# Analogy between the formation of spiral arms of galaxies and density waves in a rotating laboratory plasma

A. G. Morozov, V. G. Faĭnshteĭn, and A. M. Fridman

*Institute of Terrestrial Magnetism, the Ionosphere, and the Propagation of Radio Waves, Siberian Branch, USSR Academy of Sciences*
(Submitted March 25, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1249–1262 (October 1976)

The possibility of analogy between the formation of the spiral structure of galaxies and density waves in a rotating laboratory plasma is discussed. It is shown in different examples that in gravitating systems one can have a Kelvin–Helmholtz instability and a flute instability with growth rates much larger than the Jeans growth rate. Under certain conditions, the growth rates of these instabilities are equal to the growth rates of the corresponding plasma instabilities.

PACS numbers: 52.35.En, 95.50.+y, 98.20.Di

## §1. INTRODUCTION

The possibility raised in the title of this paper is not new: the similarity between photographs of the spiral arms of galaxies and plasmoids in a laboratory experiment was first noted by Bostick[1] more than twenty years ago. When plasmoids are shot from two or more points into one point of space, a photograph taken when the plasmoids meet really does recall photographs of spiral galaxies. In Bostick's experiments, the analogy was not taken further than the purely superficial similarity at the time of encounter of the plasmoids, each of which Bostick identified with a "spiral" arm. Thus, the number of "arms" (according to Bostick) is exactly equal to the number of plasma guns, and of course such an analogy cannot pretend to be a serious consideration. Nevertheless, if one considers the analogy between the diverse spiral structures of galaxies and objects of more modest dimensions, one's attention is drawn first of all to rotating masses of gas and plasma: we are familiar with satellite photographs of cylones and anticylones, the spiral structure in an eddy, and photographs of density waves in a rotating plasma.[2] Figure 1 shows characteristic spiral structures of plasma density waves obtained on our VP-1 machine.

In this paper, we wish to put forward a number of arguments in favor of the idea that the striking superficial similarity in the forms of the spiral structure of galaxies and a rotating laboratory plasma (Fig. 1) may under definite conditions be due to a deep analogy between the mechanisms of formation of the spiral structures in these two apparently entirely different medii. An outline of the argument leading us to this conclusion is as follows.

First of all, we establish the existence among the different possible mechanisms of formation of spiral arms in galaxies of one that is free of the influence of gravitational effects associated with the presence of huge gravitating masses in the galaxy. If this is the instability that develops in the gaseous disk of the flat subsystem, then the condition of excitation of such an instability must resolve the main paradox of formation of spiral structure through gravitational instability associated with the contradiction between the observed width of an arm and the critical Jeans wavelength.[3] This same instability must lead to large-scale density waves in a rotating laboratory plasma. It is obvious that this "universal" instability responsible for the dynamics of a rotating continuous medium can be any of the hydrodynamic instabilities associated with the presence of velocity and density gradients in the gaseous disk of the flat subsystem of a spiral galaxy and in a rotating laboratory plasma.

This elementary scheme of proof of the existence of such an analogy was proposed some time ago by one of the authors (A. M. F.),[5] 1) but it could not be exploited since the hydrodynamic instabilities of a gravitating medium considered up to that time—the two-stream[8] and the temperature-gradient[7,9]—had maximal growth rates of the order of the Jeans growth rate.

Another distinctive feature of instabilities of non-Jeans type[9] is the difficulty of applying them to real astrophysical objects. Indeed, these instabilities (investigated in an infinitely long rotating gravitating cylinder) exist under the condition that the longitudinal wavelength of the perturbation considerably exceeds the radius of the cylinder, $\lambda_z \gg R$. In nature, such objects (for example, needle-shaped galaxies) are extremely rare.

Astronomical observations of recent years[10,11] have discovered a region of sharp decrease of the rotation velocity $V_\varphi(r)$ in the disks of flat galaxies. This fact can be explained by using the results of a calculation of a stationary model of a spiral galaxy in the form of a heterogeneous "disk + nucleus" system.[12] If the nucleus is chosen in the form of a sufficiently thin lens,

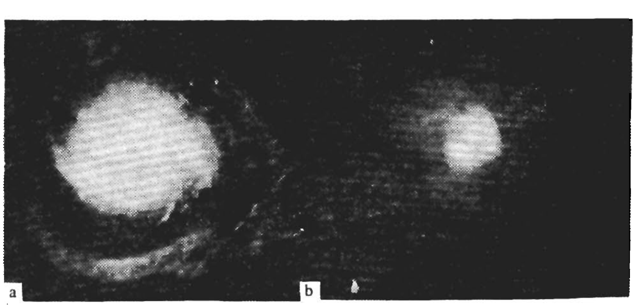

FIG. 1.

which completely corresponds to the observed forms of the nuclei of spiral galaxies,[22] then near the edge of the lens the gradient of the gravitational potential may change rather abruptly (almost discontinuously when the thickness of the lens tends to zero). It is not difficult to calculate the critical thickness of the lens at which $V_\varphi \sim 1/r$ from the equilibrium condition. If the thickness of the lens is less than the critical, then the Rayleigh instability criterion[13] is satisfied in the system; this is the necessary condition for the development of the Kelvin–Holmholtz instability. It so happens that the spiral arms extend over the region of radial growth of the surface density $\sigma$ of atomic hydrogen, i.e., the region in which $g\nabla\sigma<0$ ($g$ is the acceleration of the force of gravity).[22] In this connection, it has been conjectured[14] (see also[15]) that the spiral structure of flat galaxies is formed as a result of excitation of the Kelvin–Helmholtz instability in the region of rapid variation of $V_\varphi(r)$; a second observational fact determines the necessary condition for excitation of the flute instability (see §§2 and 4). The growth rates of these instabilities may considerably exceed the Jeans growth rate, and the conditions of development of these instabilities are not related to a critical size.

In §3 of this paper, we show that in a gravitating medium for perturbations with wavelengths shorter than the Jeans length $\lambda_J = c_s/(4\pi G\rho_0)^{1/2}$ (in a galactic spiral structure $\lambda/\lambda_J \approx 0.2$–$0.4$) the relative influence of perturbations of the gravitational field on the dynamics of the Kelvin–Helmholtz instability is rather small—the corrections to the hydrodynamic effects are of order $(\lambda/\lambda_J)^2$ (see[14,15]). With regard to the unperturbed gravitational field, it enters only into the condition of radial equilibrium and does not affect the dynamics of the perturbations.[14,15]

Because the gravitational effects are small, it is natural to consider verifying the hypothesis[14,15] under laboratory conditions.[5] However, the use of a fluid or neutral gas as experimental medium does not enable one to specify independently the necessary gradients of the rotation velocity, especially if there is a large ratio of the velocity discontinuity $\Delta v$ to the characteristic propagation velocity $c_s$ of perturbations in the medium (for galaxies[16] one usually has $\Delta v/c_s \gtrsim 5$). The fulfillment of these conditions is much simpler in a rotating (because of drift in crossed $E_r^{(0)}(r)$ and $B_z^{(0)}$ fields) plasma medium. Here, the role of the fields $E^{(0)}$ and $B^{(0)}$, like the gravitational field's, reduces merely to ensuring that the system is stationary (when $\nu_i \ll \omega_{Bi}$).

