this electron-phonon interaction which gives rise to the effect of superconductivity. The superconducting state occurs only in metals for which the electron-phonon interaction is sufficiently strong. A strong electron-phonon interaction, giving rise to a large resistivity in the normal state, contributes to the formation of the nonresistive superconducting state. This analogy is verified by the formal relation between the nonlinear Schrödinger equation (19) and the Ginzburg-Landau equation[7] for a one-dimensional model with no magnetic fields or currents, well-known in the theory of superconductivity ([6], Chap. VI).

Thus, excitations of a molecular chain in the form of solitons are very stable with respect to retaining the shape, magnitude, and size of the region involved in the excitation. This fact has been used by one of us[4] to explain the high efficiency of the transfer of energy released in the hydrolysis of ATP molecules in living organisms along the α-helical protein molecule and to explain the contraction mechanism of transversely striated animal muscles at the molecular level.[8]

1)The deformation of a molecular chain during intramolecular excitations (without taking their motion into account) was first treated by Rashba[2] (see also[3]).

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Spin susceptibility of the pseudoisotropic (*B*) phase of superfluid ^{3}He in the acoustic limit (spin waves)

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From a theory of the Larkin–Migdal type the spin-susceptibility tensor at zero temperature is calculated in the acoustic limit and in the collisionless regime. No restrictions are imposed on the effective interaction of the quasi-particles in the particle–hole channel. With regard to the effective interaction in the particle–particle channel it is assumed that only the $l = 0$ and $l = 1$ amplitudes can be close to each other. It is shown that the spin susceptibility always has two poles, corresponding to the frequencies of spin waves of different polarization. It is shown that the spin-wave frequencies depend on the Landau exchange amplitudes for $0 \le l \le 3$; the form of this dependence is found. If in the effective interaction in the particle–particle channel the $l = 1$ amplitude is greater than the $l = 0$ amplitude and these amplitudes are close to each other, the spin susceptibility also has a pole corresponding to excitations with a small gap (i.e., much smaller than the gap in the two-particle spectrum).

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The purpose of the present article is to consider the spin waves for systems with Balian-Werthamer[1] (BW)[2)] pairing without any restrictions imposed on the effective interaction of the quasi-particles in the particle-particle channel, but in the collisionless regime and at zero temperature. Interest in this topic has arisen again following the identification of the B phase of superfluid ^{3}He[3,4] with the BW state, as originally proposed by Anderson and Brinkman.[5] This identification gave rise to a number of objections, although the proposal of spin-singlet D-pairing[6] is also in disagreement with recent measurements[7,8] of the spin susceptibility, and serious objections against spin-triplet F-pairing[9] have been put forward in a paper by Mermin.[10] Therefore, the BW state is, as yet, the best state for the description of the B phase of superfluid ^{3}He.

Collective excitations of the BW state were first considered by Vdovin[2] for weak coupling. We did not impose this restriction in our previous papers,[11,12] in which, for BW pairing, we developed theories analogous to those of Larkin and Migdal[13] and Larkin,[14] which had been proposed earlier for systems with isotropic S-pairing. We[11] solved the vertex-function equations describing the scalar and vector vertices in the acoustic limit ($|\omega|$, $kv \ll \Delta$, where ω and $\mathbf{k}$ are the frequency and wave vector, v is the speed of the quasi-particles, and Δ is the energy gap). In addition, we showed[11] that all these vertex functions have a single pole at the zero-sound frequency, which coincides with the first-sound frequency obtained using the thermodynamic formulas for a Fermi liquid.[15] Our results have recently been generalized by Maki[16] to nonzero temperatures. On the other hand, our article[11] contains the erroneous statement that the spin vertex has no poles in the acoustic limit, i.e., that propagation of spin waves is impossible in a system with BW pairing.

The results of Vdovin[2] for spin waves were generalized by Combescot[17] for systems with an effective quasi-particle interaction independent of angle. Our equations for the vertex functions[11] were also derived by Gongadze, Gurgenishvili and Kharadze[18] with the tacit assumption that the spin-antisymmetric part of the effective quasi-particle interaction in the particle-particle channel vanishes. These authors solved the equations for the spin vertices by assuming, like Combescot, that the effective interaction in the particle-hole channel has no angular dependence. Although the expressions obtained by them for the spin-wave frequencies coincide with Combescot's expressions, their solutions for the vertex functions are incorrect. This will become clear from our subsequent calculations, although the erroneous character of the results of the paper by Gongadze, Gurgenishvili and Kharadze[18] can be easily understood. The equations obtained by them are a system of inhomogeneous linear equations with a degenerate matrix kernel. Such a system is equivalent to a system of inhomogeneous linear algebraic equations. Nevertheless, according to the results of their paper,[18] the solution of this system describing the response to an external magnetic field is not unique, which is physically absurd.

According to Leggett and Rice,[19] Leggett[20] and Corruccini *et al.*,[21] the $l=1$ spin-exchange Landau amplitude is a very small quantity. This is the chief physical argument for neglecting all the Landau exchange amplitudes except that for $l=0$. Nevertheless, experimental[6] and theoretical[22] estimates show that the state of affairs is greatly different from that indicated above. Moreover, the Landau amplitudes for ^{3}He should satisfy a sum rule. On the other hand, only a general solution of the problem can give us the possibility of checking whether the stability conditions[23] ensure the existence of poles of the response functions (i.e., the existence of corresponding elementary excitations). This is particularly interesting since spin waves have not yet been discovered in the B phase of superfluid ^{3}He.

