$$j = enE = (-c^{-1}\dot{A} - \nabla\varphi)en.$$

For the high-frequency limit $vk \gg T$ one can show that the role of the vector potential is unimportant and the corrections to the sound speed are given by the usual polarization operator:

$$\frac{s-s_n}{s_n} = \frac{\pi}{12}\frac{m}{M}\frac{v}{s}\int_{\eta}^{\Delta+\omega}\frac{\varepsilon(\varepsilon-\omega)-\Delta^2}{(\varepsilon^2-\Delta^2)^{1/2}[\Delta^2-(\varepsilon-\omega)^2]^{1/2}}\frac{\varepsilon}{2T}\frac{d\varepsilon}{\omega}, \qquad \eta = \Delta + (\omega-2\Delta)\theta(\omega-2\Delta); \tag{26}$$

in the limiting cases

$$\frac{s-s_n}{s_n} = -\frac{\pi^2}{48}\frac{m}{M}\frac{v}{s}\frac{\Delta}{T}\begin{cases}{}^1/_8(\omega/\Delta)^2, & \omega \ll \Delta \\ \Delta/\omega, & \Delta \ll \omega\end{cases}. \tag{27}$$

Hence it is clear that in the high-frequency limit $vk \gg T$ the temperature dependence of the longitudinal sound speed has a minimum for $(T_c - T)/T_c \sim (\omega/T)^2$ with a relative depth of the order $s\omega/vT$.

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[1] V. D. Fil', V. I. Denisenko, and P. A. Bezuglyĭ, Pis'ma Zh. Eksp. Teor. Fiz. 21, 693 (1975) [JETP Lett. 21, 329 (1975)].

[2] H. Ozaki and N. Mikoshiba, Phys. Lett. 23, 52 (1966).

[3] I. E. Bulyzhenkov and B. I. Ivlev, Zh. Eksp. Teor. Fiz. 70, 1405 (1976) [Sov. Phys. JETP 43, 731 (1976)].

[4] B. T. Geĭlikman and V. Z. Kresin, Kineticheskie i nestatsionarnye yavleniya v sverkhprovodnikakh (Kinetic and nonstationary effects in superconductors) Nauka, 1972.

[5] V. L. Gurevich, I. G. Lang, and S. T. Pavlov, Zh. Eksp. Teor. Fiz. 59, 1679 (1970) [Sov. Phys. JETP 32, 914 (1971)].

[6] E. G. Brovman and Yu. Kagan, Zh. Eksp. Teor. Fiz. 52, 557 (1967) [Sov. Phys. JETP 25, 365 (1967)].

[7] D. P. Belozorov and É. A. Kaner, Zh. Eksp. Teor. Fiz. 55, 642 (1968) [Sov. Phys. JETP 28, 334 (1969)].

[8] V. M. Kontorovich, Zh. Eksp. Teor. Fiz. 45, 1638 (1963) [Sov. Phys. JETP 18, 1125 (1964)].

[9] L. P. Gor'kov and G. M. Éliashberg, Zh. Eksp. Teor. Fiz. 54, 612 (1968) [Sov. Phys. JETP 27, 328 (1968)].

[10] G. M. Éliashberg, Zh. Eksp. Teor. Fiz. 61, 1254 (1971) [Sov. Phys. JETP 34, 668 (1972)].

[11] J. Bardeen and M. Stephen, Phys. Rev. 136, 1485 (1964).

[12] B. I. Ivlev, S. G. Lisitsyn, and G. M. Éliashberg, J. Low Temp. Phys. 10, 449 (1973).

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Energy absorption and the size effect in solid helium

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Owing to the quantum nature of the diffusion of point defects (defectons) in quantum crystals such as solid helium, energy is dissipated at the substitution defects even in the case of spatially homogeneous deformation. The absorption is due to diffusion flow of the defectons in momentum space in the absence of particle fluxes in coordinate space. Internal friction is connected with the deformation potential at low frequencies and with the inertial and activation components of the energy spectrum at high frequencies. The collision integral within the crystal is determined by quasielastic defecton–phonon scattering. It is found that the law of interaction between the vacancions and the crystal surface may be determined at low temperatures. The dissipation accompanying interaction between the defectons and the surface is determined. Diffusion-viscous flow in quantum crystals is discussed.

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1. INTRODUCTION

Internal friction is one of the characteristic manifestations of the diffusion properties of point defects in crystals. It is customarily assumed that in the case of spatially-homogeneous deformation of the crystal there is no energy dissipation at substitution defects, e.g., vacancies. The reason is that in the case of uniform deformation all the crystal lattice sites remain equivalent and no particle diffusion fluxes that lead to the absorption of energy are produced (the force acting on a point defect is proportional to the gradient of the deformation).

This reasoning is not applicable to quantum crystals of the type of solid helium with large zero-point vibration amplitudes. In such crystals[1,2] the point defects are localized and are transformed into quasiparticles—defectons—which move practically freely through the crystal. The correct quantum numbers for defectons are the values of the quasimomenta and not of the coordinates. When the crystal is deformed, the energy spectrum of the defectons is altered. As a result, even in the case of uniform deformation, diffusion fluxes of particles are produced in momentum space. Such a diffusion leads to dissipation of the energy also in the absence of particle fluxes in configuration space.

