Magnetic inversion of the Hall effect in tellurium

M. V. Glushkov, E. S. Itskevich, Yu. V. Kosichkin,
A. I. Nadezhdinskiĭ, A. N. Tolmachev, and A. M. Shirokov

Institute of High Pressure Physics, USSR Academy of Sciences;
and P. N. Lebedev Physics Institute, USSR Academy of Sciences
(Submitted March 30, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1239–1246 (September 1976)

A change in sign of the Hall coefficient following an increase of the magnetic field intensity is observed in the mixed conductivity region in tellurium. The effect of temperature and of hydrostatic pressure up to 10 kbar on the Hall coefficient, the magnetoresistance, and the resistivity is investigated. It is shown that the electron and hole densities and mobilities and their variation with temperature and pressure can be determined on the basis of the values of the inversion field strength and of the magnetoresistance at the aforementioned field intensity in conjunction with the values of the resistivity and of the Hall coefficient in weak fields.

PACS numbers: 72.15.Gd

INTRODUCTION

A characteristic feature of semiconductors having impurity conductivity of p-type is that the sign of the Hall effect is reversed with increasing temperature (temperature inversion), owing to the appearance of proper carriers in the conduction band. The position of the inversion point on the temperature scale depends in this case essentially on the intensity of the magnetic field.

It was shown in a number of papers (see, e.g., [1] for p-Ge and [2] for p-InSb) that by investigating the influence of the magnetic field on the Hall effect in the temperature inversion region and by comparing the results with an appropriate theoretical model it is possible to determine the concentrations and the mobilities for both the holes and the electrons, and also to draw certain conclusions concerning the scattering mechanisms in the investigated temperature range. A similar approach is quite useful as applied to tellurium, inasmuch as it uncovers a possibility for studying the properties of the carriers in the conduction band of this semiconductor, which is known exclusively as a p-type semiconductor. Zimmermann and Stockmann[3] have described the influence of pulsed magnetic fields on the temperature of the inversion of the Hall effect in tellurium, but drew no conclusions whatever concerning the properties of the carriers. It is obvious that this experiment can be inverted, i.e., it is possible to investigate the influence of the temperature on the dependence of the Hall coefficient on the magnetic field.

The present study is the first investigation of the magnetic inversion of the Hall effect in tellurium—the reversal of the sign of the Hall coefficient with increasing magnetic field. It is shown that the physical parameters of the inversion and of the magnetoresistance at this value of the field, in conjunction with the conductivity and the Hall coefficient in weak fields, make it possible to determine the concentrations and mobilities of the electrons and holes.

Inasmuch as it was observed earlier[4] that hydrostatic pressure has a strong influence on the band structure of tellurium, we used the method of magnetic inversion also to study the properties of the carriers at hydrostatic pressures up to 10 kbar.

FUNDAMENTAL RELATIONS FOR THE CONCENTRATIONS AND MOBILITIES OF THE CARRIERS

In the general case, the Hall coefficient and the magnetoresistance of a semiconductor are determined by the relations

$$R=-\frac{c}{H}\frac{\sigma_{12}}{\sigma_{11}^2+\sigma_{12}^2},\quad \rho=\frac{\sigma_{11}}{\sigma_{11}^2+\sigma_{12}^2}, \tag{1}$$

where σ_{11} and σ_{12} are the components of the conductivity tensor, and H is the magnetic field intensity.

