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Radiative effects in the field of an electromagnetic wave with allowance for the action of a stationary magnetic field

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Radiative effects due to the combined action of stationary and alternating electromagnetic fields are considered by the analytic continuation technique in the first order with respect to the fine structure constant $\alpha = e^2/\hbar c$. The real parts of the corrections to the electron and photon masses are determined for the case of low wave intensities ($\xi = eE_0/cm\omega < 1$) by means of the expressions for the probabilities of the multiphoton electron scattering and electron–positron pair production by photons in the wave field or magnetic field. In the overlap region the derived formulas are in agreement with results obtained in an investigation of radiative processes in stationary crossed fields and also with the results of an analysis of the mass and polarization operators in the field of an electromagnetic wave. Polarization effects are studied. It is shown that in this case the anomalous magnetic moment of the electron is a function of all the parameters that characterize the total field (ω, E_0, and H). Corrections proportional to the wave intensity are obtained for the Schwinger value of the anomalous electron moment.

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The interaction of electrons and phonons with an electromagnetic vacuum in a constant electromagnetic field has been studied in sufficient detail (see, e.g. [1–5]). Interest has arisen recently in the study of similar processes in the field of an intense electromagnetic wave, in view of the added possibilities afforded by the use of laser techniques. It turns out that the presence of a sufficiently intense electromagnetic wave in a vacuum is capable of leading to effects analogous to those that take place in constant fields. The emission of a photon by an electron and the production of electron-positron pairs by a photon in the field of a wave[6] are processes that give rise to radiative effects leading to changes in the masses of the electron and the photon. The question of the change in the photon mass in the field of a wave was considered in[7] by using the electron Green's function obtained by Schwinger.[8] An analogous method was used in[9] to calculate the polarization operator. Quite recently[10,11] the method of operator diagram technique has yielded the corrections to the mass and polarization operators in the field of a wave of rather general form.

In this paper we use the dispersion-relation method to consider radiative processes in the field of an electromagnetic wave, with account taken of the action of a constant magnetic field. The development of a method of analytic continuation as applied to the case of an investigation of radiative effects in constant crossed fields was described by Ritus.[4,12] The main deviation from[4,7,9–12] is that the external field was chosen by us to be a superposition of fields, namely a constant magnetic field of intensity H and the field of a plane electromagnetic wave propagating along a magnetic field,

$$\mathbf{H}=(0,0,H), \qquad (1)$$

$$E_0 = E_0\{e_1 \cos\varphi + g e_2 \sin\varphi\}, \quad H_0 = [n \times E_0], \tag{2}$$

where $k=(\omega, \mathbf{k})$ is the wave 4-vector ($k^2=0$), $\varphi=(kx) = k_0x_0 - \mathbf{kx} = \omega(t-z)$, $\hbar = c = 1$, $g=\pm 1$ describes the circular polarization of the wave, $\mathbf{e}_1$ and $\mathbf{e}_2$ are two-dimensional unit vectors, and $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$.

A generalization of this method of analytic continuation to the case of the joint action of a wave and a constant field has shown that the method makes it possible to treat from a unified point of view the contributions made to the masses of the electron and the photon by the presence of stationary and nonstationary fields. The introduction of a constant magnetic field $\mathbf{H}$ makes it possible to include correctly in the analysis also the electron spin and identify subsequently the additional energy, proportional to $\mu(\mathbf{H}\cdot\boldsymbol{\zeta})$ ($\boldsymbol{\zeta}$ is the electron spin), of the interaction of the electron with the magnetic field with the energy of the interaction between the anomalous magnetic moment of the electron with the magnetic field. In this case the proportionality coefficient μ depends on E_0, ωH, and ε. It turns out then that in the presence of a constant and alternating electromagnetic field the anomalous magnetic moment of the electron is a function of all the parameters that characterize the summary field ω, E_0, H, and also the electron energies ε.

It should be indicated that this approach was made possible by Redmond's[13] well-known exact solution of the Dirac equation in fields of configuration (1)+(2).

A similar procedure is used to calculate the spin-independent parts of the corrections to the self-masses of the electron and photon in a combined field consisting of a constant magnetic field and a plane electromagnetic wave. In particular, when the wave field is turned off ($E_0 \to 0$) we obtain the results of[3-5] for the values of the parameter $\chi = Hp_\perp/H_{cr}m \ll 1$ ($H_{cr}=m^2/e$, $p_\perp$ is the transverse component of the electron momentum). In the other limiting case ($H\to 0$) corresponding to turning off the magnetic field, we obtain the results of[7,9-11] for the values $\xi = eE_0/m\omega \ll 1$. In addition to the known results, we obtain here also an explicit expression for the square of the photon mass in the $\xi<1$ approximation, with allowance for the polarization effects.

We note that in modern cw lasers the parameter ξ reaches values 0.1, while the value $\xi \gg 1$ generally speaking corresponds to the constant-field limit.[6] The choice of the values of the parameter $\chi \ll 1$ ensures a real physical situation, wherein the effects of the wave and constant fields manifest themselves most brightly.

Since the imaginary part of the amplitude of elastic scattering of a photon by a photon through a zero angle (in this case by an electromagnetic wave) is connected with the total probability of pair production by a photon in the field of an electromagnetic wave, while the real part of this amplitude can be reconstructed from the dispersion relations, this information enables us to estimate the cross section for the scattering of a photon by an electromagnetic wave—an effect of fourth order in α in perturbation theory.

It should be noted that the general case of arbitrary values of the parameter ξ can be considered within the framework of the dispersion-relation technique also when several photons of the wave take part in the reaction. In this case, however, it is necessary, generally speaking, to take into account both the induced multiphoton effects, as well as effects that contribute in the higher powers of the expansion in α.