For this reason, it would be of great interest to investigate experimentally the internal friction at low-frequency compression deformations (at high frequencies the spatial inhomogeneity of the vibrations, which is connected with the finite speed of sound, is appreciable), since the very fact of observation of dissipation by substitution point defects in the case of uniform deformations would demonstrate that the defects are of the quantum type. At the present time, experiments are being carried out on the dissipation via essentially inhomogeneous vibrations[3] (for the corresponding calculations see[4]) and absorption of hypersound[5] (1.5 GHz). First indications that hypersound absorption (10 MHz) has a relaxation character were also published recently.[6] It should be noted that in addition to the mechanism proposed in the present paper for dissipation under uniform deformation, a dissipation should also appear as a result of the presence of bound pairs of point defects—"bidefectons."[7] However, the absorption lines on the bidefectons should differ because of their resonant character.[7]

The complete system of equations determining the mechanical properties of quantum crystals includes the kinetic equation for the defectons and the equations of motion for a crystal with defectons. The equations of motion are sought in the usual manner in the form that ensures satisfaction of the momentum and energy conservation laws, and also positiveness of the dissipation function. As a result, the defecton part of the energy flux, of the momentum-flux tensor, and of the dissipation function are uniquely determined. In fact, the expressions for these quantities and for the equations of motion are analogous to the corresponding expressions of electron theory of metals.[8] The concentration of point defects in a crystal is low, and dragging effects can be neglected. Therefore the equations of motions will not be needed directly in the present paper and will not be derived. It is necessary only to have the explicit form of the expression for the dissipative function in terms of the kinetic characteristics of the defectons.

In the next section of the paper we determine the absorption of energy in defecton-phonon collisions. In the last part of the paper we study the interaction of the vacancions with the surface of the crystal and calculate the corresponding value of the absorbed energy. We discuss also the diffusion-viscous flow of quantum crystals.

2. ENERGY ABSORPTION IN DEFECTON-PHONON COLLISIONS

A. Kinetic equation

The quasiparticle distribution function $n(\mathbf{p},\mathbf{r})$ is determined from the kinetic equation

$$\frac{\partial n}{\partial t}+\frac{\partial n}{\partial \mathbf{r}}\frac{\partial \varepsilon}{\partial \mathbf{p}}-\frac{\partial n}{\partial \mathbf{p}}\frac{\partial \varepsilon}{\partial \mathbf{r}}=I(n) \tag{1}$$

with collision integral $I(n)$. The Hamilton function of the defectons $\varepsilon(\mathbf{p},\mathbf{r})$ in an undeformed crystal depends only on the quasimomentum $\mathbf{p}$:

$$\varepsilon=\varepsilon_0+\varepsilon_1(\mathbf{p}). \tag{2}$$

The change of the energy spectrum following the deformation of the crystal, just as in the case of electrons in a metal,[8–10] can be described by introducing the deformation potential $\lambda_{ik}(\mathbf{p})$:

$$\varepsilon(\mathbf{p},\mathbf{r})=\varepsilon_0+\varepsilon_1+(-K\Omega_{ik}+\lambda_{ik}+p_iv_k)\,\partial u_i/\partial x_k+(p_i-mv_i)\,\dot{u}_i, \tag{3}$$

where $\mathbf{v}=\partial\varepsilon_1/\partial\mathbf{p}$ is the defecton velocity, $\mathbf{u}$ is the displacement vector, K is the hydrostatic compression modulus, Ω_{ik} is a tensor characterizing the change of the lattice volume following introduction of a point defect ($\Omega_{ll}\sim a^3$, a is the interatomic distance), and m is the mass transported by the motion of the defectons. We have taken into account the fact that the activation energy ε_0 is altered by the deformation by an amount $-K\Omega_{ik}=\partial u_i/\partial x_k$.[11] The inertial term $m\mathbf{v}\cdot\dot{\mathbf{u}}$ describes the action of the inertial force $-m\ddot{\mathbf{u}}$ on the defect. In order of magnitude we have

$$\lambda_{ik}\sim p_iv_k\sim\Delta,\quad K\Omega_{ik}\sim m_0c^2\gg\Delta, \tag{4}$$

where Δ is the width of the energy gap of the defecton, m_0 is the mass of the helium atom, and c is the speed of sound.

The collision integral $I(n)$ is made to vanish by the equilibrium distribution function $n_0(\varepsilon-\mu-\mathbf{p}\dot{\mathbf{u}})$. The chemical potential μ is reckoned from the values $\mu=0$ in the undeformed crystal with equilibrium distribution function $n_e=n_0(\varepsilon_0+\varepsilon_1)$. Since the activation energy ε_0 of point defects is large in comparison with the temperature, $\varepsilon_0\gg T$, i.e., the defecton concentration is low, the equilibrium distribution function is of the Boltzmann type:

$$n_e=\exp\{-(\varepsilon_0+\varepsilon_1)/T\},\quad n_0(\varepsilon-\mu-\mathbf{p}\dot{\mathbf{u}})=\exp\{-(\varepsilon-\mu-\mathbf{p}\mathbf{u})/T\}. \tag{5}$$

The linearized kinetic equation (1) for the nonequilibrium part of the distribution function

$$n=n_0(\varepsilon-\mu-\mathbf{p}\dot{\mathbf{u}})-\frac{\partial n_0}{\partial\varepsilon}\chi \tag{6}$$

takes the form

$$\frac{\partial\chi}{\partial t}+\mathbf{v}\frac{\partial\chi}{\partial\mathbf{r}}+I(\chi)=\frac{\partial}{\partial t}(\varepsilon-\mu-\mathbf{p}\dot{\mathbf{u}})-\mathbf{v}\frac{\partial}{\partial\mathbf{r}}(\mu+\mathbf{p}\dot{\mathbf{u}}).$$

