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Decay of the initial density discontinuity in the hydrodynamics of a collisionless plasma

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The problem is considered of the decay of the initial density and velocity discontinuity in a plasma with cold ions and hot electrons. It is shown that the equations of two-stream hydrodynamics should be used to describe the evolution of such a discontinuity. The region of initial values of the plasma parameters, $u_- - u_+$ and N_-/N_+ is found in which the self-similar solutions obtained are applicable. The stability of the resultant solutions is investigated.

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1. INTRODUCTION

In recent years, great attention has been paid in plasma physics to the description of collisionless shock waves—nonlinear waves that arise in a collisionless plasma and transform the plasma from one stationary state to another. The analog of such a problem in gas dynamics is the Riemann problem[1]—the determination of the asymptotic (as $t\to\infty$) motion of the gas, in which the gas on the right halfspace ($x>0$) is maintained (up to the initial moment) at a pressure p_+, and has a velocity u_+; in the left half space ($x<0$) these quantities have the values p_- and u_-, respectively.

The solution of the Riemann problem for a plasma in the case in which $T_e \sim T_i$ is given in the work of Gurevich *et al.*[2] with the use of the kinetic equation, with a self-consistent field for the ion distribution function. In the case in which the ions are cold ($T_i \ll T_e$) their velocity distribution function degenerates into a δ function, and in the description of the motion of the ions, a transition from the kinetic equation to equations of the hydrodynamic type becomes possible. These equations in the dimensionless variables $u_\alpha = v_\alpha (T_e/M)^{-1/2}$, $\varphi = e\psi/T_e$, $\xi = x(M/T_e)^{-1/2}$, have the form

$$\frac{\partial u_\alpha}{\partial t}+u_\alpha\frac{\partial u_\alpha}{\partial \xi}=-\frac{\partial\varphi}{\partial\xi}, \tag{1a}$$

$$\frac{\partial N_\alpha}{\partial t}+\frac{\partial}{\partial\xi}(N_\alpha u_\alpha)=0, \tag{1b}$$

$$N=N_1+N_2, \quad \alpha=1,2. \tag{1c}$$

Here N_1, N_2, v_1, v_2 are the concentrations and velocities of the two ion fluxes, ψ is the electric field potential, M is the mass of the ion, T_e is the temperature of the electrons. The system (1) is written under the assumption that all the variables depend only on the single coordinate and the time t.

The introduction of the index α corresponds to the fact that, in the absence of collisions, ions can be found at each point in space, arriving both from the right and from the left; "two-layer hydrodynamics" is required for the solution of such a system of particles. Equations (1a) express the conservation of momentum of each of the components of the ionic part of the plasma, and Eqs. (1b) express the conservation of the number of particles of these components.

The system (1) should be satisfied by the Poisson relation

$$-\frac{d^2\psi}{dx^2}=4\pi e(N_i-N_e), \tag{2}$$

where ψ, N_1, N_e are functions of the coordinate and the time.

If we consider processes that are sufficiently slow in comparison with the time of establishment of the electron distribution, then the electron density distribution is determined by the potential of the self-consistent electric field at each point at the given instant of time:

$$N_e(\xi,t)=N_e[\varphi(\xi,t)]. \quad (3)$$

It is now assumed that the characteristic sizes of the concentration inhomogeneities are much greater than the Debye radius, i. e., assuming the condition of quasi-neutrality, we obtain the equation

$$N=N_e(\varphi). \quad (4)$$

We shall use this equation in what follows for the closure of the system (1) in place of Eq. (2).

The boundary conditions to the obtained system (1), (4) in the case of the setup of the problem shown should be put in the following form:

$$\begin{gathered} N_1(\xi,t)\to N_-,\quad u_1(\xi,t)\to u_-,\quad N_2(\xi,t)\to 0\quad \text{for } \xi\to-\infty,\\ N_2(\xi,t)\to N_+,\quad u_2(\xi,t)\to u_+,\quad N_1(\xi,t)\to 0\quad \text{for } \xi\to+\infty,\\ E\sim\frac{\partial\varphi}{\partial\xi}\to 0\quad \text{for } |\xi|\to\infty. \end{gathered} \quad (5)$$

These equations mean that, sufficiently far from the source, the plasma is characterized by the unperturbed stationary values of the concentration velocity, which existed there at the initial moment and the electric field is absent in these regions.

This paper is devoted to a detailed study of the system (1), (4) with the boundary conditions (5). The corresponding motion of the plasma will be considered in two cases: 1) isothermal state of the electron gas[1)] (here $N_e(\varphi)=N_0e^{\varphi}$ and 2) with account of the possible adiabatic capture of the electrons by the electric field of the produced wave.

The stability of the resultant two-flow solution will also be studied.