1. ELECTRON MASS IN AN EXTERNAL FIELD. ANOMALOUS MAGNETIC MOMENT

The probability of emission of a photon by a polarized electron in fields (1)+(2) as a function of the invariant variables ξ, χ, $\varkappa_s = 2s(kp)/m^2$ was obtained in[4]. It is given by the formula

$$W = \frac{\alpha m}{2\pi\gamma} \sum_{s=-\infty}^{\infty} \int_0 \frac{du}{(1+u)^3} \left(\frac{u}{2\chi}\right)^{2/3} \int_{-\infty}^{\infty} d\delta \left(1+\xi^2 \frac{u^2}{\varkappa^2} b\right)^2 (F_j^s)^2, \quad j=1,2,$$

$$F_1^s = \delta_{\zeta,\zeta'}(2+u)\left\{\xi\beta\Phi J_s' - \left[\zeta' \frac{u}{2+u}\left(\Phi - a\left(\frac{2\chi}{u}\right)^{1/3}\Phi'\right) - ab\Phi + \left(\frac{2\chi}{u}\right)^{1/3}\Phi'\right] J_s\right\} + \delta_{\zeta,-\zeta'} u \left\{c\Phi - a\delta\left(\frac{2\chi}{u}\right)^{1/3}\Phi'\right\} J_s, \tag{3}$$

$$F_2^s = \delta_{\zeta,\zeta'}(2+u)\left\{c\Phi - a\delta\left(\frac{2\chi}{u}\right)^{1/3}\Phi'\right\} J_s + \delta_{\zeta,-\zeta'} u \left\{\xi\beta\Phi J_s' - \left[\zeta'\left(\Phi - a\left(\frac{2\chi}{u}\right)^{1/3}\Phi'\right) - ab\Phi + \left(\frac{2\chi}{u}\right)^{1/3}\Phi'\right] J_s\right\},$$

where

$$\delta_{\zeta,\zeta'} = \begin{cases} 1, & \zeta=\zeta' \\ 0, & \zeta\neq\zeta' \end{cases}; \quad \zeta, \zeta' = \pm 1;$$

$$a = \xi \frac{s}{\nu} \frac{2su}{\varkappa_s}\beta; \quad b = \delta^2 + (1+\xi^2)\left(1 - \frac{\varkappa_s^*}{u}\right);$$

$$c = \delta - \xi g \frac{s}{\nu}\beta; \quad \nu = g\xi \frac{2su}{\varkappa_s}\beta\left(\delta - 2g\frac{\chi}{\varkappa}\right)\left(1+\xi^2\frac{u^2}{\varkappa^2}b\right);$$

$$\beta^{-1} = \left(1 - 2g\frac{\chi}{\gamma\varkappa}\right)\left(-2g\frac{\chi}{\gamma'\varkappa}\right)\left(+\xi^2\frac{u^2}{\varkappa^2}b\right); \quad \varkappa = \varkappa_s|_{s=1} = \frac{2(kp)}{m^2};$$

$$\gamma = \frac{(kp)}{\omega m}; \quad \gamma' = \frac{(kp')}{\omega m}; \quad \varkappa_s^* = \frac{\varkappa_s}{1+\xi^2};$$

Φ and Φ' are the Airy function and its derivative with respect to the argument $y = (u/2\chi)^{2/3} b$; J_s are Bessel functions of the argument ν; p and p' are the 4-momenta of the initial and final electrons. In formula (3), the s-term of the sum is the partial probability of a process consisting of either absorption of s photons of the wave with emission of a single photon k' ($s>0$), or in the emission of $s+1$ photons, of which s are identical with the wave photons. The conservation laws for the quasi-energy and for the longitudinal component of the quasi-momentum are thereby satisfied.

Formula (3) was obtained by us in the approximation $m/\varepsilon \ll 1$ and $\xi k'/\varepsilon \ll 1$, but it is easy to show that in the case when the magnetic field is turned off and account is taken of the conservation law $(1+\xi^2)(\varkappa_s^*/u - 1) - \delta^2 = 0$, which follows directly from the properties of the Airy function, this formula goes over into the total s-quantum probability of electron emission in a field of a plane circularly-polarized wave, which agrees with the results of Nikishov and Ritus.[6]

In the case $\xi < 1$ with allowance for the terms $\sim\xi^2$ we obtain[14,15] for from formula (3) the total probability, after integrating with respect to δ,

$$W = W_0(\chi) + W_0'(\xi^2, \chi, \varkappa) + W_{-1}(\xi^2, \chi, \varkappa) + W_1(\xi^2, \chi, \varkappa), \tag{4}$$

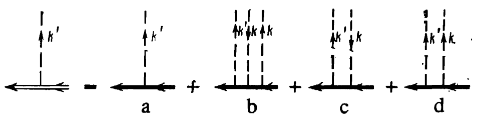

FIG. 1. Emission of an electron in a constant magnetic field and in the field of a plane electromagnetic wave.

where W_0 determines the probability of the synchrotron radiation

$$W_0=-\frac{\alpha m^2}{\sqrt{\pi}\,\varepsilon}\int_0^\infty\frac{du}{(1+u)^2}\left\{\Phi_1(y)+\frac{2}{y}\left(1+\frac{u^2}{2(1+u)}\right)\Phi'(y)-\zeta\chi\frac{y\Phi(y)}{1+u}\right\},$$

$$\Phi_1(y)=\int_y^\infty\Phi(x)\,dx,\quad y=\left(\frac{u}{\chi}\right)^{2/3},\quad \zeta=\pm1. \tag{5}$$