By definition, the chemical potential satisfies the condition

$$\int n\,d\Gamma=\int n_0(\varepsilon-\mu-\mathbf{p}\dot{\mathbf{u}})\,d\Gamma\approx\int n_e\,d\Gamma\Big\{1+\frac{\mu}{T}-\Big[T\int n_e\,d\Gamma\Big]^{-1}\int(\varepsilon-\varepsilon_0-\varepsilon_1-\mathbf{p}\dot{\mathbf{u}})\,n_e\,d\Gamma\Big\},\quad\int\chi n_e\,d\Gamma=0, \tag{7}$$

where the integration is over the momentum space $d\Gamma=d^3p/(2\pi\hbar)^3$. For electrons in metals, this condition jointly with the quasineutrality condition

$$\int n\,d\Gamma=\int n_e\,d\Gamma$$

makes it possible to relate directly the chemical potential with the change of the energy due to the crystal deformation (3). For defectons there is no additional condition, analogous to the quasineutrality condition, for the conservation of the number of particles at each point of coordinate space. The chemical potential μ and the distribution function χ should be determined in a self-consistent manner from the kinetic equation and from the condition (7)

$$\int \chi n_e \, d\Gamma = 0.$$

In our dynamic problem, as noted in the Introduction, there are no particle diffusion fluxes in coordinate space for spatially homogeneous deformations. We can then use for μ the expression

$$\mu = \left\{ -K\Omega_{ik} + \int \lambda_{ik} n_e \, d\Gamma \Big/ \int n_e \, d\Gamma \right\} \frac{\partial u_i}{\partial x_k},$$

and the kinetic equation takes the form

$$\frac{\partial \chi}{\partial t} + \mathbf{v} \frac{\partial \chi}{\partial \mathbf{r}} + I(\chi) = -v_i \left\{ -K\Omega_{lk} + \int \lambda_{lk} n_e \, d\Gamma \Big/ \int n_e \, d\Gamma \right\} \frac{\partial^2 u_k}{\partial x_i \, \partial x_l} + \frac{\partial}{\partial t} \left\{ \left[\lambda_{ik} - \int \lambda_{ik} n_e \, d\Gamma \Big/ \int n_e \, d\Gamma \right] \frac{\partial u_i}{\partial x_k} - m v_i \frac{\partial u_i}{\partial t} \right\}. \quad (8)$$

The distribution function χ obtained with the aid of this equation, as confirmed by subsequent calculations, does indeed satisfy the condition (7).

Since $K\Omega_{ik} \gg \lambda_{ik}$ [Eq. (4)], we can always neglect the term proportional to λ_{ik} in the first term of the right-hand side of (8). For the same reason the dissipation under the conditions of the Gorskiĭ effect (in the case of inhomogeneous deformations, occurring for example in flexural vibrations), it is due to the term $v_i K\Omega_{kl} \partial^2 u_k / \partial x_i \partial x_l$.[4] In this paper we are interested in the dissipations produced in homogeneous compression and tension deformations, in which the gradients of the deformation tensors are produced only as a result of the finite propagation velocity of the vibrations. In this case the right-hand terms of the kinetic equation (8), which contain the second derivatives of the displacement vector **u** with respect to time and with respect to the coordinates (inertial and activation components) are of the same order.

The entire frequency interval can be broken up into three sections. For low frequencies ω limited by

$$\omega L / c \ll \Theta / m_0 c^2, \quad (9)$$

we can regard the deformation (unilateral compression along the z axis) as completely homogeneous

$$u_z = \zeta z \sin \omega t, \quad (10)$$

and the kinetic equation takes the form

$$\frac{\partial \chi}{\partial t} + I(\chi) = \zeta \omega \Lambda \cos \omega t,$$

where L is the characteristic linear dimension of the sample, Θ is the Debye temperature, ζ is the vibration amplitude, and Λ is the quantity in the square brackets in the right-hand side of (8).

For higher frequencies$^{1)}$:

$$\Theta / m_0 c^2 \ll \omega L / c \ll 1, \quad (11)$$

the dissipation is responsible for the activation and inertial terms, and the displacement vector still is of the form (10)

$$\frac{\partial \chi}{\partial t} + v_z \frac{\partial \chi}{\partial z} + I(\chi) = -\zeta v_z \omega^2 z \mathfrak{M} \sin \omega t,$$

where $\mathfrak{M} = K\Omega_{zz}/c^2 - m$. For short-wave oscillations

$$\omega L / c \gg 1, \quad (12)$$

it is necessary to take into account explicitly in the expression for the displacement vector the spatial dispersion

$$u_z = \frac{\zeta c}{\omega} \sin \frac{\omega z}{c} \sin \omega t,$$

and the right-hand side of the kinetic equation should be expressed in the form

$$-\zeta v_z \mathfrak{M} c \omega \sin \frac{\omega z}{c} \sin \omega t.$$

B. Collision integral

The volume part of the dissipative function in a crystal with a small number of defects is connected mainly with the defecton-phonon interaction. The corresponding collision integral is of the form

$$I(n) = -\int dW \{ n f (1 + f') - n' f' (1 + f) \}, \quad (13)$$

where $f = f(\mathbf{q})$ is the phonon distribution function in terms of their momentum $\mathbf{q}$, $n' = n(\mathbf{p}')$, $f' = f(\mathbf{q}')$, dW is the probability of particle scattering wherein their momenta **p** and **q** change into $\mathbf{p}' = \mathbf{p} + \Delta$ and $\mathbf{q}' = \mathbf{q} - \Delta$, respectively. The scattering conserves the energy and momentum conservation laws:

$$\mathbf{p} + \mathbf{q} = \mathbf{p}' + \mathbf{q}', \quad \varepsilon(\mathbf{p}) - \varepsilon(\mathbf{p}') = c(q' - q). \quad (14)$$

The defecton velocity $\mathbf{v} = \partial \varepsilon_1 / \partial \mathbf{p} \sim \Delta a / \hbar$ is small in comparison with the speed of sound $v/c \sim \Delta/\Theta \ll 1$, and the phonon momentum $q \sim T/c$ is small in comparison with the defecton momentum $p \sim \hbar / a$. Consequently the scattering alters little the defecton momentum $|\Delta|/p \ll 1$ and the energy (but not the momentum) of the phonon $\delta/q = |q' - q|/q \ll 1$, the collisions being quasi-elastic. Accordingly, the collision integral (13) can be calculated in standard fashion[14] and should take the form of the divergence of a certain vector in momentum space. The probability of the scattering process can be written in the form

$$dW = (2\pi\hbar/a)^{-9} q^4 \, dq \, d\delta \, d\mathbf{e} \, d\mathbf{e}' \, d^3\Delta \, w \left(\mathbf{p} + \frac{\Delta}{2}, q + \frac{\delta}{2}; \mathbf{e}, \mathbf{e}'; \Delta, \delta \right),$$

where $\mathbf{e} = \mathbf{q}/q$, $\mathbf{e}' = \mathbf{q}'/q'$, and w is an even function of the momentum transfer Δ and the energy transfer δ.