Depending on the magnitudes of the characteristic particles of the process, the dynamics of the perturbations of such a plasma can be described either in the framework of magnetohydrodynamics ($\omega \ll \nu_i$) or in the framework of Chew–Goldberger–Low hydrodynamics[17] ($\nu_i \ll \omega \ll \omega_{Bi}$).

The simplest models convenient for investigating the Kelvin–Helmholtz and flute instabilities are: 1) a plane-parallel flow of fluid with velocity and density that vary in the direction perpendicular to the flow velocity; 2) differentially rotating cylindrical configurations of a fluid. Here it is appropriate to recall that the most general stability criterion of these models were obtained in[18,19] in the approximation of an ideal incompressible fluid. The investigation of these instabilities in a gravitating medium of necessity requires allowance for compressibility, which significantly complicates the analysis and prevents one obtaining general stability criteria in the ideal fluid approximation.

For this reason, for the original analysis of the problem (in a gravitational medium) we have chosen the simplest models: velocity and density shear layer of the gravitating medium (§2) and tangential shear between two gravitating cylinders rotating in opposite directions, their equilibrium being established by the equality of centrifugal and gravitational forces (§4). The stability of a shear layer was investigated earlier in the approximation of an incompressible fluid in an external gravitational field,[18,20] in a compressible fluid and in the magnetohydrodynamic approximation[21] in the absence of a gravitational field. In §5, besides proving that the gravitational effects have little influence on the short-wave part of the spectrum in the framework of the Kelvin–Helmholtz instability that we investigate, we obtain estimates that characterize the important role of the Kelvin–Helmholtz instability in the formation of spiral galactic structure. In §6, we consider the stability of a plasma flow with a tangential shear of the velocity in the Chew–Goldberger–Low approximation.[17] In §7, we compare the dispersion relations that describe the oscillation frequency $\omega$ as a function of the wave vector $k$ and the characteristic parameters of the plasma and gravitating media. We show that under typical conditions of the plasma experiment, the corresponding dispersion relations are identical, which demonstrates that the similarity of the spiral patterns of the rotating gravitational and plasma media is not fortuitous but a consequence of the deep analogy between the processes responsible for the formation of the spiral structure in these two very different but nevertheless "hydrodynamic" media.

## §2. VELOCITY AND DENSITY SHEAR LAYER OF A GRAVITATING MEDIUM

1. We consider the stability of a shear layer of the velocity and density in a compressible gravitating medium.

We write down the linearized equations of motion for the perturbed variables (without indices), assuming that the change of the unperturbed variables (with subscript 0) is continuous in a narrow transition region near the imagined shear:

$$\begin{gathered}
\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}_0\nabla)\mathbf{V}+V_z\frac{d\mathbf{V}_0}{dz}=-\frac{1}{\rho_0}\nabla p-\nabla\psi,\\
\frac{\partial V_z}{\partial t}+(\mathbf{V}_0\nabla)V_z=-\frac{1}{\rho_0}\frac{\partial p}{\partial z}+\frac{\rho}{\rho_0^2}\frac{dp_0}{dz}-\frac{\partial\psi}{\partial z},\\
\frac{\partial \rho}{\partial t}+(\mathbf{V}_0\nabla)\rho+\rho_0(\nabla V)+\frac{\partial}{\partial z}(\rho_0 V_z)=0,\\
\frac{\partial s}{\partial t}+(\mathbf{V}_0\nabla s)+V_z\frac{ds_0}{dz}=0,\qquad \nabla^2\psi+\frac{\partial^2\psi}{\partial z^2}=4\pi G\rho.
\end{gathered}\tag{1}$$

Here, the vector notation is two-dimensional (for the

$x$ and $y$ components) and the shear is the plane $z=0$; $G$ is the gravitational constant; $s_0 = p_0/\rho_0^\gamma$ is the unperturbed entropy; $s=(p-c^2\rho)/\rho_0^\gamma$; $c^2=\gamma p_0/\rho_0$ is the velocity of sound. Considering perturbations of the type $\exp(\{i(\mathbf{k}\cdot\mathbf{r}-\omega t)\})$, we reduce the system (1) to

$$\xi' = \frac{k^2}{\omega_*^2}\left(\frac{p}{\rho_0}+\psi\right) - \frac{p+\xi p_0'}{c^2\rho_0},$$
$$\frac{p'}{\rho_0}+\psi' = \xi\left\{\omega_*^2 - \frac{p_0'\rho_0'}{\rho_0^2} + \frac{p'^2}{\rho_0^2c^2}\right\} + \frac{p_0'p}{\rho_0^2c^2}, \tag{2}$$
$$\psi'' = k^2\psi + 4\pi G\left\{\frac{p+\xi p_0'}{c^2} - \xi\rho_0'\right\},$$
$$\omega_* = \omega - \mathbf{k}\mathbf{V}_0(z), \quad \xi = iV_z/\omega_*,$$

where the prime denotes differentiation with respect to $z$. Suppose that $\rho_0$ and $V_0$ change abruptly in the $z=0$ plane. The boundary conditions on the shear can be readily obtained from (2) by the well-known procedure of integration over a layer[26]:

$$[\xi] = \xi(z=+0) - \xi(z=-0) = 0,$$
$$[\psi'] = -4\pi G\xi[\rho_0], \quad [\psi] = 0, \quad [p] = \xi g[\rho_0]. \tag{3}$$

Here

$$g = \left.\frac{d\psi_0}{dz}\right|_{z=0},$$

and the second equation of the system (3) means that in the plane $z=0$ a simple layer of surface density $\sigma = -\xi[\rho_0]$ is formed.

We shall assume that in the regions $z>0$ and $z<0$ the unperturbed densities and velocities are constant (but different). Solving then the system (2) separately for $z>0$ and $z<0$ and matching the resulting solutions in accordance with (3), we can obtain the dispersion relation for the frequencies of small oscillations. The coefficients of the system (2) can be assumed to be independent of $z$ only for sufficiently short-wave perturbations:

$$\lambda \ll \min(\lambda_1, \lambda_2), \quad \lambda_1 = g/4\pi G\rho_0, \quad \lambda_2 = c^2/g \tag{4a}$$

or (for $g=0$) for wavelengths

$$\lambda^2 \ll \lambda_J^2 = c^2/4\pi G\rho_0. \tag{4b}$$

The dispersion relation obtained under these restrictions is given in[28]. Below, we consider some special cases.

We give the expressions for the quantities $\chi_{1,2}$ that characterize the exponential decay of the perturbed pressure along the $z$ axis (which will be needed later):

$$\chi_{1,2}^2 = k^2 - \frac{\omega_{1,2}^2 + \omega_{01,2}^2}{c_{1,2}^2} + \frac{k^2g^2}{\omega_{1,2}^2 c_{1,2}^2}, \quad \mathrm{Re}(\chi_{1,2}) \geqslant 0. \tag{5}$$

Here, the subscript 1 is appended to the variables of the region $z>0$; the subscript 2, to those of the region $z<0$; $\omega_{1,2} = \omega - kV_{01,2}$, $\omega_0^2 = 4\pi G\rho_0$.

