Connection between the properties of the system of renormalization-group equations and the symmetry of the Hamiltonian in the phase-transition problem

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(Submitted November 14, 1975; resubmitted April 12, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1434–1442 (October 1976)

On the basis of the hypothesis of the universality of the critical behavior of systems a method is proposed by means of which, for Hamiltonians of certain symmetry types, it is possible to establish a connection between the symmetry of the system and the properties of the renormalization-group equations for the invariant charges. As an example the properties of the phase transition in a two-component system with cubic symmetry are analyzed by the method. The functional equations for the Gell-Mann–Low functions are obtained for this model and, using these, it turns out to be possible to establish the positions of the fixed points of the system of renormalization-group equations on the phase plane and to draw conclusions about the nature of their stability without direct recourse to perturbation theory.

PACS numbers: 05.70.Fh

1. INTRODUCTION

In a number of papers[1–7] renormalization-group (RG) equations have been analyzed in order to study the behavior of matter near a critical point. To describe the pattern of the phase transition in many-component systems it is important to establish where the fixed points of the system of RG equations are located and to ascertain whether they are stable, since it is known that stable fixed points correspond to second-order transitions.[1)] For space of arbitrary dimensionality d the system of RG equations is very complicated and cumbersome. Simplification of the system arises only when $d=4-\varepsilon$, $\varepsilon\to 0$, when the RG equations are transformed into the Gell-Mann–Low (GML) equations of field theory. In this case, the first few terms of the Taylor series for the GML functions can be calculated by the ε-expansion method.[1,3,8,9] However, only for $\varepsilon \ll 1$ is it possible to approximate the GML functions by truncations of the series in powers of ε, and it remains unclear whether it is possible to extrapolate the results obtained for small ε to the physical point $\varepsilon=1$. It is known, e.g., that in such an extrapolation the stability character of certain fixed points is found to depend on the number of terms in the series[3] and thus we cannot obtain even a qualitative picture of the behavior of the renormalized coupling constants near a transition in three-dimensional space.

Moreover, according to the universality hypothesis, the structure of the RG equations should be connected in some way with the symmetry of the Hamiltonian. This relationship should impose certain restrictions on the general form of the GML functions and on the values of the renormalized coupling constants near the transition. If it were possible to establish this relationship, we could hope to obtain information about the properties of the phase transition to any order of perturbation theory.

In the present paper we propose a method by means of which we can elucidate the character of this relationship for Hamiltonians of certain symmetry types. The possibility of effective application of the method to a given Hamiltonian is intimately connected with the existence of a certain special group of transformations for the Hamiltonian. As will be shown below, functional equations for the GML functions follow from the symmetry of the system. Using these it turns out to be possible to establish the character of the location of the fixed points of the system of RG equations on the phase plane and to obtain information on the stability of these points. We note that, although in three-dimensional space the RG equations do not go over into the GML equations,[2)] it is clear that the form of the RG equations near the transition will depend only on the symmetry of the bare Hamiltonian and not on the values of the bare constants. It is possible to believe, therefore, that the qualitative results obtained by us in the framework of the ε-expansion and based on symmetry considerations will also remain valid for real three-dimensional systems. Since all the basic features of the method will be clear in its application to a two-component system with cubic symmetry, we shall first consider this model in detail[3)] and then give a general formulation.

