Spontaneous symmetry breaking and evolution of the universe

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A study is made of a Friedmann model of a charge-asymmetric hot universe in which particle masses arise through spontaneous symmetry breaking. It is shown that the early stages in the evolution of the universe depend strongly on the ratio of the density of γ rays to the density of the excess of neutrinos over antineutrinos. A small value of this ratio must lead to unboundedly large masses of the elementary particles near the cosmological singularity. In a universe with number of γ rays appreciably greater than the number of neutrinos, the masses of the particles are zero during the early stages of evolution and become nonzero as a result of a phase transition as the universe cools.

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In recent years, the outlines have developed in elementary particle physics for the construction of a unified renormalizable theory of electromagnetic, weak, and strong interactions. Significant successes have been achieved in the unification of electromagnetic and weak interactions (see, for example, [1,2]). The most attractive feature of these models—the renormalizability—is achieved by the high degree of symmetry of the original Lagrangians, which contain massless gauge fields and fermion fields. The masses of particles in modern models of unified field theory arise through the mechanism of spontaneous symmetry breaking. In the majority of models, the symmetry breaking is achieved by introducing scalar fields φ with nonzero vacuum expectation value, $\langle\varphi\rangle \neq 0$. Particles which interact with the scalar field acquire a mass proportional to $\langle\varphi\rangle$.

In the models of unified field theory, particles propagate and are scattered on the background of the nonzero amplitude of the classical scalar field, which in this sense is a vacuum that cannot be observed in the absence of gravitation. In the theory of gravitation, a classical scalar field with nonzero energy density is a source of the gravitational field. The possibility of a nonzero vacuum energy density can be taken into account in Einstein's equations in the form of a Λ term added to the energy-momentum tensor of the matter.

When one is considering the evolution of a hot, charge-asymmetric universe, in which the particle masses and the vacuum energy density are determined by the expectation value of the scalar field, it must be remembered that this last depends on the temperature[3] and the density of the fermion charge.[4] At a temperature comparable with the mass of the scalar particles, there is intensive production from the vacuum of quanta of the scalar field, which leads to a dependence of the expectation value of the scalar field, and with it the masses of the observed particles, on the temperature. Therefore, in models of unified field theory with spontaneous symmetry breaking there is a critical temperature (in Weinberg's model $\sim 10^2$ GeV) at which symmetry is established: $\langle\varphi\rangle = 0$ (see[3]).

The possibility of a phase transition with respect to the temperature to a state with broken symmetry during the early stages in the expansion of a hot universe was first considered by Kirzhnits and Linde.[3,5] Linde[6] investigated the time evolution of the cosmological "constant" Λ in Einstein's equations due to the temperature dependence of the energy density of the scalar field. In[4] Linde and Lebedev showed that in models of unified field theory in which massless fermions (neutrinos) interact with a gauge field, which acquires mass through the spontaneous symmetry breaking, the expectation value of the scalar field increases monotonically with increasing density of the fermion charge. Thus, during the early stages in the evolution of the universe two processes can compete: symmetry breaking with increasing density and restoration of symmetry with increasing temperature.

It is therefore of interest to consider the self-consistent variation with time of the radius of curvature of a charge-asymmetric hot universe and the expectation value of the scalar field (and with it, the masses of the elementary particles).

In writing down the Lagrangian of the scalar field in general relativity, we shall require the equations of motion to be conformally invariant in the limit of vanishing unrenormalized mass. This is achieved by adding to the Lagrangian the term $R\varphi\varphi^*/6$, where R is the scalar curvature.[7] The energy-momentum tensor corresponding to such a Lagrangian has a number of important advantages.[7,8]

We shall show that the properties of the matter near the cosmological singularity depend strongly on the ratio of the density of the neutrino excess in the universe (the difference between the numbers of neutrinos and antineutrinos) to the density n_γ of γ rays. In particular, if this ratio has a large value, there is no phase transition with respect to the temperature.[3] In this case, the masses of the elementary particles near the cosmological singularity (at the Planck time $t_g = \sqrt{G}$) exceed their modern values by the factor $(G_F/G)^{1/2} \sim 10^{16}$ (G_F is the constant of the weak interaction).

In Secs. II–III, we consider the evolution of the universe in the simplest field-theoretical scheme with spontaneous symmetry breaking—the Higgs model containing the minimal number of different particle species: scalar mesons, massless fermions (neutrinos), and vector fields. The effects due to the presence of massive fermions (leptons and hadrons) do not, as regards the evolution of the hot universe, significantly alter the conclusions of the simplest scheme, and they are considered in the Appendix, in which the behavior of Weinberg's $SU(2)\otimes U(1)$ model with the inclusion of hadrons in the limit of a high density of the fermion excess is considered.

I. EVOLUTION OF A COLD, CHARGE-ASYMMETRIC UNIVERSE

1. Spontaneous symmetry breaking and general relativity

We consider the simplest scheme of a model of unified field theory describing the gauge-invariant interaction of the vector field A_μ with a complex scalar field $\varphi=\rho\exp(i\chi)$ and a massless fermion neutrino field ψ. The corresponding Lagrangian has the form

$$L=-\frac{1}{4}F_{\mu\nu}^2+g^{\mu\nu}(D_\mu\varphi)^+(D_\nu\varphi)+\frac{R}{6}\varphi\varphi^+-U(\varphi\varphi^+)+g^{\mu\nu}\bar\psi i\gamma_\mu D_\nu\psi, \tag{I.1}$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, $D_\mu=\nabla_\mu-igA_\mu$, $g_{\mu\nu}$ is the symmetric metric tensor, R is the scalar curvature: $R=g^{\mu\nu}R_{\mu\nu}$ ($R_{\mu\nu}$ is the Ricci tensor), and ∇_μ is the covariant derivative. We take the function $U(\varphi\varphi^+)$ in the standard form

$$U(\varphi\varphi^+)=-\mu^2\varphi\varphi^++\lambda(\varphi\varphi^+)^2+\mu^4/4\lambda. \tag{I.2}$$

In the absence of a fermion current, the field φ in flat space has the anomalous vacuum expectation value

$$\rho_0^2=\mu^2/2\lambda. \tag{I.3}$$

The masses of the scalar and the vector field are

$$m_\varphi=2\sqrt{\lambda}\,\rho_0,\quad m_A=\sqrt{2}g\,\rho_0. \tag{I.4}$$

Note that in field-theory models, the choice of the last term in the expression (I.2) is arbitrary. When gravitation is included, the corresponding choice fixes the zero point for measuring the gravitational energy of the vacuum (in this connection see[6,9]). We choose the constant term in (I.2) in such a way that the vacuum energy is zero in flat space. That this energy is small follows from the experimental restrictions on the value of the cosmological constant Λ (see, for example, [10]).