In our paper[11] and in the paper by Larkin and Migdal,[13] the effective interaction of the quasi-particles in the particle-hole channel, which coincides with the effective interaction of quasi-particles for a normal system, had a completely general form. On the other hand, the effective interaction in the particle-particle channel was restricted entirely to the pairing channel, i.e., to $l=0$ in[13] and to $l=1$ in[11]. We remark that, according to these papers, this interaction can be represented by two functions $f_\varepsilon^\xi(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ ($\varepsilon=\pm1$) such that

$$f_\varepsilon^\xi(\hat{\mathbf{p}}\hat{\mathbf{p}}')=\sum_{l=0}^{\infty}(2l+1)f_lP_l(\hat{\mathbf{p}}\hat{\mathbf{p}}') \tag{1}$$

with summation over even l for $\varepsilon=1$ and over odd l for $\varepsilon=-1$. For the dimensionless interaction, the Legendre amplitudes f_l of this interaction are equal to $[\ln(2\xi/r_1)]^{-1}$, where ξ denotes the energy cutoff and the r_l are certain non-negative constants (cf.[13]). In formula (1), $\hat{\mathbf{p}}$ denotes the unit vector parallel to the vector $\mathbf{p}$. In the pairing channel, i.e., $l=0$ for S-pairing and $l=1$ for BW pairing, $r_l=\Delta$; in the following we shall denote l for the pairing channel by l_0. Using the equations for the vertex functions[11,13], it is easy to verify that if

$$\min|\ln(\Delta/r_l)|,\quad l\neq l_0,$$

is of order unity or greater, the harmonics with $l\neq l_0$ can be neglected in the acoustic limit. On the other hand, if

$$\min|\ln(\Delta/r_l)|\ll 1,\quad l\neq l_0,$$

and the harmonic satisfying this inequality occurs in the equations for the vertex functions, the known solutions[11,13] cease to be valid if the inequality

$$|\omega|,kv\ll\Delta[\min|\ln(\Delta/r_l)|]^{1/2},\quad l\neq l_0$$

is not fulfilled.

We note that it is only in the equations for the spin vertices of systems with BW pairing that both functions $f_\varepsilon^\xi(\varepsilon=\pm1)$ appear simultaneously (cf.[11]). Moreover, in the equation for the spin vertex of a system with S-pairing, $\ln(2\xi/\Delta)$ and $f_{-1}^\xi(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ enter simultaneously. Therefore, for the scalar and vector vertices of systems with BCS and BW pairing, Legendre amplitudes with minimum difference $\Delta l=2$ are encountered. Above a certain l, the amplitudes f_l should fall off as a result of the action of the centrifugal force.[13] Hence, it would be surprising if the condition $|\ln(\Delta/r_l)|\ll1$ were fulfilled for $l=2, 4, 6, \ldots$ (for BCS pairing) or for $l=3, 5, 7, \ldots$ (for BW pairing). On the other hand, the condition $|\ln(\Delta/r_0)|\ll1$ seems very natural, since the tendency for f_l to decrease with increase of l acts here against the tendency for f_l to be a maximum in the pairing channel. Therefore, it is interesting to solve the equations for the spin vertices of systems with BW pairing by assuming that $|\ln(\Delta/r_1)|$ can be much smaller than unity only for $l=0$. If the inequality $|\ln(\Delta/r_0)|\ll1$ is not fulfilled, we obtain the solution of the equations with an excess of accuracy, and this can be avoided by taking a simple limit in the expression for the solution.

1. DISCUSSION OF THE BASIC EQUATIONS

We shall not derive anew the equations for the spin vertices and expressions for the spin susceptibility obtained by us earlier[11] by means of methods developed by Larkin and Migdal.[13] The derivation of the equivalent equations for the spin vertices and of the expression for the spin susceptibility by the methods worked out by Larkin[14] can be found in the Appendix.

We shall choose the usual phase for the BW Δ-matrix, i.e., $\hat{\Delta}=\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}i\sigma^y\Delta$. (Here a hat over a character denotes a spin matrix, except in the case of bold-face characters, when it denotes the unit vector parallel to the vector under the hat.) With this choice of $\hat{\Delta}$ the anomalous vertices $\hat{\tau}_1$ and $\hat{\tau}_2$ (i.e., with two incoming and two outgoing particle lines) can be expressed by the formula

$$\hat{\tau}_1=(\hat{\tau}+\lambda)i\sigma^y,\quad \hat{\tau}_2=i\sigma^y(\hat{\tau}-\lambda),$$

where $\hat{\tau}$ is a matrix with zero trace. The equations for the normal vertex $\hat{J}_a$ and the vertices $\hat{\tau}_a$ and λ_a will have the form (cf. [11])

$$\hat{J}_a(\hat{\mathbf{p}})=\hat{J}_a^{\omega}+\langle B(\hat{\mathbf{p}}\hat{\mathbf{p}}')\{L\hat{J}_a(\hat{\mathbf{p}}')+O(\sigma\hat{\mathbf{p}}')\hat{J}_a(-\hat{\mathbf{p}}')(\sigma\hat{\mathbf{p}}') - M[\hat{\tau}_b(\hat{\mathbf{p}}'),\sigma\hat{\mathbf{p}}']_- - 2M\lambda_a(\hat{\mathbf{p}}')\sigma\hat{\mathbf{p}}'\}\rangle_{\mathbf{p}'},\tag{2}$$

$$\hat{\tau}_a(\hat{\mathbf{p}})=\langle f_{-1}^{\xi}(\hat{\mathbf{p}}\hat{\mathbf{p}}')\{[N+\ln(2\xi/\Delta)]\hat{\tau}_a(\hat{\mathbf{p}}')-O(\sigma\hat{\mathbf{p}}')\hat{\tau}_a(\hat{\mathbf{p}}')(\sigma\hat{\mathbf{p}}') + M[\hat{J}_a(\hat{\mathbf{p}}'),\sigma\hat{\mathbf{p}}']_-\}\rangle_{\mathbf{p}'},\tag{3}$$

$$\lambda_a(\hat{\mathbf{p}})=\langle f_1^{\xi}(\hat{\mathbf{p}}\hat{\mathbf{p}}')\{[N+O+\ln(2\xi/\Delta)]\lambda_a(\hat{\mathbf{p}}')+M[\hat{J}_a(\hat{\mathbf{p}}'),\sigma\hat{\mathbf{p}}']_+\}\rangle_{\mathbf{p}'}.\tag{4}$$