2. MODEL OF TWO COUPLED SCALAR FIELDS

The Hamiltonian of the model has the form[4)]

$$\mathcal{H}=(\nabla\varphi_1)^2+(\nabla\varphi_2)^2+\tau(\varphi_1^2+\varphi_2^2)+\lambda_1(\varphi_1^4+\varphi_2^4)+\lambda_2\varphi_1^2\varphi_2^2. \quad (1)$$

Assuming the dimensionality d of space to be close to 4, we can write the RG equations for the invariant charges Λ_1 and Λ_2 in the form

$$\partial\Lambda_1/\partial t=\Psi_1(\Lambda_1,\Lambda_2),\quad \Lambda_1|_{t=0}=\lambda_1,$$
$$\partial\Lambda_2/\partial t=\Psi_2(\Lambda_1,\Lambda_2),\quad \Lambda_2|_{t=0}=\lambda_2. \quad (2)$$

Here Ψ_1 and Ψ_2 are the GML functions and $t=-\ln\varkappa^2$, where $\varkappa^2$ is the "physical mass." We shall consider those transformations K of the fields $\{\varphi_i\}$ under which the Hamiltonian (1) preserves its form but has different values λ_1' and λ_2' of the constants. It is known that, from the group of rotations, rotations of the fields $\{\varphi_i\}$ through angles equal to $\pi(1+2n)/4$ where n is an integer (and only these rotations) will be such transformations. These transformations carry $\mathcal{H}$ into $\mathcal{H}'$,

$$\mathcal{H}'=(\nabla\varphi_1')^2+(\nabla\varphi_2')^2+\tau(\varphi_1'^2+\varphi_2'^2)+\lambda_1'(\varphi_1'^4+\varphi_2'^4)+\lambda_2'\varphi_1'^2\varphi_2'^2, \tag{3}$$

where

$$\lambda_1'=\alpha\lambda_1+\beta\lambda_2,\quad \alpha={}^1/_2,\quad \beta={}^1/_4,$$
$$\lambda_2'=\gamma\lambda_1+\delta\lambda_2,\quad \gamma=3,\quad \delta=-{}^1/_2. \tag{4}$$

The system of RG equations describes the critical behavior, which, according to the universality hypothesis, depends only on the symmetry of the Hamiltonian. Therefore, it is natural to assume that the form of the system of RG equations for $\mathcal{H}'$ is the same as for $\mathcal{H}$, if $\mathcal{H}$ and $\mathcal{H}'$ have the same symmetry. The symmetry of $\mathcal{H}'$ will coincide with the symmetry of $\mathcal{H}$ in the case when there are, as before, two independent invariants of the symmetry in the expression for $\mathcal{H}'$, i.e., when the values of the constants λ_1' and λ_2' are independent and nonzero. Then for the system with Hamiltonian $\mathcal{H}'$ we shall have

$$\partial\Lambda_1'/\partial t=\Psi_1(\Lambda_1',\Lambda_2'),\quad \Lambda_1'|_{t=0}=\lambda_1',$$
$$\partial\Lambda_2'/\partial t=\Psi_2(\Lambda_1',\Lambda_2'),\quad \Lambda_2'|_{t=0}=\lambda_2'. \tag{5}$$

We shall establish the relation between the Λ_i and the Λ_i'. The invariant charges Λ_1 and Λ_2 are proportional to the renormalized vertices

$$\Gamma_1=\Gamma_{\alpha\alpha\alpha\alpha}(0,0,0,\varkappa^2)\ \text{and}\ \Gamma_2=3\Gamma_{\alpha\alpha\beta\beta}(0,0,0,\varkappa^2),\quad \alpha\neq\beta.$$

The coefficients of proportionality do not change under a transformation K, since they are related to the Green functions, the matrix $G_{ij}=G\delta_{ij}$ of which does not change under transformations rotating the fields. It is sufficient, therefore, to find out how Γ_1 and Γ_2 transform. Taking into account that Γ_1 and Γ_2 are simply expressed in terms of irreducible correlation functions (whose transformation law is easily found if we know the transformation of the fields $\{\varphi_i\}$), we obtain

$$\Lambda_1'=\alpha\Lambda_1+\beta\Lambda_2,\quad \Lambda_2'=\gamma\Lambda_1+\delta\Lambda_2. \tag{6}$$