The introduction into the Lagrangian (I.1) of the term $R\varphi\varphi^+/6$ ensures conformal invariance of the theory in the limit $\mu\to 0$. The equations of motion of the scalar and vector fields corresponding to the Lagrangian (I.1) have the form

$$g^{\mu\nu}(\partial_\nu\rho)_{;\mu}=-\left[-\frac{R}{6}+U'(\rho^2)\right]\rho+\rho g^{\mu\nu}V_\mu V_\nu, \tag{I.5}$$

$$F^{\mu\nu}_{;\nu}=g^{\mu\nu}(2g\rho^2V_\nu-g\bar\psi\gamma_\nu\psi). \tag{I.6}$$

where $V_\mu\equiv\partial_\mu\chi-gA_\mu$ is a gauge-invariant combination of fields. The covariant differentiation (I.6) gives a law of conservation of the current on the right-hand side of this equation.

To derive the equations of gravitation, we use the principle of least action. We choose the action in the standard form

$$S=\int d^4x\sqrt{-\Delta}\{\tilde L+L-R/16\pi G\}. \tag{I.7}$$

where $\Delta=\det g_{\mu\nu}$, L corresponds to (I.1), $\tilde L$ is the Lagrangian of the fields that are not included in (I.1) and do not interact with the scalar field and the massive vector field. Varying (I.7) with respect to $g_{\mu\nu}$, we find

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\left\{\frac{1}{8\pi G}-\frac{1}{3}\varphi\varphi^+\right\}^{-1}\theta_{\mu\nu}. \tag{I.8}$$

where $\theta_{\mu\nu}$ is the modified energy-momentum tensor

$$\theta_{\mu\nu}=\tilde T_{\mu\nu}+T_{\mu\nu}-\frac{1}{3}\{(\partial_\mu\varphi\varphi^+)_{;\nu}-g_{\mu\nu}(\partial^\rho\varphi\varphi^+)_{;\rho}\}, \tag{I.9}$$

where $\tilde T_{\mu\nu}$ and $T_{\mu\nu}$ are the standard energy-momentum tensors of the fields calculated from the Lagrangians $\tilde L$ and L, respectively, without allowance for the term $R\varphi\varphi^+/6$ in (I.1).

The equations of gravitation in the form (I.8) were derived for the first time in[8]. In the same paper, it was shown that the energy-momentum tensor modified by the addition of the last term in (I.9) has finite matrix elements in all orders of renormalizable perturbation theory in g and λ. Note (see[8] for more details) that this modification of the Einstein equations does not lead to consequences that contradict experiments.

We shall assume that the matter described by the energy-momentum tensor $\tilde T_{\mu\nu}$ has the equation of state $P=\varepsilon/3$, and therefore the only constant dimensional parameters of the theory are μ and G. In this case, contraction of (I.9) in conjunction with use of the equations of motion of the fields gives a simple expression for the scalar curvature:

$$R=16\pi G\mu^2\rho^2. \tag{I.10}$$

Substitution of (I.10) into Eq. (I.5) redefines the constant of the self-interaction of the scalar field (cf. [8]):

$$\lambda\to\lambda'=\lambda-\frac{4}{3}\pi G\mu^2. \tag{I.11}$$

Since $\mu^2/\lambda\sim G_F^{-1}$ in models of unified field theory, [11] it is easy to see that $(\lambda-\lambda')/\lambda\sim G/G_F\sim 10^{-23}$. Therefore, the inclusion of the term $R\varphi\varphi^+/6$ into the Lagrangian of the scalar field modifies the equations of gravitation but hardly affects the equations of motion.

2. Friedmann universe with spontaneous symmetry breaking

In what follows, we shall consider the evolution of a homogeneous and isotropic charge-asymmetric Friedmann universe with the metric

$$ds^2=dt^2-a^2(t)\,dl^2, \tag{I.12}$$

where $a(t)$ is the radius of curvature of the universe, and we shall assume that the main excess of fermion charge is in the neutrinos. In such a universe, there is a nonzero density of the charge J_0 interacting with the field A_μ,

$$J_0=g\langle\bar{\psi}\gamma_0\psi\rangle=gn_\nu, \tag{I.13}$$

where n_ν is the density of the neutrino–antineutrino excess. In a homogeneous and isotropic universe, only the classical parts of the fields $\rho(t)$ and $V_0(t)$ can be nonzero in accordance with Eqs. (I.5) and (I.6).