Here $B(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ denotes the spin-exchange part of the dimensionless effective interaction in the particle-hole channel, $\langle\cdots\rangle_{\mathbf{p}'}$ denotes averaging over the solid angles associated with the vector $\mathbf{p}'$, and σ is the pseudovector of the Pauli matrices.

In the accoustic limit it is sufficient to put

$$O=-L={}^1/_2,\quad 2M=-\omega-\mathbf{kv}',\quad N=-{}^1/_2+\omega^2-(\mathbf{kv}')^2,$$

where $\mathbf{v}'=v\hat{\mathbf{p}}'$, and ω and kv are measured in units of 2Δ; the definitions of these functions are given in the papers by Larkin and Migdal[13,14] (cf. also[11]). We have chosen the vertex functions here in such a way that the $\hat{J}_{a,\lambda}^{\omega}$ which in a system without pairing are the vertices $\hat{J}_a$, are equal to σ_a when $|\omega|\gg kv$. For such vertex functions, the paramagnetic-susceptibility tensor is given by the formula

$$\chi_{ab}=-{}^1/_2\mu_B^2\nu\,\mathrm{Sp}\,\langle\sigma_a\{L\hat{J}_b+O(\sigma\hat{\mathbf{p}})\hat{J}_b(\sigma\hat{\mathbf{p}})-M[\hat{\tau}_b,\sigma\hat{\mathbf{p}}]_- - 2M\lambda_b\sigma\hat{\mathbf{p}}\}\rangle_{\mathbf{p}},\tag{5}$$

where μ_B is the Bohr magneton, ν is the density of states at the Fermi surface, and the trace is taken over the spin indices. The kernel B will be determined here by its Legendre amplitudes (i.e., by the Landau amplitudes). In order to avoid over-complicated denominators, these have a form such that

$$B(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')=\sum_{l=0}^{\infty}(2l+1)b_lP_l(\hat{\mathbf{p}}\hat{\mathbf{p}}').\tag{6}$$

We note that the new b_l are equal to $b_l/(2l+1)$ in our old notation[11] and $Z_1/4(2l+1)$ in Leggett's notation.[24]

In the following we shall restrict ourselves to

$$f_{-1}^{\xi}(\hat{\mathbf{p}}\hat{\mathbf{p}}')=3\hat{\mathbf{p}}\hat{\mathbf{p}}'[\ln(2\xi/\Delta)]^{-1},\quad f_1^{\xi}(\hat{\mathbf{p}}\hat{\mathbf{p}}')=[\ln(2\xi/r)]^{-1},$$

i.e., we shall neglect all the other Legendre amplitudes of the interaction in this channel. We note here that all the earlier concrete calculations were carried out under stronger restrictions. On the other hand, we shall not impose any restrictions on $B(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$.

As follows from (2) and (3), $\hat{J}_a$ and $\hat{\tau}_a$ are the a-components of a pseudo-vector and vector with vanishing traces. As a result of our restriction on the quantity f_{-1}^{ξ}, the matrix $\hat{\tau}_a$ should be a linear function of the vector $\hat{\mathbf{p}}$, i.e., $\hat{\tau}_a=\tau_{abc}\sigma_b\hat{p}_c$, where the third-rank pseudotensor τ_{abc} depends only on ω and $\mathbf{k}$. We note that summation over repeated vector indices is implied throughout. We can also write $\hat{J}_a=J_{ab}\sigma_b$, where J_{ab} is a tensor depending on $\hat{\mathbf{p}}$, $\mathbf{k}$ and ω. The most general form of such a tensor can be written as

$$J_{ab}=A\delta_{ab}+B\hat{p}_a\hat{p}_b+C\hat{k}_a\hat{p}_b+D\hat{p}_a\hat{k}_b+E\hat{k}_a\hat{k}_b,\tag{7}$$

where the functions $A,\ldots,E$ depend only on $\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}\equiv w$, k and ω. The most general p-independent third-rank pseudotensor can be represented in the form

$$\tau_{abc}=iR\varepsilon_{abc}+i(X-R)\varepsilon_{dbc}\hat{k}_a\hat{k}_d+iY\varepsilon_{adc}\hat{k}_b\hat{k}_d+iZ\varepsilon_{abd}\hat{k}_c\hat{k}_d,\tag{8}$$

where ε_{abc} is the Levi-Civita pseudotensor. As a result of the identity

$$\varepsilon_{abc}=\hat{k}_d(\hat{k}_a\varepsilon_{dbc}+\hat{k}_b\varepsilon_{adc}+\hat{k}_c\varepsilon_{abd})\tag{9}$$

the pseudotensor τ_{abc} actually depends on only three combinations of the variables R, X, Y and Z, and hence we can put $Y=0$ with no loss of generality.