W_0' are the corrections to W_0, due to the influence of the wave:

$$W_0'=-\xi^2\frac{\alpha m^2}{\sqrt{\pi}\,\varepsilon}\int_0^\infty\frac{du}{(1+u)^2}\left\{\frac{u^2}{2y}\left[2y^2\Phi+2r\Phi'+y\Phi_1\right]\right.$$

$$\left.+\frac{u+1}{y}\left[y^2\Phi+2r\Phi'+y(r+1)\Phi_1\right]-\zeta\frac{u}{\sqrt{y}(1+u)}(r\Phi+y\Phi')\right\}, \tag{6}$$

$$\Phi=\Phi(y),\quad \Phi'=\Phi'(y),\quad \Phi_1=\Phi_1(y),\quad y=\left(\frac{u}{\chi}\right)^{2/3},$$

$$r=2\frac{u^2}{\varkappa^2}\left(1-4\frac{\chi^2}{\varkappa^2}\right).$$

W_{-1} corresponds to the probability of the Compton scattering with allowance for the magnetic field, W_1 determines the probability of the stimulated Compton effect in a magnetic field:

$$W_{-1,1}=\xi^2\frac{\alpha m^2}{4\sqrt{\pi}\,\varepsilon\varkappa^2}\int_0^\infty\frac{u^2\,du}{(1+u)^3}\left\{\varkappa^2\Phi_1^{\mp}+8\left(\frac{\chi}{u}\right)^{4/3}u^2\left(1\mp2\frac{u}{\varkappa}\right)\Phi_{\mp}\right.$$

$$-4\left(\frac{\chi}{u}\right)^{2/3}u^2\left(1+4\frac{\chi^2}{\varkappa^2}\right)\Phi_{\mp}'+4(1+u)\left[\left(1\mp\frac{\varkappa}{u}+\frac{\varkappa^2}{2u^2}-4\frac{\chi^2}{\varkappa^2}\right)\Phi_1^{\mp}\right.$$

$$\left.+4\left(\frac{\chi}{u}\right)^{4/3}\left(1\mp\frac{u}{\varkappa}\right)\Phi_{\mp}-8\left(\frac{\chi}{u}\right)^{2/3}\frac{\chi^2}{\varkappa^2}\Phi_{\mp}'\right]+4u\zeta\left(\frac{\chi}{u}\right)^{1/3}\left[\left(1\right.\right.$$

$$\left.\left.\left.\mp\frac{\varkappa}{2u}+4\frac{\chi^2}{\varkappa^2}\right)\Phi_{\mp}\pm4\frac{\chi}{\varkappa}\left(\frac{\chi}{u}\right)^{1/3}\Phi_{\mp}'\right]\right\}, \tag{7}$$

$$\Phi_{\mp}=\Phi(y^{\mp}),\quad y^{\mp}=\left(\frac{u}{\chi}\right)^{2/3}\left(1\mp\frac{\varkappa}{u}\right). \tag{8}$$

The processes (5)–(8) correspond to the Feynman diagrams in Fig. 1.

The diagram on left represents the summary process (4), while on the right side diagram a corresponds to the probability (5), b to (6), and c and d to (7) and (8). As seen from (5)–(8), the partial probabilities depend essentially on the ratio $\chi/\varkappa$, and as resonance is approached ($2\chi/\varkappa\to\varepsilon/m$) the principal terms in the summary probability cancel each other in such a way that W takes the form[14,15]

$$dW=\xi^2\left(\frac{\varepsilon}{m}\right)^4\left(\frac{u}{2\chi}\right)^2 dW_0, \tag{9}$$

where it is implied that $\xi^2(\varepsilon/m)^4<1$, and W_0 is the probability of the synchrotron radiation.

This circumstance is obvious beforehand, for if the frequency of the wave is close to the frequency of revolution of the electron in the magnetic field (eH/ε), the radiated series of frequencies becomes dependent on a single parameter, $\omega\sim eH/\omega$, in contrast to the case when ω and eH/ε differ greatly from each other. We shall therefore investigate here in greater detail the case $\chi/\varkappa<1$, when the increments to the probability W_0 are not trivial.

According to the unitarity relations for the S matrix, the amplitude T_{ii} forward scattering of an electron in an external electromagnetic field, in order e^2 in the quantized photon field, is connected with the probability of photon emission by a polarized electron by the relation (see, e.g., [4,16])

$$2\,\mathrm{Im}\,T_{ii}=W(\chi,\varkappa,\xi^2). \tag{10}$$

By regarding the probability W as a function of the complex variables χ and $\varkappa$ (ξ^2 is fixed in the corresponding probabilities), we can write for each of them, at a fixed value of the other, dispersion relations that connect the real and imaginary parts of the amplitude T_{ii}. Recognizing that

$$T_{ii}=-\frac{m}{\varepsilon}\Delta m, \tag{11}$$

where m and ε are the mass and energy of the electron while Δm is the correction to the self-energy of the electron, the dispersion relations can be written directly for $\mathrm{Re}\,\Delta m$. Since each of the variables χ and $\varkappa$ contains the dynamic characteristics of the electron $p_\perp$ and p, the changes of the variables χ and $\varkappa$ can be carried out fixed values of $p_\perp$ and p. Thus, we use the dispersion relations to investigate the properties of the electron self-energy as a function of the magnetic field H and the wave frequency ω.

Using the analytic-continuation procedure proposed by Ritus (see[4,12]), we can reconstruct from the imaginary part of the mass operator [this part is determined by formulas (4)–(8), (10), and (11)] a real part having the necessary analytic properties, by changing over from the Airy functions to the functions

$$f(z)=\frac{i}{\sqrt{\pi}}\int_0^\infty e^{-i(zt+t^3/3)}\,dt, \tag{12}$$

the real and imaginary parts of which on the real axis are connected by the Hilbert transformations

$$f(z)=\Upsilon(z)+i\Phi(z),\quad \Upsilon(z)=-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\Phi(z')\,dz'}{z'-z}. \tag{13}$$

This holds true because the analytic properties of the mass operator with respect to the variables χ and $\varkappa$ are determined in the main by the properties of the functions $f(z_s)$, $f'(z_s)$, and $f_1(z_s)$, where in our case $z_s=(u/\chi)^{2/3}\cdot(1+s\ \ /u)$, and $s=0,\pm1$. A study of the analytic properties of the functions f, f', and f_1 of different arguments was carried out in detail in[4,12].