In this case, we obtain from (I.6)

$$J_0^{(V)}+J_0=0, \tag{I.14}$$

where $J_0^{(V)}$ is the charge whose existence is due to the nonzero amplitude of the scalar field:

$$J_0^{(V)}=-2g\langle\rho^2V_0\rangle. \tag{I.15}$$

In other words, in the model of unified field theory the weak charge J_0 (in our case it is proportional to the fermion charge) induces in the scalar vacuum a charge equal to it in magnitude but opposite in sign.[4]

The energy density ε of the matter can be found by averaging θ_0^0 with respect to the ground state with $n_\nu=\langle\psi^*\psi\rangle\neq 0$ (in this connection, see[12]). With allowance for the equations of motion (I.5) and (I.6) and the relation (I.14), ε takes the form

$$\varepsilon=\tilde{\varepsilon}+\dot{\rho}^2+2\frac{\dot{a}}{a}\rho\dot{\rho}+U(\rho^2)+\frac{n_\nu^2}{4\rho^2}+\frac{3}{4}(6\pi^2)^{1/3}n_\nu^{4/3}, \tag{I.16}$$

where $\tilde{\varepsilon}=\langle\tilde{T}_0^0\rangle$. The last term in this expression corresponds to the energy of the degenerate neutrino gas. The term

$$U_{eff}=n_\nu^2/4\rho^2 \tag{I.17}$$

is the energy of the effective interaction of the fermions through the massive gauge field. Such an interaction was considered for the first time by Zel'dovich[13] (see also[12]), who assumed that the mass of the vector field m_A is constant, and then $U_{eff}=g^2n_\nu^2/2m_A^2$. In models of unified field theory, m_A depends on the expectation value of the scalar field, which, in its turn, is a function of n_ν, and therefore the dependence $U_{eff}(n_\nu)$ in the model we consider is more complicated than n_ν^2.

In a closed model of the universe (with volume $2\pi^2a^3(t)$) $n_\nu=N_\nu/2\pi^2a^3$, where N_ν is the difference between the total number of neutrinos and antineutrinos. In an open universe $n_\nu=Aa^{-3}$; the constant A, however, in this case does not have such a clear physical meaning as for the closed model, and in what follows we shall therefore consider a closed model of the universe for convenience. We shall see that all results for physical quantities depend only on the densities of the charges, and are therefore valid for a model of any type.

The equation that describes the variation with time of the radius of curvature of the Friedmann universe has in accordance with (I.8) and (I.16)

$$\left(\frac{\dot{a}}{a}\right)^2=\frac{1}{3}\left\{\frac{1}{8\pi G}-\frac{1}{3}\rho^2\right\}^{-1}\varepsilon. \tag{I.18}$$

Averaging the equation of motion (I.5) with respect to the ground state and using (I.10) and (I.14), we find

$$\ddot{\rho}+3\frac{\dot{a}}{a}\dot{\rho}+\frac{1}{2}\frac{\partial\varepsilon}{\partial\rho}=0. \tag{I.19}$$

The system of equations (I.18) and (I.19) describes the self-consistent variation with time of the radius of curvature $a(t)$ of the universe and the expectation value of the scalar field $\rho(t)$.

It is easy to see that in a charge-symmetric cold universe ($N_\nu=0$) the anomalous expectation value (and therefore the masses of the fields) does not depend on the time, $\rho=\rho_0$. The presence of an uncompensated number of neutrinos in the universe necessarily leads to a time dependence of the expectation value of the scalar field.

Near the cosmological singularity, when

$$a\ll a_0=(N_\nu\lambda/2\pi^2)^{1/3}\mu^{-1}\sim\lambda^{-1/2}G_F^{1/2}n_\nu^{(m)}a_m, \tag{I.20}$$

where $n_\nu^{(m)}$ and a_m are the contemporary neutrino density and radius of curvature, Eqs. (I.18) and (I.19) describe solutions with $\rho\gg\rho_0$. Note that for values of λ which do not contradict experiment ($\lambda>10^{-6}$; see[14] and also[15], in which the restriction $\lambda>10^{-3}$ is found) and all plausible n_ν the inequality $a_0\ll a_m$ holds.

In the limit $\rho\gg\rho_0$, when we can set $\mu=0$ in the equations, the equation of motion (I.19) for the expectation value of the scalar field becomes scale invariant. Therefore, for the early stage in the evolution of the universe ($a\ll a_0$) there exists an exact solution of the system (I.18) and (I.19):

$$\rho a=\rho_0a_0, \tag{I.21}$$

$$\frac{a}{l_g}=\left(\frac{t}{t_g}\right)^{1/2}\left(\frac{48}{\pi^3}\right)^{1/12}\left\{1+\left(\frac{\lambda}{6\pi^2}\right)^{1/3}\right\}^{1/4}N_\nu^{1/3}, \tag{I.22}$$

where $l_g=\sqrt{G}$ is the characteristic gravitational length ($t_g=l_g$). In Eq. (I.22), we have ignored the contribution of $\tilde{\varepsilon}$ to the matter energy. Allowance for this energy merely changes the numerical coefficient in the curly brackets.

Note that for the scale-invariant solution (I.21) the equation of gravitation (I.18) takes the customary form

$$\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\varepsilon_0, \tag{I.23}$$

where ε_0 is the matter energy density, not including the kinetic term ($\varepsilon_0=\varepsilon-\dot{\rho}^2-2(\dot{a}/a)\rho\dot{\rho}$). It is also easy to see that for $a\sim t^{1/2}$ (I.22) the sum of the time derivatives in (I.19) is zero. Thus, inclusion of the term $R\varphi\varphi^+/6$ in the Lagrangian (I.1) leads to a negligibly small contribution of the time derivatives of the classical part of the scalar field, both in the Einstein equations (I.18) and in the equation of motion (I.19).

For $\lambda=0$, Eq. (I.22) describes the ordinary Friedmann evolution of a universe filled with a degenerate Fermi gas of neutrinos. Note that for $\lambda<1$ the spontaneous symmetry breaking realized by the scalar fields does not lead to a significant change in the Hubble constant.

For $a \gg a_0$, the treatment of the homogeneous classical fields $\rho(n_\nu)$ is hardly valid. Therefore, all the results of the paper refer to the hadron stage in the evolution of the universe. From a certain mass on, the characteristic neutrino density depends strongly on the radius of curvature of space: $n_\nu \sim G_F^{-3/2} \sim 10^{48}$ cm^{-3} (see[4]).