Hence we have

$$\frac{\partial\Lambda_1'}{\partial t}=\alpha\frac{\partial\Lambda_1}{\partial t}+\beta\frac{\partial\Lambda_2}{\partial t},\quad \frac{\partial\Lambda_2'}{\partial t}=\gamma\frac{\partial\Lambda_1}{\partial t}+\delta\frac{\partial\Lambda_2}{\partial t}. \tag{7}$$

Then for the GML functions Ψ_1 and Ψ_2 we obtain

$$\Psi_1(\Lambda_1',\Lambda_2')=\alpha\Psi_1(\Lambda_1,\Lambda_2)+\beta\Psi_2(\Lambda_1,\Lambda_2),$$
$$\Psi_2(\Lambda_1',\Lambda_2')=\gamma\Psi_1(\Lambda_1,\Lambda_2)+\delta\Psi_2(\Lambda_1,\Lambda_2). \tag{8}$$

These equalities are functional equations for Ψ_1 and Ψ_2. If, as is usually done, we expand the Ψ_i in series in Λ_1, Λ_2, relations for the expansion coefficients will follow from (8). In n-th order there will be n independent linear relations for $2n$ unknown expansion coefficients, since, knowing one of the functions Ψ_i, it is easy to find the second from (8). In particular, the expansions obtained in[5] for the GML functions for four-dimensional space satisfy these relations.

Starting from Eqs. (6) and (8), we can determine the location of the fixed points of the system (2) on the phase plane and obtain information about their stability. We shall regard the equalities (6) as a transformation of the point with coordinates Λ_1, Λ_2 in the phase plane to a point with coordinates Λ_1', Λ_2'. Then, generally speaking, the stable fixed points should also transform into each other. We shall assume, however, that the coordinates $\Lambda_i^{(0)}$ of such points satisfy the relations

$$\Lambda_i^{(0)\prime}=\Lambda_i^{(0)}, \tag{9}$$

i.e., stable fixed points remain stationary under the transformation (6). It is obvious that if the system has only one such point the condition (9) is fulfilled for it. Below, using rather natural assumptions, we shall show that, apparently, it is precisely this case which occurs in the system under consideration. At the same time we shall adduce arguments from which it follows that the stable fixed points should be stationary for certain other systems too. The stationary points are the eigenvectors, with eigenvalue +1, of the matrix of the linear transformation (6). This matrix has two eigenvectors: $\Lambda_2=2\Lambda_1$ and $\Lambda_2=-6\Lambda_1$, with eigenvalues +1 and −1 respectively. Consequently, the stationary fixed points of the system (2) can lie only on the straight line $\Lambda_2=2\Lambda_1$, i.e., they are the fixed points of the isotropic Heisenberg model.

The arguments adduced cannot be extended to those values of the bare constants λ_1 and λ_2 for which the symmetry of the Hamiltonian changes under the transformation K. Such special values do exist. In fact, if the bare constants λ_1 and λ_2 lie on one of the straight lines $y=0$ or $y=6$ ($y\equiv\lambda_2/\lambda_1$), the transformation K carries them on to the straight lines $\lambda_2'=6\lambda_1'$ and $\lambda_2'=0$. In this case the Hamiltonians $\mathcal{H}$ and $\mathcal{H}'$ will have different numbers of invariants and, consequently, their symmetry will be different. Therefore, in these cases we cannot, of course, use the universality hypothesis, and the form of the system of RG equations for the Hamiltonian $\mathcal{H}'$ will differ from (2). Thus, we arrive at the conclusion that, apart from points that are stationary under the transformation (6), points lying on the straight lines $\Lambda_2=6\Lambda_1$ and $\Lambda_2=0$ can also be fixed points of the system (2). It is obvious that the fixed points on these straight lines are the fixed points of the Ising model.

We see, then, that (if the assumption (9) is valid) all the fixed points in our model lie on only three straight lines: the straight lines $y=0$ and $y=6$ (Ising fixed points) and the straight line $y=2$ (Heisenberg (XY-model) fixed points).