Guiding ourselves by the indicated scheme, we can write for the real part of the spin-independent increment to the electron mass

$$\operatorname{Re}\Delta m=\frac{\alpha m}{2\sqrt{\pi}}\operatorname{Re}\int_0^\infty\frac{du}{(1+u)^3}\left\{\frac{5+7u+5u^2}{3z}f'\right.$$
$$+\xi^2\left[\left[\frac{u^2}{4}+(1+u)\left(\frac{1}{2}+\frac{u^2}{\varkappa^2}-4\frac{u^2\chi^2}{\varkappa^4}\right)\right](2f_1-f_1^--f_1^+)\right.$$
$$+(u^2+u+1)\left[fz+4\frac{u\chi^2}{\varkappa^3}(z_-f_--z_+f_+)\right]$$
$$+\frac{u^2+2u+2}{z}\left[f'r+4\frac{u^2\chi^2}{\varkappa^4}(f_-'+f_+')\right]$$
$$\left.\left.+\frac{u(1+u)}{\varkappa}(f_1^--f_1^+)+2u^2z\frac{\chi^2}{\varkappa^2}(f_-+f_+)+\frac{u^4}{\varkappa^2 z}(f_-'+f_+')\right]\right\},\qquad(14)$$

where

$$f=f(z),\quad f'=f'(z),\quad f_1=f_1(z)=\int_z^\infty dx\left(f(x)-\frac{1}{x}\right),$$
$$f_\mp=f(z_\mp),\quad f_\mp'=f'(z_\mp),\quad f_1^\mp=f_1(z_\mp),\quad z_\mp=z\left(1\mp\frac{\varkappa}{u}\right),$$
$$z=\left(\frac{u}{\chi}\right)^{2/3},\quad r=2\frac{u^2}{\varkappa^2}\left(1-4\frac{\chi^2}{\varkappa^2}\right).$$

For the spin part we have respectively

$$\operatorname{Re}\Delta m_\zeta=-\zeta\chi\frac{\alpha m}{2\sqrt{\pi}}\operatorname{Re}\int_0^\infty\frac{z\,du}{(1+u)^3}\left\{f+\xi^2\left[rf+zf'\right.\right.$$
$$\left.\left.+\frac{u^2}{\varkappa^2}\left(1+4\frac{\chi^2}{\varkappa^2}\right)(f_-+f_+)-\frac{u}{2\varkappa}(f_--f_+)-4z\frac{u\chi^2}{\varkappa^3}(f_-'-f_+')\right]\right\}.\qquad(15)$$

Since we are interested mainly in the corrections to $\operatorname{Re}\Delta m$ and $\operatorname{Re}\Delta m_\zeta$, in the case when the contribution from the wave and from the constant field are comparable in magnitude, we shall carry out the calculations under the assumption that $\chi\ll 1$. In this limit, expressions (14) and (15) become much simpler and, using the asymptotic expansion of the functions

$$\Upsilon(z_\mp)=\frac{1}{z\sqrt{\pi}}\sum_{k=0}^\infty\frac{\Gamma(3k+1)}{k!}\left(\frac{\chi^2}{3u^2}\right)^k\left(\frac{z}{z_\mp}\right)^{3k+1},\qquad(16)$$

where

$$z=(u/\chi)^{2/3},\quad z_\mp=z(1\mp\varkappa/u),\quad \Gamma(m)=(m-1)!,$$

we can obtain a series in powers of χ. Retaining in the expansion the terms $\sim\chi^0$, χ, χ^2, and also ξ^0, ξ^2, $\chi\xi^2$ (recognizing that $\xi<1$) and neglecting $\sim\xi^2\chi^2$, we obtain for the regularized value of $\operatorname{Re}\Delta m_\xi$

$$\operatorname{Re}\Delta m=\frac{\alpha m}{2\pi}\left\{\frac{8}{3}\chi^2\left(\ln\frac{1}{\chi}+C+\frac{\ln 3}{2}-\frac{33}{16}\right)\right.$$
$$+\xi^2\left[\frac{\varkappa^2-8}{4\varkappa^2}\left(\frac{\pi^2}{3}+F(\varkappa-1)+F(-\varkappa-1)\right)\right.$$
$$\left.\left.+\frac{F(\varkappa-1)-F(-\varkappa-1)}{\varkappa}-\frac{\varkappa^4-2\varkappa^2+2}{2(1-\varkappa^2)^2}\right]+\ldots\right\}.\qquad(17)$$

Here

$$F(x)=\int_0^x\frac{\ln(1+t)\,dt}{t}$$

is the Spence function and $C=0.577\ldots$.