2. We consider first effects due solely to the velocity shear (Kelvin–Helmholtz instability). Suppose

$$\rho_{01} = \rho_{02}, \; c_1^2 = c_2^2, \quad V_{01} = -V_{02} = V_0, \quad g = 0.$$

We shall describe the solution by means of the dimensionless parameters

$$M = |\mathbf{V}_0|/c, \quad \beta = M\cos\alpha, \quad \cos\alpha = (\mathbf{k}\mathbf{V}_0)/(|k|\cdot|\mathbf{V}_0|), \; \nu = \omega_0/kc.$$

In the limit of short-wave perturbations $\omega_0 \ll kc$ in the zeroth approximation, we readily obtain from the dispersion relation the well-known result of the theory of a compressible fluid.[21]

$$\omega = ikc\,\beta\gamma, \quad \gamma = \{(1+4\beta^2)^{1/2} - (1+\beta^2)\}^{1/2}/\beta. \tag{6}$$

Let us here make a remark which will be used below in §4. It can be seen from (6) that a tangential shear in this approximation is unstable for all $M \neq 0$. However, for $M < 2^{1/2}$ perturbations with any direction of the wave vector (except purely transverse) are unstable, while only perturbations for which $\cos^2\alpha < 2/M^2$ are unstable when $M > 2^{1/2}$. The growth rate is maximal, $\omega = ikc[(1+4M^2)^{1/2} - (1+M^2)]^{1/2}$ when $M < 3^{1/2}\cdot 2^{-1}$ for purely longitudinal ($\mathbf{k} \parallel \mathbf{V}_0$) perturbations, and for $M > 3^{1/2}\cdot 2^{-1}$ the maximum of the growth rate ($\omega = ikc/2$) is achieved when $\cos^2\alpha = 3/4M^2$.

In the following approximation in $\nu = \omega_0/kc$,

$$\omega = ikc\beta\gamma\{1 - \nu^2 A(\gamma)\}, \quad A(\gamma) = \frac{(1-\gamma)(1+\gamma^2)[2\gamma + (1+\gamma)^2]}{4\gamma(1+\gamma)(3-\gamma^2)}. \tag{7}$$

It is easy to see that $A(\gamma) > 0$ and $A$ is a monotonically increasing function of $\beta$. Thus, perturbations of the shear surface are subject to an additional stabilization at longer wavelengths due to the gravitational properties of the medium. However, it must be borne in mind that in accordance with (4) the results (7) apply only for wavelengths $\lambda \ll \lambda_J = c/\omega_0$. At the same time, as can be seen from (7), the growth rate of the instability is much greater than the Jeans growth rate: $\mathrm{Im}(\omega) \gg \omega_0$ (Fig. 2).

3. We now consider the effects associated with the change of the density. Assuming that $V_{01} = V_{02} = 0$ and that the magnitude of the change in the density is not too small compared with $\rho_0$, we obtain

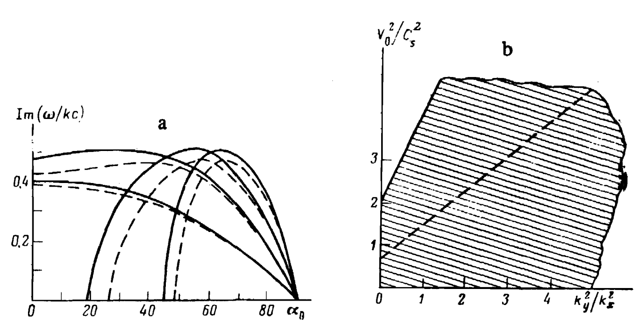


FIG. 2. a) Growth rate of the instability of a plane tangential shear as a function of the Mach number $M$ and the direction of the wave vector. The continuous curves are the growth rates for $\omega_0^2/k^2c^2=0$; the dashed curves are the same for $\omega_0^2/k^2c^2=0.2$. b) Region of the Kelvin–Helmholtz instability in an ideal compressible fluid. The dashed line shows the line of the maximal growth rate.

$$\omega=\left(kg\frac{\rho_{02}-\rho_{01}}{\rho_{02}+\rho_{01}}\right)^{1/2}\left\{1+\frac{\pi G}{kg}(\rho_{01}-\rho_{02})+\frac{\rho_{01}\rho_{02}g(\rho_{01}c_1^2+\rho_{02}c_2^2)}{kc_1^2c_2^2(\rho_{01}-\rho_{02})(\rho_{01}+\rho_{02})^2}\right\}. \quad (8)$$

In the approximation $k\to\infty$, we obtain from (8) the well-known result of the theory of an incompressible fluid.[20]

## §3. ABSENCE OF INFLUENCE OF GRAVITATIONAL FORCES ON THE SHORT-WAVE PART OF THE OSCILLATION SPECTRUM

Thus, we have shown (§2) that hydrodynamic instabilities can develop in a gravitating medium. The Jeans instability characteristic of such a medium is stabilized by thermal spread in the region of short, $k^2c^2\gtrsim\omega_0^2$, wavelengths. The hydrodynamic instabilities, in contrast to the gravitational, are not stabilized by the thermal spread in the short-wave region.[2)] Moreover, in accordance with (6) and (8), the growth rates of the hydrodynamic instabilities increase with decreasing wavelength of the perturbation.[3)] This unique property of the Kelvin–Holmholtz and flute instabilities distinguishes them from the previously investigated hydrodynamic instabilities of a gravitating medium.

If one assumes that the gravitating medium is in equilibrium, $\nabla p_0+\rho_0\nabla\psi_0=0$, then from the original system of equations with allowance for the gradients of the unperturbed quantities one can readily see that $|\nabla p|$ is greater than $|\rho\nabla\psi_0|$ by $kL$ times ($L$ is the characteristic inhomogeneity scale, $kL\gg 1$) and $|\rho_0\nabla\psi|$ is smaller than $|\rho\nabla\psi_0|$, also by $kL$ times. Thus, the influence of the "external" gravitational field can be regarded as a small correction to the hydrodynamic effects. The influence of "self-gravitation" is even smaller.

## §4. CYLINDRICAL TANGENTIAL SHEAR OF THE VELOCITY AND DENSITY OF A GRAVITATING MEDIUM

1. We now consider a cylindrical tangential shear, assuming that equilibrium is established by equality of the centrifugal and gravitational forces ($p_0=\text{const}$, $\Omega^2=\omega_0^2/2$ is the square of the angular velocity of rotation of the cylinders, $R$ is the radius of the shear, and $\Omega(r>R)=-\Omega(r<R)$).

To investigate the stability of perturbations of the type $\exp\{i(m\varphi+kz-\omega t)\}$ of the shear surface, we readily obtain the dispersion relation (see[29]), but because it is cumbersome we shall not give it here in full.