2. DECAY OF THE INITIAL DISCONTINUITY IN THE CASE OF A BOLTZMANN DISTRIBUTION OF THE ELECTRONS

1. *Converging plasma flows.* Considering the system (1), (4), (5), we first note that it does not have two-velocity stationary continuous solutions. However, by noting that a parameter of the length dimension is lacking, we can seek self-similar two-stream continuous solutions. (The self-similar variable is $\tau=\xi/t$.) The method of construction of such solutions is given below.

After the introduction of the self-similar variable, the set (1), (4) takes the form

$$(u_\alpha-\tau)\frac{du_\alpha}{d\tau}=-\frac{d\varphi}{d\tau}, \quad (6a)$$

$$(u_\alpha-\tau)\frac{dN_\alpha}{d\tau}+N_\alpha\frac{du_\alpha}{d\tau}=0, \quad (6b)$$

$$N=N_1+N_2,\quad \alpha=1,2, \quad (6c)$$

$$N(\tau)=N_0e^{\varphi} \quad (6d)$$

for a Boltzmann distribution of the electrons. The system (6) has solutions differing from a constant value only upon satisfaction of the condition

$$\frac{N_1}{(u_1-\tau)^2}+\frac{N_2}{(u_2-\tau)^2}=N. \quad (7)$$

The set of equations (6) and the identity (7) were first described by Alexeff *et al.* [4] and an approximate solution was found for the system (6) for the case $N_2\ll N_1$ and $u_1\gg 1$.

From (1c) and (7), we find N_1 and N_2, expressed in terms of the flow velocities

$$\begin{aligned} N_1(\tau)&=N(\tau)\frac{(u_1-\tau)^2(1-(u_2-\tau)^2)}{(u_1-\tau)^2-(u_2-\tau)^2},\\ N_2(\tau)&=-N(\tau)\frac{(u_2-\tau)^2(1-(u_1-\tau)^2)}{(u_1-\tau)^2-(u_2-\tau)^2}. \end{aligned} \quad (8)$$

The condition of non-negative character of N_1 and N_2 here means that two-stream solutions exist only when the velocities of these flows satisfy one of the following systems of inequalities

$$(u_1-\tau)^2\geqslant 1,\quad (u_2-\tau)^2\leqslant 1, \quad (9a)$$

$$(u_1-\tau)^2\leqslant 1,\quad (u_2-\tau)^2\geqslant 1. \quad (9b)$$

The force acting on the ion from the electric field can also be expressed in terms of one of the flow velocities

$$\begin{aligned} F(\tau)&=\frac{d\varphi}{d\tau}=\frac{1}{N}\frac{dN}{d\tau}\\ &=2\left(\frac{N_1/N}{(u_1-\tau)^3}+\frac{N_2/N}{(u_2-\tau)^3}\right)\Big/\left[1-3\left(\frac{N_1/N}{(u_1-\tau)^4}+\frac{N_2/N}{(u_2-\tau)^4}\right)\right]\\ &=\frac{2(u_1-\tau)(u_2-\tau)[(u_1-\tau)^2+(u_1-\tau)(u_2-\tau)+(u_2-\tau)^2-1]}{[(u_1-\tau)+(u_2-\tau)][(u_1-\tau)^2(u_2-\tau)^2-3((u_1-\tau)^2+(u_2-\tau)^2-1)]}. \end{aligned} \quad (10)$$

The remarks that we have made allow us, by introducing the new variables $w_\alpha=u_\alpha-\tau$, to reduce the solution of the system (6) to a solution of the single differential equation

$$\frac{dw_1}{dw_2}=\frac{(w_1+w_2)\{w_1^2w_2^2-3(w_1^2+w_2^2-1)\}+2w_2(w_1^2+w_1w_2+w_2^2-1)}{(w_1+w_2)\{w_1^2w_2^2-3(w_1^2+w_2^2-1)\}+2w_1(w_1^2+w_1w_2+w_2^2-1)}. \quad (11)$$

The dependence of τ and N_α on w_2 can be expressed in terms of quadratures by the solution (11). The investigation of the integral curves of the Eq. (11) is conveniently carried out with the use of the phase plane (w_1, w_2). In view of the symmetry of the equations relative to the exchange of variables $w_2\rightleftarrows w_1$, it is sufficient to consider only the halfplane $w_1\geqslant w_2$. The conditions (9) show that the integral curves of interest to us should be located inside the band

$$w_1\geqslant 1,\quad |w_2|\leqslant 1, \quad (12a)$$

$$|w_1|\leqslant 1,\quad w_2\leqslant -1. \quad (12b)$$

The curves on the indicated phase plane, on which the ratio $N_1/N_2=[w_1^2(1-w_2^2)]/[w_2^2(1-w_1^2)]=\text{const}$, are shown in Fig. 1. It is seen from these curves that for the satisfaction of the boundary conditions (5), the integral curves should begin either on the line $w_2=0$ or on the lines $w_1=\pm 1$, and end either on the line $w_1=0$ or on the lines $w_2=\pm 1$. If we also require that the change in the value of the electric potential in the transition through

