Features of the phase transition in nonequilibrium superconductors with optical pumping

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(Submitted April 4, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1490–1502 (October 1976)

The behavior of a nonequilibrium superconductor near the phase-transition point is investigated. Nonequilibrium quasi-particles are created as a result of absorption of an electromagnetic field with frequency considerably greater than the superconducting gap. The energy distribution of the quasi-particles at zero temperature and at temperatures close to the transition temperature is studied by means of the kinetic equation describing the energy relaxation of the excess quasi-particles with emission of phonons. The dependence of the order parameter on the power of the source and on the temperature is found. It is shown that, above a certain critical power at a fixed temperature (or above a certain temperature at a fixed power of the source), this dependence becomes double-valued, i.e., apart from the usual solution, according to which the gap decreases with increase of pumping power, there appears a second solution, describing an increase of the gap with increased pumping. The existence of the second solution is connected with the coherent character of the interaction of the quasi-particles of the superconductor with the phonons. The stability of the state of a superconductor with optical pumping against small fluctuations is considered and it is postulated that the double-valued dependence of the order parameter on the pumping power may be responsible for the experimentally observed gradual increase in the resistivity of a superconductor with increased pumping.

PACS numbers: 74.30.Hp

INTRODUCTION

In recent years papers have appeared that are devoted to the experimental study of superconductors subjected to the action of an electromagnetic field with frequency considerably greater than the magnitude of the gap Δ (so-called optical pumping).[1,2] In this case the absorption of the field leads to the formation of extra quasi-particles, in excess of the thermal quasi-particles.

It was discovered that the state of the superconductors and, in particular, the magnitude of the order parameter Δ are exceptionally sensitive to the number of excess quasi-particles and to the form of their distribution function, and the phase transition from the superconducting to the normal state possesses, in nonequilibrium conditions, a number of important distinctive features.

In 1972 Owen and Scalapino[3] postulated that the distribution function of the excess quasi-particles is described by a Fermi function n_F with a nonzero chemical potential (a quasi-equilibrium function). At zero temperature this function becomes equal to unity in a certain range of energies. This means that there is an inverted population for the quasi-particles ($n > \frac{1}{2}$). An inverted population would lead to a number of unusual properties—in particular, to a first-order transition to the normal state when the gap reaches the value $\Delta_0/3$ (Δ_0 is the gap in the absence of pumping).

However, it was shown in a paper by the author[4] that the distribution function of the optically excited quasi-particles in ordinary superconductors (in which the gap Δ is much smaller than the Debye frequency ω_D) cannot exceed $\frac{1}{2}$. The reason is that the scattering and recombination of the quasi-particles as a result of one-phonon processes proceed with almost equal probability. In addition, it was found that the order parameter does not vanish discontinuously at $\Delta_0/3$, but decreases and goes to zero at a certain critical power β_c of the source. Subsequent experimental investigations[5–7] have confirmed this behavior of the order parameter. At the same time, new interesting phenomena were observed near the phase-transition point. In particular, it was found that the appearance of finite resistance of the nonequilibrium superconductor occurs not discontinuously at $\Delta = 0$ (as in the equilibrium case) but smoothly, starting from a certain critical value of the power of the pumping source.[6,7]

Despite the considerations at hand in[6–8], the causes of the smeared-out transition for the resistance of a superconductor with optical pumping and also of certain other phenomena analyzed in[9] remain, as yet, insufficiently clear.

The purpose of the present paper is to investigate the behavior of a superconductor with optically excited quasi-particles near the phase-transition point, when the order parameter is small owing to the action of the source ($T = 0$) or of the source and the temperature ($T \lesssim T_c$; T_c is the temperature of the transition in the equilibrium superconductor). For this we have obtained a more accurate (as compared with[4]) solution of the kinetic equation and, on the basis of this, have found the dependence of the order parameter Δ on the pumping power and temperature. The principal result is that the dependence of Δ on the pumping and temperature (in the presence of pumping) turns out to be nonunique in the region of small Δ. In other words, above a certain critical pumping power β_c at a fixed temperature (or above a certain temperature at a fixed pumping power), there exist two solutions for Δ: one describes an increase of the gap with pumping and the other describes a decrease of Δ with increase of the power of the source. There is a definite analogy between the dependence of Δ

on β and T and the dependence obtained by Éliashberg[10] for superconductors irradiated by a high-frequency field with $\omega \ll \Delta_0$. However, the physical reason for the appearance of the growing solution is different, and is as follows. The coherent character of the interaction of the quasi-particles with the phonons (the interaction with phonons is responsible for the relaxation of the nonequilibrium quasi-particles) leads to an increase in the probability of recombination of quasi-particles, which is proportional to $\Delta^2/\varepsilon\varepsilon'$. Therefore, with increase of pumping power a decrease occurs in the number of quasi-particles near the Fermi surface (which make the principal contribution to the equation for the gap) and, consequently, the gap increases.