We consider a number of limiting cases for the most interesting modes $m\geq 2$. In the limit of an incompressible fluid, we obtain from the dispersion relation

$$\omega=\omega^{(0)}=\Omega\{-1+i(m^2-1)^{1/2}\}. \quad (9)$$

In the weakly compressible case ($M\ll 1$), where $M=\Omega R/c$, and in the long-wave ($kR\ll m$) approximation

$$\omega=\omega^{(0)}+\frac{\varkappa^2\Omega}{m^2-1}-\frac{i\Omega}{(m^2-1)^{1/2}}\left\{\frac{\varkappa^2}{m^2-1}+m^2M^2\right\} \quad (10)$$

we see that allowance for compressibility, as also in the shear layer case, partly stabilizes the instability. In the opposite limiting case ($kR\gg m$, no restrictions imposed on $M$), using the asymptotic behavior of the Bessel functions $I_m$ and $K_m$, we can readily obtain the stability condition for such perturbations:

$$m^2M^2/k^2R^2\geqslant 2-4/m^2, \quad (11)$$

which for $m\gg 1$ agrees with the stability condition for perturbations of a planar tangential shear (in the same approximation $k_\parallel\ll k_\perp$):

$$\beta^2=(k_\parallel^2/k_\perp^2)M^2\geqslant 2. \quad (12)$$

In the limit of very short waves $kR\gg m$, the growth rate tends asymptotically to $\Omega(m^2-2)^{1/2}$.

The dispersion relation for the largest scale modes $m=2$ and $m=3$ was investigated on a computer. The results are as follows: The dependence of the growth rate of the instability (in units of $\Omega$) on the Mach number $M$ and the dimensionless wavelength $kR$ are shown in Fig. 3. Note the rather complicated dependence of the growth rate in the region $1\lesssim kR\lesssim 10$ for supersonic shears.

2. We now investigate the possibility of exciting a flute instability in a gravitating cylinder. For this we consider a model of an infinitely long cylinder with abrupt change of the density $\rho_0$ at a distance $R$ from the axis of the cylinder, assuming that equilibrium is established by the resultant effect of the centrifugal and gravitational forces and the pressure force, so that $g=d\varphi_0/dr-\Omega^2r\neq 0$. Considering short-wave (compared with the Jeans length) oscillations, for which the influence of the perturbed gravitational potential is negligibly small, we obtain the following equations for the perturbations of the pressure $p$ and the displacement $\xi=iv_r/(\omega-m\Omega)$ of the shear surface:

$$\begin{aligned}p'&=\frac{2m\Omega}{r\omega_*}p-g\left[\frac{p+\xi p_0'}{c^2}-\xi\rho_0\right]-(\chi^2-\omega_*^2)\rho_0\xi,\\ \xi'&=\frac{\tilde{k}^2p}{\omega_*^2\rho_0}-\frac{(\omega_*-2m\Omega)}{\omega_*r}\xi-\frac{p+\xi p_0'}{\rho_0c^2},\end{aligned} \quad (13)$$

where

$$\omega_*=\omega-m\Omega,\quad \chi^2=2\Omega(2\Omega+r\Omega'),\quad \tilde{k}^2=k^2+m^2/r^2,$$

$c^2=\gamma p_0/\rho_0$ is the square of the velocity of sound. The

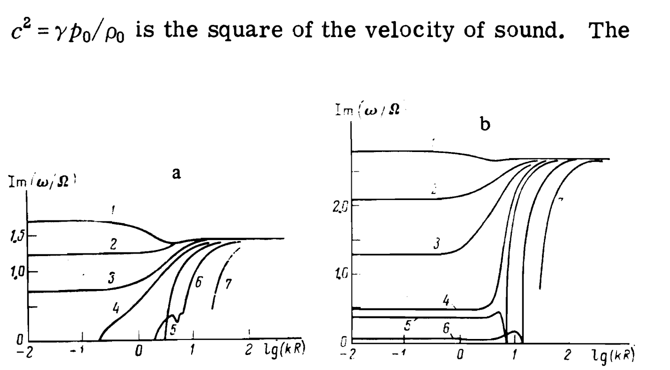


FIG. 3. Growth rate of the instability of a cylindrical tangential shear as a function of the Mach number $M$ and the wavelength $kR$ for the modes $m=2$ (a) and $m=3$ (b). The values of $M$ for the curves of Fig. 3a: 1) 0.1, 2) 0.6, 3) 1.0, 4) 1.2, 5) 2.0, 6) 3.2, 7) 10.0; the values of $M$ for the curves of Fig. 3b: 1) 1.0, 2) 0.6, 3) 1.0, 4) 1.7, 5) 2.0, 6) 4.5, 7) 10.0.

prime denotes differentiation with respect to $r$. From (13), we obtain boundary conditions for matching the solutions at $r=R$:

$$[\xi]=\xi(R+0)-\xi(R-0)=0, \quad [p]=\xi g[\rho_0]. \tag{14}$$

Solving the system (13)–(14) in the limit $m^2 \ll k^2R^2$, we obtain the dispersion relation

$$\sum_{n=1}^{2}\rho_{0i}(-1)^n\left\{\frac{[(-1)^n\alpha_n+(\omega_*+2m\Omega)/\omega_*R-g/c_n^2]}{(k^2/\omega_*^2)-(1/c_n^2)}-g\right\}=0, \tag{15}$$

$$\alpha_n^2=k^2\left(1+\frac{g^2}{c_n^2\omega_*^2}\right)-\frac{\omega_*^2}{c_n^2}-\frac{g}{c_n^2}\frac{\omega_*+4m\Omega}{R\omega_*}+\frac{2m\Omega(\omega_*+2m\Omega)}{R^2\omega_*^2}. \tag{16}$$

Since we are only interested in the basic possibility of exciting the flute instability, we consider the case of a fairly hot ($c^2\to\infty$) medium. Then from (15) for $\rho_0 = \rho_{01}(r>R) \neq \rho_{02} = \rho_0(r<R)$ we obtain the growth rate

$$\gamma\approx\left\{kgA+\frac{(2-A^2)m^2\Omega^2}{k^2R^2}\right\}^{1/2}, \quad A=(\rho_{01}-\rho_{02})/(\rho_{01}+\rho_{02}). \tag{17}$$

As follows from the expression (17), the necessary condition for instability is

$$gA>0. \tag{18}$$

This means that for $g=(\partial\psi_0/\partial r)-\Omega^2 r>0$ the flute instability develops for $\rho_{01}>\rho_{02}$ while for $g<0$ it develops for $\rho_{02}>\rho_{01}$. The second term in (17) is much smaller than the first and plays the role of a small correction due to the cylindrical symmetry. Since $|A|<1$, the curvature has a destabilizing influence on the flute instability. Note that, as follows from Eq. (9), this same curvature effect has a stabilizing influence in the case of a tangential velocity shear.