In the rest system we can rewrite $\operatorname{Re}\Delta m_\xi$ in the form

$$\operatorname{Re}\Delta m_\zeta=-\mu'(\zeta\mathbf{H}),$$
$$\frac{\mu'}{\mu_0}=\frac{\alpha}{2\pi}\left\{1-12\chi^2\left(\ln\frac{1}{\chi}+C+\frac{\ln 3}{2}-\frac{37}{12}\right)\right.$$
$$\left.+\xi^2\left[-1+\frac{1+3\varkappa^2}{(1-\varkappa^2)^2}+\frac{2\varkappa^4+6\varkappa^2}{(1-\varkappa^2)^3}\ln\varkappa\right]+\ldots\right\}\quad \mu_0=\frac{e}{2m}.\qquad(18)$$

From (17) and (18), when the wave is turned off ($\xi\to 0$), we obtain the results of[3-5] in the limit $\chi\ll 1$. Accurate to terms linear in χ, expression (17) agrees with the result of[10] in the case $\xi\ll 1$. Formula (18) in this limit ($\chi\to 0$) determines the contribution of the plane electromagnetic wave to the correlation term $\zeta\mu H$. This term can be identified with the interaction energy of the vacuum magnetic moment of the electron with the magnetic field H, and the coefficient $\mu(\xi)$ thus determines the increment to the anomalous magnetic moment of the electron, due to the presence of a plane electromagnetic wave:

$$\mu(\xi)=\xi^2\frac{\alpha}{2\pi}\mu_0\left[-1+\frac{1+3\varkappa^2}{(1-\varkappa^2)^2}+\frac{2\varkappa^4+6\varkappa^2}{(1-\varkappa^2)^3}\ln\varkappa\right].\qquad(19)$$

Breakdown of the total probability into partial probabilities makes it possible to trace the contribution of each partial process to the values of $\operatorname{Re}\Delta m$ and μ. Thus, in the approximation linear in χ, the first term of (18) stems from the probability W_0', and the remainder from W_{-1} and W_1. Thus, in an alternating field the anomalous magnetic moment of the electron depends on the amplitude and frequency of this field. We call attention to the fact that the real part of the electron mass contains pole term s of the type $1/(1-\varkappa^2)$. It is easily seen that these poles, however, lie in the unphysical region, since the condition $\varkappa=1$ is equivalent to condition $2(kp)=m^2$ or $k-p=0$, which leads to a patently incorrect conclusion that the isotropic vector k is equal to the timelike vector p.

In the limits $\varkappa\ll 1$ and $\varkappa\gg 1$ we have for $\mu(\xi)$ respectively

$$\mu=-\xi^2\mu_0\frac{3\alpha}{\pi}\varkappa^2\ln\frac{1}{\varkappa},\quad \varkappa\ll 1,$$
$$\mu=-\xi^2\mu_0\frac{\alpha}{2\pi}\left(1+2\frac{\ln\varkappa}{\varkappa^2}\right),\quad \varkappa\gg 1.\qquad(20)$$

We note that the radiative corrections in the wave field are nonlinear in the wave amplitude.

2. SELF-MASS OF PHOTON IN AN EXTERNAL FIELD. SCATTERING OF PHOTON BY AN INTENSE WAVE

The correction of order α to the photon mass in an external electromagnetic field of the chosen configuration (1) + (2) can also be obtained by the dispersion-relation method, using the explicit form of the probabilities of the process of electron-positron pair production by an external (non-wave) photon k'. The total probability of this process is a function of the variables $\eta=Hk_0'/H_{cr}m$, $\lambda_s=2s(kk')/m^2$, and ξ.

In the case when the photon k' propagates in a plane perpendicular to the magnetic-field intensity vector, this probability is equal to[17]

$$W=\frac{\alpha m^2}{2\pi k_0'\eta}\sum_{s=-\infty}^{\infty}\int_4^{\infty}\frac{du}{u[u(u-4)]^{1/2}}\left(\frac{2\eta}{u}\right)^{2/3}\times\int_{-\infty}^{\infty}dv\left(1+\xi^2\frac{u^2}{\lambda^2}b\right)^2(F_j^s)^2,\quad j=1,2;\tag{21}$$

$$F_1^s=\delta_{\zeta^-,\zeta^+}\sqrt{u(u-4)}\left\{\xi\beta\Phi J_s'+\left[\zeta^+\sqrt{\frac{u}{u-4}}\left(\Phi-a\left(\frac{2\eta}{u}\right)^{1/3}\Phi'\right)+ab\Phi-\left(\frac{2\eta}{u}\right)^{1/3}\Phi'\right]J_s\right\}+\delta_{\zeta^-,-\zeta^+}u\left[c\Phi-av\left(\frac{2\eta}{u}\right)^{1/3}\Phi'\right]J_s;$$

$$F_2^s=\delta_{\zeta^-,\zeta^+}\sqrt{u(u-4)}\left[c\Phi-av\left(\frac{2\eta}{u}\right)^{1/3}\Phi'\right]J_s+\delta_{\zeta^-,-\zeta^+}u\times\left\{\xi\beta\Phi J_s'+\left[\zeta^+\left(\Phi-a\left(\frac{2\eta}{u}\right)^{1/3}\Phi'\right)+ab\Phi-\left(\frac{2\eta}{u}\right)^{1/3}\Phi'\right]J_s\right\};$$

$$a=\xi\frac{s}{v}\frac{2su}{\lambda_s}\beta;\quad b=v^2+(1+\xi^2)\left(1-\frac{\lambda_s^*}{u}\right);\quad c=v+g\xi\frac{s}{v}\beta;$$

$$\nu=-g\xi\frac{2su}{\lambda_s}\beta\left(v+2g\frac{\eta}{\lambda}\right)\left(1+\xi^2\frac{u^2}{\lambda^2}b\right);$$

$$\beta^{-1}=\left(1-2g\frac{\eta}{\gamma^-\lambda}\right)\left(1+2g\frac{\eta}{\gamma^+\lambda}\right)\left(1+\xi^2\frac{u^2}{\lambda^2}b\right);$$

$$\zeta^-\zeta^+=\pm1;\quad\lambda=\lambda_s|_{s=1}=\frac{2(kk')}{m^2};\quad\lambda_s^*=\frac{\lambda_s}{1+\xi^2}.$$

Here p^- and p^+ are respectively the 4-momenta of the electron and positron, $\gamma^-=(kp^-)/\omega m$, $\gamma^+=(kp^+)/\omega m$, Φ and J_s are Airy and Bessel functions, respectively, of the arguments $y=(u/2\eta)^{2/3}b$ and ν.