We now consider the question of the stability of the fixed points. We introduce the matrix A:

$$A\equiv\begin{pmatrix}\partial\Psi_1/\partial\Lambda_1 & \partial\Psi_1/\partial\Lambda_2\\ \partial\Psi_2/\partial\Lambda_1 & \partial\Psi_2/\partial\Lambda_2\end{pmatrix}\equiv\begin{pmatrix}a & b\\ c & d\end{pmatrix}. \tag{10}$$

The characteristic equation at the fixed point then has the form

$$\rho^2-(a+d)\rho+ad-bc=0. \tag{11}$$

Points which are carried into each other by the transformation (6) will be called conjugate points. By taking differentials of both sides of the equalities (8) we obtain the following relations between the first derivatives of the GML functions at conjugate points:

$$\alpha a'+\gamma b'=\alpha a+\beta c,\quad \beta a'+\delta b'=\alpha b+\beta d,$$
$$\alpha c'+\gamma d'=\gamma a+\delta c,\quad \beta c'+\delta d'=\gamma b+\delta d. \tag{12}$$

Here a', b', c' and d' are the elements of the matrix A at the conjugate point. As is easily checked, it follows from (12) that at conjugate points

$$a'+d'=a+d,\quad a'd'-b'c'=ad-bc, \tag{13}$$

i.e., the roots ρ_1 and ρ_2 of the characteristic equation (11) are the same at conjugate Ising points, as they should be. Therefore, it is sufficient to ascertain the stability of the Ising points on the straight line $\Lambda_2=0$. It is clear that for points on this straight line, $c=0$ and $a=\Psi'_{n=1}$ is the derivative of the GML function for the Ising model. Thus the matrix A has the form

$$\begin{pmatrix} \Psi'_{n=1} & b \\ 0 & d \end{pmatrix} \tag{14}$$

and the roots of the characteristic equation are $\rho_1=\Psi'_{n=1}$ and $\rho_2=d$. Consequently, the stability of the Ising point depends only on the sign of the function $d\equiv\partial\Psi_2/\partial\Lambda_2$ at this point. For the Heisenberg (stationary) point it is clear that

$$\partial\Psi_i/\partial\Lambda_k'=\partial\Psi_i/\partial\Lambda_k, \tag{15}$$

and from (12) we find that the matrix A has the form

$$\begin{pmatrix} a & {}^1/_4(a-d) \\ 3(a-d) & d \end{pmatrix}, \tag{16}$$

with $a+2b=3a/2>d/2=\Psi'_{n=2}$, where $\Psi'_{n=2}$ is the derivative of the GML function for the XY-model. The roots of the characteristic equation are $\rho_1=\Psi'_{n=2}$ and $\rho_2=3d/2-a/2$, and, consequently, the Heisenberg point is stable if

$$0<\Psi'_{n=2}/d<4. \tag{17}$$

Finally, for the point $\Lambda_1=\Lambda_2=0$ both (14) and (16) are fulfilled, whence it follows that the matrix A has the form

$$\begin{pmatrix} \Psi'_{n=1} & 0 \\ 0 & \Psi'_{n=2} \end{pmatrix} \tag{18}$$

and the equality

$$\Psi'_{n=1}=\Psi'_{n=2} \tag{19}$$

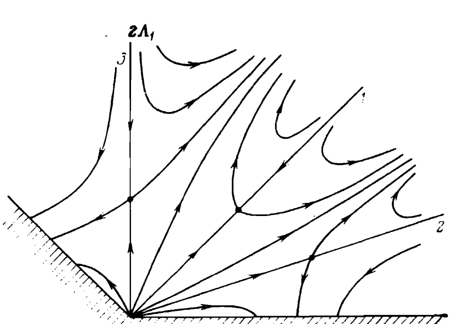

FIG. 1.