In the case opposite to the one considered in this subsection, i.e., wavelengths much greater than the width $a$ of the transition layer, we consider a different limiting case: $\lambda \ll a$. For perturbations of the type $\exp[i(kr+m\varphi-\omega t)]$, we obtain instead of (17) the following growth rate of the flute instability:

$$\gamma=\left[g\frac{d\ln\rho_0}{dr}\frac{m^2}{k^2r^2}\right]^{1/2}. \tag{19}$$

Naturally, the instability condition is analogous to (18). The growth rate (19) is much greater than the Jeans growth rate when $m/kr \gg 1$.

## §5. SPIRAL STRUCTURE OF GALAXIES AS A CONSEQUENCE OF HYDRODYNAMIC INSTABILITIES

Hitherto, it has been assumed that the maximal growth rate of instabilities that can develop in gravitating systems is the Jeans growth rate $\gamma\approx(4\pi G\rho_0)^{1/2}$. The attempt to explain the formation of the spiral arms in our Galaxy by the Jeans instability led, as is well known,[3] to the paradox of a contradiction between the critical Jeans wavelength and the width of a spiral arm. All the remaining hitherto known instabilities of a gravitating medium have growth rates less than the Jeans growth rate.

The investigation in the preceding sections of hydrodynamic instabilities of gravitating systems with growth rates appreciably greater than the Jeans opens up a new possibility for eliminating the paradox and explaining the origin of the spiral structure if the conditions observed in spiral galaxies correspond to the conditions of development of these instabilities. In the present section we bring forward arguments for the existence in spiral galaxies of the necessary conditions for the development of the hydrodynamic instabilities.

The recent investigations of the rotation curve of the nearest spiral galaxy to us—the Andromeda Nebula[11]—has revealed the presence of a region of abrupt change in the rotation velocity of the flat subsystem. In the region 0.4 kpc$\lesssim r \lesssim$2 kpc there is a section of rapidly decreasing (from the center) rotation velocity $V_\varphi(r)$, in which $(d/dr)(\alpha^2/2\Omega)$ changes sign. Such a distribution of the rotation velocity is unstable in accordance with the Rayleigh criterion[13] in the approximation of an ideal incompressible fluid unstable, and the finite compressibility of the medium evidently cannot significantly alter this result. According to the Rayleigh criterion,[13] the Kelvin–Helmholtz instability can be excited in rotating systems if over a certain interval $\Delta r$ the rotation velocity $V_\varphi(r)$ decreases faster than $r^{-1}$. A sufficiently detailed study of the rotation curves of the gaseous subsystems of flat galaxies has made it possible to find such regions in M 31 (see[11]) and apparently in NGC 4736 (see[22]). The reasons for this behavior of $V_\varphi(r)$ are to be found in the strong oblateness of the dense nuclear regions of flat galaxies.[22] This may also be the case for barred galaxies. For example, in NGC 4027 (see[23]) the ratio of the semiaxes of the bar are $b/a\approx 0.6$, $c/a\approx 0.2$ (see[24]) (almost elliptic disk).

We now show how the number of spirals is determined in the case when a Kelvin–Helmholtz instability develops in the system. As follows from §§ 2 and 4, the growth rates of the Kelvin–Helmholtz instability of gravitating systems with cylindrical and plane shears of the velocity for modes $m \geq 2$ have similar dependences on the wave numbers. Using, for simplicity, the results of §2 and then making a transition to cylindrical coordinates ($k_\parallel \to k_\varphi$, $k_\perp \to k_z$), we can readily estimate the number of spiral arms. Indeed, let us set $k_\varphi = m/R_s$ ($m$ is the number of spirals and $R_s$ is the radius of the shear), $k_z=\pi/h$ (on the basis of the observational data, we assume that approximately half a wavelength fits into the thickness $h$ of the disk). For the Andromeda nebula, the magnitude of the discontinuity of the rotation velocity (see[11]) $\Delta v\approx 150$ km/sec is much greater than the turbulent velocities of the gas and gas clouds, $v_T\approx 20$ km/sec, and therefore perturbations excited with the maximal growth rate (satisfying the relation $\Delta v k_\parallel \approx 3^{1/2}v_T(k_\parallel^2+k_\perp^2)^{1/2}$) necessarily have $k_\perp \gg k_\parallel$ (see Fig. 2a, §2). Thus, the number $m$ of spiral arms is

$$m\approx 3^{1/2}\pi(v_T/\Delta v)(R_s/h).$$

Since $R_s\approx 0.5$ kpc, $h\approx 0.1$ kpc (see[11]), the number of spirals is of the order of a few units.

It is also easy to establish the direction of winding of the spirals. For this (see §2.2), we go over to a frame of reference moving in the direction $V_{01}$ with velocity $V_s > V_0$, so that $V'_{01} = V_0 + V_s$, $V'_{02} = V_s - V_0$. In such a system, the oscillation frequency is $\omega' = -k_\parallel V_s + ikc\beta\gamma$ (see (6)). Identifying $z > 0 \to r > R_s$ ($R_s$ is the radius of the velocity shear $\Delta v = 2V_0$) and substituting $\omega$ into (5), we find that for perturbations with the maximal growth rate in the region $r > R_s$ the relation $\mathrm{Im}(\chi_1) < 0$ necessarily holds. It can be seen from this that the equation of constant phase in the $(r, \varphi)$ plane, $m\varphi - \mathrm{Im}(\chi_1)r = \mathrm{const}$, describes a trailing sprial ($m > 0$, $V_\varphi > 0$).

A mystery of galactic astronomy is, in particular, the multiple (double, more seldom triple) spiral systems.[22] For example, in the Andromeda Nebula the very carefully measured $V_\varphi(r)$ distribution reveals an inner section of two dust spirals[22] between the inner[25] and outer[11] abrupt changes of the velocity. In our scheme, this has a natural explanation for rotation curves with several shears.

The distribution of the density in the gas disks of the flat subsystems of spiral galaxies has, as is known from observations, a bell-shaped form (with a point at which the density is maximal). Therefore, the presence of even a small radial gradient of the gas temperature (in the neighborhood of the extremum of the density) may lead to the development of the flute instability. Indeed, in this region $|(1/p_0)\nabla p_0/(1/\rho_0)\nabla\rho_0| > 1$ and on either side of the extremum point of the density $\nabla p_0/\nabla\rho_0 < 0$.