We postulate that the nonunique dependence of the order parameter on the pumping power is able to explain the smooth growth of the d.c. resistance of a superconducting film. In fact, the existence of two solutions for the gap can lead to instability of the uniform state of the superconductor and to partition into regions corresponding to the different solutions (including the solution $\Delta = 0$); in its turn, the nonuniform state leads to a finite resistance.

In the paper we find the solutions of the kinetic equation and the equation for the gap at $T=0$ (Secs. 2, 3) and at temperatures close to the critical temperature T_c (Sec. 4). In Sec. 5 we discuss the stability of the nonequilibrium state of the superconductor. For simplicity we consider a spatially uniform system and assume that the phonons are in equilibrium, inasmuch as allowance for nonuniformity and phonon heating will not change the qualitative results.

1. BASIC EQUATIONS AND FORMULATION OF THE PROBLEM

We shall consider a model in which the energy relaxation of the excess quasi-particles proceeds as a result of interaction with phonons. As shown in[11,12], owing to the small value of the effective interaction constant, electron–electron collisions have only a weak influence on the form of the quasi-particle distribution function. They lead, however, to a substantial renormalization of the quasi-particle source, and we shall assume that this has been carried out. The kinetic equation for the distribution function n and the equation for the gap in the nonequilibrium state have the form[10]

$$\frac{\pi\lambda}{2\omega_D^{k+1}}[nS(n')-S^+(n')]=Q(\varepsilon),$$

$$S(n')=\int_\xi^{\omega_D}(n'+N_{\varepsilon'-\varepsilon})\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon-\varepsilon')^{k+1}d\xi'$$
$$+\int_0^\xi(1-n'-N_{\varepsilon-\varepsilon'})\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon-\varepsilon')^{k+1}d\xi'$$
$$+\int_0^{\omega_D}(n'+N_{\varepsilon+\varepsilon'})(\varepsilon+\varepsilon')^{k+1}\left(1+\frac{\Delta^2}{\varepsilon\varepsilon'}\right)d\xi', \quad (1)$$

$$S^+(n')=\int_\xi^{\omega_D}n'(1+N_{\varepsilon'-\varepsilon})\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon-\varepsilon')^{k+1}d\xi'$$
$$+\int_0^\xi n'N_{\varepsilon-\varepsilon'}\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon-\varepsilon')^{k+1}d\xi'$$
$$+\int_0^{\omega_D}(1-n')N_{\varepsilon+\varepsilon'}\left(1+\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon+\varepsilon')^{k+1}d\xi',$$

$$N_\varepsilon=(e^{\varepsilon/T}-1)^{-1},\quad \varepsilon=(\xi^2+\Delta^2)^{1/2},\quad \xi=p^2/2m-\mu,$$
$$n(\xi')\equiv n',\quad \varepsilon(\xi')\equiv\varepsilon',\quad \hbar=1,\quad \lambda=g^2\frac{mp_0}{2\pi^2},\quad \mu=\frac{p_0^2}{2m},$$

$$1=\lambda\int_0^{\omega_D}\frac{1-2n(\xi)}{\varepsilon}d\xi, \quad (2)$$

where N_ε is the equilibrium distribution function of the phonons, k is the power in the dependence of the square of the matrix element of the electron-phonon interaction on the wave vector q ($k=0,\pm1$),

$$M_q^2\sim q^k, \quad (3)$$

ω_D is the Debye frequency, and μ is the Fermi level of the superconductor in equilibrium. The right-hand side $Q(\varepsilon)$ of Eq. (1) describes the interaction of the quasi-particles with the electromagnetic field. This interaction leads to a change in the energy distribution of the quasi-particles and to the creation of new quasi-particles ($\omega > 2\Delta_0$). It can be shown (cf. [4]) that in the case of interest to us, i.e., the case of optical pumping, when the frequency of the field is considerably greater than the gap (the case of a broad source), the principal effect is the creation of quasi-particles, since the ratio of the probabilities of the two effects is proportional to ω/Δ_0. In this approximation the expression for $Q(\varepsilon)$ is simplified and takes the form

$$Q(\varepsilon)=2\tilde{\alpha}\theta(\omega-\varepsilon),\quad \tilde{\alpha}=\frac{E^2e^2lp_0}{\omega^2}\left(1+\frac{c_1}{6\cdot2^{2/3}}\frac{\omega^3}{\mu\omega_D^2}\right), \quad (4)$$

where E is the amplitude of the field, l is the electron mean free path, and c_1 is the ratio of the electron–electron to the electron-phonon constant.[11] It is convenient to supplement Eq. (1) by the condition

$$\int_0^\infty d\xi\int_0^\infty d\xi'(nn'-N_{\varepsilon+\varepsilon'}(1-n-n'))\left(1+\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon+\varepsilon')^{k+1}$$
$$=\int_0^\infty d\xi Q(\varepsilon)=\frac{4\tilde{\alpha}\omega\omega_D^{k+1}}{\pi\lambda}, \quad (5)$$

which is obtained if we integrate (1). The point is that, in the case of a broad source, it is sufficient to solve the homogeneous equation (1) and then, with the aid of (5), find the coupling of the resulting solution with the pumping source.