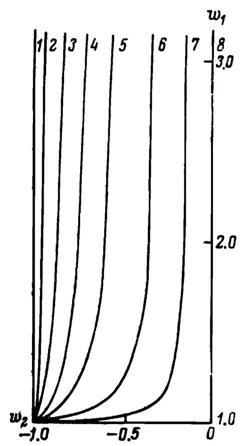

FIG. 1. Lines of constant ratios of the densities in two flows of ions N_1/N_2 = const, $\varepsilon = N_1/N_2$, M is the value of the maximum (in k) increment of growth of the perturbation

Curve	1	2	3	4	5	6	7	8
ε	0	0.1	0.3	0.5	0.7	0.9	0.98	1
$\tilde{M}$	0	0.336	0.350	0.354	0.350	0.336	0.327	0

the region of two-flow current is finite, there is left only one possibility: the beginning of the integral curves on the line $w_2 = 0$ and the end on the line $w_1 = 0$ within the limits of the bands shown in (12).

Several such integral curves are shown in Fig. 2. The arrows indicate the direction along the curve in the case of increase in τ. All these curves pass through the point ($w_1 = 1$, $w_2 = -1$), which is a singular point of the knot type for Eq. (11). At this point, the value of N_1/N_2 for each curve is determined by the angle of entrance of the integrated curve $(dw_1/dw_2)|_{w_1=1, w_2=-1}$. All the integrated curves necessarily intersect the line $w_1^2 + w_1 w_2 + w_2^2 = 1$, shown as dashed in Fig. 2. The quantity $F(\tau)$ goes to zero on this line, which corresponds to the extremum $N(\tau)$.

The solutions of the system (6) corresponding to this integral curve are shown in Fig. 3. Qualitatively, they have the following character. At τ less than some τ_-, there are only particles N_1 with constant density $N = N_1 = N_-$ and velocity $u_1 = u_-$. In the region $\tau_- < \tau < \tau_+$, there is two-velocity motion, in which $u_2 = \tau$ at $\tau = \tau_-$ and $u_1 = \tau$ at $\tau = \tau_+$. The dependence of N on τ in this region is described by a curve with a single maximum. At the point $\tau = \tau_+$, $N_1 = 0$ and at $\tau > \tau_+$ there are only particles N_2 with constant velocity $u_2 = u_+$ and density $N = N_2 = N_+$. The graphs of the functions $N(\tau)$, $u_1(\tau)$, $u_2(\tau)$ are shown for several values of $u_- - u_+$ and N_-/N_+.

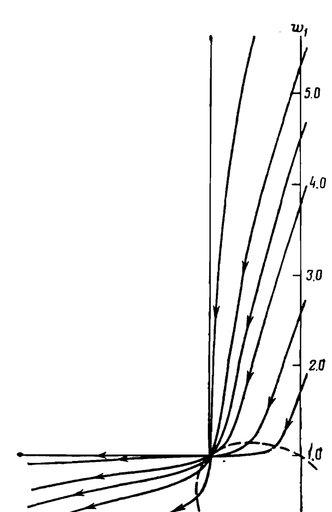

FIG. 2. Integrated curves of Eq. (11) on the phase plane.

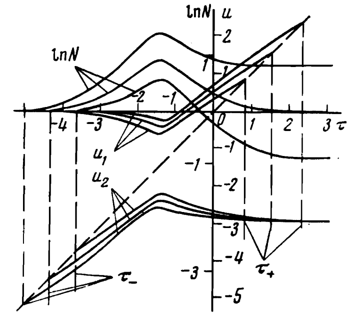

FIG. 3. Distribution of the total density and velocity of two flows of ions in the region of their mutual penetration for smooth solutions of the set of equations (6).

The solutions thus found are determined by the specification of a single arbitrary constant, as an example of which we can choose the value τ_-. This selection corresponds to a choice of the value w_1 on the line $w_2 = 0$ on the phase plane (here and below, we disregard the inessential arbitrariness connected with the possibility of multiplication of N_1 and N_2 by an arbitrary constant, and with the possibility of an arbitary choice of the origin for measuring the velocities). This means that the values of N_-/N_+ and $u_- - u_+$ in solutions of the considered type are not independent, but are connected by some relation. In other words, the values of N_-/N_+ and $u_- - u_+$ for these solutions should lie on some curve (the solid line in Fig. 4). However, since, physically, the jumps in the velocity and density can be given independently, more general solutions should exist which depend on two arbitrary constants.