## §6. TANGENTIAL VELOCITY SHEAR OF A MAGNETIZED PLASMA MEDIUM

In this case, we believe it is convenient to use the system of equations of Chew–Goldberger–Low hydrodynamics.[17]

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\mathbf{V} = -\frac{1}{\rho}\operatorname{div}\overleftrightarrow{P} + \frac{1}{4\pi\rho}[\operatorname{rot}\mathbf{B}\times\mathbf{B}],$$
$$\frac{\partial \mathbf{B}}{\partial t} = \operatorname{rot}[\mathbf{V}\times\mathbf{B}],$$
$$\frac{\partial p_\parallel}{\partial t} + \mathbf{V}\nabla p_\parallel + p_\parallel \operatorname{div}\mathbf{V} + 2p_\parallel \boldsymbol{\tau}(\boldsymbol{\tau}\nabla)\mathbf{V} = 0, \tag{20}$$
$$\frac{\partial p_\perp}{\partial t} + \mathbf{V}\nabla p_\perp + 2p_\perp \operatorname{div}\mathbf{V} - p_\perp\boldsymbol{\tau}(\boldsymbol{\tau}\nabla)\mathbf{V} = 0,$$
$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho\mathbf{V}) = 0,$$
$$\boldsymbol{\tau} = \mathbf{B}/|\mathbf{B}|, \quad \operatorname{div}\overleftrightarrow{P} = \nabla p_\perp + (p_\parallel - p_\perp)(\boldsymbol{\tau}\nabla)\boldsymbol{\tau} + \boldsymbol{\tau}\operatorname{div}(\boldsymbol{\tau}(p_\parallel - p_\perp))$$

As was shown in §3, for a gravitating system in equilibrium the influence of the gravitational field on the stability of the system against short waves ($kL \gg 1$, $k$ is the wave number and $L$ the characteristic inhomogeneity scale) is negligibly small.

On the other hand, it is known from the theory of plasma instabilities[26] that the maximum of the growth rate of the Kelvin–Helmholtz instability lies in the short-wavelength part of the spectrum, $kL \gg 1$. For a plasma cylinder of radius $R$, this condition corresponds to the condition $kR \gg 1$; in this case, to terms $\sim 1/kR$, a cylindrical plasma velocity shear can be replaced by a velocity shear layer in a plasma with homogeneous magnetic field $B_{0y}$. Assuming that the variation of $V_{0x}$ near the plane $z = 0$ is smooth and linearizing Eqs. (20) for perturbations of the type $\exp\{i(k_x x + k_y y - \omega t\}$, we obtain

$$(\omega_*^2 - k_y^2 V_{A\parallel}^2)\xi = c_{s\perp}^2 p' - k_y^2 c_\perp^2 (c_{s\perp}^2 - c_\perp^2)\left[\frac{p}{\omega_*^2 - k_y^2(c_{s\parallel}^2 - c_\perp^2)}\right]', \tag{21}$$

$$\xi' = p\left\{k_x^2 \frac{c_{s\perp}^2 - k_y^2 c_\perp^2 (c_{s\perp}^2 - c_\perp^2)/[\omega_*^2 - k_y^2(c_{s\parallel}^2 - c_\perp^2)]}{\omega_*^2 - k_y^2 V_{A\parallel}^2} + k_y^2 \frac{c_\perp^2}{\omega_*^2 k_y^2 (c_{s\parallel}^2 - c_\perp^2)} - 1\right\}, \tag{22}$$

where $\omega_* = \omega - k_x V_{0x}(z)$, $\xi = iV_{z1}/\omega_*$ is the displacement of the plasma in the $z$ direction, $p = \rho_1/\rho_0$ is the ratio of the perturbed to the unperturbed density, $V_{A\parallel}^2 = V_A^2 + (p_{\perp 0} - p_{\parallel 0})/\rho_0$, $V_A^2 = B_0^2/4\pi\rho_0$, $c_\perp^2 = p_{\perp 0}/\rho_0$, $c_{s\perp}^2 = V_A^2 + 2c_\perp^2$, $c_{s\parallel}^2 = 3\rho_{\parallel 0}/\rho_0$, and the prime denotes differentiation with respect to $z$. Integrating Eqs. (21) and (22) over a narrow surface layer (as in[26]), we obtain the conditions for matching $\xi$ and $p$ in the plane $z = 0$:

$$c_{s\perp}^2[p] = k_y^2 c_\perp^2 (c_{s\perp}^2 - c_\perp^2)\left[\frac{p}{\omega_*^2 - k_y^2(c_{s\parallel}^2 - c_\perp^2)}\right], \tag{23}$$

$$[\xi] = 0, \quad [A] \equiv A(z = +0) - A(z = -0). \tag{24}$$

Matching in accordance with (23) and (24) the solutions of the system (21) and (22) that do not increase away from the velocity shear, we obtain the dispersion relation

$$\chi_1(\omega_{*2}^2 - k_y^2 V_{A\parallel}^2) + \chi_2(\omega_{*1}^2 - k_y^2 V_{A\parallel}^2) = 0; \tag{25}$$

the subscripts 1 and 2 are appended to the variables of the regions $z > 0$ and $z < 0$, respectively, and

$$\chi_{1,2}^2 = k^2 + \frac{(k_y^2 c_{s\parallel}^2 - \omega_{*1,2}^2)(\omega_{*1,2}^2 - k_y^2 V_{A\parallel}^2)}{c_{s\perp}^2[\omega_{*1,2}^2 - k_y^2(c_{s\parallel}^2 - c_\perp^4/c_{s\perp}^2)]}. \tag{26}$$

It can be seen from this that in the investigated case we can expect a dependence $\omega = \omega(k_x, k_y)$ like the one observed in ordinary hydrodynamics[14] when one of the following two conditions is satisfied:

$$V_{A\parallel}^2 \simeq c_{s\parallel}^2 - c_\perp^4/c_{s\perp}^2, \tag{27}$$
$$c_\perp^4 \ll c_{s\parallel}^2 c_{s\perp}^2. \tag{28}$$

The condition (27) corresponds to a plasma with $\beta \sim 1$. The solution of the dispersion relation (25) in this case,

$$\omega^2 \simeq k_x^2 V_0^2 + \tilde{k}^2 c_{s\perp}^2 - \{(k_x^2 c_{s\perp}^2 + k_y^2 c_\perp^4/c_{s\parallel}^2)^2 + 4k_x^2 V_0^2 \tilde{k}^2 c_{s\perp}^2\}^{1/2}, \tag{29}$$

where $\tilde{k}^2 c_{s\perp}^2 = k_x^2 c_{s\perp}^2 + k_y^2 c_{s\parallel}^2$, shows that instability ($\omega^2 > 0$) will occur for $V_0^2 < c_{s\perp}^2$ for perturbations with

$$\cos^2\alpha = \frac{k_x^2}{k^2} > \cos^2\alpha_1 = \frac{(c_{s\parallel}^2 c_{s\perp}^2 - c_\perp^4)}{(c_{s\parallel}^2 c_{s\perp}^2 - c_\perp^4) + V_0^2 c_{s\perp}^2}$$

and for $V_0^2 > 2c_{s\perp}^2$ for perturbations with

$$\cos^2\alpha_1 < \cos^2\alpha < \cos^2\alpha_2 = \frac{(c_{s\parallel}^2 c_{s\perp}^2 + c_\perp^4)}{(c_{s\parallel}^2 c_{s\perp}^2 + c_\perp^4) + c_{s\perp}^2(V_0^2 - 2c_{s\perp}^2)}.$$

The condition (28) corresponds to a plasma with $\beta \ll 1$. The solution of the dispersion relation (25) in this case:

$$\omega^2 = k_x^2 V_0^2 + k^2 V_A^2 - \{k_x^4 V_A^4 + 4k_x^2 V_0^2 k^2 V_A^2\}^{1/2} \tag{30}$$