3. Production of particles and the equation of state of matter in unified field theory near the cosmological singularity

We now consider small deviations of the amplitudes of the scalar and vector fields from their mean values:

$$\varphi=\rho+(\varphi_1+i\varphi_2)/\sqrt{2}, \quad \langle\varphi_1\rangle=\langle\varphi_2\rangle=0, \tag{I.24}$$

$$A_\mu=A_0\delta_{0\mu}+a_\mu, \quad \langle a_\mu\rangle=0. \tag{I.25}$$

The massive fields φ_1 and a_μ are observable excitations of quasiparticle type. In vacuum, the masses of the fields φ_1 and a_μ are determined by the expressions (I.4). In the presence of an uncompensated fermion charge, the fields φ_1 and a_μ begin to interact not only with the classical scalar field but also with the classical part of the vector field. In this case (as in the absence of fermion charge) the masses of the fields can be found by linearizing the equations of motion:

$$m_\varphi=2\sqrt{\lambda}\,\rho(a), \quad m_A=\sqrt{2}\,g\rho(a). \tag{I.26}$$

In the region of small radii (I.20), the masses of the quasiparticles decrease monotonically ($\sim a^{-1}$) to their vacuum values as the universe expands.

We emphasize that these results apply to a cold universe. In a hot universe, there may be a phase transition,[3] at which the mass of the vector field becomes zero, near the cosmological singularity. As will be shown below, the dependence (I.26) also remains valid in a hot universe if there is a sufficiently large neutrino excess.

Note that in the region (I.20) the only dimensional parameter in the model of unified field theory is the gravitational length $l_g=\sqrt{G}$, and therefore a rapid variation of the masses of the elementary particles ($\sim a^{-1}$) near the cosmological singularity can be "observed" only in gravitational effects.

An important influence on the evolution of the universe (and, in particular, on the equation of state of the matter near the cosmological singularity) may be exerted by particle creation by the gravitational field (see, for example,[16]). In our model however, particle creation effects are almost completely absent because of the conformal invariance of the equations of motion during the early stage of evolution (in this connection, see[17]). More precisely, the creation of particles by the metric is suppressed by the smallness of $G\mu^2$. Let us illustrate this for the example of scalar particles with mass (I.26), which are described in the Friedmann metric by the equation

$$\varphi_{1;\mu}^{;\mu}+m_\varphi^2(t)\varphi_1=0. \tag{I.27}$$

We make a conformal transformation of the Friedmann metric to the Minkowski metric:

$$g_{\mu\nu}\to\eta_{\mu\nu}=a^{-2}g_{\mu\nu}$$

and, transforming the fields simultaneously in accordance with

$$\varphi\to\tilde{\varphi}=a\varphi$$

we rewrite (I.27) in the form

$$\Box\tilde{\varphi}_1+4\lambda(\rho_0a_0)^2\tilde{\varphi}_1=0. \tag{I.28}$$

This equation describes the propagation of scalar particles with constant mass in a pseudo-Euclidean space with a time-independent amplitude of the new expectation value $\tilde{\rho}=\rho_0a_0$ of the scalar field. In this case, particles are not created. Therefore, particles described by Eq. (I.27) are not created either in the real Friedmann universe whose conformal image is pseudo-Euclidean space. The above arguments are valid of course not only for the linearized equation but also for the complete equation (I.19) when the unrenormalized mass μ is ignored.

The equation of state of fermions interacting through a massive vector field was considered for the first time by Zel'dovich.[13] At high densities, the repulsion of fermions with energy $U_{\rm eff}\sim n^2/m_A$ (n is the fermion density) leads for $m_A=$ const to the hardest equation of state possible: $P=\varepsilon$ (see[13]). It is easy to see that in our model $U_{\rm eff}$ (I.17) is suppressed by the growth of the mass of the vector field, $m_A\sim\rho\sim a^{-1}$, and does not exceed the kinetic energy of the fermions ($\sim a^{-4}$). Therefore, in the models of unified field theory, when the mass of the vector field is acquired through interaction with the scalar field, the maximally hard equation of state is not realized for any fermion densities.

By means of (I.9) and the equations of motion, we find

$$\theta_\mu^{\ \mu}=\varepsilon-3P=-2\mu^2\rho^2. \tag{I.29}$$

Near the singularity, when $\varepsilon\sim a^{-4}$, $\rho\sim a^{-1}$, the equation of state for the matter has the form $P=\varepsilon/3$.

II. EVOLUTION OF CHARGE-ASYMMETRIC MATTER IN A HOT UNIVERSE

1. Applicability of the thermodynamic approach

Hitherto, we have considered a universe populated only by excess neutrinos at $T=0$. The presence of γ rays capable of producing pairs of particles of various species can appreciably alter the results of the preceding section. We restrict our treatment to a universe with a minimal number of particle species: scalar φ mesons, massive A_μ and massless (γ rays) vector particles, and massless fermions (neutrinos). Such a model, which reflects the main features of models of unified field theory, enables one to draw qualitative conclusions about the evolution of a universe with spontaneous symmetry breaking of the vacuum.[1)]

At a high temperature, the production of scalar and massive vector particles reduces the expectation value of the scalar field. In the absence of a fermion excess, there exists a critical temperature above which the symmetry of the vacuum is restored.[3] In a charge-asymmetric universe, the situation is complicated by the competition between two mechanisms—the reduction of the expectation value of the scalar field with increasing T and its increase with increasing density of fermion excess.