For construction of such solutions, it is essential that the equality (7), considered as an equation for τ in the case of given values of u_α and N_α, always has two roots, which satisfy the condition $u_1 > \tau > u_2$. Therefore, by choosing any point τ, in the two-velocity region of solutions, it is possible to assume that, in the case $\tau > \tau_1$, N_α and u_α have constant values, equal to their values at $\tau = \tau_1$. Such a plateau in the solution should extend to the value $\tau = \tau_2$, which is the second root of the equality (7) at $N_\alpha = N_\alpha(\tau_1)$ and $u_\alpha = u_\alpha(\tau_1)$. Further, at $\tau > \tau_2$, all the

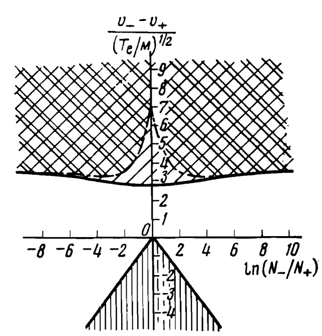

FIG. 4. Region of possible values of the discontinuities of the velocity and density.

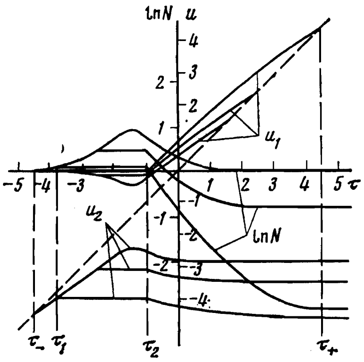

FIG. 5. Distribution of the total density and velocities of two streams of ions in the region of their mutual penetration for solutions of the set of Eqs. (6), including the plateau.

quantities begin to change according to the two-velocity solution.

In the consideration of the picture of the phase density, the introduction of a plateau corresponds to the transition from the point (w_1^*, w_2^*), achieved in the motion along the integral curve at $\tau = \tau_1$, to the point $(w_1^* - \Delta, w_2^* - \Delta)$ on another integral curve with the same value of N_1/N_2 as at the point $(w_1^* \cdot w_2^*)$. In this transition, the quantity Δ is determined as a root of the equation

$$\Delta^3-2\Delta^2(w_1^*+w_2^*)+\Delta(w_1^{*2}+w_2^{*2}+4w_1^*w_2^*-1) - \frac{2w_1^*w_2^*}{w_1^*+w_2^*}(w_1^{*2}+w_2^{*2}+w_1^*w_2^*-1)=0, \tag{13}$$

which satisfies the condition $0 < \Delta < w_1^*$. Motion along the new integral curve begins with the value $\tau = \tau_2 = \tau_1 + \Delta$.

We note that in order that the described transition be possible, τ_1 must lie to the left of the maximum density N for the two-velocity solution. Therefore, in solutions with a plateau, the maximum value of N is achieved on the plateau itself. We note that at the points τ_1 and τ_2, the solution has a weak discontinuity—at these points, the continuity of the first derivatives of N_α and u_α is disrupted. Several solutions with a plateau are shown in Fig. 5.

In view of the arbitrariness of τ_1, the solution with the plateau is characterized by two essential arbitrary constants (τ_1 and τ_+), which must be so chosen that N_-/N_+ and $u_- - u_+$ are equal to the specified initial conditions at the discontinuity. However, it is easy to show that the possible values of N_-/N_+ and $u_- - u_+$ corresponding plane lie above the curve obtained for the smooth solutions. Thus the described hydrodynamic method of decay of the initial discontinuity can exist only for a jump, the initial conditions on which lie in the obliquely shaded regions on Fig. 4. In particular, it is impossible to construct a solution of the demonstrated form in which both regions of the plasma would be quiescent relative to one another at the initial moment. The positiveness of $u_- - u_+$ for the obtained solutions means that both regions of the plasma move toward one another.

2. *Diverging plasma flows.* If $u_- - u_+$ was negative at the initial instant, i.e., both regions diverged, then the ions located to the left and to the right, cannot be in one region of the halfspace, and two-stream motion does not arise. In this case, for the description of the motion of the system, we must use the equations of single-velocity hydrodynamics. In place of the system (6) and the condition (7), we how have, respectively, the system

$$(u-\tau)\frac{du}{d\tau} = -\frac{d\varphi}{d\tau}, \quad (u-\tau)\frac{dN}{d\tau} + N\frac{du}{d\tau} = 0, \quad N(\tau) = N_0 e^{\varphi(\tau)} \tag{14}$$

and the condition

$$(u-\tau)^2 = 1. \tag{15}$$

The system (14), (15) has two forms of solutions:

$$u=\tau+1, \quad N=N_0e^{-\tau}, \tag{16a}$$
$$u=\tau-1, \quad N=N_0e^{\tau}. \tag{16b}$$

The solution of the problem with boundary conditions of the form (5) can belong to one of two types, described below:

$$\begin{array}{lll} u=u_-, & N=N_-, & \tau<\tau_-, \\ u=\tau+1, & N=N_0e^{-\tau}, & \tau_-<\tau<\tau_+, \\ u=u_+, & N=N_+, & \tau>\tau_+. \end{array} \tag{17a}$$

$$\begin{array}{lll} u=u_-, & N=N_-, & \tau<\tau_-, \\ u=\tau-1, & N=N_0e^{\tau}, & \tau_-<\tau<\tau_+, \\ u=u_+, & N=N_+, & \tau>\tau_+. \end{array} \tag{17b}$$