We note that formula (21) leads, by a transition to the limit as $\eta\to0$ with allowance for $v^2/(1+\xi^2)=\lambda_s^*/u-1$, to the formulas of Nikishov and Ritus[6] for the case of pair production by a photon interacting with a plane electromagnetic wave, when s photons of the wave take part in the reaction.

Just as in the preceding section, we consider the case of small ξ ($\xi<1$). We then obtain from (21)[17,18]

$$W=W_0(\eta)+W_0'(\xi^2,\eta,\lambda)+W_{-1}(\xi^2,\eta,\lambda)+W_1(\xi^2;\eta,\lambda),\tag{22}$$

where the partial probabilities have the following meaning: W_0 is the probability of production of a pair by a photon k' in a magnetic field for the case of a linearly-polarized photon:

$$W_0^{\parallel,\perp}=\frac{2\alpha m^2}{\pi^{1/2}k_0'}\int_4^{\infty}\frac{du}{u[u(u-4)]^{1/2}}\left[\Phi_1(y)+\frac{2}{y}\left(1\pm\frac{1}{2}-\frac{u}{2}\right)\Phi'(y)\right],$$
$$y=\left(\frac{u}{\eta}\right)^{2/3},\quad\Phi_1(y)=\int_y^{\infty}\Phi(x)\,dx;\tag{23}$$

W_0' are the corrections to W_0, due to the wave

$$W_0'^{\parallel,\perp}=-\xi^2\frac{\alpha m^2}{\pi^{1/2}k_0'}\int_4^{\infty}\frac{du}{y[u(u-4)]^{1/2}}\left\{2r\Phi'+2y^2\Phi+y\Phi_1-\frac{2}{u}[(2\pm1)r\Phi'+(1\pm1)y^2\Phi+y(r+1)\Phi_1]\right\},$$
$$\Phi=\Phi(y),\quad y=\left(\frac{u}{\eta}\right)^{2/3},\quad r=2\frac{u^2}{\lambda^2}\left(1-4\frac{\eta^2}{\lambda^2}\right);\tag{24}$$

W_{-1} is the probability of production of a pair by two photons (by an external linearly-polarized photon k' and wave photon k):

$$W_{-1}^{\parallel,\perp}=\xi^2\frac{\alpha m^2}{2\pi^{1/2}k_0'\lambda^2}\int_4^{\infty}\frac{du}{[u(u-4)]^{1/2}}\left\{\lambda^2\Phi_1^-+8\left(\frac{\eta}{u}\right)^{4/3}u^2\left(1-2\frac{u}{\lambda}\right)\Phi_- -4\left(\frac{\eta}{u}\right)^{2/3}u^2\left(1+4\frac{\eta^2}{\lambda^2}\right)\Phi_-'-4u\left[\left(1-\frac{\lambda}{u}+\frac{\lambda^2}{2u^2}-4\frac{\eta^2}{\lambda^2}\right)\Phi_1^- +4\left(\frac{\eta}{u}\right)^{4/3}\left(1\pm\frac{1}{2}-\frac{u}{\lambda}(1\pm1)\right)\Phi_- -8\left(\frac{\eta}{u}\right)^{2/3}\left(\frac{\eta^2}{\lambda^2}\left(1\pm\frac{1}{2}\right)\mp\frac{1}{8}\right)\Phi_-'\right]\right\},$$
$$\Phi_-=\Phi(y_-),\quad y_-=\left(\frac{u}{\eta}\right)^{2/3}\left(1-\frac{\lambda}{u}\right).\tag{25}$$

The probability $W_1^{\parallel,\perp}$ corresponds to the process of pair production by the photon k', accompanied by emission of the photon k which is identical with the wave photon. This probability can be easily obtained from formula (25) by replacing λ with $-\lambda$.

For the same reasons as above, we confine ourselves to the cases far from resonance.

By considering elastic scattering of a photon in an external field in first order in α at $k'>2m$, we find that the photon mass becomes different from zero, with the following relation for the imaginary part of the mass:

$$\operatorname{Im}k'^2=-k_0'^2\operatorname{Im}n_F^2=-k_0'W.\tag{26}$$

Here W is the total probability for pair production by the photon k', n_F is the refractive index in the given field (1) + (2), and

$$k'^2=k_0'^2-\mathbf{k}'^2=k_0'^2(1-n_F^2).\tag{27}$$

The real part of k'^2 is reconstructed from the dispersion relations. The contribution to the amplitude of the elastic scattering of the photon comes in this case from the diagrams of Fig. 2. The solid line corresponds here to the motion of the electron in the magnetic field, the cross marks the external field of the wave, and the dashed line represents the motion of the external nonwave photon while the vertical lines denote the dissection of the diagrams.

It is easily seen that the square of the summary amplitude of the last three diagrams (Figs. 2b, 2c, 2d) is proportional to the cross section of the scattering of a photon by an electromagnetic wave through zero angle.