It is well known (see, for example, [18]) that in thermodynamic equilibrium the temperature can be uniquely expressed in terms of the density n_γ of the γ rays:

$$T=[\pi^2 n_\gamma/2\zeta(3)]^{1/3}=[N_\gamma/4\zeta(3)]^{1/3}a^{-1}, \tag{II.1}$$

where the constant N_γ for a closed model is the total number of γ rays and $\zeta(x)$ is the zeta function. During the expansion of the universe from the cosmological singularity, the number N_γ is not constant. As the temperature cools to $T_i < m_i$ (m_i is the mass of the particles of species i), this number rapidly increases by an amount of the order of itself because of the annihilation of pairs of particles of species i right down to the exponentially small "freezing" density. This last is the density at which the distance between particles in the expanding universe increases faster than collisions occur (see[16] for more details). For constant masses, during the period of evolution when $T(t)\sim m_i$, the various particle species are not in thermodynamic equilibrium. Their entropy changes because of the change in the total number of particles of given species.[2)] In the model we consider, when the particle masses depend on the time, the thermodynamic approach can also be valid when $T\sim m_i$ because of the constancy of the ratio $m_i(t)/T(t)$ (see (I.26) and (I.1)).

If the total entropy of the universe is constant during evolution, the number

$$N=N_\gamma+\sum_i N_i, \tag{II.2}$$

where N_i is the number of particle pairs of species i, remains unchanged. Therefore, at a definite stage of evolution N_γ can be assumed constant if the N_i's are constant during this stage. Taking as an example the total number of massive vector particles, we have

$$N_A=2\pi^2 a^3\int\frac{3d^3p}{(2\pi)^3}\left\{\exp\frac{\varepsilon_A(p)}{T}-1\right\}^{-1},$$
$$\varepsilon_A(p)=(p^2+m_A^2)^{1/2}. \tag{II.3}$$

By means of (II.1), we find from this expression

$$N_A=\frac{3J(K_A)}{4\zeta(3)}N_\gamma,\qquad J(K_A)=\int_0^\infty dx\,x^2\{\exp(x^2+K_A^2)^{1/2}-1\}^{-1},$$
$$K_A\equiv m_A(a)/T(a). \tag{II.4}$$

We see that the number of γ rays may be assumed constant for $T\gg m_i$ and for $m_i\sim a^{-1}$. During the late stage of evolution (as $T\to 0$), when virtually all pairs have been annihilated, N becomes equal to the contemporary number of γ rays.

In what follows, we shall assume that N_γ does not depend on the time, but we remember that in reality N_γ in (II.1) is a step function whose values are almost always constant but different in different stages of evolution of the universe.

As we saw in our treatment of the cold universe, the masses of the scalar and vector mesons increase unboundedly as the cosmological singularity is approached. Because of the scale invariance of the equations of motion of the particles during the early stages of evolution, their interaction cross sections are proportional to $\sigma\sim m^{-2}(a)$ and they tend to zero as $a\to 0$. In this connection, we may expect that the heavy particles are not in thermodynamic equilibrium as the universe emerges from the cosmological singularity. Therefore, the methods of quantum statistics (which, in particular, enable one to speak about the dependence $m(a)$) do not become applicable immediately after the emergence from the singularity.

With the expansion of the universe and the increase in the cross sections (proportional to a^2) there comes a time t_S at which thermodynamic equilibrium is established. Leaving aside for one moment the question of the symmetry of the vacuum for $t>t_S$, let us estimate when this time occurs. Thermodynamic equilibrium begins when the cosmological time t exceeds the mean free time τ of the particles:

$$t>\tau=1/\sigma n, \tag{II.5}$$

where n is the equilibrium density of heavy particles,

$$n=(g_S/2\pi^2)T^3 J(K), \tag{II.6}$$

and g_S is the number of degrees of freedom of the field which describes the given species of particles. Using the expressions (I.22), (I.26), and (II.1), we find for scalar mesons[3)]

$$t_S\sim\lambda^{-2/3}K_0^6 J^{-2}(K_0)\,t_g, \tag{II.7}$$

$$K_0=\sqrt{2}\,\pi^{-2/3}[2\zeta(3)\lambda N_\nu/N_\gamma]^{1/3}. \tag{II.8}$$

For $\lambda\sim 10^{-3}$ and $K_0\sim 10$, which corresponds to the maximal possible value of the ratio N_ν/N_γ that does not contradict experiments (see below), we have

$$t_S\sim 10^{14}\,t_g. \tag{II.9}$$

It follows from the above expressions that for matter with low specific entropy ($N_\nu\gg N_\gamma$) the applicability of the thermodynamic approach ($t>t_S$) begins much later than t_g. Note that this restriction on the applicability

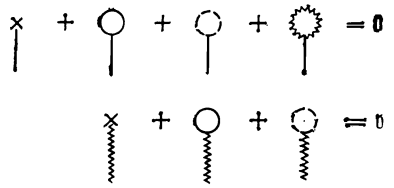

FIG. 1.

of the thermodynamic description from below (and not above, as in calculations in the framework of traditional schemes; see, for example, [16]) is due to the rapid decrease in the cross sections for interactions of particles ($\sigma \sim t$) as t_g is approached. Note that much more stringent restrictions on the applicability of the thermodynamic description of the evolution of the universe in models of unified field theory arise in homogeneous anisotropic metrics. For example, for the Kasner solution,

$$t_S \sim \lambda^{-1/2} K_0^{9/2} J^{-3/2}(K_0)\,(\theta_0/t_g)^{1/4} t_g, \tag{II.10}$$

where θ_0 is the characteristic isotropization time. For the values $\theta_0 \lesssim 1$ sec and $K_0 < 10$, which do not contradict experiment, we find

$$t_S \leqslant 10^{21}\, t_g. \tag{II.11}$$

For $K_0 > 1$, the applicability of the thermodynamic treatment is restricted to the region of times ($t > t_S$) in which the time derivatives in the equation of motion (I.19) are small compared with the other terms. Indeed, it is easy to see that the terms with derivatives ($\sim t^{-5/2}$) become less than the remaining terms ($\sim t^{-3/2}$) at the time $t \sim \lambda^{-2/3} t_g < t_S$ for $K_0 > 1$.

In what follows, in our examination of the evolution of a hot universe we shall ignore the time derivatives of the classical fields. In this case, the determination of the Gibbs expectation of the scalar field reduces to finding the minimum of the thermodynamic potential Ω. The explicit expression for Ω also enables one to find the effective masses of the particles and determine the energy of the matter which occurs on the right-hand side of the evolution equation (I.18).