Solutions of type (17a) and (17b) include two weak discontinuities and arise if the given initial conditions on the jump satisfy the equations

$$\ln\frac{N_-}{N_+} = -(u_- - u_+), \quad u_- - u_+ < 0; \tag{18a}$$

$$\ln\frac{N_-}{N_+} = (u_- - u_+), \quad u_- - u_+ < 0, \tag{18b}$$

respectively. There are also solutions that are a splicing of a solution of type (17b), a plateau, and a solution of the type (17a), having four weak discontinuities. Such solutions arise if the initial conditions at the jump fall into a region denoted by the vertically shaded area in Fig. 4. We note that precisely these regions correspond to jumps for which the Riemann solution in gasdynamics with isothermal sound velocity gives two diverging rarefaction waves.

3. DECAY OF THE INITIAL DISCONTINUITY WITH ADIABATIC COLLISIONLESS ELECTRON CAPTURE

The solutions constructed above for the case of moving towards each other plasma regions are characterized by the presence of a region of positive spike in the electric potential. It is evident that such a region is a potential well for electrons; the motion of the electrons trapped in such a well become finite. In the presence of trapping of particles and in the absence of collisions, they no longer have a Boltzmann distribution function. As was shown earlier,[5,6] the electron distribution functions (in the case in which the potential well changes sufficiently slowly with time) is determined from the condition of the adiabaticity of the trapping of the electrons by the field of the wave, and the distribution of concentration of electrons in the potential will have the form:

$$N_e(\varphi)=N_0(e^{\varphi}\operatorname{erfc}\varphi^{1/2}+2\varphi^{1/2}/\pi^{1/2}),\quad \varphi>0, \tag{19}$$

where N_0 is the concentration of electrons at $\varphi=0$. The first term in this formula corresponds to electrons that carry out infinite motion, and the second, to the trapped electrons.

Obtaining from this the value of $d\varphi/d\xi$, we have

$$\frac{d\varphi}{d\xi}=\frac{d\varphi}{dN_e}\frac{dN}{d\xi}=\frac{1}{N_{inf}}\frac{dN}{d\xi}=\frac{N}{N_{inf}}\frac{1}{N}\frac{dN}{d\xi}. \tag{20}$$

The use of this expression for the electric field in the set of equations (1) again allows us to obtain equations of the type of the equations of hydrodynamics, where the role of the gasdynamic pressure is played by the pressure of the electron gas. It is seen from this comparison that $c=(N/N_{inf})^{1/2}$ has the meaning of the local sound velocity. For a Boltzmann distribution of the electrons $c(N)=1$. (Or, in dimensional units, $c=(T_e/M)^{1/2}$—the isothermal ion sound velocity.)

In the presence of captured particles, the temperature of the electron gas is no longer constant, but changes in the region $\varphi>0$ according to the law

$$\frac{T_e(\varphi)}{T_{e0}}=\frac{2}{3}\varphi+\frac{N_{e0}}{N_e(\varphi)}\left\{\left(1-\frac{2}{3}\varphi\right)e_{\varphi}\operatorname{erfc}\varphi^{1/2}+2\varphi^{1/2}/\pi^{1/2}\right\}, \tag{21}$$

and its pressure will be determined in this region by the expression

$$\frac{p_e(\varphi)}{p_{e0}}=e^{\varphi}\operatorname{erfc}\varphi^{1/2}+(2\varphi^{1/2}/\pi^{1/2})\left(1+\frac{2}{3}\varphi\right), \tag{22}$$

where T_{e0} and p_{e0} are the temperature and pressure of the electron gas at $\varphi=0$. It is seen from the expressions (21), (22) that the state of the electron gas at $0<\varphi\ll 1$ (as also for $\varphi\leq 0$) can again be described by the isothermal equation of state, since at $\varphi\gg 1$ we can use the polytropic equation of state with the polytropic exponent equal to 3.

The solution of the set of equations (6) with account of (19) and (20) is carried out by a method similar to that developed in Sec. 2.1. After transformation to the variables $u_\alpha=u_\alpha/c-\tau$, we obtain the set of equations

$$\frac{d\mu_1}{d\mu_2}=\frac{[(\mu_1+\mu_2)\{\mu_1^2\mu_2^2-3(\mu_1^2+\mu_2^2-1)\}+2\mu_2(\mu_1^2+\mu_1\mu_2+\mu_2^2-1)]+G(\varphi)\mu_1^2\mu_2(\mu_1^2-1)}{[(\mu_1+\mu_2)\{\mu_1^2\mu_2^2-3(\mu_1^2+\mu_2^2-1)\}+2\mu_1(\mu_1^2+\mu_1\mu_2+\mu_2^2-1)]+G(\varphi)\mu_1\mu_2^2(\mu_1^2-1)}, \tag{23a}$$