In the region of mixed (two-band) conductivity we have

$$\sigma_{11}=\frac{en\mu_n A_{1n}}{1+(\mu_n H/c)^2}+\frac{ep\mu_p A_{1p}}{1+(\mu_p H/c)^2}, \tag{2a}$$

$$\sigma_{12}=\frac{H}{c}\left(\frac{en\mu_n^2 A_{2n}}{1+(\mu_n H/c)^2}-\frac{ep\mu_p^2 A_{2p}}{1+(\mu_p H/c)^2}\right), \tag{2b}$$

where p, n, μ_p, and μ_n are respectively the concentrations and mobilities of the holes and electrons, A_{ij} ($i=1,2$; $j=n,p$) are integral quantities that depend little on the magnetic field:

$$A_{ij}=\left\langle\frac{(\mu_j(\varepsilon))^i}{[1+(\mu_j(\varepsilon)H/c)^2]}\right\rangle\left(1+\left(\frac{\langle\mu_j(\varepsilon)\rangle H}{c}\right)^2\right)\Big/\langle\mu_j(\varepsilon)\rangle^i, \tag{3}$$

where $\mu_j(\varepsilon)=e\tau(\varepsilon)/m_j$ are the mobilities of the carriers with energy ε, and the averaging was carried out in accordance with the formula

$$\langle g(\varepsilon)\rangle=\frac{4}{3\sqrt{\pi}}\int_0^\infty g(\varepsilon)\varepsilon^{3/2}e^{-\varepsilon}\,d\varepsilon.$$

In the derivation of (2) and (3) we used a Maxwell–Boltzmann distribution, inasmuch as tellurium is a nondegenerate semiconductor in the considered temperature interval. In addition, it was assumed that the valence band and the conduction band are described by a quadratic dispersion law, and the carrier relaxation time τ does not depend on the magnetic field and is a

Copyright © 1977 American Institute of Physics

power-law function of the energy $\tau = \tau_0 \varepsilon^s$, where s is determined by the scattering mechanism.

The sign of the Hall coefficient depends on the sign of σ_{12}, i.e., on the ratio of the terms in expression (2b). In weak magnetic fields, R is negative, inasmuch as $\mu_n > \mu_p$. In strong fields, the sign of the Hall coefficient depends on the type of the impurity conductivity, and in the case of a p-type semiconductor we have $R > 0$. Consequently, there should exist a magnetic field $H_{R=0}$ (inversion field) at which the Hall coefficient R vanishes.

At the inversion point, expression (1) greatly simplifies and it becomes possible to obtain relations that connect the concentrations and mobilities of the electrons and holes with the easily measured quantities $H_{R=0}$ (magnetoresistance and inversion field):

$$H_{R=0}^2 = c^2 \frac{n\mu_n^2 A_{2n}' - p\mu_p^2 A_{2p}'}{\mu_n^2 \mu_p^2 (pA_{2p}' - nA_{2n}')}, \tag{4a}$$

$$\rho_{R=0} = \frac{(\mu_n^2 - \mu_p^2) A_{2p}' A_{2n}'}{e\mu_n \mu_p (nA_{2p}' - pA_{2n}')(A_{1p}' A_{2n}' \mu_n + A_{1n}' A_{2p}' \mu_p)}. \tag{4b}$$

Here all the integral quantities are taken in the inversion field $A_{ij}' = A_{ij}(H_{R=0})$. In addition, we have expressions for the resistivity and for the Hall coefficient in a weak magnetic field:

$$\rho_0 = (ep\mu_p + en\mu_n)^{-1}, \tag{4c}$$

$$R_0 = -\frac{A_2}{e} \frac{(n\mu_n^2 - p\mu_p^2)}{(n\mu_n + p\mu_p)^2}, \tag{4d}$$

where $A_2 = \langle \mu^2(\varepsilon) \rangle / \langle \mu(\varepsilon) \rangle$.

Solving the system (4) for some concrete scattering mechanism, we can determine p, n, μ_p, and μ_n at all temperatures and pressures at which the magnetic inversion of the Hall effect can be observed.

EXPERIMENTAL RESULTS

We have investigated single-crystal tellurium samples with Hall density $p_A = (1.5\text{–}4) \times 10^{15}$ cm^{-3}. The samples were made of ingots grown by the Czochralski method. These ingots were cut in the direction perpendicular to the trigonal axis C_3 with a filament moistened in a chromium etchant, and were cleaved in the C_3 direction. The contacts were soldered with bismuth. The dimensions of the samples were limited by the diameter of the high-pressure chamber channel and amounted to $\sim 6 \times 1 \times 1$ mm^3.