Neglecting the dragging effects, we assume an equilibrium phonon distribution function:

$$f(q) = \{ e^{cq/T} - 1 \}^{-1}. \quad (15)$$

We expand the integrand of (13) in powers of the small parameters $\boldsymbol{\Delta}$ and δ accurate to terms of second order, and integrate by parts the terms that contain $\partial(q^4 w)/\partial q$. The integration with respect to $d^3\Delta$ and $d\delta$, with allowance for the corresponding number of δ functions that reflect the conservation laws (14), is elementary. After substituting the distribution functions (6) and (15), the collision integral takes the form

$$I(n)=\frac{D_{ik}}{T}\frac{\partial}{\partial p_i}\left(n_e\frac{\partial\chi}{\partial p_k}\right),$$

$$D_{ik}=\frac{8!\zeta(8)}{(2\pi)^9}\left(\frac{T}{\Theta}\right)^9\int d\mathbf{e}\,d\mathbf{e}'\,(e_i'-e_i)\,(e_k'-e_k)\frac{w(q\to0)}{q^2}. \tag{16}$$

In the calculation of w, the defect can be regarded here as localized. The function w contains the square of the matrix element of the phonon interaction with a point defect, proportional to q at a small phonon momentum.

The diffusion tensor in momentum space D_{ik}, as expected,[1] is proportional to T^9:

$$D_{ik}=\frac{\Theta\hbar}{a^2}\left(\frac{T}{\Theta}\right)^9\alpha_{ik},$$

where α_{ik} is a certain dimensionless positive-definite matrix, the calculation of which calls for the solution of the exact microscopic problem of the defecton-phonon interaction (see[15]). A collision integral similar to (16) was used previously[16,17] in the study of defecton diffusion. The kinetic equation with such a collision integral takes the form of a Fokker–Planck equation.

C. Energy dissipation

The energy absorption is determined by the dissipation function of the system R. It can be shown that just as in the known case of ultra-sound absorption in metals,[8,19,18] the dissipation function is equal to

$$R=-\int d\Gamma\chi I(n).$$

It is convenient to continue the calculations separately for particles with narrow energy bands $\Delta\ll T$ and particles with broad bands $\Delta\gg T$. The first inequality is valid for impuritons ($\Delta\sim10^{-4}$ K) and for vacancions at high pressures. For vacancions at not too high pressures, on the other hand, Δ is apparently of the order of several degrees and we can use the condition $\Delta\gg T$. We calculate below not the dissipation function R, but the corresponding energy H absorbed during one period of the oscillations in a unit volume. With the aid of (16) we can express H in the following quadratic form:

$$H=\frac{D_{ik}}{T}\int_0^{2\pi/\omega}dt\int d\Gamma\,n_e\frac{\partial\chi}{\partial p_i}\frac{\partial\chi}{\partial p_k}. \tag{17}$$

For defectons with narrow bands $\Delta\ll T$ we can neglect in the kinetic equation and in the collision integral the terms $v/c\sim\Delta/\Theta$ in comparison with

$$\frac{T}{c}\frac{\partial}{\partial\mathbf{p}}\sim\frac{T}{\Theta}.$$

At $\Delta\ll T$, the band is uniformly filled and it is necessary to replace n_e [Eq. (5)] in the integrand of (17) by $\exp\{-\varepsilon_0/T\}$. Expanding the distribution function χ, the velocity v_z and the deformation potential Λ in Fourier series in terms of the lattice translation vectors (we assume for simplicity that the lattice contains one atom per unit cell):

$$\chi=\sum_{\mathbf{a}}\chi_{\mathbf{a}}e^{i\mathbf{pa}/\hbar},\qquad v_z=\sum_{\mathbf{a}}v_{\mathbf{a}}e^{i\mathbf{pa}/\hbar},\qquad \Lambda=\sum_{\mathbf{a}}\Lambda_{\mathbf{a}}e^{i\mathbf{pa}/\hbar},$$

we obtain for low frequencies [(9), (10)]

$$\chi_{\mathbf{a}}=\frac{1}{2}\zeta\omega\Lambda_{\mathbf{a}}\left\{\frac{e^{i\omega t}}{i\omega+D_{ik}a_ia_k/\hbar^2}+\text{c.c.}\right\}$$

and, neglecting the terms containing the higher orders of the defecton velocity, we obtain for higher frequencies [Eq. (11)]

$$\chi_{\mathbf{a}}=-\frac{z}{2i}\zeta\mathfrak{M}\omega^2v_{\mathbf{a}}\left\{\frac{e^{i\omega t}}{i\omega+D_{ik}a_ia_k/\hbar^2}-\text{c.c.}\right\}.$$