Using arguments similar to those in the procedure of Sec. 1, we can obtain for the real part of the photon mass the following expression, which depends on the polarization of the external photon (the σ and π components):

$$\operatorname{Re}k_{\parallel,\perp}'^2=\frac{\alpha m^2}{\pi^{1/2}}\operatorname{Re}\int_4^{\infty}\frac{du}{[u(u-4)]^{1/2}}\left\{\frac{2(2u+1\mp3)}{3zu}f'+2\xi^2\left[\left[\frac{1}{4}-\frac{1}{u}\left(\frac{1}{2}+\frac{u^2}{\lambda^2}-4\frac{u^2\eta^2}{\lambda^4}\right)\right](2f_1-f_1^--f_1^+)-\frac{1}{\lambda}(f_1^--f_1^+)+\frac{u-2\mp1}{uz}\left[f'r+4\frac{u^2\eta^2}{\lambda^4}(f_-'+f_+')\right]+\frac{u(u\mp1)}{\lambda^2z}(f_-'+f_+')+\frac{u-1\mp1}{u}\left[fz+4\frac{u\eta^2}{\lambda^3}(z_-f_--z_+f_+)\right]+\frac{u\mp1}{u}z\frac{\eta^2}{\lambda^2}(f_-+f_+)\right]\right\},\tag{28}$$

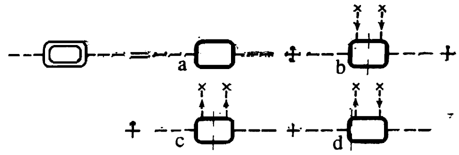

FIG. 2. Elastic scattering of a photon in a constant magnetic field and in the field of a plane electromagnetic wave.

where

$$f=f(z),\quad f_{\mp}=f(z_{\mp}),\quad z=\left(\frac{u}{\eta}\right)^{2/3},\quad z_{\mp}=z\left(1\mp\frac{\lambda}{u}\right),$$
$$r=2\frac{u^2}{\lambda^2}\left(1-4\frac{\eta^2}{\lambda^2}\right).$$

We carry out the calculation in analogy with Sec. 1. Then at $\eta\ll 1$ the renormalized real part of the photon mass is

$$\operatorname{Re}k'^2_{\mp}=-\frac{\alpha m^2}{\pi}\left\{\frac{11\mp 3}{90}\eta^2+\xi^2\left[-3+\frac{\lambda-4}{\lambda}\sqrt{\frac{\lambda+4}{\lambda}}\ln\left(\sqrt{\frac{\lambda}{4}}+\sqrt{\frac{\lambda}{4}+1}\right)+\sqrt{\frac{\lambda-4}{\lambda}}\frac{\lambda+4}{\lambda}\ln\left(\sqrt{\frac{\lambda}{4}}+\sqrt{\frac{\lambda}{4}-1}\right)-\left(1-\frac{4}{\lambda}-\frac{8}{\lambda^2}\right)\times\ln^2\left(\sqrt{\frac{\lambda}{4}}+\sqrt{\frac{\lambda}{4}+1}\right)-\left(1+\frac{4}{\lambda}-\frac{8}{\lambda^2}\right)\ln^2\left(\sqrt{\frac{\lambda}{4}}+\sqrt{\frac{\lambda}{4}-1}\right)\right]\right\}. \tag{29}$$

In particular, as $\eta\to 0$ we obtain from (29) the contribution made by the wave field to $\operatorname{Re}k'^2$ and averaged over the photon polarization:

$$\operatorname{Re}k'^2=-\xi^2\frac{\alpha m^2}{\pi}\left\{-3+\ln\left(\sqrt{\frac{\lambda}{4}}+\sqrt{\frac{\lambda}{4}+1}\right)\left[\sqrt{\frac{\lambda+4}{\lambda}}\frac{\lambda-4}{\lambda}-\left(1-\frac{4}{\lambda}-\frac{8}{\lambda^2}\right)\ln\left(\sqrt{\frac{\lambda}{4}}+\sqrt{\frac{\lambda}{4}+1}\right)\right]+\ln\left(\sqrt{\frac{\lambda}{4}}+\sqrt{\frac{\lambda}{4}-1}\right)\times\left[\sqrt{\frac{\lambda-4}{\lambda}}\frac{\lambda+4}{\lambda}-\left(1+\frac{4}{\lambda}-\frac{8}{\lambda^2}\right)\ln\left(\sqrt{\frac{\lambda}{4}}+\sqrt{\frac{\lambda}{4}-1}\right)\right]\right\}. \tag{30}$$

We present the values of $\operatorname{Re}k'^2(\lambda)$ in the limiting cases $\lambda\ll 1$ and $\lambda\gg 1$. At $\lambda\ll 1$, obviously, the external photon produces practically no electron-positron pairs as it propagates in the field of a plane wave of low intensity. In this case we have

$$\operatorname{Re}k'^2=-\xi^2\frac{\alpha m^2}{\pi}\lambda^2\frac{11}{360}. \tag{31}$$

This result agrees with that of[11] (see also[7]). Thus, in the absence of absorption $\operatorname{Re}k'^2<0$ and $\operatorname{Re}n_F^2>1$.

In the case $\lambda\gg 1$ the contribution of the imaginary part of the photon mass to the refractive index becomes different from 0, and the real part of the photon mass is equal to

$$\operatorname{Re}k'^2=\xi^2\frac{\alpha m^2}{2\pi}(\ln^2\lambda-2\ln\lambda-\pi^2+6). \tag{32}$$

Knowledge of the polarization operator (Π) makes it possible to obtain an asymptotic formula for the scattering of the photon by a photon through zero angle at $\omega\gg m$. Indeed, noting that the density of the incident wave photons is $j=E_0^2/4\pi\omega$, and the density of the external photons is equal to unity, we obtain for the cross section

$$d\sigma=\frac{\omega^2}{E_0^{\,2}}\{(\operatorname{Re}\Pi)^2+(\operatorname{Im}\Pi)^2\}\,d\Omega. \tag{33}$$

In particular, in the c. m. s.

$$d\sigma\approx\frac{\alpha^4}{\pi^2\omega^2}\ln^4\frac{\omega}{m}\,d\Omega,$$

which coincides exactly with the corresponding expression given in[16] p. 80. At $\omega\ll m$ we can obtain from formula (33) an estimating formula for the scattering of light by light in the limit of low frequencies. Using the asymptotic form of $\operatorname{Re}\Pi$ at $\omega\ll m$, we obtain

$$d\sigma\sim\alpha^2\left(\frac{11}{180}\right)^2 r_e^2\left(\frac{\omega}{m}\right)^6 d\Omega. \tag{34}$$