$$\frac{d\varphi}{d\mu_2}=-\frac{2\mu_1\mu_2c^2(\varphi)(\mu_1^2+\mu_1\mu_2+\mu_2^2-1)}{2\mu_1(\mu_1^2+\mu_1\mu_2+\mu_2^2-1)+(\mu_1+\mu_2)\{\mu_1^2\mu_2^2-3(\mu_1^2+\mu_2^2-1)\}+G(\varphi)\mu_1\mu_2^2(\mu_2^2-1)} \tag{23b}$$

$$G(\varphi)=\begin{cases} d(c^2(\varphi))/d\varphi, & \varphi>0,\\ 0, & \varphi\leq 0,\end{cases} \tag{24}$$

with the additional condition $\varphi=0$ at $\mu_2=0$. The dependence of τ on μ_2 is expressed in quadratures through the solution (23), and N_α are found from equations of the type (8), where μ_α plays the role of $u_\alpha-\tau$.

The solutions thus obtained do not differ qualitatively from those described in Sec. 2.1. The solutions with a plateau are constructed as described earlier.

The region of values of N_-/N_+ and u_--u_+ achievable for the given solutions at the jump is shown in Fig. 4 by the doubly-hatched area. The lower boundary of this region, shown by the dot-dash curve, approaches the solid line at $|n(N_-/N_+)|\gg 1$; this solid curve represents the similar boundary for solutions with Boltzmann electrons. The reason for this approach is the fact that the jumps with $|n(N_-/N_+)|\gg 1$ include only a small part of the region of the positive potential, in which there arises only capture of electrons and the difference, associated with the capture of the "trapped" solutions from the solutions with a Boltzmann electron distribution.

4. STABILITY OF TWO-STREAM SOLUTIONS

The motions of the plasma described above, that arise from the state with oppositely directed flows, can be regarded as the motion of a beam in the plasma, which can naturally lead to instability. The possibility of the formation of an ion-ion beam instability for the solutions constructed in Secs. 2.1 and 3 is investigated below.

In view of the fact that the arising disturbance can not propagate along the velocity of the flows, we must write down the system of equations of the plasma motion with account of other components of the velocity. Such a generalization of the system (6) has the form

$$\begin{gathered}\frac{\partial u_\alpha}{\partial t}+u_\alpha\frac{\partial u_\alpha}{\partial\xi}+v_\alpha\frac{\partial u_\alpha}{\partial y}+c^2(N)\frac{1}{N}\frac{\partial N}{\partial\xi}=0,\\ \frac{\partial v_\alpha}{\partial t}+u_\alpha\frac{\partial v_\alpha}{\partial\xi}+v_\alpha\frac{\partial v_\alpha}{\partial y}+c^2(N)\frac{1}{N}\frac{\partial N}{\partial y}=0,\\ \frac{\partial N_\alpha}{\partial t}+N_\alpha\left(\frac{\partial u_\alpha}{\partial\xi}+\frac{\partial v_\alpha}{\partial y}\right)+u_\alpha\frac{\partial N_\alpha}{\partial\xi}+v_\alpha\frac{\partial N_\alpha}{\partial y}=0,\\ N=N_1+N_2,\quad \alpha=1,\ 2.\end{gathered} \tag{25}$$

In the unperturbed motion, there were the self-similar solutions $u_\alpha=u_{a\alpha}(\xi/t)$, $v_\alpha=0$, $N_\alpha=N_{a\alpha}(\xi,t)$, and the perturbations u'_α, v'_α, N'_α were assumed to be dependent on two coordinates and the time. Assuming the perturbation to be small ($|f'|\ll f$) and the condition of quasiclassical approximation $|\partial f'/\partial\xi|\gg\partial f/\xi$ to be satisfied, we obtain, in first approximation,

$$\begin{gathered}\frac{\partial u_\alpha'}{\partial t}+u_{a\alpha}\frac{\partial u_\alpha'}{\partial\xi}+c^2(N_a)\frac{1}{N_a}\frac{\partial N'}{\partial\xi}=0,\\ \frac{\partial v_\alpha'}{\partial t}+u_{a\alpha}\frac{\partial v_\alpha'}{\partial\xi}+c^2(N_a)\frac{1}{N_a}\frac{\partial N'}{\partial y}=0,\\ \frac{\partial N_\alpha'}{\partial t}+N_{a\alpha}\left(\frac{\partial u_\alpha'}{\partial\xi}+\frac{\partial v_\alpha'}{\partial y}\right)+u_{a\alpha}\frac{\partial N_\alpha'}{\partial\xi}=0.\end{gathered} \tag{26}$$