The measurements were performed with both transverse ($I \perp C_3 \parallel H$) and longitudinal ($I \parallel C_3 \perp H$) samples in

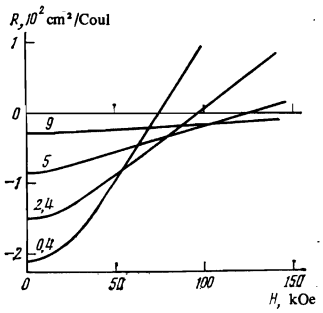

FIG. 1. Dependence of the Hall coefficient on the magnetic field intensity at various pressures (kbar), $T = 290$ °K.

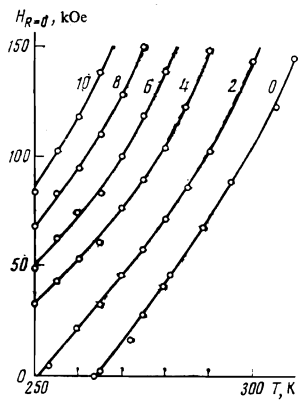

FIG. 2. Influence of temperature on the inversion field at various pressures (kbar).

stationary magnetic fields up to 150 kOe[5] at pressures up to 10 kbar and temperatures in the interval 240–320 °K. The sample resistance was measured by a null method. The Hall voltage was determined from measurements made at opposite directions of the magnetic field, followed by averaging. To produce the pressure we used an autonomous high-pressure cell[6] made of nonmagnetic materials. The pressure-transmitting medium was a mixture of transformer oil and kerosene.

The first four figures show the baric and temperature dependences of those experimentally determined quantities which are needed to solve the system (4). The data presented in these figures were measured on one and the same transverse sample and are typical of all the samples investigated by us. The curves of Fig. 1 illustrate the inversion of the Hall effect in a magnetic field for several values of the hydrostatic pressure at constant temperature. The inversion field depends significantly on the pressure and temperature, owing to the change of the mobility and density of the carriers (Fig. 2). Inasmuch as in tellurium the width of the forbidden band decreases under pressure, the density of the majority carriers will increase with increasing pressure (as well as with increasing temperature), and therefore the inversion point should shift towards stronger magnetic fields (see formula (4a)).

The complicated dependence of the magnetoresistance in the inversion field $\rho_{R=0}$ on the pressure (Fig. 3) is due to the simultaneous change of the concentration and mobility of the carriers with changing pressure.

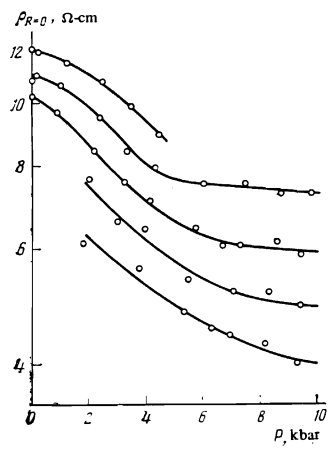

FIG. 3. Dependence of the magnetoresistance in the inversion fields on the pressure at the temperatures 290, 280, 270, 260, and 250 °K (reading downward).

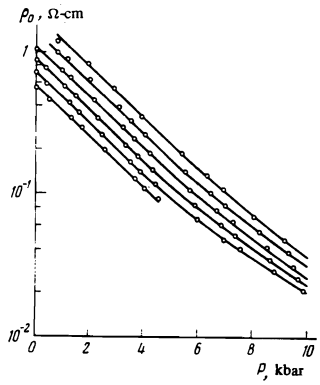

FIG. 4. Effect of pressure on the resistivity of the sample at the temperatures 250, 260, 270, 280, 290, and 300 °K (reading downward).

