0\sin\psi+H_A'), \tag{3}$$

$$(\omega_{20}/\gamma)^2=(2H_E\cos 2\psi+2H_D\sin 2\psi+H_0\sin\psi)(H_A+H_0\sin\psi+H_D\sin 2\psi-2H_E\sin^2\psi). \tag{4}$$

Plots of $\omega_{n0}(H_0)$ for different $d\equiv H_D/2H_E$ are shown in Fig. 1.

The value of $\omega_{20}(2H_E)$ is very sensitive to the quantity H_D[7]:

$$\omega_{20}^2(2H_E)\approx 3\gamma^2 H_A H_E(H_D/H_E)^{1/3}, \tag{5}$$

so that its measurement can serve as a means of determining very small H_D.

It follows from (2) that at $H_D\neq 0$ no collapse is possible: $\psi\neq\pi/2$ in any field. On the other hand if $H_D=0$, then the spectrum of the sf mode is of the form[3]

$$(\omega_{2k}/\gamma)^2=(H_0-2H_E+\alpha k^2)(H_0-2H_E+\alpha k^2+H_A). \tag{6}$$

It is easily seen that the sf mode is the limit of the

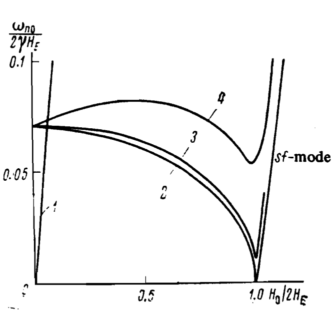

FIG. 1. Dependence of the antiferromagnetic resonance frequencies on the magnetic field intensity ($d=H_D/2H_E$). Curve 1—asymptote ($n=1$, $d=0$), curves 2, 3, 4—at $n=2$ respectively for $d=0$, 0.0004, and 0.0068 ($MnCO_3$).

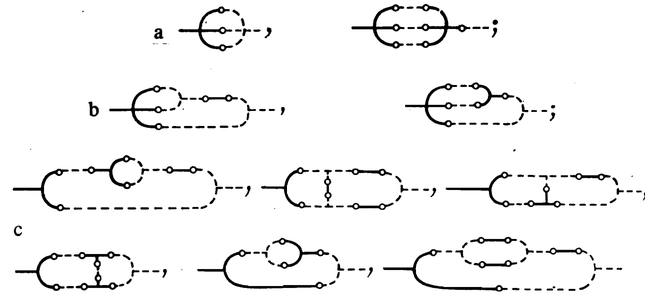

FIG. 2. Diagrams that determine the contribution to the sf-mode relaxation frequency from scattering processes in which four magnons take part.

quasi-antiferromagnet mode ω_{2k} at $H_0\geq 2H_E$ and as $H_D\to 0$. Only the variables λ_y and λ_z take part in the oscillations of this mode, and these variables are not connected linearly with the alternating magnetic field $h(t)$, since $l_x=0$ (at $H_0<2H_E$, the linear excitation of the quasi-antiferromagnetic mode by a field $\mathbf{h}\parallel\mathbf{H}_0$ is weaker the closer H_0 is to $2H_E$).

Since anisotropy of H_A leads to ellipticity of the precessions of $\mathbf{M}_1$ and $\mathbf{M}_2$ in the sf mode, one can expect its parametric excitation to be realizable by parallel pumping ($\mathbf{h}\parallel\mathbf{H}_0$) if $H_A\neq 0$. Indeed, calculation of the threshold amplitude h_c by any of the known methods leads to the expression

$$h_c=4\eta_{2k}\omega_{2k}/\gamma^2 H_A,\qquad 2\omega_{2k}=\omega_p, \tag{7}$$

where ω_p is the pump frequency and η_{2k} is the frequency of the relaxation of the sf-mode waves. For a biaxial crystal characterized by two anisotropy fields for the AF vector, the denominator of the expression for h_c will contain one of these fields or their difference—depending along which of the principal axes of the crystal h_0 is oriented.

For a concrete estimate of h_c it is necessary to calculate the sf-magnon relaxation frequency. At temperatures T that are small in comparison with the Néel frequency T_N, the sf-mode relaxation is determined by the scattering of the spin waves by one another. Magnon-phonon interactions are either absent by virtue of the symmetry properties of the mode oscillations (processes with participation of one magnon and two phonons; processes with participation of three magnons and one phonon), or, in the case of practical interest when T_N is small in comparison with the Debye temperature, make a vanishingly small contribution to the damping (processes with participation of two magnons and one or two phonons). The decisive role in the onset of damping of the sf-magnon mode is played by processes with participation of four magnons of this mode, which are due to exchange interactions.