2. Phase transition in a hot, charge-asymmetric universe

At a nonzero temperature, the density of the thermodynamic potential of a system including a scalar and a massive vector field and also γ rays and neutrinos has the form

$$\Omega = \Omega_e - \frac{\pi^2}{45} T^4$$

$$-\frac{T^4}{2\pi^2} \int_0^\infty dx\, x^2 \left\{ \ln\left[1 + \exp\left(-x + \frac{\eta}{T}\right)\right] + \ln\left[1 + \exp\left(-x - \frac{\eta}{T}\right)\right] \right\}, \tag{II.12}$$

where Ω_e is the thermodynamic potential of the interacting fields φ and A_μ, and η is the chemical potential of the neutrinos determined from the condition of constancy of the fermion excess

$$N_\nu = \frac{N_\gamma}{4\zeta(3)} \int_0^\infty dx\, x^2 \left\{ \left[\exp\left(x - \frac{\eta}{T}\right) + 1\right]^{-1} - \left[\exp\left(x + \frac{\eta}{T}\right) + 1\right]^{-1} \right\} \tag{II.13}$$

(we have borne in mind that the temperature is related uniquely to the γ density by (II.1)).

The matter energy density in the gravitational equation (I.18) is determined from Ω in the standard manner:

$$\varepsilon = \Omega - T\frac{\partial \Omega}{\partial T} - \eta \frac{\partial \Omega}{\partial \eta}. \tag{II.14}$$

The potential Ω_e for neutral matter was calculated in [20,21]. In gauge theories, the final expression for Ω_e depends on the choice of the gauge. As was shown earlier, [20,22] the correct expression for the thermodynamic potential is obtained in renormalizable gauges that contain only physical particles.

Near the phase transition, the masses of the elementary excitations are small compared with their values at $T = 0$ and at the transition point $T = T_0$ they vanish. [3] In the limit of small deviations $K_i = m_i/T$, the equations for the expectation values of the fields:

$$\partial\Omega/\partial\rho = 0, \quad \partial\Omega/\partial A_0 = 0 \tag{II.15}$$

do not depend on the choice of the gauge for certain renormalizable types of gauges (Feynman, Coulomb, and a number of others).

In the single-loop approximation, Eqs. (II.15) graphically have the form shown in Fig. 1. The solid and the dashed lines correspond to the components of the scalar field φ_1 and φ_2 (I.24), and the wavy line to the vector field a_μ (I.25). As has been shown by one of the authors (A. L.; detailed calculations will be published elsewhere), calculation of the diagrams under the assumption[4)] $g^4 \ll \lambda$ and $T \ll 1$ gives

$$2\rho\left\{-\mu^2\left(1 - \frac{T^2}{T_0^2}\right) + 2\lambda\rho^2 - gA_0^2\right\} = 0, \tag{II.16}$$

$$2g^2 A_0\left(\rho^2 + \frac{T^2}{12}\right) + gn_\nu = 0, \tag{II.17}$$

where T_0 is the temperature of the phase transition of the second kind to charge-symmetric matter[20,21,23]:

$$T_0^2 = 12\mu^2/(3g^2 + 4\lambda). \tag{II.18}$$

If $n_\nu \neq 0$, the phase transition temperature T_c is determined in accordance with (II.16) and (II.17) by

$$T_c^4[(T_c^2/T_0^2) - 1] = 36 n_\nu^2/\mu^2. \tag{II.19}$$

With allowance for (II.1), Eq. (II.19) can be written in the form

$$T_c^{-2} = T_0^{-2} - [12\zeta(3)/\pi^2\mu]^2 (n_\nu/n_\gamma)^2. \tag{II.20}$$

We see that in a charge-asymmetric universe, there exists a maximal ratio $\alpha \equiv n_\nu/n_\gamma$ at which a phase transition is still possible:

$$\alpha_c = \pi^2(3g^2 + 4\lambda)^{1/2}/24 \cdot 3^{1/2}\zeta(3). \tag{II.21}$$

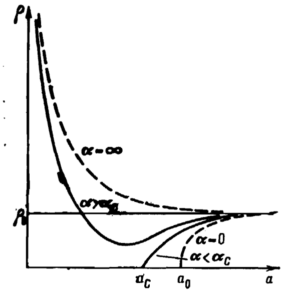

FIG. 2.

The dependence of the expectation value of the scalar field on the radius of curvature of the universe for different values of α is shown in Fig. 2. The dashed curves in Fig. 2 correspond to the cases of cold charge-asymmetric model ($\alpha = \infty$) and a hot universe filled with neutral matter ($\alpha = 0$). The masses of the particles are nonzero in the region $\rho \neq 0$ and repeat the profile of the curves for the expectation value of the scalar field.

For $\alpha > \alpha_c$, when a phase transition does not occur, the expectation value of the scalar field in the limit of small radii ($a \ll a_c$, where $a_c = a(T_c)$; see (II.1)) decreases monotonically with the expansion of the universe, $\rho \sim a^{-1}$. In this case, the evolution of the universe is qualitatively similar to the case $T=0$ considered earlier. In accordance with (II.16) and (II.17), the dependence $\rho(a)$ in the limit of small a and $\alpha > \alpha_c$ has the form

$$\rho a = \rho_0 a_0 \beta^{1/2}, \tag{II.22}$$

where β is a positive root of the equation

$$\left\{\beta + \left(\frac{2\lambda}{3g^2+4\lambda}\right)^{1/3}\left(\frac{\alpha_c}{\alpha}\right)^{2/3}\right\}^2\left\{\beta + \frac{(3g^2+4\lambda)^{1/6}}{(2\lambda)^{2/3}}\left(\frac{\alpha_c}{\alpha}\right)^{2/3}\right\} = 1. \tag{II.23}$$

Equations (II.16) and (II.17) are qualitatively correct if the parameters $K_i = m_i/T$ are small. Using the explicit expressions for the masses and the expressions (II.1) and (II.22), we find that the applicability of Eqs. (II.16) and (II.23) is restricted to the region of α values

$$\alpha < \alpha_b \sim \alpha_c/\lambda(3g^2+4\lambda). \tag{II.24}$$

For $\alpha \gg \alpha_b$, the temperature corrections to the expressions obtained for $T=0$ are always small.