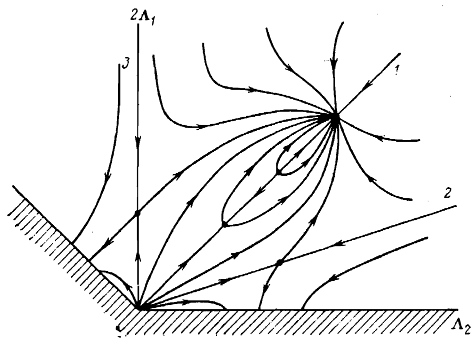

FIG. 2.

should be fulfilled at $\Lambda_1=\Lambda_2=0$. This equality is consistent with the fact that the form of the linear term in the expansion of the GML function in the invariant charge for the n-component Heisenberg model does not depend on the number of components.[4]

Thus, the point $\Lambda_1=\Lambda_2=0$ is an unstable zero, as we should expect. Taking account of this fact and the fact that we know the positions of the other fixed points, we can analyze qualitatively the behavior of the phase trajectories. It is then not difficult to convince oneself that the Ising points cannot be stable zeros but are saddle points.

Of course, the stability of the Heisenberg point nearest to the origin cannot be determined unambiguously from symmetry arguments alone. We shall discuss, therefore, both the available possibilities.

A. The Heisenberg point is a saddle point. Then, if the GML function of the two-component Heisenberg model has only one nontrivial zero, then, in our model, there is no power-law solution at all and, consequently, no scaling, since the phase trajectories go away to infinity for practically all values of the bare constants (see Fig. 1). A power-law solution can be obtained only by assuming that the GML function of the Heisenberg model has no fewer than three nontrivial zeros (see Fig. 2). In this case, depending on the magnitude of the bare interaction, we should observe no fewer than two different (generally speaking) types of critical indices. However, while not rejecting this case entirely, on the basis of the arguments cited we regard it as improbable.

B. The Heisenberg point is a stable zero. This situation seems to us to be more natural. The critical indices in this case are the same as those for the Heisenberg model. The qualitative pattern of the behavior of the phase trajectories is depicted in Fig. 3.

In the figures the numeral 1 denotes the phase trajectory $\Lambda_2=2\Lambda_1$ (the Heisenberg fixed points (stationary) lie on this straight line) and the numerals 2 and 3 denote the phase trajectories $\Lambda_2=6\Lambda_1$ and $\Lambda_2=0$ (the Ising points (conjugate to each other) lie on these straight lines). The region of instability of the Hamiltonian (1) is shaded.

We return to the condition (9). If the Heisenberg point is stable, there will evidently be no other stable points. The point is that, if this were not so, there would be no fewer than three different stable fixed

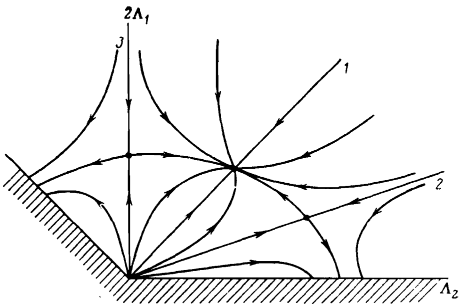

FIG. 3.

points, and integral curves which would separate the regions of stability corresponding to these points would exist. It seems natural to assume that the existence of such separatrices should be associated with certain changes in the symmetry of the Hamiltonian, i.e., with definite values of the ratio $y = \lambda_2/\lambda_1$ of the bare constants. Therefore, such separatrices would be straight lines passing through the coordinate origin. However, for any value of y other than 0, 2, or 6, the symmetry of the Hamiltonian is the same (there are two independent invariants). Thus, there are evidently no such separatrices, and the stable Heisenberg point is the only stable fixed point.