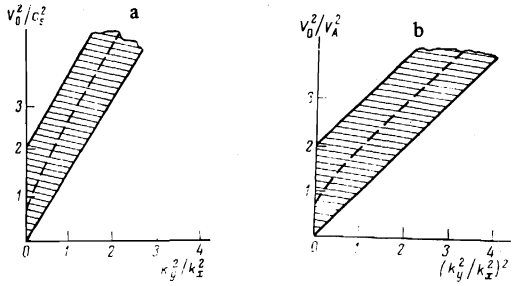


FIG. 4. Region of the Kelvin-Helmholtz instability in a plasma (hatched). a) The case $V_{A\parallel}^2 = c_{s\parallel}^2 - c_\perp^4/c_{s\perp}^2$; b) $c_\perp^4 \ll c_{s\perp}^2 c_{s\perp}^2$ ($\beta \ll 1$). The line of the maximal growth rate is the dashed curve.

predicts instability when $V_0^2 < 2V_A^2$ for perturbations with $\cos^2\alpha > \cos^2\alpha_1 = V_A^2/(V_A^2 + V_0^2)$ and when $V_0^2 > V_A^2$ for perturbations with $\cos^2\alpha_1 < \cos^2\alpha < \cos^2\alpha_2 = V_A^2/(V_0^2 - V_A^2)$.

## §7. ANALOGY BETWEEN THE DISPERSION RELATIONS DESCRIBING THE KELVIN-HELMHOLTZ INSTABILITY AND THE FLUTE INSTABILITY IN A GRAVITATING MEDIUM AND A PLASMA

For the typical conditions of the plasma experiment, $E_{0_2} \ll B_0^2/c(4\pi n_0 M)^{1/2}$, which corresponds to the inequality

$$V_0^2 \ll V_A^2 \sim c_{s\perp}^2. \tag{31}$$

The instability regions described by Eqs. (29) and (30) are shown in Fig. 4. It can be seen that under the condition (31) $\gamma \to \gamma_{max}$ in an $\varepsilon$ neighborhood of the straight line

$$k_y/k_x \approx 0. \tag{32}$$

Thus, under the conditions of the plasma experiment the dispersion relations of nonelectrostatic oscillations of the plasma with tangential velocity shear have the form

$$\omega^2 = k_x^2 V_0^2 + k_x^2 c_{s\perp}^2 - (k_x^4 c_{s\perp}^4 + 4k_x^4 V_0^2 c_{s\perp}^2)^{1/2}, \quad \beta \sim 1. \tag{33}$$

$$\omega^2 = k_x^2 V_0^2 + k_x^2 V_A^2 - (k_x^4 V_A^4 + 4k_x^4 V_0^2 V_A^2)^{1/2}, \quad \beta \ll 1. \tag{34}$$

Since $c_{s\perp}^2 \sim V_A^2$, the two dispersion relations (33) and (34) are identical under the condition of instability (32).

In the case of a tangential velocity shear in a gravitating medium, the following dispersion relation was obtained for short wavelengths in §2 (see (6)):

$$\omega^2 = k_x^2 V_0^2 + k^2 c_s^2 - (k^4 c_s^4 + 4k_x^2 V_0^2 k^2 c_s^2)^{1/2}. \tag{35}$$

In Fig. 4a, the dashed curve shows the region $\gamma = \gamma_{max}$. In this region

$$k_y \approx 2 \cdot 3^{-1/2} V_0 k_x / c_s. \tag{36}$$

In the case $V_0 \ll c_s$, $k_y \ll k_x$, Eq. (35) is identical with Eqs. (33) and (34).

In the case $V_0 \gg c_s$ (which corresponds to the observations of the investigated galaxies)

$$\gamma_{max} \approx 3^{-1/2} k_x V_0. \tag{37}$$

We find a similar dependence of $\gamma_{max}$ on $k_x$ from Eqs. (33) and (34) under the condition (31).[4)]

We now consider a plasma cylinder in which the electrons and ions drift in crossed electric $E_r$ and magnetic $B_z$ fields. Suppose that in a plasma with $\beta \ll 1$, $\omega_{pi} \gg \omega_{Bi}$, $V_0 \ll r\omega_{Bi}$ ($\omega_{pi}$ and $\omega_{Bi}$ are the plasma and cyclotron frequencies, respectively, and $V_0$ is the velocity of rotation of a particle about the axis) oscillations with $\omega \ll \omega_{Bi}$, $k_z = 0$ are excited. One can show[27] that in the case of a radially decreasing plasma density these oscillations are unstable and that for $l \gg 1$ the growth rate is

$$\gamma = \left(\frac{\partial n_0}{n_0 \partial r}\frac{1}{r}\right)^{1/2} V_E \left(\frac{k_\varphi}{k}\right)^{1/2}, \tag{38}$$
$$V_E = cE_r/B_z, \quad k_\varphi = l/r.$$

Denoting $V_E^2/r = g$, this growth rate is identical to the growth rate (19).

We are very grateful to A. B. Mikhaĭlovskiĭ for a helpful discussion of the results and valuable advice.

1)Recently, Cotaldo and Skalafuries[6] have put forward a similar suggestion about the role of the Kelvin-Helmholtz instability in the formation of spiral structure in galaxies and plasmas. However, they associate the Kelvin-Helmholtz instability in the galactic disk with a density gradient, which contradicts the paper[7], in which it is shown that a density gradient of a gravitating medium does not by itself lead to instability.[7]

2)This result is obvious if one recalls that the shear model considered in §2 is also unstable in the approximation of an incompressible fluid,[20] in which the thermal spread is by definition infinite.

3)This assertion is true at least for wavelengths that are greater or of order of the thickness of the transition layer: role.

4)In the framework of two-fluid hydrodynamics one can show that in a plasma with $\beta \ll 1$ and inhomogeneous velocity profile electrostatic oscillations can be excited. If certain conditions are satisfied, the equation for the perturbed potential is identical to the equation of the oscillations of a plane-parallel flow of an ideal fluid.[27] Therefore, in a plasma described by the equations of two-fluid hydrodynamics in the case of a tangential velocity shear, the growth rate of the instability associated with the excitation of electrostatic oscillations is $\gamma \approx k_x V_0$.


1W. H. Bostick, Rev. Mod. Phys. 30, 1090 (1958).

2M. A. Vlasov, Pis'ma Zh. Eksp. Teor. Fiz. 2, 274 (1965) [JETP Lett. 2, 174 (1965)].

3A. Toomre, Astrophys. J. 139, 1217 (1964).

4A. M. Fridman, Itogi Nauki Tekh., Astron. 10, 61 (1975).

5V. G. Eselevich, G. N. Kichigin, V. I. Koroteev, V. L. Polyachenko, V. G. Faĭnshtein, and A. M. Fridman, in: Dinamika Galaktik i Zvezdnykh Skopleniĭ, (Dynamics of Galaxies and Star Clusters), Izd-vo KazSSR, Alma-Ata, Doklad (Report), October (1972).

6J. C. Cotaldo and A. J. Skalafuries, Astrophys. and Space Sci. 32, L11 (1975).