An important property of the function n in the case $\Delta_0 \ll \omega_D$ usually realized (which is considered below) is the absence of an inverted population on optical pumping, i.e.,

$$n(\xi)<{}^1/_2. \quad (6)$$

This property was proved in[4] for $T=0$. It can be shown that it also remains valid at finite temperatures.

2. THE ORDER PARAMETER NEAR THE TRANSITION POINT FOR $T=0$, $k=-1$

We shall consider a superconductor at $T=0$, in which the order parameter vanishes on account of the action of optical pumping. For a clear idea of the features of the problem it is convenient to study first a model in which the dependence (3) for the matrix element has the form $M_q^2 \sim 1/q$, i.e., $k=-1$. Substituting $k=-1$ into Eq. (1) and putting $T=0$, $N=0$, we obtain the following equation:

$$-(1-n)\int_{\xi}^{\omega_D} d\xi' n'\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)+n\int_0^{\xi} d\xi'(1-n')\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)$$
$$+n\int_0^{\omega_D} d\xi' n'\left(1+\frac{\Delta^2}{\varepsilon\varepsilon'}\right)=0. \tag{7}$$

The normalization condition (5) takes the form

$$\frac{1}{\Delta_0^2}\int_0^{\infty} d\xi\int_0^{\infty} d\xi' nn'\left(1+\frac{\Delta^2}{\varepsilon\varepsilon'}\right)=\beta^0, \quad \beta^0=\frac{4\tilde{a}\omega}{\pi\lambda\Delta^2}. \tag{8}$$

We seek the solution of (7) in the form

$$n(\xi)=n_0(\xi)+n_1(\xi), \tag{9}$$

where the functions n_0 and n_1 respectively satisfy the equations

$$-(1-n_0)\int_{\xi}^{\omega_D} d\xi' n_0'+n_0\int_0^{\xi} d\xi'(1-n_0')+n_0\int_0^{\omega_D} d\xi' n_0'=0, \tag{10}$$

$$n_1\Gamma(\xi)-P(\xi)\int_{\xi}^{\omega_D} n_1'\, d\xi'=\psi(\xi), \tag{11}$$

$$\Gamma(\xi)=a+\int_0^{\xi}(1-2n_0')\,d\xi', \quad P(\xi)=1-2n_0, \quad a=2\int_0^{\omega_D} n_0\, d\xi; \tag{12}$$

$$\psi(\xi)=-\frac{\Delta^2}{\varepsilon}\left[\int_{\xi}^{\omega_D}\frac{d\xi' n_0'}{\varepsilon'}-n_0\int_0^{\xi}\frac{d\xi'(1-2n_0')}{\varepsilon'}\right].$$

We are interested in the behavior of the order parameter Δ at values of the pumping power for which Δ becomes small compared with Δ_0. In this case, the quantity a characterizing the energy interval which is occupied by nonequilibrium quasi-particles is approximately equal to Δ_0.[4] It can be seen from (11), (12) that the function n_1 is proportional to the small parameter Δ/a, so that the terms $n_1\Delta^2$ in Eq. (7) have a higher order of smallness and, for this reason, are omitted in (11). In the approximation taken, the normalization condition has the form

$$a^2/4+a\int_0^{\infty} n_1\, d\xi+\Delta^2\left(\int_0^{\infty}\frac{n_0\, d\xi}{\varepsilon}\right)^2=\Delta_0^2\beta^0. \tag{13}$$

Equation (10) is solved exactly (cf.[4]):

$$n_0(\xi)=\frac{1}{2}\left(1-\frac{\xi}{(\xi^2+a^2)^{1/2}}\right). \tag{14}$$

The constant a is found from the condition (13) and, as will be shown, is equal to

$$a=2\Delta_0(\beta^0)^{1/2}. \tag{15}$$

With the aid of (14) we can find the critical pumping power at which the gap vanishes. Substituting (14) into (2) and putting $\Delta=0$, we find

$$a_c=\Delta_0, \quad \beta_c^0={}^1/_4, \tag{16}$$

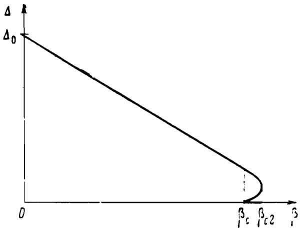

FIG. 1. Dependence of the order parameter on pumping power at $T=0$.

which coincides with formula (21) of the paper.[4] But, as we shall see, this solution is not unique. If $\Delta\neq 0$ it is necessary to take n_1 into consideration. Solving Eq. (11), we obtain for the function n_1 the following expression:

$$n_1=\frac{\psi}{\Gamma}+\frac{P(\xi)}{\Gamma^2}\int_{\xi}^{\omega_D}\psi(\xi')\,d\xi',$$
$$P(\xi)=\xi/(\xi^2+a^2)^{1/2}, \quad \Gamma(\xi)=(\xi^2+a^2)^{1/2}. \tag{17}$$