For both cases (A and B) and for bare constants λ_1, λ_2 such that $-2 < y < 0$ or $6 < y < \infty$ the phase trajectories leave the region of stability. This indicates the possibility of the existence of a first-order phase transition in the system. The properties of first-order transitions have been calculated recently by Lyuksyutov and Pokrovskiĭ[5] in first order in ε. We note that the results obtained in[3,8] by approximate methods do not contradict ours and give the case B.

3. GENERAL FORMULATION AND DISCUSSION OF THE METHOD

We shall formulate our principal assumption in general form. Suppose that we have the Hamiltonian $\mathscr{H} \equiv \mathscr{H}(\{\varphi_i\}, \{\lambda_k\})$ of an n-component field $\{\varphi_i\}$ with m coupling constants λ_k and that it possesses a given symmetry. Let there exist a transformation of the field, $K\{\varphi_i\} = \{\varphi'_i\}$, such that the Hamiltonian $\mathscr{H}$ preserves its symmetry under this transformation, i.e., the same number of the same symmetry invariants appear in the expression for the transformed Hamiltonian $\mathscr{H}'$ as in $\mathscr{H}$. We then assert that the form of the RG equations will be the same for the systems with Hamiltonians $\mathscr{H}$ and $\mathscr{H}'$.[5)] This postulate is based on the idea that the GML functions describe the behavior near the phase-transition point, and this behavior, by the universality hypothesis, depends only on the symmetry of the Hamiltonian and is, consequently, the same for systems with the Hamiltonians $\mathscr{H}$ and $\mathscr{H}'$. It follows from this that such symmetry considerations are applicable to the system of RG equations near T_c not only for small ε but also for the real three-dimensional case (when, strictly speaking, there are no GML equations of the form (2)). Here the conclusion that there is a possible increase in the symmetry of the system at the transition point evidently remains in force.

It is clear that the set of K-transformations forms a group. We shall call it the group of covariance of the Hamiltonian. Inasmuch as a transformation K depends, obviously, on the concrete form of $\mathscr{H}$, the structure of this group is related only to the Hamiltonian (for a given degree of nonlinearity of $\mathscr{H}_{\text{int}}$), since the actual form of the Hamiltonian is completely determined by the symmetry of the system. At the same time the covariance group imposes certain conditions on the GML functions, i.e., on the structure of the RG. In fact, if we express the invariant charges Λ'_i in terms of the Λ_i, we obtain certain functional equations of the type (8) for the GML functions. From these equations we can obtain certain information on the stability of the fixed points, as was done in the example cited above. Thus, the structure of the RG is found to be connected with the symmetry of the Hamiltonian.

In general, in the approach expounded the basic problem concerns whether covariant transformations that are nontrivial (i.e., not identity transformations, with respect to the invariant charges) exist for the given Hamiltonian, and how to find them. Inasmuch as the symmetry group of the system is defined by a certain set of rotation and reflection operations, the covariance transformations must also be sought primarily amongst the linear transformations. It is fairly clear that such linear transformations exist for systems with low symmetry. For example, if we consider a system with triclinic symmetry, the interaction Hamiltonian is in general a complete homogeneous polynomial with independent coefficients. Any rotation leaves the form of the Hamiltonian unchanged, i.e., the covariance group is continuous. Therefore, if a second-order phase transition occurs in such a system, the position of the corresponding stable fixed point (in the space of the invariant charges) cannot be arbitrary. Indeed, otherwise an infinitesimal K-transformation would carry this point into an arbitrarily close point, which ought also to be stable. Thus, the fixed point would turn out not to be isolated.

As the symmetry of the system is increased (i.e., as the bare coupling constants obey an ever larger number of conditions), the number of nontrivial covariant transformations decreases, and for sufficiently high symmetry there can be none at all. For example, it can be shown that there are no nontrivial linear covariant trans-transformations for systems with cubic symmetry and $n > 2$ components. If we assume that the covariance group of the RG equations is exhausted by the linear transformations, the absence of such transformations for systems with sufficiently high symmetry makes possible, for such systems, the existence of fixed points of a type that is characteristic just of the given symmetry. The appearance of the specific fixed point (cubic) for $n > 2$, obtained by the ε-expansion method in[3], is not surprising from this point of view.