In the calculation of the relaxation frequency η_{2k} by the self-consistent-field method,[5-8] their contribution is taken into account by the diagrams of Fig. 2, which correspond to second-order perturbation theory. In the diagrams, the block of order n, in this case of fourth (a, b) and third (b, c) order, is represented by a point with n outgoing lines, each of which corresponds to a definite frequency and momentum satisfying the conservation law in each block. The blocks of the first and second sublattices are shown by solid and dashed lines, respectively. To each point relating the solid or dashed lines there corresponds an effective interaction tensor $\hat{\Lambda}_{12q}(\omega, -\omega)$. The summation is carried out over all the internal frequencies and momentum.

The expression obtained for η_{2k} by summing the transformed diagrams takes the form (we neglect exchange inside each sublattice, i.e., $\alpha' = -\alpha$)

$$\eta_{2k} = \frac{\pi}{8\hbar N^2} n_{2k}^{-1} \sum_{q,k} \{(J_q + J_p + J_{k-q} + J_{k-p})^2 n_{2q} n_{2p} [1 + n_{2p+q-k}] \times \delta[\varepsilon_{2q} + \varepsilon_{2p} - \varepsilon_{2\,p+q-k} - \varepsilon_{2k}] + 2(J_q - J_p + J_{k-q} - J_{k-p})^2 \times n_{2q} n_{1p} [1 + n_{1\,p+q-k}] \delta[\varepsilon_{2q} + \varepsilon_{1p} - \varepsilon_{1\,p+q-k} - \varepsilon_{2k}]\}, \qquad (8)$$

where $n_{1,2k}$ is the Bose distribution function for spin waves with energies ε_{1k} and ε_{2k}; J_k is the Fourier component of the exchange integral, see[8]. At low temperatures ($T \ll T_N$) and at small quasi-momenta ($(\varepsilon_{2k}/\varepsilon_{20} - 1) \ll 1$) we obtain

$$\eta_{2k} = \frac{1}{4\pi^3} \left(\frac{v_0}{R_0^3}\right)^2 \frac{4\gamma H_E}{s^2} \left(\frac{k_B T}{2\mu_B H_E}\right)^2 F\left(\frac{\varepsilon_{20}}{k_B T}\right), \qquad (9)$$

$$F\left(\frac{\varepsilon_{20}}{k_B T}\right) = \int_0^{n_{20}} \frac{\ln(x+1)}{x(x+1)} dx,$$

where $(v_0/R_0^3)^2 = 1/27$ when account is taken of only the interaction between the nearest neighbors. We have

$$F\left(\frac{\varepsilon_{20}}{k_B T}\right) \leqslant 1 \text{ and } F\left(\frac{\varepsilon_{20}}{k_B T}\right) \approx \exp\left\{-\frac{\varepsilon_{20}}{k_B T}\right\} \text{ at } \frac{\varepsilon_{20}}{k_B T} \gg 1.$$

It is realistic in practice to obtain a state with collapsed spins in AF with small T_N. At $2H_E \sim 50$ kOe ($T_N \sim 10$ °K) and $T \sim 1$ °K we obtain $\eta_{2k}/\gamma \sim 1$ Oe. It follows then from (7) at $H_A \sim 3$ kOe and $\omega_p/\gamma \sim 13$ kOe (the 8-mm microwave band) that $h \sim 10$ Oe, which is perfectly attainable. We note in conclusion that a curious feature of the sf mode is that it has no linear connection with the phonons.

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Effect of hydrostatic pressure on the magnetization of the alloy MnSb

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Variation of the magnetization of an MnSb alloy, induced by hydrostatic pressure up to 8 kbar, is measured at temperatures of $T_1 = 83$°K and $T_2 = 294$°K. It is shown that the magnetization decreases under pressure: $\Delta\sigma/\Delta p = -(0.34 \pm 0.13)$ G-cm^3/g-kbar and $\Delta\sigma/\Delta p = -(0.45 \pm 0.13)$ G-cm^3/g-kbar respectively for each of the temperatures. The temperature dependence of the spontaneous magnetization of MnSb measured in the range between 83 and 358°K and at atmospheric pressure is satisfactorily described by the Stoner quadratic law. The experimental results obtained are analyzed on basis of the theory of band ferromagnetism.

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INTRODUCTION

The question of the nature of exchange interactions in the MnSb alloy has not been answered to this day. To describe the exchange mechanisms in MnSb, models were proposed based on the concept of interaction of localized spins (Lotgering and Gorter,[1] de Gennes[2]). Edwards and Bartel[3,4] have recently attempted to apply the model of collectivized electrons to a description of the pressure-induced change of the Curie temperature of the alloys MnSb and $MnSb_{1-x}As_x$.

The Stoner theory of band ferromagnetism was developed by Wohlfarth[5] for the particular case of compounds having close values of the Curie temperature and of the magnetization. A classical example of such weak band ferromagnets is $ZrZn_2$. Within the framework of Wohlfarth's theory of weak band ferromagnetism, relations were obtained between the pressure-induced change of the Curie temperature and the change of the magnetization. These relations were confirmed by experiment.

Edwards and Bartel,[3] following Wohlfarth's theory,