7G. S. Bisnovatyĭ-Kogan and A. B. Mikhaĭlovskiĭ, Astron. Zh. 50, 312 (1973) [Sov. Astron. 17, 205 (1973)].

8A. B. Mikhaĭlovskiĭ and A. M. Fridman, Zh. Eksp. Teor. Fiz. 61, 457 (1971) [Sov. Phys. JETP 34, 243 (1972)].

9V. L. Polyachenko and I. G. Shukhman, Astron. Zh. 50, 649 (1973) [Sov. Astron. 17, 413 (1973)].

[10]G. W. Rougoor and J. H. Oort, Proc. Nat. Acad. Scien. 46, 1 (1960).
[11]V. C. Rubin and W. K. Ford, Jr., Astrophys. J. 159, 379 (1970).
[12]A. G. Morozov and A. M. Fridman, Astron. Zh. 50, 1028 (1973) [Sov. Astron. 17, 651 (1974)].
[13]C. Hunter, Ann. Rev. Fluid Mech. 4, 219 (1972).
[14]A. G. Morozov, V. G. Faĭnshteĭn, and A. M. Fridman, in: Dinamika i Évolyutsiya Zvezdnykh Sistem (Dynamics and Evolution of Star Systems), Leningrad Branch, All-Union Astronomical and Geodetic Society (1975).
[15]A. G. Morozov and A. M. Fridman, in: Report at All-Union Conference on Hidden Mass in the Universe, Tallin, January (1975).
[16]R. H. Sanders and G. T. Wrixon, Astron. and Astrophys. 26, 365 (1973).
[17]G. F. Chew, M. L. Goldberger, and F. E. Low, "The Boltzmann equation and the one-fluid hydromagnetic equations in the absence of particle collisions," Proc. Roy. Soc. (London) A236, 112 (1956).
[18]S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Oxford, Clarendon Press (1961).
[19]A. V. Timofeev, Usp. Fiz. Nauk 102, 185 (1970) [Sov. Phys. Usp. 13, 632 (1971)].
[20]L. D. Landau and E. M. Lifshits, Mekhanika Sploshnykh Sred, Gostekhizdat, Moscow (1954); translated as: Fluid Mechanics, Pergamon, Oxford (1959).
[21]S. I. Syrovatskiĭ, Tr. Fiz. Inst., Akad. Nauk SSSR 8, 13 (1956).
[22]B. A. Vorontsov-Vel'yaminov, Vnegalakticheskaya Astronomiya (Extragalactic Astronomy), Nauka (1972).
[23]G. de Vauvouleurs, A. de Vaucouleurs, and K. C. Freeman, Mon. Not. Roy. Astron. Soc. 139, 425 (1968).
[24]A. G. Morozov, V. L. Polyachenko, and I. G. Shukhman, Astron. Zh. 51, 75 (1974) [Sov. Astron. 18, 44 (1974)].
[25]A. Lalleman, N. Duchesne, and M. Walker, Publ. A. S. P. 72, 76 (1960).
[26]A. B. Mikhaĭlovskiĭ, Teoriya Plazmennykh Neustoĭchivosteĭ, T. 1, Atomizdat (1970); translated as: Theory of Plasma Instabilities, Vol. 1, Consultants Bureau, New York (1974).
[27]A. B. Mikhaĭlovskiĭ, Teoriya Plazmennykh Neustoĭchivosteĭ, T. 2, Atomizdat (1971); translated as: Theory of Plasma Instabilities, Vol. 2, Consultants Bureau, New York (1971).
[28]A. G. Morozov, V. G. Faĭnshteĭn, and A. M. Fridman, Dokl. Akad. Nauk SSSR 231, No. 4 (1976).
[29]A. G. Morozov, V. L. Polyachenko, V. G. Faĭnshteĭn, and A. M. Fridman, Astron. Zh. 53, No. 5 (1976) [Sov. Astron. 20, No. 5 (1976)].

Translated by Julian B. Barbour

# The $\gamma\to\nu\bar{\nu}$ and $\nu\to\gamma\nu$ reactions in strong magnetic fields

V. V. Skobelev

*Moscow Institution of Geodesy, Aerial-Photography, and Cartography Engineers*
(Submitted February 28, 1976)
Zh. Eksp. Teor. Fiz. 71, 1263–1267 (October 1976)

The $\gamma\to\nu\bar{\nu}$ and $\nu\to\gamma\nu$ decay probabilities in a strong magnetic field are found by employing an effectively two-dimensional representation of the electron Green's function. The contribution of the $\gamma\to\nu\bar{\nu}$ photodecay to the neutrino luminosity of pulsars is estimated. The contributions of other diagrams with vacuum loops are discussed. Previous results obtained in the frequency range $\omega \gg m$, in which the crossed-field approximation is valid, are confirmed. In fields $\sim 10^{16}$ G the $\gamma\to\nu\bar{\nu}$ process competes with the $n+n\to n+p+e^-+\bar{\nu}$ reaction, so that vacuum polarization effects may influence the cooling of neutron stars in their initial evolution stage.

PACS numbers: 95.60.+d, 97.70.Qq, 98.60.Gi

In connection with the possible existence of ultrastrong magnetic fields $\sim B_0 = 4.41\times 10^{13}$ G in the vicinity of a neutron star, calculations of various electrodynamic and weak processes in constant high-intensity electromagnetic fields have become quite timely. Thus, for example, in[1] they considered the processes $\gamma\to\nu\bar{\nu}$ and $\nu\to\gamma\nu$ in a strong crossed field $(\mathbf{E}\cdot\mathbf{B}) = E^2 - B^2 = 0$. Obviously, these calculations are of practical significance only in the energy region where the crossed-field approximation is equivalent to a constant and homogeneous magnetic field, since the possible realization of constant fields $\sim B_0$ occurs precisely in the case of a magnetic field.[1)] This is reached at photon and neutrino energies $\omega \gg m$. However, if the reactions $\gamma\to\nu\bar{\nu}$ and $\gamma\to\gamma\nu$ are considered in the sense of their contribution to the neutrino luminosity of pulsars, then it is the frequencies $\omega \lesssim m$ that are significant, since they receive the greater part of the energy radiated by the stars (with the exception of x-ray pulsars), and then the crossed field approximation is not suitable. It should be noted that in this region the reaction $\gamma\to\nu\bar{\nu}$ is suppressed in part, since a photon in a strong magnetic field acquires at $\omega \ll m$ an imaginary mass $(\omega < |\mathbf{q}|)$.[3] This effect, however, can be compensated for by the interaction of the radiation with a plasma, and at a sufficiently large electron density the photon will have a time-like momentum (see below), and the decay $\gamma\to\nu\bar{\nu}$ will be allowed. On the other hand, the difficulties in the calculation of diagrams with electron loops in a magnetic field, with exact allowance of the interaction with the field, were due to the absence of a convenient representation of the electron Green's function suitable for practical applications, so that it became necessary to use the crossed-field approximation.

In this paper we use the method developed by us to calculate diagrams in a strong magnetic field $B\sim B_0$ with the aid of an effectively two-dimensional represen-

  