To prove the identity (9) we note that the two sides of (9) have the same tensor character, and, in a coordinate frame with the z-axis parallel to the vector $\hat{\mathbf{k}}$, (9) takes the form

$$\varepsilon_{abc}=\delta_{3a}\varepsilon_{3bc}+\delta_{3b}\varepsilon_{a3c}+\delta_{3c}\varepsilon_{ab3}.\tag{10}$$

This relation can be verified directly. We note that (10) is equivalent to (9); our proof is thus completed.

Substituting $\hat{J}_a$ and $\hat{\tau}_a$ in the form indicated above into (3), we find

$$\langle\hat{p}_f\{U[R\varepsilon_{abc}+(X-R)\varepsilon_{dbc}\hat{k}_d\hat{k}_a]\hat{p}_c+UZ\varepsilon_{abd}\hat{k}_dw-2OZ\hat{p}_bw\varepsilon_{acd}\hat{p}_c\hat{k}_d\}\rangle_{\mathbf{p}} =-2\langle\hat{p}_fM[A\varepsilon_{adb}\hat{p}_d+D\varepsilon_{cdb}\hat{p}_a\hat{p}_d\hat{k}_c+E\varepsilon_{cdb}\hat{k}_a\hat{k}_c\hat{p}_d]\rangle_{\mathbf{p}},\tag{11}$$

where $U=N+O$ and $w=\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}$. We note that the region of applicability of Eq. (11) is not restricted to the acoustic limit and that the variables R, X and Z do not depend on $\hat{\mathbf{p}}$.

With our assumption concerning the quantity f_1^{ξ}, λ_a is equal to $\lambda\hat{k}_a$, where λ does not depend on $\hat{\mathbf{p}}$. Substituting λ_a in this form into (4), we obtain

$$\lambda[\ln(\Delta/r)-\langle U\rangle_{\mathbf{p}}]=2\langle M\hat{k}_aJ_{ab}\hat{p}_b\rangle_{\mathbf{p}}.\tag{12}$$

The paramagnetic-susceptibility tensor (5) is expressed in terms of J_{ab}, X, R, Z and λ and has the form

$$\chi_{ab}=\mu_B^2\nu\langle(O-L)J_{ba}-2O\hat{p}_a\hat{p}_cJ_{bc}-2M[R(\delta_{ab}-\hat{p}_a\hat{p}_b) +(X-R)\hat{k}_b(\hat{k}_a-\hat{p}_aw)+Zw(w\delta_{ab}-\hat{k}_a\hat{p}_b)-\lambda\hat{p}_a\hat{k}_b]\rangle_{\mathbf{p}}.\tag{13}$$

Taking the symmetry properties into consideration and using formula (7), we can rewrite (13) in the form

$$\chi_{ab}=\chi_\perp(\delta_{ab}-\hat{k}_a\hat{k}_b)+\chi_\parallel\hat{k}_a\hat{k}_b;$$
$$\chi_\perp=\mu_B^2\nu\langle A(-L+Ow^2)-{}^1/_2B(O+L)(1-w^2) -Dw(O+L-Ow^2)-M[R(1+w^2)+2Zw^2]\rangle_{\mathbf{p}},\tag{14}$$

$$\chi_\parallel=\mu_B^2\nu\langle(A+Dw+E)(O-L-2Ow^2)-(Bw^2+Cw)(O+L) -2M[X(1-w^2)-\lambda w]\rangle_{\mathbf{p}}.\tag{15}$$

2. TRANSFORMATION AND SOLUTION OF THE EQUATIONS IN THE ACOUSTIC LIMIT

In the acoustic limit, $0\gg|U|$, whence it follows, from Eq. (11), that $|X|$, $|R|\gg Z$. Therefore, the term proportional to UZ in (11) and the terms proportional to Z in (13) and (15) must be neglected, together with the terms of order Z in $\hat{\tau}_a$ when it is substituted into (2). This equation, written in terms of J_{ab} and R, X and λ, has, in the acoustic limit, the form

$$\begin{aligned} J_{ab}=\delta_{ab}-\langle B(\omega+\mathbf{kv}')[R(\delta_{ab}-\hat{p}_a'\hat{p}_b')+(X-R) \\ \times\hat{k}_a(\hat{k}_b-\hat{p}_b'w')-\lambda\hat{k}_a\hat{p}_b']\rangle_{\mathbf{p}'}-\langle B[{}^1/_2(1+\tilde{P})J_{ab}-\tilde{P}\hat{p}_b'J_{ac}\hat{p}_c']\rangle_{\mathbf{p}'} \end{aligned} \tag{16}$$

with B dependent on $\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}'$, and J_{ab} dependent on $\hat{\mathbf{p}}'$ inside the brackets $\langle\cdots\rangle$ or $\mathbf{p}$ outside them; the operator $\tilde{P}$ converts $\hat{\mathbf{p}}'$ to $-\hat{\mathbf{p}}'$.