We seek solutions (26) for the perturbations f' in the form

$$f'(\xi,y,t)=f_0'\exp[i(k\xi+ry-\omega t)].$$

We obtain here

$$\begin{gathered}u_{\alpha0}'i(ku_{a\alpha}-\omega)=-c^2(N_a)N_0'ik/N_a,\\ v_{\alpha0}'i(ku_{a\alpha}-\omega)=-c^2(N_a)N_0'ir/N_a,\\ N_{\alpha0}i(ku_{a\alpha}-\omega)+N_{a\alpha}i(ku_{\alpha0}'+rv_{\alpha0}')=0.\end{gathered} \tag{27}$$

The system (27) is written down under the assumption that the perturbations preserve the quasineutrality of the plasma. To take into account possible departures from the quasineutrality, the substitution

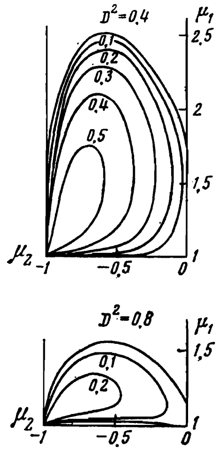

FIG. 6. Level lines of the surface $\operatorname{Im}\gamma = \text{const}$. The value of $\operatorname{Im}\gamma$ is shown on the curves. The outer unmarked curves are the boundaries of the regions of possible growth of the perturbation.

$$ik \to \frac{ik}{1+a^2(k^2+r^2)}, \quad ir \to \frac{ir}{1+a^2(k^2+r^2)}, \tag{28}$$

must be carried out in Eqs. (27), where $a=(M/4\pi N_a e^2)^{1/2}$ is the Debye radius.

With account of the remarks that have been made, the characteristic equation for the system (27) will be the following:

$$(ku_{a1}-\omega)(ku_{a2}-\omega)\left\{(ku_{a1}-\omega)^2(ku_{a2}-\omega)^2 - \frac{c^2(N_a)(r^2+k^2)}{N_a[1+a^2(k^2+r^2)]}[N_{a2}(ku_{a1}-\omega)^2+N_{a1}(ku_{a2}-\omega)^2]\right\}=0. \tag{29}$$

The instability of the motion, which is of interest to us, arises if there exists a root ω^* of Eq. (29) which has a positive imaginary part $\Omega=\operatorname{Im}\omega^*>0$; therefore, the analysis of the roots of Eq. (29) is equivalent to the analysis of the roots of the equation

$$\frac{N_{a1}/N_a}{(u_{a1}/c-\omega/kc)^2}+\frac{N_{a2}/N_a}{(u_{a2}/c-\omega/kc)^2}=\cos^2\theta+(ka)^2\equiv D^2. \tag{30}$$

The angle θ, which enters in Eq. (30), is the angle between the direction of propagation of the disturbance and the direction of the unperturbed velocity of flow of the plasma, so that $\cos^2\theta = k^2/(r^2+k^2)$.

As was to be expected, the usual dispersion relation is obtained for the two-stream plasma, which consists of cold ions and hot electrons. However, the flux densities are not independent, and are related in our solutions with the relative velocity of their motion by equations of the type (8). This fact turns out to be very important.

It is convenient to represent the results of the investigation on the phase plane $\mu_1 = u_1/c-\tau$, $\mu_2=u_2/c-\tau$ introduced earlier. The imaginary parts of the roots ω/k of Eq. (30) coincide with the imaginary parts of the roots of the equation for γ (γ differs from ω/k only by the selection of the reference system for the velocities)

$$\gamma^4-2(\mu_1+\mu_2)\gamma^3+\left\{[(\mu_1+\mu_2)^2+2\mu_1\mu_2]-\frac{1}{D^2}\right\}\gamma^2 -2\mu_1\mu_2(\mu_1+\mu_2)\left[1-\frac{1}{D^2}\frac{1+\mu_1\mu_2}{(\mu_1+\mu_2)^2}\right]\gamma+\mu_1^2\mu_2^2\left(1-\frac{1}{D^2}\right)=0. \tag{31}$$

It is easy to show that at values $D\geq 1$, Eq. (31) has only real roots. This means that under the condition $\cos^2\theta + (ka)^2 \geq 1$, the constructed self-similar motion is stable. In particular, it is stable at any k relative to increase in the perturbations which are propagating along the x axis.

At values $D<1$, a pair of complex conjugate roots appear. The lines of the surface level $\operatorname{Im}\gamma=\text{const}$ are shown in Fig. 6 for several values of D in this region. At a given value $\delta=\mu_1-\mu_2$ and relative concentrations N_{1a}/N_a and N_{2a}/N_a the satisfaction of the condition

$$D<D_{cr}=(1/\delta)\left[(N_{1a}/N_a)^{1/3}+(N_{2a}/N_a)^{1/3}\right]^{3/2},$$

is necessary for the appearance of the complex root, i.e., the perturbations are increasing with the wave vector k, which satisfies the inequality

$$ka \leq k_{cr}a = (D_{cr}^2-\cos^2\theta)^{1/2}. \tag{32}$$

The level lines of the surface $D_{cr}(\mu_1, \mu_2)=\text{const}$ are shown in Fig. 7.