With the aid of (14) the function ψ (12) can be calculated in explicit form:

$$\psi(\xi)=-\frac{\Delta^2}{2\varepsilon}\left[\ln\frac{a+\Delta}{\xi+\varepsilon}+\frac{\xi}{(\xi^2+a^2)^{1/2}}\ln\frac{\varepsilon+(\xi^2+a^2)^{1/2}}{a+\Delta}\right]. \tag{18}$$

It is not difficult to see that the function n_1 does not exceed the small quantity $(\Delta/\Delta_0)\ln(\Delta_0/\Delta)\ll 1$, and, therefore, the approximations we have made are justified.

Since the function ψ is negative, the correction n_1 to the distribution function, arising on account of the coherent terms $\Delta^2/\varepsilon\varepsilon'$, also turns out to be negative. This important property, which is conserved in the general case, gives rise to the nonunique dependence of the gap on the pumping power.

Substituting (9), (14), (17) and (18) into (2), we obtain the equation for the gap

$$\frac{\Delta}{\Delta_0}\left[\ln\frac{\Delta_0}{\Delta}-\frac{2}{\pi}(1+G)\right]=\frac{2}{\pi}\frac{a-a_c}{a_c}, \tag{19}$$
$$G=\int_1^{\infty}\frac{dz\ln z}{z^2+1}\approx 0.915, \quad \frac{2}{\pi}(1+G)\approx 1.8$$

to within terms $(\Delta/\Delta_0)^2$. We note that the principal contribution from the function n_1 for $\xi\sim\Delta$ is made by the first term in (17).

The solution of Eq. (19) is depicted in Fig. 1. It can be seen from Fig. 1 that, for $a<a_c$, the gap decreases monotonically with increase of pumping power β^0, to a value $\Delta/\Delta_0\approx 0.16$. If the pumping power exceeds the critical value a_c there appears a second solution, according to which the order parameter Δ increases with increase of a. The two solutions merge at pumping $a_{c2}/a_c=1.1$, $\Delta_2/\Delta_0=0.06$, above which the gap vanishes. (We note that in the interval $1<a/a_c<1.1$ the gap $\Delta/\Delta_0<0.16$, so that the parameter $(\Delta/2\Delta_0)\ln(\Delta_0/\Delta)\approx 1.5$ remains small in the region of interest to us.)

The growing solution arises because of the negative sign of the correction n_1 to the distribution function. In its turn, the sign of the correction is due to the sign of the terms $\Delta^2/\varepsilon\varepsilon'$ in Eq. (1), which take into account the coherent character of the interaction of the quasi-particles of the superconductor with the phonons. These terms $\Delta^2/\varepsilon\varepsilon'$ lead to an increase in the recombination and a decrease in the scattering of quasi-particles. (In an excitonic insulator the sign of the correction is positive, since the coherence factors decrease the recombination and increase the scattering of quasi-particles, and, consequently, the growing solution is absent.) For small Δ, when this effect is important, a decrease of the quasi-particle distribution function in the interval $\xi\sim\Delta$ occurs, giving rise to the increase of the gap.

We draw attention to a certain analogy between the dependence of Δ on a and the dependence obtained in[10] for superconductors irradiated by a high-frequency field with $\omega\ll\Delta_0$. In the latter case the growing solution arises not from the coherent character of the relaxation processes but from the action of the field, which changes the energy distribution of the quasi-particles.

The model solution $n_1(\xi)$ that we have found possesses certain properties that are conserved in the general case. As has been noted, the principal contribution at small values of ξ is made by the first term in (17); this term is obtained from Eq. (11) if in it we neglect the integral term. Taking into account that

$$\int_0^\infty n_1\,d\xi=\frac{1}{a}\int_0^\infty \psi(\xi)\,d\xi=-\left(\frac{\Delta}{2\Delta_0}\ln\frac{\Delta_0}{\Delta}\right)^2, \tag{20}$$

we can omit the integral term in (11) in our approximation. If we substitute (9), (17) and (20) into (13), we arrive at the relation (15), which is obtained immediately if we omit the last two terms in (13).