It was noted earlier[4] that the resistivity ρ_0 decreases with changing pressure at helium temperatures, i.e., in the region of the impurity conductivity; ρ_0 decreased by approximately a factor of 4 when the pressure was increased to $P=10$ kbar, while the variation of $\ln\rho_0(P)$ was linear. In the present study, ρ_0 decreased by an approximate factor of 30 at a pressure 10 kbar (Fig. 4), and the behavior of $\ln\rho_0(P)$ deviated from linear. So large a change of ρ_0 is due primarily to the increase of the density of the proper carriers, due to the decrease of the width of the forbidden band.

DISCUSSION OF RESULTS

Equations (4) become quite simple if it is assumed that the relaxation time is independent of the energy ($s=0$). Then the integral quantities A_{ij} become equal to unity, and Eq. (4d) can be replaced by

$$\frac{R(H)}{R_0}=1-\left(\frac{\mu_n\mu_p H^2_{R=0}}{c^2}\right)^{-1}\left(\frac{\rho(H)-\rho_0}{\rho_0}\right). \tag{5}$$

The solution of the obtained system of equations makes it possible to determine all the quantities (n, p, μ_n, μ_p). The dependence of $R(H)/R_0$ on $\Delta\rho(H)/\rho_0$ (see (5)) should be linear at $s=0$. The experimentally obtained dependence (Fig. 5), however, deviates from linearity in weak fields. This indicates that it is necessary to take into account the dependence of the relaxation time on the energy. We note in this connection that extrapolation of the linear sections of the plot of $R(H)/R_0$ against $\Delta\rho(H)/\rho_0$ to zero leads to an approximately constant value $R_{\Delta\rho=0}/R_0=0.75$ in the entire range of employed pressures. Neglecting the terms $(p_A/n)^2$, we can obtain from (2)

$$R_{\Delta\rho=0}/R_0\approx\langle\mu^{-1}(\varepsilon)\rangle^{-2}\langle\mu^2(\varepsilon)\rangle^{-1}. \tag{6}$$

The value of $R_{\Delta\rho=0}/R_0$ calculated from (6) was ~0.60 and ~0.71 respectively for the cases $s=-\frac{1}{2}$ (scattering by acoustic phonons) and $s=\frac{1}{2}$. It can be assumed that the true exponent of the energy dependence of τ is close to $|s|=\frac{1}{2}$.

We indicate, however, that in the case $s=0$ we can obtain a very simple expression for the impurity concentration of the holes $p_A=p-n$:

$$p_A=\frac{R(H)H^2_{R=0}}{ec^2(\rho(H)-\rho_{R=0})(\rho_{R=0}-\rho_0)}. \tag{7}$$

The value of p_A calculated from formula (7) turned out to be independent of temperature and pressure within 15%. In addition, this quantity coincides, accurate to 10%, with the value of p_A obtained by measuring the Hall effect in the same samples in the impurity conductivity region (77 °K). Thus, the value of p_A is determined with sufficient accuracy also for $s=0$. Assuming that $A'_{in}=A'_{ip}$ and considering the data for the region $(\mu H_{R=0}/c)^2>1$, we can estimate, by using (4), the difference between the values of p_A for the cases $s=0$, $s=-\frac{1}{2}$, and $s=\frac{1}{2}$. The estimate has shown that the difference does not exceed 5%. Therefore, even though the system (4) can be solved completely at any value of s, we have subsequently used the value of p_A obtained for $s=0$, and by the same token decreased the number of equations.

We report below the results of the reduction of the experimental data assuming scattering by acoustic phonons.

An important characteristic of a semiconducting material in the mixed-conductivity region is the ratio of the electron mobility to the hole mobility, $b=\mu_n/\mu_p$. Calculation has shown that b increases weakly with increasing temperature (by approximately 10% when heated from 240 to 300 °K) in the entire investigated pressure interval. In the investigated temperature interval, b does not depend on the pressure within the range ~5%; thus, for example, for 280 °K we have obtained $b=1.40\pm0.05$ for two transverse samples and 1.8 ± 0.1 for a longitudinal sample in the entire pressure interval from 0 to 9 kbar. We note that the absolute value of b is determined with lower accuracy, owing to the low accuracy with which the distances between the contacts are determined.