Applying the relations

$$\begin{aligned} \langle B\rangle_{\mathbf{p}'}=b_0,\quad \langle B\hat{p}_a'\rangle_{\mathbf{p}'}=b_1\hat{p}_a,\quad \langle B\hat{p}_a'\hat{p}_b'\rangle_{\mathbf{p}'}={}^1/_3(b_0-b_2)\delta_{ab}+b_2\hat{p}_a\hat{p}_b, \\ \langle B\hat{p}_a'\hat{p}_b'\hat{p}_c'\rangle_{\mathbf{p}'}={}^1/_5(b_1-b_3)(\hat{p}_c\delta_{ab}+\hat{p}_b\delta_{ac}+\hat{p}_a\delta_{bc})+b_3\hat{p}_a\hat{p}_b\hat{p}_c \end{aligned} \tag{17}$$

(cf. (6)), we can represent the second term of the right-hand side as

$$\begin{aligned} &-\delta_{ab}R[{}^1/_3\omega(2b_0+b_2)+{}^1/_5kv(4b_1+b_3)w]+\hat{p}_a\hat{p}_bR(\omega b_2+kvb_3w) \\ &+\hat{k}_a\hat{p}_b[{}^1/_5Xkv(b_1-b_3)+(X-R)(\omega b_2w+kvb_3w^2)+\lambda(\omega b_1+kvb_2w)] \\ &-\hat{k}_a\hat{k}_b\{[{}^1/_3\omega(2b_0+b_2)+{}^1/_5kv(3b_1+2b_3)](X-R) \\ &-{}^1/_3\lambda kv(b_0-b_2)\}+{}^1/_5\hat{p}_a\hat{k}_bRkv(b_1-b_3). \end{aligned} \tag{18}$$

Comparing (16) and (18) with (7), we can see that the following terms, at least, should appear in the solution of Eq. (16): 1) the terms of A, B and E with $l=0$, 1; 2) the terms of C with $l=0$, 1, 2; 3) the $l=0$ term of D; here, l is the order of the Legendre polynomial $P_l(w)$ in the functions $A,\ldots,E$. Taking account of how the terms J_{ab} transform into each other under the multiplication $\hat{p}_bJ_{ac}\hat{p}_c$ in (16), we can see that the terms 1)–3) will also be *sufficient* for the solution of Eq. (16).

We shall determine the amplitudes of

$$F(w)=\sum_{n=0}^{m(F)}F_nw^n, \tag{19}$$

where F is one of the functions $A,\ldots,E$. Substituting (7) into (16), we remark that we have enough formulas (17) to calculate all the integrals appearing there. Hence, equating all the linearly independent terms to each other (*sic!*), we find directly

$$\begin{aligned} A_0=[1-R\omega(S-1)]S^{-1},\quad B_0=b_2(1+\omega R)S^{-1}, \\ C_1=b_2[1+\omega R+\lambda kv(1+b_1)]S^{-1}, \\ E_0=-\omega(X-R)(1-S^{-1})+{}^1/_3\lambda kv(b_0-b_2)S^{-1}, \end{aligned} \tag{20}$$

where

$$S\equiv1+{}^2/_3b_0+{}^1/_3b_2. \tag{21}$$

In addition,

$$C_2+b_3G=b_3(X-R)kv,\quad C_2-G[1+{}^1/_5(b_1-b_3)]={}^1/_5(X-R)kv(3b_1+2b_3), \tag{22}$$

where $G\equiv E_1+C_2$, and

$$\begin{aligned} A_1+{}^1/_5(b_1-b_3)F=-{}^1/_5Rkv(4b_1+b_3),\quad B_1+b_3F=b_3Rkv, \\ A_1+B_1-F[1+{}^1/_5(b_1-b_3)]=-{}^1/_5Rkv(b_1-b_3), \end{aligned} \tag{23}$$

where $F\equiv A_1+B_1+D_0$. We also have

$$C_0={}^1/_5(b_1-b_3)(Xkv-F-G)+\lambda\omega b_1(1+b_1)^{-1}. \tag{24}$$

Solving the systems of equations (22) and (23) and substituting the results into (24), we find

$$\begin{aligned} A_1=-{}^1/_5Rkv(b_1^2+4b_1b_3+4b_1+b_3)P^{-1},\quad B_1=Rkvb_3(1+b_1)P^{-1}. \\ C_0={}^1/_5Xkv(b_1-b_3)P^{-1}+\lambda\omega b_1(1+b_1)^{-1}, \\ C_2=(X-R)kvb_3(1+b_1)P^{-1},\quad D={}^1/_5Rkv(1+b_1)(b_1-b_3)P^{-1}, \\ E_1=-(X-R)kv[{}^3/_5(b_1-b_3)+b_3(1+b_1)]P^{-1}, \end{aligned} \tag{25}$$

where

$$P=1+{}^2/_5b_1+{}^3/_5b_3. \tag{26}$$

We return again to Eq. (11) in the acoustic limit. Taking into account that

$$\langle\hat{p}_a\hat{p}_b\hat{p}_c\hat{p}_d\rangle_{\mathbf{p}}={}^1/_{15}(\delta_{ab}\delta_{cd}+\delta_{ad}\delta_{bc}+\delta_{ac}\delta_{bd}), \tag{27}$$

we can find

$$\begin{aligned} &(\omega^2-{}^1/_5k^2v^2)[R\varepsilon_{abc}+(X-R)\hat{k}_a\hat{k}_d\varepsilon_{dbc}]+{}^1/_5Z\hat{k}_b\hat{k}_d\varepsilon_{adc} \\ &+{}^1/_5(2k^2v^2R+Z)\hat{k}_c\hat{k}_d\varepsilon_{abd}=-(\omega A_0+{}^1/_5kvA_1)\varepsilon_{abc} \\ &-[\omega E_0+{}^1/_5kv(D+E_1)]\hat{k}_a\hat{k}_d\varepsilon_{dbc}+{}^1/_5kv(D-2A_1)\hat{k}_c\hat{k}_d\varepsilon_{abd}. \end{aligned} \tag{28}$$

According to the assumption made by Gongadze, Gurgenishvili and Kharadze[18] concerning the form of the anomalous vertex functions, X should be equal to R, and $Z=0$. It is perfectly obvious that with this assumption Eq. (28) is not satisfied.