3. THE ORDER PARAMETER FOR $T=0$, $k=1$

We now consider the more realistic case when the dependence of the matrix element has the form $M_q^2\sim q$. Equation (1) takes the form

$$-(1-n)\int_\xi^{\omega_D} n'\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon-\varepsilon')^2\,d\xi'$$
$$+n\int_0^\xi (1-n')\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon-\varepsilon')^2\,d\xi'+n\int_0^{\omega_D} n'\left(1+\frac{\Delta^2}{\varepsilon\varepsilon'}\right)(\varepsilon+\varepsilon')^2\,d\xi'. \tag{21}$$

It is not difficult to show that the distribution function n and its derivative at the point $\xi=0$ are respectively equal to

$$n(\xi=0)=\frac{1}{2}\left(1-\frac{3\Delta a_1}{a_2}\right),\quad \left.\frac{dn}{d\varepsilon}\right|_{\xi=0}=-\frac{a_1}{2a_2}, \tag{22}$$

$$a_m=\int_0^\infty d\xi\,\varepsilon^m n(\xi),\qquad m=0,1,2. \tag{23}$$

We shall seek the solution of (21) in the form $n=n_0(\varepsilon)+n_1(\xi)$; then the equations for the functions n_0 and n_1 are found to be the following:

$$-(1-n_0)\int_\xi^{\omega_D} n_0'(\varepsilon-\varepsilon')^2\,d\xi'+n_0\int_0^\xi (1-n_0')(\varepsilon-\varepsilon')^2\,d\xi'$$
$$+n_0\int_0^{\omega_D} n_0'(\varepsilon+\varepsilon')^2\,d\xi'=0; \tag{24}$$

$$n_1 S(n_0,0)+\hat{S}\{n_1\}=\psi, \tag{25}$$

$$\hat{S}\{n_1\}=4n_0\varepsilon\int_0^\infty n_1'\varepsilon'\,d\xi'-(1-2n_0)\int_\xi^{\omega_D} n_1'(\varepsilon-\varepsilon')^2\,d\xi', \tag{26}$$

$$\psi=-\frac{\Delta^2}{\varepsilon}\left[\int_\xi^{\omega_D}\frac{n_0'}{\varepsilon'}(\varepsilon-\varepsilon')^2\,d\xi'+4n_0\varepsilon a_0-n_0\int_0^\xi\frac{d\xi'}{\varepsilon'}(1-2n_0')(\varepsilon-\varepsilon')^2\right]. \tag{27}$$

Here $S(n_0,0)$ is given by Eq. (1), in which the terms with $\Delta^2/\varepsilon\varepsilon'$, N_ε are put equal to zero. Taking into account that the integral term in (25) gives a correction $(\Delta/\Delta_0)^2$, for n_1 we find

$$n_1=\psi(\xi)/S(n_0,0), \tag{28}$$

and the gap equation (2) takes the form

$$\frac{1}{\lambda}=\int_0^{\omega_D}\frac{1-2n_0(\varepsilon)}{\varepsilon}d\xi-2\int_0^\infty\frac{\psi(\xi)\,d\xi}{\varepsilon S(n_0,0)}. \tag{29}$$

We shall confine ourselves to calculating the terms linear in Δ. Therefore, we can put $\Delta=0$ in the first term of (29), since it is not difficult to show that the corrections will be proportional to $(\Delta/\Delta_0)^2$. Taking into account that the principal contribution to the last term is made by small $\xi\sim\Delta$, we obtain

$$\frac{1}{\lambda}-\int_0^{\omega_D}\frac{1-2n_0(\xi,a)}{\xi}d\xi=\frac{\pi a_1}{2a_2}\Delta, \tag{30}$$

where $n_0(\xi,a)$ is the distribution function for $\Delta=0$ and for pumping power corresponding to a. In its turn, a characterizes the energy interval in which the quasi-particles are concentrated ($a\sim\Delta_0\sim a_0\sim a_1^{1/2}\sim a_2^{1/3}$). The function $n_0(\xi,a)$ possesses the following properties:

$$n_0(\xi,a)=n_0(\xi/a)\equiv n_0(x);$$
$$1-2n_0(x)\sim x,\quad x\to0,\quad n_0(x)\sim x^{-4},\quad x\to\infty. \tag{31}$$

Using these we transform the left-hand side of (30) to the form

$$\frac{1}{\lambda}-\int_0^{\omega_D}\frac{1-2n_0(\xi,a)}{\xi}d\xi=\int_{\omega_D/a_c}^{\omega_D/a}\frac{1-2n_0(x)}{x}dx=\frac{a-a_c}{a_c};$$

finally we obtain

$$\Delta\frac{\pi a_1}{2a_2}=\frac{a-a_c}{a_c}, \tag{32}$$

where $a_c\sim\Delta_0\zeta$ is the critical pumping power; ζ is a numerical factor of order unity.

Thus, we again obtain a solution according to which the order parameter grows with increase of pumping power. It is obvious that the growing solution is bound to merge with the ordinary decreasing solution, and we arrive at a dependence of the gap on pumping that coincides qualitatively with the dependence depicted in Fig. 1.