Calculation has shown that the mobility of the electrons and the holes depends exponentially on the pressure in the entire investigated temperature interval (Fig. 6), $\mu=\mu_0 e^{\beta P}$. The baric exponent β is the same for both types of carrier: $\beta=0.062\pm0.007$, and is the same within 20% for transverse and longitudinal samples. Koma *et al.*[7] have established that the mobility of the holes varies exponentially with pressure, but in the region of the impurity conductivity (77 °K). In the pressure interval investigated by us, the electron mobility decreases with increasing temperature like $T^{-2.5}$, and the hole mobility like $T^{-2.7}$, the exponent being determined accurate to about 10%.

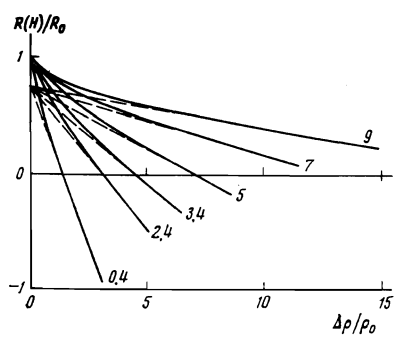

FIG. 5. Dependence of the relative Hall coefficient on the relative magnetoresistance at various pressures (kbar).

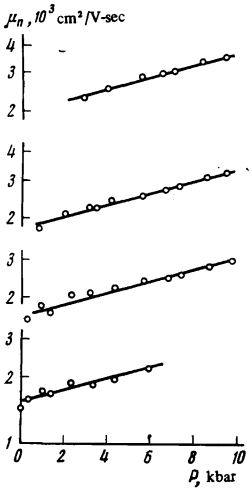

FIG. 6. Effect of pressure on the electron conductivity at various temperatures: 260, 270, 280, and 290 °K (reading downward).

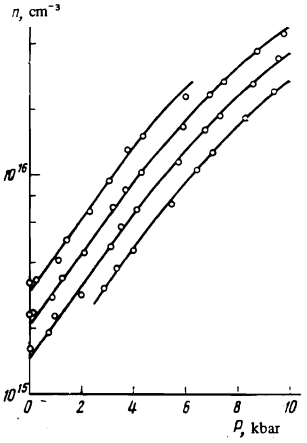

FIG. 7. Dependence of the free-carrier density on the pressure at the temperatures 290, 280, 270, and 260 °K (reading downward).

The concentration of the proper carriers (Fig. 7) turned out to be a complicated function of the pressure. The growth of the density with increasing pressure is due mainly to the decrease of ε_g. These data make it possible to determine the change of the width of the forbidden band of tellurium with changing pressure. A similar reduction of the experimental data for the case $s = +\frac{1}{2}$ has shown that no qualitative differences are observed in the behavior of the densities and mobilities of the carriers. At the same time, the numerical values differ by approximately 10–20%. Using the known relations between the density and the mobility, on the one hand, and the effective mass, on the other, we can calculate, within the limits of our assumptions concerning the band structure and scattering by acoustic phonons, the effective masses of the electrons and holes. Our calculations have yielded the following results at room temperatures ($\varepsilon_{g0} = 325$ MeV):

$$m_n m_p = (0.076 \pm 0.005)\, m_0^2.$$
$$m_n = 0.25 m_0. \quad m_{\perp n} = 0.18 m_0, \quad m_{\parallel n} = 0.13 m_0.$$
$$m_p = 0.30 m_0. \quad m_{\perp p} = 0.20 m_0. \quad m_{\parallel p} = 0.18 m_0.$$

where $m_j = (N^2 m_{1j}^2 m_{\parallel j})^{1/3}$ and $N = 2$.