For narrow bands the only terms significant in the sums are the harmonics corresponding to the nearest neighbors, for which the quantity

$$1/\tau_{\mathbf{a}}=D_{ik}a_ia_k/\hbar^2$$

is an invariant independent of $\mathbf{a}$ in cubic crystals. Since $\chi_{\mathbf{a}}=\chi^*_{-\mathbf{a}}$ and $\mathbf{v}$ is an odd function of $\mathbf{p}$, the absorbed energy H for the frequencies (9) and (11) is respectively equal to

$$H=\frac{\pi\zeta^2}{VT}\exp\left\{-\frac{\varepsilon_0}{T}\right\}\sum_{\mathbf{a}}\frac{\omega\tau_{\mathbf{a}}}{1+\omega^2\tau_{\mathbf{a}}^2}|\Lambda_{\mathbf{a}}|^2,\qquad\frac{\omega L}{c}\ll\frac{\Theta}{m_0c^2}, \tag{18a}$$

$$H=\frac{\pi\zeta^2}{3VT}\exp\left\{-\frac{\varepsilon_0}{T}\right\}\mathfrak{M}^2\omega^2L^2\sum_{\mathbf{a}}\frac{\omega\tau_{\mathbf{a}}}{1+\omega^2\tau_{\mathbf{a}}^2}|v_{\mathbf{a}}|^2,\qquad\frac{\Theta}{m_0c^2}\ll\frac{\omega L}{c}\ll1, \tag{18b}$$

where V is the volume of the unit cell.

The absorbed energy for short-wave oscillations (12) is determined by a formula similar to (18b)

$$H=\frac{\pi\zeta^2}{2VT}\exp\left\{-\frac{\varepsilon_0}{T}\right\}\mathfrak{M}^2c^2\sum_{\mathbf{a}}\frac{\omega\tau_{\mathbf{a}}}{1+\omega^2\tau_{\mathbf{a}}^2}|v_{\mathbf{a}}|^2,\qquad\frac{\omega L}{c}\gg1. \tag{18c}$$

We note that, for example, in a primitive cubic lattice the quantities Λ_a are equal to

$$\Lambda_{\mathbf{a}}=\frac{1}{4}\frac{(\mathbf{an})^2}{a^2}\left\{a\frac{\partial\Delta}{\partial a}+\frac{1}{2}\Delta\right\},$$

and the diffusion tensor is diagonal $D_{ik}=D_0\delta_{ik}$ ($\mathbf{n}$ is a unit vector in the deformation direction).

For particles with broad bands, the condition $\Delta\gg T$ means that the particles are located near the bottom of the band, where their spectrum is quadratic[4]:

$$\varepsilon_1(p)=p^2/2M,\quad M\approx2\hbar^2/a^2\Delta, \tag{19}$$

and the quantity Λ depends on the momentum in the form

$$\Lambda=\Lambda_0-\frac{\Lambda_1 p_z^2}{2M},\qquad \Lambda_1=\frac{\partial M}{\partial a}\frac{a}{M}.$$

The solutions of the kinetic equation for the frequencies (9) and (11), with the same accuracy as above, are

$$\chi=\frac{1}{2}\zeta\omega\left\{e^{i\omega t}\left[\frac{\Lambda_0}{i\omega}-\frac{\Lambda_1}{2M}\left(p_z^2+\frac{2D}{i\omega}\right)\Big/\left(i\omega+\frac{2D}{MT}\right)\right]+\text{c.c.}\right\};$$
$$\chi=-\frac{1}{2i}\zeta\omega^2\frac{\mathfrak{M}}{M}\left\{e^{i\omega t}\frac{p_z z}{i\omega+D/MT}-\text{c.c.}\right\},$$

where $D=D_{zz}$. In this case the relaxation time τ_z is proportional to T^{-3}

$$1/\tau_z=2D_{zz}/MT. \tag{20}$$

As a result of integration of (17) we obtain for the absorbed energy the expressions

$$H=\frac{\Delta\zeta^2}{2a^3\pi^{1/2}}\exp\left\{-\frac{\varepsilon_0}{T}\right\}\left(\frac{T}{\Delta}\right)^{5/2}\frac{\omega\tau_z}{1+\omega^2\tau_z^2}\Lambda_1^2,\qquad \frac{\omega L}{c}\ll\frac{\Theta}{m_0c^2}, \tag{21a}$$
$$H=\frac{\zeta^2}{6a^3\pi^{1/2}}\exp\left\{-\frac{\varepsilon_0}{T}\right\}\left(\frac{T}{\Delta}\right)^{3/2}\frac{\omega\tau_z}{1/4+\omega^2\tau_z^2}\frac{\mathfrak{M}^2}{M}\omega^2L^2,\qquad \frac{\Theta}{m_0c^2}\ll\frac{\omega L}{c}\ll 1. \tag{21b}$$

For short-wave oscillations (12) we get

$$H=\frac{\zeta^2}{4a^3\pi^{1/2}}\exp\left\{-\frac{\varepsilon_0}{T}\right\}\left(\frac{T}{\Delta}\right)^{3/2}\frac{\omega\tau_z}{1/4+\omega^2\tau_z^2}\frac{\mathfrak{M}^2}{M}c^2,\qquad \frac{\omega L}{c}\gg 1. \tag{21c}$$

Formulas (18) and (21) describe the energy absorption in helium crystals following compression deformation in a wide range of frequencies and temperatures. We emphasize once more that only at low frequencies (9) can the deformation be regarded as purely uniform in investigations of the internal friction (18a) and (21a). At high frequencies, the main contribution to the absorption is made by the spatial dispersion of the oscillations. The dissipation is then connected with the action of the inertia force on the defect and with a change of the activation energy upon deformation.

3. INTERACTION OF VACANCIONS WITH A CRYSTAL SURFACE

With decreasing temperature, the defecton mean free path, which is determined by the defecton-phonon scattering, increases rapidly like T^{-9} (or T^{-8} at $\Delta\gg T$ (20)). At the same time, there is a well-developed experimental procedure for growing practically idean single crystal of helium, in which the particle mean free path can reach the dimensions of the sample. Under these conditions, the main dissipative mechanism is the scattering of the defectons by the surface of the sample or by the boundaries of the individual crystallites in a polycrystal, and a unique size effect can arise. In this section we consider only thermally activated vacancies with a diagonal tensor $\Omega_{ik}=-\Omega\delta_{ik}$, $\Omega>0$.