If in (28) we express $\hat{k}_b\hat{k}_d\varepsilon_{adc}$ using (9), we obtain three equations for the variables R, X and Z. An equivalent method is to choose $\hat{\mathbf{k}}$ along the third axis and define an order 1, 2, 3 for the indices a, b, c. Hence we obtain

$$(\omega^2-{}^1/_5k^2v^2)X=-\omega(A_0+E_0)-{}^1/_5kv(A_1+D+E_1); \tag{29}$$

$$\begin{aligned} (\omega^2-{}^3/_5k^2v^2)R-{}^1/_5Z=-\omega A_0-{}^3/_5kvA_1+{}^1/_5kvD, \\ (\omega^2-{}^1/_5k^2v^2)R+{}^1/_5Z=-\omega A_0-{}^1/_5kvA_1. \end{aligned} \tag{30}$$

From the formulas (30) we have the equation

$$(\omega^2-{}^2/_5k^2v^2)R=-\omega A_0-{}^2/_5kvA_1+{}^1/_{10}kvD. \tag{31}$$

Substituting A_0, A_1 and D, given by formulas (20) and (25), into (31), we obtain

$$\begin{aligned} R=-\omega[\omega^2-{}^2/_5k^2v^2(1+{}^2/_3b_0+{}^1/_3b_2)(1+b_1)(1+{}^1/_4b_1 \\ +{}^3/_4b_3)(1+{}^2/_5b_1+{}^3/_5b_3)^{-1}]^{-1}. \end{aligned} \tag{32}$$

Analogously, we obtain from (29)

$${}^1/_3\lambda\omega kv(b_0-b_2)+X(\omega^2-v_1^2k^2)=-\omega, \tag{33}$$

where

$$v_1^2\equiv{}^1/_5v^2(1+{}^2/_3b_0+{}^1/_3b_2)(1+b_1)(1+b_3)(1+{}^2/_5b_1+{}^3/_5b_3)^{-1}. \tag{34}$$

The right-hand side of (34) is positive by virtue of the Fermi-liquid stability conditions.[23]

We turn now to the analysis of Eq. (12) in the acoustic limit. Substituting J_{ab} from (7) into it, we obtain

$$\lambda[\ln(\Delta/r)+{}^1/_3k^2v^2-\omega^2]=-{}^1/_3kv(A_0+B_0+C_1+E_0)$$
$$-{}^1/_3\omega(A_1+B_1+3C_0+C_2+E_1+D). \quad (35)$$

Having expressed $A_n, \ldots, E_n$ by (20) and (25), we can rewrite Eq. (35) in the form

$$\lambda(a+k^2v_2^2-\omega^2)-{}^2/_9Xkv(b_0-b_2)(1+b_1)S^{-1}$$
$$=-{}^1/_3kv(1+b_1)(1+b_2)S^{-1}, \quad (36)$$

where

$$a=(1+b_1)\ln(\Delta/r), \quad (37)$$
$$v_2^2={}^1/_3v^2(1+b_0)(1+b_1)(1+b_2)(1+{}^2/_3b_0+{}^1/_3b_2)^{-1}. \quad (38)$$

Solving the system of equations (33), (36), we find

$$X=\omega[(\omega^2-u_1)(\omega^2-u_2)]^{-1}[a+{}^1/_3k^2v^2(1+b_1)(1+b_2)-\omega^2],$$
$$\lambda={}^1/_3kv(1+b_1)[(\omega^2-u_1)(\omega^2-u_2)]^{-1}[\omega^2-{}^1/_5k^2v^2(1$$
$$+b_1)(1+b_2)(1+b_3)(1+{}^2/_5b_1+{}^3/_5b_3)^{-1}], \quad (39)$$

where

$$u_{1,2}={}^1/_2(a+k^2v_0^2\pm\operatorname{sign} a\sqrt{Q}), \quad (40)$$

with

$$v_0^2=v_1^2+v_2^2+v_3^2, \quad v_3^2={}^2/_{27}v^2(b_0-b_2)^2(1+{}^2/_3b_0+{}^1/_3b_2)^{-1},$$
$$Q=(a+k^2v_0^2)^2-4ak^2v_1^2-4v_1^2v_2^2k^4>0. \quad (41)$$

It can be proved that u_1 and u_2 are increasing functions of the variable k^2 ($u_2 \geq 0$), and for $k^2 \ll |a|$ we have $u_2 \approx k^2v_1^2$. In addition, the sign of u_1 coincides with the sign of a. Here, therefore, we always have an acoustic mode. If $a>0$, i.e., if the effective interaction in the pairing channel predominates over the interaction in the $l=0$ channel, an optical mode also appears. Since for $k^2 \ll |a|$ we have $u_1 \approx a+k^2(v_2^2+v_3^2)$, it corresponds to excitations with a gap $2\Delta(1+b_1)^{1/2}[\ln(\Delta/r)]^{1/2} \ll 2\Delta$. If $a<0$, the optical mode becomes a diffusion mode. We remark that, in all our previous formulas, ω and kv are measured in units of 2Δ. We must therefore substitute $\omega/2\Delta$ and $kv/2\Delta$ in place of ω and kv, in order to remove this stipulation. If $|\omega/2\Delta|$, $(kv/2\Delta) \ll |a|^{1/2}$, then

$$X=-\omega[\omega^2-k^2v_1^2]^{-1}, \quad \lambda=0. \quad (42)$$

The solution has precisely this form if the inequality $|\ln(\Delta/r)| \ll 1$ is not fulfilled. We note that formulas (20) and (25), together with (32) and (38), now serve as a compact form for writing the solutions for the spin vertex.