We note that we can determine reliably only the quantity $m_n m_p$, while the remaining quantities can only be estimated. The point is that in the calculations of the quantity $m_n m_p$ we can use the experimental values for only transverse samples and calculated values that depend little on the scattering mechanism. On the other hand, when it comes to calculating the components of the effective-mass tensor, an important role is played by the scattering mechanism, and in this case the results used are obtained with longitudinal samples and are less accurate, and furthermore, the assumptions used in the calculations are not sufficiently correct in this case. Nonetheless, our result for $m_{\perp p}$ is in satisfactory agreement with the result obtained in[8] from measurements of the Faraday rotation at 300 °K.

We have also calculated the baric dependences of the effective masses. From the baric dependences of the mobilities, recognizing that within 15%, we found that $m^{-1}\partial m/\partial P = (-2.5 \pm 0.5) \times 10^{-2}$ kbar^{-1} for both the longitudinal and the transverse mass. The results of an investigation of the magnetic inversion of the Hall effect enable us to estimate the baric dependences of the effective density of state and of the width of the forbidden band. From these data, which will be published separately, we obtained $m^{-1}\partial m/\partial P = (-1.8 \pm 0.6) \times 10^{-2}$ kbar^{-1}, which is in good agreement with the preceding results.

Thus, an investigation of the inversion of the Hall effect in a magnetic field makes it possible to determine the principal characteristics of the carriers in the valence band and in the conduction band, as well as their dependence on the temperature and pressure.

The authors thank Academician L. F. Vereshchagin and Academician A. M. Prokhorov for interest in the work and for support.

[1]R. K. Willardson, T. C. Harman, and A. C. Beer, Phys. Rev. 96, 1512 (1954).
[2]D. J. Howarth, R. H. Jones, and E. H. Putley, Proc. Phys. Soc. Lond. 70B, 124 (1957).
[3]W. Zimmermann and F. Stockmann, Phys. Status Solidi 28, K101 (1968).
[4]V. B. Anzin, M. S. Bresler, I. I. Farbstein, E. S. Itskevich, Yu. V. Kosichkin, W. A. Sukhoparov, A. S. Telepnev, and V. G. Veselago, Phys. Status Solidi [b] 48, 531 (1971).
[5]V. G. Veselago, L. P. Maksimov, and A. M. Prokhorov, Prib. Tekh. Eksp. 4, 192 (1968).
[6]E. S. Itskevich, A. N. Voronovskiĭ, A. F. Gavrilov, and V. A. Sukhoparov, Prib. Tekh. Eksp. No. 6, 161 (1966).
[7]A. Koma, T. Tani, and S. Tanaka, Phys. Status Solidi [b] 66, 669 (1974).
[8]M. Lutz, H. Stolze, and P. Grosse, Phys. Status Solidi [b] 62, 665 (1974).

Erratum: Diagram technique and gas approximation in the Hubbard model [Sov. Phys. JETP 43, 574–579 (1976)]

R. O. Zaitsev

Zh. Eksp. Teor. Fiz. **71**, 1247 (1976)

PACS numbers: 99.10.+g, 75.10.Lp

1. At a concentration close to unity, the amplitude for magnon scattering by holes was calculated in the Born approximation. If we replace the Born amplitude by the exact amplitude, i.e., solve the Bethe-Salpeter equation for the scattering of a magnon by a hole, then the diagram method yields the same result as the Nagaoka theory. For fcc and hcp lattices, we can proceed further and find the critical concentration c_0, starting with which the ferromagnetic phase becomes unstable, $1-c_0=a\exp(-\pi^2/4)$ (a is a constant on the order of unity).

2. The density of states $mp_0/2\pi^2$ used in the ferromagnetic condition (formula (36)) should be replaced in the case of bcc and fcc lattices by $p_0/8t\pi^2$ and $p_0/16t\pi^2$, since the volume of the unit cell is, respectively, $\frac{1}{2}$ and $\frac{1}{4}$ of the unit cell of the primitive cubic lattice. As a result, despite the increase in the number of nearest neighbors, the critical concentration is large because of the strong decrease of the density of state, and ferromagnetism in the gas phase is impossible.

 Copyright © 1977 American Institute of Physics