Localized vacancies that diffuse in ordinary crystals are always in a state of thermodynamic equilibrium with the lattice. The vacancies can either absorb or be produced on the surface of the crystal and on the boundaries of the crystallites. Consequently, the concentration of the localized vacancies in ordinary crystals near the surface is always in equilibrium and is determined by the vacancy-formation energy $\Omega\sigma_{nn}$ ($\sigma_{nn}=\sigma_{ik}n_in_k$ are the normal stresses on the surface, and $\mathbf{n}$ is a vector normal to the surface). The vacancy concentration at the surface is equal to

$$C=C_0\exp(-\Omega\sigma_{nn}/T), \tag{22}$$

C_0 is the concentration at the unloaded surface. The energy of vacancy production $\Omega\sigma_{nn}$ is equal to the activation term in the particle energy (3)

$$-K\Omega_{ik}\frac{\partial u_i}{\partial x_k}=\frac{1}{3}\Omega\sigma_{ik}\delta_{ik}$$

only in the case of hydrostatic uniform compression, when the stress tensor is diagonal $\sigma_{ik}\propto\delta_{ik}$.

Delocalized vacancions in quantum crystals move by tunneling and not by activation jumps. As they move, they need not necessarily be in equilibrium with the lattice. Thermalization takes place at distances on the order of the mean free path, which increases rapidly with decreasing temperature. The distribution function of the vacancions moving away from the surface, which is determined by the distribution of the emitted and reflected particles, may turn out to be in disequilibrium. It will be shown below that at low temperatures the reflection coefficient of the vacancions from the surface is close to unity, and the absorption and emission coefficients are small, i.e., the accommodation of the vacancions by the surface is insignificant.

The interaction of vacancions with the wall is conveniently described in terms of scattering theory, just as in the case of interaction of vacancions with other point defects.[17,19] Thus, at low temperatures the vacancion spectrum is close to quadratic (19), and from among all the reaction channels there is separated a channel corresponding to the conservation of the projection of the quasimomentum of the particle on the surface and to a reversal of the sign of the component normal to the surface. The probability of such an elastic scattering, i.e., specular reflection, will be designated $P(\mathbf{p})$. In the course of reflection, owing to random roughnesses of the surface, there is a probability $Q(\mathbf{p})$ of losing the information concerning the direction of the quasimomentum of the incident particles. The inelastic processes correspond also to absorption of particles by the surface, and the absorption probability is equal to the emission probability $S(\mathbf{p})$. The emitted particles have an equilibrium distribution function $e^{-\varepsilon/T}$, and the expression for the energy includes the vacancion formation energy $\Omega\sigma_{nn}$.

The law governing the interaction of the vacancions with the surface can be expressed in the form of a boundary condition that connects the distribution functions n_+ and n_- of the particles arriving at the surface and leaving the surface:

$$n_-(\mathbf{p})=P(\mathbf{p})n_+(\mathbf{p})+Q(\mathbf{p})\langle n_+(\mathbf{p})\rangle+S(\mathbf{p})e^{-\varepsilon/T},\quad P+Q+S=1, \tag{23}$$

where the angle brackets denote averaging over the directions of the quasimomentum at constant energy.

At low temperatures, it becomes possible to determine the absorption coefficient $S(\mathbf{p})$ in general form, without using model representations concerning the structure of the surface and concerning the form of the interaction potential. The condition $T \ll \Delta$ means that the vacancions are near the bottom of the band (19), where their velocity is low and the wavelength is large $pa/\hbar \ll 1$. It is known that the problem of scattering of slow particles in quantum mechanics admits of a solution at any scattering potential that tends sufficiently rapidly to zero at infinity. The absorption and emission of vacancions is due to processes that develop over atomic distances, and do not depend on the macroscopic roughnesses of the surface. It is easy to show that the probability of inelastic processes of this scale, in the case of scattering by a one-dimensional potential, is low[20]:

$$S(\mathbf{p}) = \alpha p_n a/\hbar \ll 1, \tag{24}$$

where α is a certain constant of the order of unity and p_n is the vacancion-momentum component normal to the surface.

The deviation of the specularity coefficient $P(\mathbf{p})$ from unity for a surface with macroscopic roughnesses is determined not by the absorption and emission of the particles (24), but by the fact that the direction of the normal at each point of the surface is in practice a random quantity. For long-wave particles $P \approx 1 - Q$ can be expressed in terms of the statistical properties of the surface.[20] For a sufficiently smooth surface and low temperatures, the specularity coefficient P is always close to unity ($Q \ll 1$).

The energy absorbed in elastic collisions of a single crystal with a large mean free path of the particles depends essentially on the actual geometry of the sample. For an infinite plate of thickness L and perpendicular to the direction of the deformation (10), the boundary condition (23) becomes

$$\chi_- = P(\mathbf{p})\chi_+ + Q(\mathbf{p})\langle\chi_+\rangle - \zeta S(\mathbf{p})\frac{\Omega E(1-\sigma)}{(1+\sigma)(1-2\sigma)}\sin\omega t. \tag{25}$$

Here E is Young's modulus, σ is the Poisson coefficient, and χ determines the deviation of the distribution function from n_e (5):

$$n = n_e(1 + \chi/T).$$

In the kinetic equation, at not too high frequencies $\omega L/c \ll \Theta(T\Delta)^{-1/2}$, we can neglect the right-hand side and, at sufficiently low temperatures we can neglect the collision integral

$$\frac{\partial\chi}{\partial t} + v_z\frac{\partial\chi}{\partial z} = 0. \tag{26}$$

The solution of the kinetic equation (26) with the boundary condition (25), accurate to small terms proportional to $Q(\mathbf{p})S(\mathbf{p})$, takes the form

$$\chi_- = -\zeta S(\mathbf{p})\frac{\Omega E(1-\sigma)}{(1+\sigma)(1-2\sigma)}\frac{\sin\omega t - P(\mathbf{p})\sin(\omega t + \omega\vartheta)}{1+P^2-2P\cos\omega\vartheta}, \quad \chi_+ = \chi_-(t-\vartheta),$$

where we have introduced the vacancy on time of flight through the plate $\vartheta(\mathbf{p}) = L/v_z$.