From the arguments adduced it follows that second-order phase transitions which would correspond to "low-symmetry" fixed points, i.e., points at which the symmetry admits a continuous covariant transformation, do not exist. Therefore, in a system with "low" symmetry, a second-order phase transition can occur only with an increase of symmetry at the transition point. Moreover, an increase of symmetry at the transition point will also occur for systems possessing a discrete covariance group if the system of RG equations has only one stable fixed point.

For such systems the same critical indices as for systems with the higher symmetry should be observed experimentally. The structure of the critical fluctuations and, consequently, the increase in the symmetry of the system as $T \to T_c$ can be investigated by means of electron paramagnetic resonance (EPR), light-scattering experiments, acoustic methods, etc. In particular, data which apparently indicate a change in the symmetry of the field of the critical fluctuations in the structural phase transition in the cubic crystal $SrTiO_3$ have recently been obtained by the method of EPR spectroscopy.[11]

In conclusion I express my gratitude to A. I. Sokolov for numerous discussions and useful criticism, and also to S. L. Ginzburg and S. V. Maleev for a discussion of the results of the work. I am sincerely grateful to D. E. Khmel'nitskiĭ and A. A. Migdal. Discussions with them on the structure of the RG equations have been of great benefit to me.

1)It is implied that the power-law asymptotic form has already been separated out from the expressions for the invariant charges.

2)The GML equation in[4] was obtained in the framework of a certain self-consistent scheme and is not, strictly speaking, an exact equation for the renormalized coupling constants. It is possible, however, to adduce arguments that the values of the critical indices calculated using such an equation will be close to the true values.

3)The behavior of such a model in the framework of the ε-expansion was investigated earlier in[3,5,8]. It is used to describe structural phase transitions from tetragonal to rhombic symmetry and also applies to the formation of superstructures in alloys forming a body-centered cubic lattice.[5,10]

4)The notation is the same as in[5].

5)We note that for certain special values of the constants $\{\lambda_k\}$ the transformation K can lead to couplings between different λ'_k or can make some of the λ'_k vanish. In this case the symmetry of $\mathcal{H}'$ will not coincide with the symmetry of $\mathcal{H}$ and our assertion does not apply to these special cases.

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Translated by P. J. Shepherd

Phase diagram and domain-boundary structure in a uniaxial ferrimagnet near the compensation point

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(Submitted February 20, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1443–1452 (October 1976)

The phase diagram of a quasi-uniaxial mixed rare-earth iron garnet has been investigated by magneto-optical methods. The lines of stability loss were determined for the low- and high-temperature collinear phases and the noncollinear phase, together with their ranges of coexistence. Near the triple point of the phase diagram, there was observed a broadening of the domain boundary between collinear phases, with subsequent transformation of the boundary to the noncollinear phase. It was shown that phase segregation in the specimen occurs over a wide range of temperatures and of magnetic fields, located within the single-domain range for the "Weiss" domains that are due to the demagnetizing fields. The structure of the transition regions between coexisting magnetic phases was investigated.

PACS numbers: 75.50.Gg, 75.60.Fk, 78.20.Ls

The continued interest in investigation of the behavior of ferrimagnets in the vicinity of their magnetic compensation point has recently increased significantly because of the discovery, in this region, of the phenomenon of coexistence of several magnetic phases; that is, of a distinctive domain structure, which exists over a quite wide range of variation of the temperature and of the external magnetic field, including fields that appreciably exceed the field for "technical" saturation of the material.[1–10] In experiments on iron garnets, which have good optical transparency in the visible and infrared ranges of wavelength, broad use is made of visual methods of investigation, based on the use of the Faraday and Cotton–Mouton magneto-optic effects.[2–11]