The increment of the growth of the perturbation in the given point of the solution is a function of k and θ. The maximum value of the increment is achieved at $\theta=\pi/2$ and a value $k^*<k_{cr}$, which satisfies the equation

$$\frac{\partial}{\partial k}(k\operatorname{Im}\gamma(k))=0. \tag{33}$$

It can be shown that this maximum value is a function only of the ratio N_{1a}/N_{2a} at the given point of the solution. The values of $M=k^*\operatorname{Im}\gamma(k^*)$ calculated from (32), (33) are given on the curves of Fig. 1. It is seen that the greatest increment of growth of the perturbation on each solution is achieved at the point of equal densities of flows of the ions ($N_{1a}=N_{2a}=N_a/2$) and amounts to $\tilde{M}=8^{-1/2}=0.354$, i.e., the increase in the perturbation by a factor e takes place within 2.8 periods of the plasma oscillations.

It must be noted, however, that our consideration is entirely applicable only for angles that are not too close to $\pi/2$, and the given value of $\tilde{M}$ is only the upper boundary

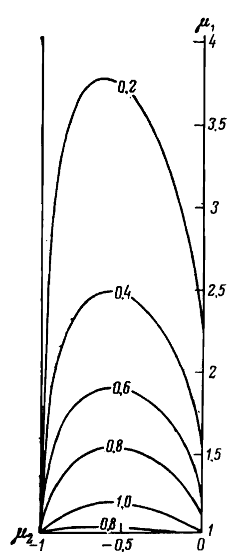

FIG. 7. The boundaries of the regions of instability on the phase plane for different values of the parameter D^2 (shown in the curves). The region of instability lies between the curves corresponding to the given D. (The lower curves with $D^2 < 0.8$ are close to the abscissa.)

of the maximum increments.

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[1)]A short exposition of the results for this case is contained in the work of Pitaevskiĭ and the author.[3] We note that a different normalization of the flow velocities is used in this case.

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Frequency of inelastic collisions in plasma

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An analysis is given of the influence of polarization plasma effects on the frequency of inelastic atomic transitions induced by the combined electron and ion fields. The inelastic collision frequency W is expressed in terms of the Born matrix element and the longitudinal component of the plasma permittivity at the frequency $\omega_0 = \Delta E/h$ of the atomic transition (ΔE is the transition energy). For low-density plasma, the frequency W is equal to $N\langle v\sigma_{01}\rangle$, where σ_{01} is the cross section for the excitation of the particular transition, averaged over the velocity distribution function of the exciting charged particles in the plasma, and N is the density of the particles. The dependence of collision frequency on plasma temperature and density is investigated in detail for optically allowed transitions with a small resonance defect ω_0. It is shown that polarization effects play an important role at temperatures for which the main contribution to the inelastic collision frequency is due to electron-atom collisions. Whilst inelastic transitions in the plasma are largely due to interactions with ions, polarization effects are not appreciable, even at high densities that are close to those of a solid body. Conditions are formulated for the validity of first-order perturbation theory as applied to the frequency of inelastic transitions with a small resonance defect.

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The polarization properties of plasma modify the character of interactions between charged particles. The spectroscopic consequences of this effect have been discussed in connection with the emission of forbidden spectral lines[1] and the theory of broadening of spectral lines in plasma.[2] By polarization properties, we understand two physically clear effects, namely, the screening of Coulomb forces and the interaction between charged particles and plasma oscillations. It is obvious that both these effects should have an important influence on the rate of relaxation of excited states in dense plasma. The present paper is concerned with this question. By dense plasma, we shall understand plasma in which the Langmuir frequency exceeds the frequency of the atomic transition. This condition is realized, for example, for transitions between highly excited atomic states and for a number of transitions between the energy levels of multiply-charged ions in dense laser plasma.

1. FORMULATION OF THE PROBLEM

In low-density plasma, the frequency of inelastic transitions between atomic levels 0 and 1 due to collisions with electrons is given by

$$\nu = N\langle v\sigma_{01}\rangle = \int v\sigma_{01}(v)F(\mathbf{v})\,d\mathbf{v}, \tag{1}$$

where $F(\mathbf{v})$ is the electron velocity distribution function, σ_{01} is the cross section for the excitation of the 0–1 transition, and N is the electron density.

In dense plasma, the interaction between an atom and electrons can no longer be looked upon as the result of successive independent collisions. An incident electron interacts with the ambient electrons and ions, and induces a dipole moment in the plasma. This means that the resultant field acting on the atom is made up of fields produced by the electron and by the dipole moment induced by it in the plasma.

Let us therefore consider a more general formulation of the problem. Suppose that the atom is located in a random field $V(\mathbf{r}, t)$ due to all the charged particles in the plasma. In first-order perturbation theory, the probability of a transition from state 0 to state 1 at time t is then given by the expression[1)]