The energy absorbed by the surface is determined by the difference between the energy fluxes of the particles reaching the surface and moving away from it. The energy absorbed during one period of the oscillations, normalized to unit volume of the sample, is given, when (24) is taken into account, by

$$H = \frac{2\alpha T}{\pi a^3\Delta^2}\left[\frac{\zeta\Omega E(1-\sigma)}{(1+\sigma)(1-2\sigma)}\right]^2 \exp\left\{-\frac{\varepsilon_0}{T}\right\} \times \frac{1}{\eta}\int e^{-x^2}\frac{(1+P)(1-\cos\eta x)x^2\,dx}{1+P^2-2P\cos\eta x}, \tag{27}$$

where $x = p_z/(2MT)^{1/2}$, $\eta = \omega L\hbar/(a^2T\Delta)^{1/2}$ is the ratio of the time of flight of the vacancion with thermal velocity through the plate to the period of the oscillations.

If the time of flight is short in comparison with the period of the oscillations, and the reflection coefficient P is not too close to unity, $\eta \ll 1 - P$, then the energy absorbed per unit volume is proportional to the plate thickness L:

$$H = \frac{\alpha T}{\pi a^3\Delta^2}\left[\frac{\zeta\Omega E(1-\sigma)}{(1+\sigma)(1-2\sigma)}\right]^2 \exp\left\{-\frac{\varepsilon_0}{T}\right\}\eta\int\frac{1+P}{(1-P)^2}e^{-x^2}x^4\,dx. \tag{28}$$

In the opposite limiting case of large flight times $\eta \gg 1$ the absorption does not depend on the reflection coefficient:

$$H = \frac{\alpha T}{2\pi^{1/2}a^3\Delta^2}\left[\frac{\zeta\Omega E(1-\sigma)}{(1+\sigma)(1-2\sigma)}\right]^2\frac{\exp\{-\varepsilon_0/T\}}{\eta}. \tag{29}$$

For polycrystalline samples the expressions for the absorbed energy (27)–(29) should be averaged over the dimensions L of the crystallites. We note that the wall between the blocks constitutes a set of parallel dislocations, and the characteristic dimension of the inhomogeneity of the wall, which is equal to the distance between the dislocations, is expressed in terms of the block disorientation angle φ by the relation a/φ.

The "classical" boundary condition (22), i.e., complete accommodation of the vacancions to the surface, corresponds to the following relation between the reflection coefficients in the boundary condition (23):

$$P = Q = 0, \quad S = 1.$$

The value obtained in this case for the absorbed energy at low temperatures is greatly overestimated and is characterized by a different temperature dependence in comparison with expression (27)–(29).

The foregoing approach to the interaction of vacancions with the surface should be used also in another extensive group of problems, connected with diffusion-viscous flow of crystalline bodies. In this case the diffusion equations and the classical boundary condition (22) (see[21, 22]) should also be replaced by the kinetic equation and by the boundary condition (23). Just as in the case of internal friction, many of the considered effects turn out in this case to be proportional to the absorption coefficient.

Let us consider, for example, the motion of a macroscopic foreign inclusion frozen into solid helium, which is a problem of practical interest.[23,24] If an external force **F** is applied to such an inclusion, in the form of a sphere or radius r_0, then the stresses produced in the surrounding helium are ($\mathbf{n}=\mathbf{r}/r$)

$$\sigma_{ik}=-\frac{1}{8\pi r^2(1-\sigma)}\Big\{(1-2\sigma)(F_in_k+F_kn_i)+2\sigma\mathbf{F}\mathbf{n}\delta_{ik}$$
$$+(3n_in_k-\delta_{ik})\mathbf{F}\mathbf{n}+\frac{3r_0^2}{r^2}(F_in_k+F_kn_i+\delta_{ik}\mathbf{F}\mathbf{n}-5n_in_k\mathbf{F}\mathbf{n})\Big\}.$$

Just as above, we neglect the collision integral in the kinetic equation. The distribution function of the vacancions arriving at the sphere can be easily expressed with the aid of the known integrals of motion in terms of the distribution function of the particles at infinity n_e (5):

$$n_+=n_e\exp\{-{}^1/_3\Omega\sigma_{ik}\delta_{ik}/T\}.$$

The boundary condition (23) at $r=r_0$ leads to the following value of the distribution function of the outgoing particles:

$$n_-=n_++S(\mathbf{p})n_e\exp\left\{\frac{\Omega}{T}\left(\frac{1}{3}\sigma_{ik}\delta_{ik}-\sigma_{ik}n_in_k\right)\right\}.$$

The normal component of the velocity of the surface element of the sphere is determined by the normal component of the vacancy flux on the surface of the sphere:

$$\Omega\int d\Gamma(n_+-n_-)v_n,\qquad v_n=\frac{\partial\varepsilon}{\partial\mathbf{p}}\mathbf{n}.$$

As a result, the sphere as a unit has a velccity

$$\mathbf{U}=\frac{\alpha\mathbf{F}\Omega(1+\sigma)}{6\pi^{5/2}r_0^2T(1-\sigma)}\exp\left\{-\frac{\varepsilon_0}{T}\right\}\frac{\Delta a}{\hbar}\left(\frac{T}{\Delta}\right)^{5/2}\frac{\Omega}{a^3}.$$

This expression has a simple physical meaning: the excess vacancion concentrations on the surface of the sphere is of the order of

$$\frac{\alpha F\Omega(1+\sigma)}{r_0^2T(1-\sigma)}\exp\left\{-\frac{\varepsilon_0}{T}\right\}\left(\frac{T}{\Delta}\right)^2,$$

and the vacancion velocity is

$$v\sim\frac{\Delta a}{\hbar}\left(\frac{T}{\Delta}\right)^{1/2}.$$

In this case, just as in the study of internal friction, the use of the condition of total accommodation of the vacancions $S=1$ leads to a significant overestimate of the transport velocity of the sphere at low temperatures.