4. THE ORDER PARAMETER AT NEAR-CRITICAL TEMPERATURES, $k=1$

We shall generalize the results obtained to the case of finite temperatures. We shall find the equation for the gap in the situation when the lattice temperature is close to T_c (T_c is the temperature of the superconducting transition in the equilibrium state) and the pumping power is small ($\beta \ll 1$). We seek the solution of Eq. (1) in the form

$$n=n_T(\varepsilon)+n_0(\varepsilon)+n_1(\xi), \tag{33}$$

where $n_T(\varepsilon)$ is the distribution function of the thermal quasi-particles:

$$n_T(\varepsilon)=(e^{\varepsilon/T}+1)^{-1}, \tag{34}$$

and n_0, n_1 are the distribution functions of the excess quasi-particles created by the source. The functions n_0 and n_1 satisfy the equations

$$n_0 S(n_T',0)+\hat{S}\{n_0\}=\frac{2\omega_D^2}{\pi\lambda}Q(\varepsilon), \tag{35}$$

$$n_1 S(n_T',0)+\hat{S}\{n_1\}=\psi(\xi), \tag{36}$$

where $\hat{S}$ is the integral operator

$$\hat{S}\{n_i\}=4\varepsilon n_T\int_0^{\omega_D} n_i'\varepsilon'\,d\xi'-(1-2n_T)\int_\xi^{\omega_D} n_i'(\varepsilon-\varepsilon')^2\,d\xi' + \int_0^{\omega_D} n_i'[N_{|\varepsilon-\varepsilon'|}(\varepsilon-\varepsilon')^2-N_{\varepsilon+\varepsilon'}(\varepsilon+\varepsilon')^2]d\xi', \tag{37}$$

$$\psi(\xi)=-\frac{\Delta^2}{\varepsilon}\left\{\int_0^\infty\frac{n_0'}{\varepsilon'}[(\varepsilon-\varepsilon')^2(1+N_{|\varepsilon-\varepsilon'|})+N_{\varepsilon+\varepsilon'}(\varepsilon+\varepsilon')^2]d\xi' + (2n_T-1)\int_0^\xi\frac{n_0'}{\varepsilon'}(\varepsilon-\varepsilon')^2\,d\xi'+4\varepsilon n_T(\varepsilon)\int_0^\infty n_0'\,d\xi'\right\}. \tag{38}$$

The function n_0 is nonzero in the interval of energies $\varepsilon \sim T$ and possesses the following properties:

$$n_0(\varepsilon,T)=n_0(\varepsilon/T)\equiv n_0(x);\quad n_0(x)\sim x,\quad x\to 0;\qquad n_0(x)\sim x^{-4},\quad x\to\infty. \tag{39}$$

For the function n_1 we again obtain (28), neglecting the integral term, which is of order (Δ/T_c^2). As a result we arrive at the equation for the gap:

$$0=-\frac{1}{\lambda}+\int_0^{\omega_D}\frac{1-2n_T(\varepsilon)}{\varepsilon}d\xi-2\int_0^\infty\frac{n_0(\varepsilon)}{\varepsilon}d\xi-2\int_0^\infty\frac{\psi(\xi)\,d\xi}{\varepsilon S(n_T,0)}. \tag{40}$$

The first two terms in (40) give the usual terms of the Ginzburg–Landau equation. In the zeroth approximation in Δ, the third term has, with allowance for the normalization (5), the form[1)]

$$2\int_0^\infty\frac{n_0(\xi)}{\xi}d\xi=2\beta\zeta_2,\qquad \beta=\frac{4\tilde{a}\omega\omega_D^2}{\pi\lambda T_c^4}, \tag{41}$$

where ζ_1 and ζ_2 are numerical factors of order unity:

$$\zeta_1=\int_0^\infty dx\int_0^\infty dx'\,y(x')[n_T(x)+N_{x+x'}],\qquad \zeta_2=\frac{1}{\zeta_1}\int_0^\infty\frac{dx}{x}y(x),\quad y(x)=\frac{\zeta_1}{\beta}n_0(x). \tag{42}$$

It can be shown that the next term in the expansion in

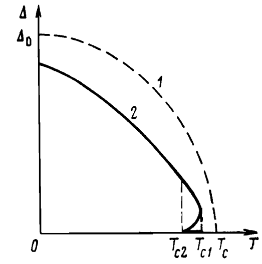

FIG. 2. Dependence of the order parameter on temperature in the absence of pumping ($\beta=0$, curve 1) and at fixed pumping ($\beta=0$, curve 2).

Δ is proportional to $(\Delta/T_c)^2\beta$. Since $\beta\ll 1$ this term is small, and we omit it. For a similar reason we confine ourselves to terms linear in Δ in the last term $\Delta\beta\zeta_3/T_c$ of Eq. (40), in which

$$\zeta_3=\frac{\pi}{\zeta_1\cdot 7\zeta(3)}\int_0^\infty y(x)x(1+2N_x)\,dx.$$

Collecting the results we obtain the gap equation

$$\frac{T-T_c}{T_c}+2\beta\zeta_2=-b\left(\frac{\Delta}{T_c}\right)^2+\frac{\Delta}{T_c}\beta\zeta_3,\qquad b=\frac{7\zeta(3)}{8\pi^2}, \tag{43}$$

which, for a fixed temperature, possesses the features that were discussed above. From the solution of (43):