3. DISCUSSION OF RESULTS

The foregoing radiative corrections to the motion of an electron and a photon in an external electromagnetic field of the form (1) + (2) should in principle lead to observable effects. A classical example of effects of this type, observed in experiments, is the level shift of a bound electron in an external Coulomb field (the Lamb shift). A characteristic feature of the corrections to the electron and photon masses at high energies ($\varkappa\gg 1$ for the electron and $\lambda\gg 1$ for the photon) is their growth, which is proportional to the squares of the logarithms, with increasing invariant variables $\varkappa$ and λ. This dependence on the variables $\varkappa$ and λ is typical of radiative corrections in vacuum (for example, for the form factor of the electron $f(t)$, see[16], p. 80). The existence of a similar analogy was indicated by Ritus.[4] In contrast to the radiative corrections in a constant field, where at large values of the invariant variables the real and imaginary parts of these corrections are practically the same in absolute value and increase with increasing variables in power-law fashion,[4] in an alternating field at $\varkappa\gg 1$ and $\lambda\gg 1$ the real parts of these corrections are decisive and are equal to

$$\operatorname{Re}\Delta m=\frac{\alpha}{8\pi}\frac{e^2E_0^2}{m\omega^2}\ln^2\varkappa,\qquad \operatorname{Re}k'^2=\frac{\alpha}{2\pi}\frac{e^2E_0^2}{\omega^2}\ln^2\lambda.$$

In the region of small values of $\varkappa,\lambda$ ($\varkappa,\lambda\ll 1$) the dependence on the invariant variables is substantially different. It is interesting that the radiative corrections to the electron mass and to the photon mass in a weak constant magnetic field and in the field of a plane electromagnetic wave ($\xi<1$), at the values $\lambda\ll 1$ and $\varkappa\ll 1$ of the invariant variables, can be written in the following symmetrical forms:

$$\operatorname{Re}\Delta m=\frac{4}{3}\frac{\alpha m}{\pi}\left\{\chi^2\ln\frac{1}{\chi}+\chi_1^2\ln\frac{1}{\varkappa}\right\},$$
$$\operatorname{Re}k'^2=-\frac{11}{90}\frac{\alpha m^2}{\pi}(\eta^2+\eta_1^2),$$
$$\mu=\mu_0\frac{\alpha}{2\pi}\left\{1-12\left(\chi^2\ln\frac{1}{\chi}+2\chi_1^2\ln\frac{1}{\varkappa}\right)\right\}.$$

Here

$$\chi_1=\frac{E_0}{H_{\text{кр}}}\frac{p_0-p_3}{m},\qquad \eta_1=\frac{E_0}{H_{\text{кр}}}\frac{\omega'}{m}.$$

These formulas are asymptotic: they are valid in regions where not only the arguments of the logarithms but the logarithms themselves are large $\ln(1/\chi)\gg 1$, $\ln(1/\varkappa)\gg 1$. For experiments with laser beams it appears that greatest interest can attach to just this region of small values of $\varkappa$.

In conclusion, let us estimate the contributions made to the anomalous moment of the electron by the presence of the constant and alternating fields. Thus, for record large stationary magnetic fields, in the case $\varepsilon/m=10^3$, we have the parameter $\chi\sim 10^{-5}$, whereas in modern lasers it appears that fields can be obtained with values $\xi=0.1$ at $\omega\sim 10^{15}$ sec^{-1}. For the values given above, a contribution to the anomalous magnetic moment of the electron, due to the electromagnetic wave, can exceed the contribution from the constant magnetic field by two orders of magnitude. When account is taken of the contribution to the anomalous moment of the vacuum corrections in the next higher approximations in α ($\sim\alpha^2, \alpha^3$), it turns out that the corrections obtained here generally speaking make a smaller contribution than the term $\sim\alpha^2$, but a larger one than the term $\sim\alpha^3$.[19]

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Quasi-classical dynamics of symmetric molecules

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We study the vibronic problem in the simplest Jahn–Teller and Renner–Teller systems in the quasi-classical energy and momentum range. The method developed in this paper leads to a transcendental equation for the levels, which includes the action in the adiabatic terms, the interference phase, and the probability for non-adiabatic transitions. We discuss the consequences and possible generalizations of the theory.

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The basic features of the nuclear motion in symmetric molecules are caused by the existence of those surfaces in the nuclear coordinate configuration space on which the molecular electronic terms are degenerate. The adiabatic approximation is violated near the corresponding symmetric configurations and the motion of the nuclei on any one of the degenerate potential surfaces becomes coupled with the motion on the others.

These features of the nuclear dynamics manifest themselves strongly in the electronic-vibrational spectra of Jahn-Teller and Renner-Teller molecules. Optical transitions connecting electronic terms in the regions of adiabatic nuclear motion are collected in relatively wide bands with a simple Franck-Condon structure, while anamolous spectra arise for transitions between terms in symmetric nuclear configurations. The data from spectral studies at those frequencies, which give information about the dynamical coupling of electronic states, cannot be understood without a detailed study of the dynamics of the nuclear motion in the appropriate regions of space. Meanwhile all calculations of electronic-vibrational wavefunctions and energy levels performed for symmetric molecules up to the present[1-4] refer to low-lying excitations corresponding to initial values of the series of quantum numbers m and n (see Figs. 1 and 2 below). In those states the nuclear motion of real molecules is localized close to stable molecular configurations which are far from being completely symmetric. Completely symmetric configurations are reached only in states with large quantum numbers when the motion of the nuclei along connected potential surfaces is complicated.

Notwithstanding the complexity of such a motion the conditions $m\gg 1$ and $n\gg 1$ give us the possibility of ap-