In connection with the existence of a critical neutrino density above which a phase transition does not occur, it is of interest to estimate the contemporary value of α_0—the ratio of the neutrino density to the γ density. We point out immediately that a small value of α_0 compared with α_c (II.21) does not by itself indicate that there was a phase transition to a hadronic era of evolution. As was pointed out above, the contemporary number of γ's may be appreciably greater than the number during the early stages of evolution, when pairs of different species of massive particles were excited. Allowance for these particles can however appreciably change the temperature of the phase transition and thus increase the value of α_c (II.21). Although these effects, which work in different directions, increase the accuracy of our treatment insofar as certain particle species have been ignored, they do not permit us to make a unique estimate for the critical ratio n_ν/n_γ.

At the present time, there are not sufficiently accurate experimental data on the value of α_0. The main restriction on the density of the neutrino–antineutrino excess is given by the measured rate of expansion of the universe (the Hubble constant)[5]: $\alpha_0 < 10^5$. A more stringent restriction follows from the assumption that an appreciable fraction of the He^4 observed in the universe has a primordial origin. In this case, $\alpha_0 < 10$ (see, for example[16], in which restrictions on the excess of both electronic and muonic neutrinos is found).

In our simple model for $\lambda \sim g^2 \sim 0.1$, the critical ratio n_ν/n_γ at which a phase transition does not yet occur is in order of magnitude 0.1. Therefore, the existing restrictions on the contemporary value of α_0 do not enable us to establish uniquely whether there was a phase transition during an early stage of the evolution. In this connection, a more accurate experimental value for the lepton charge of the universe would be of great interest for the reconstruction of the main stages of evolution to the hadronic era.

The large masses of the elementary particles during the early stages in the expansion of the universe could lead to new consequences for the abundances of massive primordial particles, in particular, free stable quarks. In the absence of a phase transition, the ratio

$$m_q/T \equiv K_q \sim (h_q/\lambda^{1/6})(n_\nu/n_\gamma)^{1/3} \tag{II.25}$$

(m_q is the quark mass, $m_q = h_q\rho$) for quarks interacting with the scalar field (with constant h_q) in the region (I.20) does not depend on the time. Using Eqs. (II.1) and (II.6), we can readily find that in this region the ratio of the quark density n_q to the γ density for $K_q \gg 1$ is in order of magnitude

$$n_q/n_\gamma \sim K_q^{3/2} e^{-K_q}. \tag{II.26}$$

If at the time the universe left the region (I.20) the number of quarks in accordance with (II.26) was small compared with the "freezing" density,[16] then Eq. (II.26) in the presence of thermodynamic equilibrium determines the abundance of quarks not only during early but also late stages in the expansion of the universe. The traditional hot universe model predicts a relative abundance of stable primordial quarks equal to $n_q/n_\gamma \sim 10^{-19}$, which appreciably exceeds the experimental bounds.[26]

In our model, a small abundance (which does not contradict experiment) of quarks is obtained for large values of K_q ($K_q \gg 10$). By virtue of the existing restrictions on the ratio n_ν/n_γ (see above) large values in accordance with (II.25) are possible however only if there is a strong interaction between the quarks and the scalar field ($h_q > 10$).

We are aware that this model may be far from the definitive version of a model of unified field theory.

However, if the main features of models with spontaneous symmetry breaking are preserved in the definitive model, our conclusions will be qualitatively correct. During the evolution of the universe, two mechanisms will compete: symmetry breaking with increase in the density of the fermion charge and restoration of symmetry through the variation of the temperature.

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APPENDIX

In unified models of weak and electromagnetic interactions there are, in addition to massless neutrinos, massive fermions—leptons and hadrons. During the early stages in the expansion of the universe, the number of neutrinos is not conserved because of $\nu \rightleftarrows e$ transitions in the reactions $\nu n \to pe$. The total lepton, L, and baryon B, charges are conserved. We shall also assume electrical neutrality of the universe. Since a weak charge J_0 of fermions interacting with a neutral vector field Z_μ cannot be constructed from conserved charges, it must be found from a condition of minimum of the thermodynamic potential. Thus, in thermodynamic equilibrium, the ratio of the number of leptons and hadrons with different helicity for given L and B is determined. We consider Weinberg's $SU(2)\otimes U(1)$ model[11] with the inclusion of hadrons.[24] For such a model, in the limit of a high density of fermions at $T=0$, the effective potential of the electrically neutral matter in the quasiclassical approximation in the unitary gauge has the form

$$\Omega = -(g'^2+g^2)\rho^2 Z_0^2 - \mu^2\rho^2 + \lambda\rho^4 - Z_0 J_0 + \sum_{\substack{L=e,\,\nu\\ j=l,\,r}} \int^{k_L^{(j)}} \frac{d^3k}{(2\pi)^3}(\mathbf{k}^2 + h_L^2\rho^2)^{1/2} + \sum_{\substack{B=p,\,n\\ j=l,\,r}} \int^{k_B^{(j)}} \frac{d^3k}{(2\pi)^3}(\mathbf{k}^2 + h_B^2\rho^2)^{1/2}, \qquad \text{(A.1)}$$

where $g'/g = \tan\theta_W$, θ_W is Weinberg's mixing angle, $h_{L,B}\rho$ are the masses of the leptons and hadrons ($h_\nu \equiv 0$, $h_p = h_n = h$), the indices l and r are appended to fermions with helicity ∓ 1; $k_{L,B}^{(j)}$ are the Fermi momenta of the degenerate leptons and hadrons with different helicity:

$$k_{L,B}^{(j)} = [6\pi^2 n_{L,B}^{(j)}]^{1/3}, \qquad \text{(A.2)}$$

where $n_{L,B}^{(j)}$ are their densities. The neutral weak charge of the fermions is determined by the expression

$$J_0 = (g'^2+g^2)^{1/2}\left\{ n_\nu - n_n^{(r)} + \frac{g'^2-g^2}{g'^2+g^2}[n_e^{(l)} - n_p^{(l)}] + \frac{2g'^2}{g'^2+g^2}[n_e^{(r)} - n_p^{(r)}]\right\}. \qquad \text{(A.3)}$$