$$\frac{\Delta}{T_c}=\frac{\beta\zeta_3}{2b}\pm\left[\left(\frac{\beta\zeta_3}{2b}\right)^2+\frac{T_c-T}{bT_c}-\frac{2\beta\zeta_2}{b}\right]^{1/2} \tag{44}$$

it follows that in the temperature interval $T_{c2}<T<T_{c1}$ at fixed pumping (and in the pumping-power interval $\beta_{c2}<\beta<\beta_{c1}$ at fixed temperature), with

$$T_{c2}/T_c=1-2\beta\zeta_2, \tag{45}$$

$$T_{c1}/T_c=1-2\beta\zeta_2+\beta^2\zeta_3^2/4b \tag{46}$$

there exist two solutions (Fig. 2). Generally speaking, it should be necessary to add a term quadratic in β to (43); however, allowance for this would lead only to a shift of T_{c2} and T_{c1} without changing the temperature interval in which the two solutions exist.

5. INVESTIGATION OF THE STABILITY OF THE STATE OF THE SUPERCONDUCTOR

We shall consider the stability of the system against small fluctuations of the gap and of the quasi-particle distribution function over times during which collision processes are unimportant. Linearizing the system consisting of the equations for the gap $\Delta(\mathbf{r},t)$ and the collisionless equation for the distribution function $n(\mathbf{r},t)$ we obtain the following criterion for stability against perturbations with frequency $\Omega=0$, $\mathbf{q}\to 0$:

$$J=\Delta^2\int_0^\infty\frac{d\xi}{\varepsilon^2}\left(\frac{1-2n}{\varepsilon}+2\frac{dn}{d\varepsilon}\right)>0. \tag{47}$$

This criterion is contained in the paper,[13] if we take into account, in the limit of small Ω and $\mathbf{q}$, the first term omitted there. It can also be obtained from the results of[14] in the limit $\mathbf{q}\to 0$, $\Omega/q\to 0$. After integration by parts the criterion (47) reduces to the form

$$J=1+2\int_0^\infty \frac{\partial n}{\partial \varepsilon}d\xi>0, \quad (48)$$

which expresses the fact that it is necessary for stability that the superconductor remain diamagnetic (to avoid misunderstandings, we recall that we are concerned with an ordinary superconductor with attraction, with $\Delta_0 \ll \omega_D$). Calculating (48) using the distribution functions found in Secs. 2–4, we obtain

$$J=\frac{2\Delta}{\Delta_0}\left(\frac{\pi}{2}\ln\frac{\Delta_0}{\Delta}-2G+\frac{1}{2}\right), \quad T=0, \quad k=-1; \quad (49)$$

$$J=\Delta\frac{\pi a_1}{2a_2}, \quad T=0, \quad k=1; \quad (50)$$

$$J=2b\left(\frac{\Delta}{T_c}\right)^2+\frac{\Delta}{T_c}\beta\zeta_3, \quad T\leqslant T_c, \quad k=1. \quad (51)$$

It can be seen that in all cases (in (49) within the limits of applicability) the current is positive, i.e., according to the criterion (48) a nonequilibrium superconductor is stable against small fluctuations.

It is also of interest to ascertain the sign of the current in an external electric field $\mathbf{E}$ with frequency Ω_1. It can be shown (cf. [15]) that, with allowance for scattering by impurities, the expression for the current has the form

$$\mathbf{j}=i\frac{\pi\Delta}{\Omega_1}(1-2n(\xi=0))\sigma_N\mathbf{E}, \quad (52)$$

where σ_N is the conductivity of the normal metal. By virtue of the condition (6), the sign of the current remains the same as the sign characteristic of the equilibrium situation. We draw attention to the possibility of checking the relation (6) experimentally by measuring the sign of the current (52).

It must be noted that the criterion (48) is extremely sensitive to the form of the distribution function. If, e.g., we substitute into (48) a Fermi function with a nonzero chemical potential, the sign can become negative. Thus, at $T=0$ the function n_F is a single step, and it is easy to see that $J<0$ for all values of the chemical potential (i.e., of the pumping), which is in disagreement with (49), (50). From this follows the necessity of using quasi-particle distribution functions satisfying a kinetic equation.

We note that if $2\Delta_0>\omega_D$, the function n becomes, in accordance with (1), a Fermi function, and $J<0$, as was shown in[16] in a discussion of nonequilibrium superconductivity with repulsion. However, in ordinary superconductors ($\Delta_0 \ll \omega_D$) the distribution functions of the excess quasi-particles differ substantially from equilibrium functions in the cases of interest to us.[12,4]

Thus, according to the criterion (48) for $\mathbf{q}\to 0$, which obviously does not exhaust the possible instabilities, a superconductor with optical pumping remains stable. Consequently, a more detailed investigation of the stability of the different nonequilibrium situations is necessary. This investigation is a complicated problem, lying outside the scope of the present paper. However, it is well known that a nonunique dependence of the characteristics of a system on the external parameters usually leads to instability and to a transition to a nonuniform state. For example, in[17,18] it was shown that a nonunique dependence of the order parameter Δ on the internal magnetic field (in a certain range of magnetic fields), analogous in form to the dependence depicted in Figs. 1 and 2, leads to a second-order phase transition to a nonuniform state with finite $\mathbf{q}$.