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Note added in proof (20 *July* 1976). Data on the damping of ultrasound (5–10 MHz) in He^4 at $T=1.7$ °K in a molar volume 20.5 cm^3 were recently published by Y. Hiki and F. Tsukuoka (Phys. Lett. **56A**, 484, 1976). For vacancions under these conditions we have $\varepsilon_0\sim 10$ °K and $\Delta\sim T$ (K. O. Keshishev, Zh. Eksp. Teor. Fiz., in press), and the experimental value of the absorbed energy agrees fully with the damping decrement calculated for the vacancions (18c), (21c) at $\mathfrak{M}\sim m_0$. An estimate $b\sim 4$ is obtained for the relaxation time $\tau\sim(\hbar/\Theta)(\Theta/bT)^9$.

[1)] The ratio Θ/m_0c^2, which is similar to the known de Boer parameter[12] is small in ordinary crystals; this quantity characterizes the role of the quantum effects and shows the degree to which the considered crystal is of the quantum type.[13] In solid helium at large molar volumes, Θ/m_0c^2 is close to unity and the frequency region (11) is quite narrow.

[1] A. F. Andreev and I. M. Lifshitz, Zh. Eksp. Teor. Fiz. **56**, 2057 (1969) [Sov. Phys. JETP **29**, 1107 (1969)].

[2] R. A. Guyer and L. I. Zane, Phys. Rev. Lett. 24, 660 (1970).

[3] E. L. Andronikashvili, I. A. Gachechiladze, and V. A. Melik-Shakhnazarov, Fiz. Nizkikh Temp. 1, 635 (1975) [Sov. J. Low Temp. Phys. 1, 305 (1975).

[4] A. É. Meĭerovich, Zh. Eksp. Teor. Fiz. 67, 744 (1974) [Sov. Phys. JETP 40, 368 (1975)].

[5] S. Hunklinger, P. Leiderer, and P. Berberich, Proc. Second Intern. Conf. on Light Scattering in Solids, ed. M. Balkanski, Flammarion, Paris, 1971, p. 453; P. Leiderer, P. Berberich, S. Hunklinger, and K. Dransfeld, Low Temp. Phys., LT 13, 2, Plenum, New York, 1974, p. 53; P. Berberich, P. Leiderer, and S. Hunklinger, J. Low Temp. Phys. 22, 61 (1976).

[6] Y. Hiki and F. Tsuruoka, Low Temp. Phys. LT 14, 1, Helsinki, 1975, p. 479.

[7] A. É. Meĭerovich, Zh. Eksp. Teor. Fiz. 69, 1325 (1975) [Sov. Phys. JETP 42, 676 (1976)].

[8] V. M. Kontorovich, Zh. Eksp. Teor. Fiz. 45, 1638 (1963) [Sov. Phys. JETP 18, 1125 (1964)].

[9] A. I. Akhiezer, M. I. Kaganov, and G. Ya. Lyubarskiĭ, Zh. Eksp. Teor. Fiz. 32, 837 (1957) [Sov. Phys. JETP 5, 685 (1957)].

[10] V. L. Gurevich, Zh. Eksp. Teor. Fiz. 37, 1680 (1959) [Sov. Phys. JETP 10, 1190 (1960)].

[11] A. M. Kosevich, Osnovy mekhaniki kristallicheskoĭ reshetki (Principles of Crystal Lattice Mechanics), Nauka, 1972, Sec. 13.

[12] J. de Boer, Physica 14, 139 (1948).

[13] A. F. Andreev, Usp. Fiz. Nauk 118, 251 (1976) [Sov. Phys. Usp. 19, 137 (1976)].

[14] L. D. Landau, Zh. Eksp. Teor. Fiz. 7, 203 (1937) (Sobr. Trudov [Collected Works] 1, Nauka, 1969, p. 199).

[15] D. I. Pushkarov, Zh. Eksp. Teor. Fiz. 59, 1755 (1970): 68, 1471 (1975) [Sov. Phys. JETP 32, 954 (1971); 41, 735 (1975)].

[16] Yu. Kagan and L. A. Maksimov, Zh. Eksp. Teor. Fiz. 65, 622 (1973) [Sov. Phys. JETP 38, 307 (1974)].

[17] A. F. Andreev and A. E. Meĭerovich, Zh. Eksp. Teor. Fiz. 67, 1559 (1974) [Sov. Phys. JETP 40, 776 (1975)].

[18] E. I. Blount, Phys. Rev. 114, 418 (1959).

[19] A. É. Meĭerovich, Zh. Eksp. Teor. Fiz. 68, 1477 (1975) [Sov. Phys. JETP 41, 738 (1975)].

[20] A. F. Andreev, Usp. Fiz. Nauk 105, 113 (1971) [Sov. Phys. Usp. 14, 609 (1972)].

[21] C. Herring, J. Appl. Phys. 21, 437 (1950).

[22] I. M. Lifshitz, Zh. Eksp. Teor. Fiz. 44, 1349 (1963) [Sov. Phys. JETP 17, 909 (1963)].

[23] A. Andreev, K. Keshishev, L. Mezhov-Deglin, and A. Shal'nikov, Pis'ma Zh. Eksp. Teor. Fiz. 9, 507 (1969) [JETP Lett. 9, 306 (1969)].

[24] V. B. Shikin, Zh. Eksp. Teor. Fiz. 61, 2053 (1971) [Sov. Phys. JETP 34, 1095 (1972)].

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