The first four terms in (A.1) correspond to the effective potential in the tree approximation; the last terms are the energy density of the degenerate gas of left- and right-handed fermions. Note that in the presence of a P-parity breaking interaction of fermions with the massive gauge field Z_μ the Fermi surfaces of particles with different helicity do not coincide. In the equilibrium state, their densities can be found from the condition of the minimum of Ω under additional conditions: electrical neutrality

$$n_e^{(l)} + n_e^{(r)} = n_p^{(l)} + n_p^{(r)}, \qquad \text{(A.4)}$$

conservation of the lepton charge:

$$L = n_\nu + n_e^{(l)} + n_e^{(r)} \qquad \text{(A.5)}$$

and conservation of the baryon charge:

$$B = n_p^{(l)} + n_p^{(r)} + n_n^{(l)} + n_n^{(r)}. \qquad \text{(A.6)}$$

Varying Ω with respect to all independent variables, we readily obtain a system of six equations that, in conjunction with (A.4)–(A.6), determine the dependence of ρ, Z_0, and $n_{L,B}^{(j)}$ on L and B. The investigation of this system shows that in the case of compression of matter with lepton–hadron ratio in the interval

$$\left|\frac{L}{B} - \frac{5}{9}\right| < \frac{29\sqrt{5}}{108\pi}h, \qquad \text{(A.7)}$$

there exists a critical density above which symmetry of the vacuum is restored. (Note that the numerical coefficient $\frac{5}{9}$ in (A.7) is due to the particular way in which the weak interaction of hadrons is taken into account.[24])

If the opposite inequality holds, a phase transition does not occur in cold matter. In the limit of a high density of fermions, $n \gg (\mu/h)^3$, the expectation value of the scalar field increases monotonically with increasing density. For a universe with large neutrino excess, $L \approx n_\nu \gg B$, the form of the dependence $\rho(n_\nu)$ agrees with the result of the simple model investigated in detail in the paper.

1) We do not consider here the Hagedorn model with infinite number of particle species, which encounters a number of serious objections.[19] However, the qualitative results remain the same in the Hagedorn model.[25]

2) Throughout the present paper we assume that in the absence of particle creation effects by the gravitational field the total entropy of the universe is conserved. This is the case if matter viscosity can be ignored.

3) Because of the existing restrictions on the constants λ and g, the analogous time for vector mesons does not differ significantly from t_S.

4) In this case the phase transition is of the second kind. Note also that when $g^4 > \lambda$ the quantum corrections to the effective potential lead to instability of the vacuum with $\langle\varphi\rangle = 0$ (see[15]).

5) The presence of a degenerate neutrino gas with such a density may explain the difference between the mass of the universe determined from the Hubble constant and from the mean density of baryons in the universe.

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Multiregge processes in the Yang–Mills theory

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For nonabelian gauge theories with the Higgs mechanism of mass generation the scattering amplitudes have been calculated in the multiregge kinematics in the leading logarithmic approximation. The reggeization of the vector particle is proved in this approximation.

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1. INTRODUCTION

The hypothesis that all observed hadrons are reggeons turned out to be so fruitful that it gave the possibility of constructing a phenomenological theory of hadron interactions at high energies, based on Gribov's reggeon diagram technique.[1] Testing this theory in the framework of a local field theory presents great interest. As is known, in an abelian gauge theory—quantum electrodynamics (QED)—the spinor particle reggeizes,[2] but the vector particle remains elementary.[3] More realistic models for the strong interactions will probably be based on nonabelian gauge theories with Yang–Mills vector mesons.[4] In such theories the interaction vanishes at small distances, so that in distinction from QED[5] they exhibit approximate scale-invariance.[6]

Moreover, the Higgs mechanism[7] allows the vector mesons in nonabelian gauge theories to acquire a mass without destroying the renormalizability of the theory.[8] In a spontaneously broken theory which arises in this manner the necessary conditions for the reggeization of the vector meson are satisfied.[9] One of the authors has shown by direct calculation of the scattering amplitudes to sixth order of perturbation theory that for the gauge group $SU(2)$ the reggeization of the vector meson does indeed take place.[10] Later this result was generalized to other models.[11] In our preceding short note, based on calculations up to eighth order of perturbation theory, we have shown that the hypothesis of reggeization to any order is self-consistent and have also determined the form of the Pomeranchuk singularity.[12]

In the present paper we give the details of these calculations to eighth order for the group $SU(2)$. Our computation method which is based on the dispersion theory approach, also allows one to obtain the inelastic amplitudes in the multiregge kinematics (Eq. (55)). Making use of the expression (55) for these amplitudes we obtain an equation of the Bethe–Salpeter type for the $2\to 2$ partial wave amplitudes with isospins $T=0$, 1, 2 in the t-channel, Eq. (64). The solution of the equation with $T=1$ is the Regge pole (66). The Appendix contains a generalization of these results to the group $SU(N)$.

2. THE MODEL AND THE RESULTS OF CALCULATIONS OF THE TWO-PARTICLE AMPLITUDES IN LOWEST ORDERS

Following[10], we consider the simplest nonabelian gauge theory whose Lagrangian, after spontaneous symmetry breaking and removal of the unphysical degrees of freedom by means of a gauge transformation has the