In our case the role of the internal magnetic field is played by the pumping power β. Attention was drawn to this fact in the paper.[19] There it was found that the critical power β_c (at which $\Delta=0$) decreases monotonically with increase of $\mathbf{q}$ (in contrast with the situation with an internal magnetic field[17,18]) and it was concluded that a second-order transition to a nonuniform state is impossible. As can be seen from the results of our work, this is valid if the power is less than the critical power β_c. In the interval $\beta_c<\beta<\beta_{c2}$ a first-order phase transition to a nonuniform state[2)] with finite $\mathbf{q}$ is possible. (To find the interval of $\mathbf{q}$ for which a transition occurs a numerical calculation is required, as in[17,18].) In this case partition occurs into regions with $\Delta\neq 0$ and $\Delta=0$, and the latter give rise to the appearance of a finite d.c. resistance of the sample.

We postulate that the nonunique dependence, found in this paper, of the order parameter on the pumping power, and the associated transition of the superconductor to a nonuniform state, can explain the experimental results on the smooth increase in the resistance of a sample with increase of the pumping power above the critical value.[6,7] An argument in favor of the proposed interpretation is provided by the analogy with the situation in superconductors with high-frequency pumping ($\Delta \ll \omega$). In fact, in the experiments of Rose and Sherrill[20] it was discovered that, above a certain critical power, a film acquired a finite d.c. resistance, which increased gradually, up to the normal value. On the other hand, Éliashberg showed[10] that the gap in a superconductor with high-frequency pumping ($\Delta \ll \omega$) has a nonunique dependence on the pumping power.

The author is grateful to Yu. V. Kopaev for discussion of the work and useful comments.

[1)] A term of a similar type has been obtained earlier.[12]

[2)] We note that, in the model of Owen and Scalapino,[3] in which, at $T=0$, a nonunique dependence exists for any pumping power, a transition to a nonuniform state is possible even for small q.[8]

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Translated by P. J. Shepherd.

Electron properties of amorphic and crystalline ytterbium films

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(Submitted April 7, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1503–1513 (October 1976)

The electron properties (electric conductivity, magnetoresistance and Hall effect) of amorphous and crystalline ytterbium films are investigated at low temperatures. Some peculiarities are observed, especially a reduction of the resistivity of amorphous Yb films with decrease of their thickness. Information is obtained regarding the number of carriers, and their mobility and mean free path in the films. The differences in the properties of amorphous and crystalline Yb are ascribed to a shift of the conduction band relative to the valence band.

PACS numbers: 73.60.Fw

1. INTRODUCTION

The last 10–15 years have seen an intensive development of experimental and theoretical investigations of noncrystalline substances, particularly amorphous metals and alloys.[1] These new metallic modifications frequently exhibit interesting and unexpected physical properties, such as superconductivity of the amorphous films of Bi, Ga, and Be.[2,3] This, however, is not the only reason for interest in amorphous metals. A quantum-mechanical explanation of the electronic properties of metals and semiconductors, which was based on periodicity of the potential, on the presence of long-range order in the arrangement of the atoms, is recently being substantially reconsidered.[4,5] It appears that not the long-range order but the short-range order is responsible for the main electronic properties. The experimental data on the properties of amorphous metals are extremely scanty.

We report here a comprehensive investigation of the electric and galvanomagnetic properties of low-temperature films of a rare-earth metal, ytterbium, both in the amorphous and in the crystalline state. It was observed earlier[6] that condensation of ytterbium vapor in ultrahigh vacuum on a substrate cooled with liquid helium leads to the formation of a new modification of this metal. It was shown by electron diffraction[1)] that the low-temperature modification of ytterbium is an amorphous state. Amorphous ytterbium films are metastable and undergo an irreversible transition into the crystalline states ($a-c$ transitions) when heated to a definite temperature T_{tr} and when they reach a critical thickness d_{cr} in the course of the condensation.

The value of d_{cr} obtained in preliminary investigations[6] was ~ 3000 Å. It is known, however,[1] that impurities greatly increase the stability of amorphous metallic films, increasing both their crystallization temperature and their critical thickness. The value of d_{cr} for the purest ytterbium films obtained recently is 500–1000 Å.[7] It has also been shown[7,8] that the crystallization temperature of amorphous ytterbium films increases in accordance with a hyperbolic law with decrease of their thickness. T_{tr} of subcritical thickness is approximately 14 °K. The amorphous ytterbium layers whose thickness is smaller by several percent than critical undergo an $a-c$ transition also when a magnetic field of definite intensity is applied perpendicular to the layer.[9]

In this paper we report an investigation of the electronic properties of amorphous Yb films in the interval of thicknesses, temperatures, and fields that limit their stability.