3. DISCUSSION OF THE SOLUTIONS AND CONCLUDING REMARKS

First we shall find the expression for the spin susceptibility. Substituting the solutions into (13), we have

$$\chi_{ab}=\chi_{st}[\delta_{ab}+(\delta_{ab}-\hat{k}_a\hat{k}_b)\omega R+\hat{k}_a\hat{k}_b\omega X-{}^1/_2\hat{k}_a\hat{k}_bkv\lambda(1+b_2)], \quad (43)$$
$$\chi_{st}=2\mu_B^2\nu/3(1+{}^2/_3b_0+{}^1/_3b_2);$$

the static quantity χ_{st} was obtained in our paper[11] for $M=0$ (cf. (2)–(4)). We have

$$\lim_{k\to0}R=\lim_{k\to0}X=-\omega^{-1} \text{ and } \lim_{k\to0}\lambda=0 \quad \text{for } \omega\neq0$$

and, therefore,

$$\lim_{k\to0}\chi_{ab}=0 \quad \text{for } \omega\neq0.$$

This result is fully comprehensible in the acoustic limit; if the inequality $|\ln(\Delta/r)| \ll 1$ is violated, then, in a uniform field that is periodic in time, the quantity χ_{ab} should be equal in order of magnitude to $(\omega/2\Delta)^2$ for $|\omega| \ll 2\Delta$, and this is in excess of the acoustic-limit accuracy. We may add that even the presence of excitations with a gap much smaller than 2Δ does not have the result that $\chi_{ab}(k=0, \omega)$ contains terms of order ω^2/a, which are contained within the limits of our accuracy. This property has an analog in the theory of the normal Fermi liquid. It is well known that

$$\lim_{k\to0}S(k,\omega)=0 \quad \text{for } \omega\neq0,$$

where S is any correlation function of conserved quantities. Moreover, with the usual accuracy, the theories considered by us neglect the quantity $(\omega/\mu)^2$, where μ is the chemical potential of the system.

On the other hand,

$$\lim_{\omega\to0}X=\lim_{\omega\to0}R=0, \quad \lim_{\omega\to0}\lambda=-kv(1+b_2)/[12\Delta^2\ln(\Delta/r)(1+{}^2/_3b_0+{}^1/_3b_2)$$
$$+(kv)^2(1+b_0)(1+b_2)]$$

whence

$$\lim_{\omega\to0}\chi_{ab}=\chi_{st}\{\delta_{ab}+\hat{k}_a\hat{k}_b(kv)^2(1+b_2)^2[24\Delta^2\ln(\Delta/r)(1+{}^2/_3b_0+{}^1/_3b_2)$$
$$+2(kv)^2(1+b_0)(1+b_2)]^{-1}\}. \quad (44)$$

If the inequality $|\ln(\Delta/r)| \ll 1$ is violated, the second term in the curly brackets must be neglected and we obtain $\chi_{st}\delta_{ab}$. This property has a simple physical meaning, since the static limit corresponds to a static field that is slightly nonuniform, e.g., as a consequence of the finite size of the sample. If, however, $|\ln(\Delta/r)| \ll 1$, to get $\chi_{st}\delta_{ab}$ it is not sufficient to take only the one limit $\omega\to0$. This means that, if $|a| \ll 1$, to obtain the static limit it is not sufficient that the field vary weakly over a distance equal to the coherence length; the field should also vary weakly over a distance $\sim\hbar v/2\Delta|a|^{1/2}$.

According to (43), the pole of R corresponds to the transverse spin waves and the poles of X and λ to the longitudinal spin waves. It is obvious that the transverse waves are doubly degenerate while the longitudinal waves are nondegenerate (cf. [17]). All factors containing amplitudes b_l in R and X (for $|\omega|$, $kv \ll 2\Delta|a|^{1/2}$ in the latter case) can be represented in the form

$$\frac{m}{n}(1+b_l)+\frac{n-m}{n}(1+b_{l'}),$$

where $0 \leq m \leq n$, and l, l' are respectively equal to 0, 2 or 1, 3. Since the stability conditions for the Landau spin amplitudes defined as in this paper have the form $1+b_l>0$, [23] all these factors are positive. From this we obtain that, if the stability conditions are fulfilled, (43) always contains two poles for $|\omega|$, $kv \ll 2\Delta|a|^{1/2}$.

In addition, the stability conditions then also guarantee that the transverse excitations set in at energies above those of the longitudinal excitations at the same values of k, i.e., that the transverse excitations are faster than the longitudinal ones. We note that, for $|\omega|$, $kv \ll 2\Delta|a|^{1/2}$, the Landau amplitudes b_0 and b_2 appear in (43) in only the same combination as in the static susceptibility.

Our calculations possess a property characteristic of all theories based on a sufficiently general phenomenological approach. This can be defined as a principle of maximum freedom of the physical system. We shall elucidate this principle using the example of the spin susceptibility of ^{3}He. The static susceptibility of the normal system determines b_0, while that of the B phase also determines b_2.[11] In addition, observation of the longitudinal and transverse waves for $|\omega|$, $kv \ll 2\Delta|a|^{1/2}$ gives us b_1 and b_3. From this point of view, here there is no cross-check on the theory; the independent measurements of[19–21] can be regarded as an exception confirming the general rule.

In all our calculations we have neglected both temperature effects and the spin-nonconservation effects associated with the dipole-dipole interaction. The first of these can be taken into account by the methods developed by Leggett,[25] and the second lead to serious difficulties in a similar formulation of the theory (cf.[26]).

It must be noted that collective excitations with a gap for a superfluid (superconducting) Fermi liquid have been considered in the literature (see, e.g.,[27] and[2]), but collective excitations with a gap much smaller than 2Δ have not been considered. The appearance of these excitations is a characteristic feature of the Larkin-Migdal approach,[13] if the $l=l_0$ amplitude and an $l=l'\neq l_0$ amplitude of the effective interaction in the particle-particle channel are close to each other.

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APPENDIX

We shall transform the equation for the spin vertices and spin susceptibility in such a way that only the integration over the Fermi surface becomes important, in analogy with the transformation performed by Larkin.[14] We note that only the transformation of the equation for the normal vertex will be of interest for us. Applying the procedure from Larkin's paper[14] to this equation, we find

$$\hat{J}_a(\hat{\mathbf{p}}) = \hat{J}_a{}^k + \langle g(\hat{\mathbf{p}}\hat{\mathbf{p}}')\{S\hat{J}_a(\hat{\mathbf{p}}') + O(\sigma\hat{\mathbf{p}}')\hat{J}_a(-\hat{\mathbf{p}}')(\sigma\hat{\mathbf{p}}') - M[\hat{\tau}_a(\hat{\mathbf{p}}'), \sigma\hat{\mathbf{p}}']_- - 2M\lambda_a(\hat{\mathbf{p}}')\sigma\hat{\mathbf{p}}'\}\rangle_{\mathbf{p}'}, \tag{45}$$

where $\hat{J}_a^k$ denotes the spin vertex, taken in the k-limit, for the normal system, and $g(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')$ is the spin-exchange part of the dimensionless quasi-particle scattering amplitude; $g_1 = b_1/(1+b_1)$, $S = L+1$. The equations for λ_a and $\hat{\tau}_a$ will coincide with the corresponding equations of our paper.[11] According to the work of Kondratenko[28] (cf. also[29]), if the spin vertex is chosen such that $\hat{J}_a^\omega = \sigma_a$, then

$$\hat{J}_a{}^k = \sigma_a(1-g_0) = \sigma_a/(1+b_0).$$

We turn now to the transformation of the spin susceptibility. According to our paper,[11] the latter can be written in the form

$$\chi_{ab} = -\left(\frac{\mu_B}{z}\right)^2 \int \frac{d^4p}{(2\pi)^4 i} \operatorname{Sp}\{\hat{J}_a{}^0[G\hat{J}_bG - F\hat{D}\hat{J}_b{}^- D^+F - G\hat{\tau}_{1b}\hat{D}^+F + F\hat{D}\hat{\tau}_{2b}G]\}, \tag{46}$$

where $\hat{J}_a^0 = z\sigma_a$ (as a result of which, $\hat{J}_a^\omega$ at the Fermi surface is equal to σ_a), z denotes the discontinuity in the occupation numbers at the Fermi surface for a normal system, $D = \hat{\Delta}/\Delta = \boldsymbol{\sigma}\cdot\hat{\mathbf{p}}i\sigma^y$, and the normal ($G$) and anomalous ($F$) Green functions standing before and after the vertex are taken with the variables $\mathbf{p}+\mathbf{k}/2$, $\varepsilon+\omega/2$ and $\mathbf{p}-\mathbf{k}/2$, $\varepsilon-\omega/2$, respectively; $d^4p = d^3\mathbf{p}\,d\varepsilon$ and $\hat{J}^-(\mathbf{p}) \equiv J^T(-\mathbf{p})$.

Carrying out in (45) the transformations originally proposed by Larkin,[14] using Eq. (44) and expressing $\hat{\tau}_{1b}$, $\hat{\tau}_{2b}$ in terms of $\hat{\tau}_b$ and λ_b, we find that

$$\chi_{ab} = \chi_{ab}^k - {}^1\!/_2\mu_B^2\nu \operatorname{Sp}\langle\hat{J}_a^k\{S\hat{J}_b + O(\sigma\hat{\mathbf{p}})\hat{J}_b(\sigma\hat{\mathbf{p}}) - M[\hat{\tau}_b, \sigma\hat{\mathbf{p}}] - 2M\lambda_b\sigma\hat{\mathbf{p}}\}\rangle_{\mathbf{p}}, \tag{47}$$

where

$$\chi_{ab}{}^k = -\left(\frac{\mu_B}{z}\right)^2 \int \frac{d^4p}{(2\pi)^4 i} \operatorname{Sp}[\hat{J}_a{}^0(GG)^k\hat{J}_b{}^k], \tag{48}$$

with

$$(GG)^k = \lim_{k\to 0}\lim_{\omega\to 0}\lim_{\Delta\to 0} GG.$$

Taking into account the results of the work of Kondratenko[28] (see also[29]), we find that χ_{ab}^k is the static susceptibility of the normal system, i.e., $\delta_{ab}\mu_B^2\nu/(1+b_0)$. Moreover, $\hat{J}_a^k$ in the second term of formula (47), i.e., $\hat{J}_a^k$ at the Fermi surface, is equal to $\sigma_a/(1+b_0)$. It is clear that formula (47) is equivalent to (13) and can also be written in the form (14), (15). We note that, in the Appendix, according to (46) χ_{ab} and ν correspond to quantities calculated per unit volume.

Note added in proof (August 3, 1976). If we take into account the non-negativity of the static autocorrelation functions, which follows from their spectral representation, it is not difficult to prove that $a > 0$ always. Therefore, when the $l=0$ and $l=1$ amplitudes of the effective interaction in the particle-particle channel are close to each other, a pole with a small gap is always present.

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