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Electron properties of amorphic and crystalline ytterbium films

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The electron properties (electric conductivity, magnetoresistance and Hall effect) of amorphous and crystalline ytterbium films are investigated at low temperatures. Some peculiarities are observed, especially a reduction of the resistivity of amorphous Yb films with decrease of their thickness. Information is obtained regarding the number of carriers, and their mobility and mean free path in the films. The differences in the properties of amorphous and crystalline Yb are ascribed to a shift of the conduction band relative to the valence band.

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1. INTRODUCTION

The last 10–15 years have seen an intensive development of experimental and theoretical investigations of noncrystalline substances, particularly amorphous metals and alloys.[1] These new metallic modifications frequently exhibit interesting and unexpected physical properties, such as superconductivity of the amorphous films of Bi, Ga, and Be.[2,3] This, however, is not the only reason for interest in amorphous metals. A quantum-mechanical explanation of the electronic properties of metals and semiconductors, which was based on periodicity of the potential, on the presence of long-range order in the arrangement of the atoms, is recently being substantially reconsidered.[4,5] It appears that not the long-range order but the short-range order is responsible for the main electronic properties. The experimental data on the properties of amorphous metals are extremely scanty.

We report here a comprehensive investigation of the electric and galvanomagnetic properties of low-temperature films of a rare-earth metal, ytterbium, both in the amorphous and in the crystalline state. It was observed earlier[6] that condensation of ytterbium vapor in ultrahigh vacuum on a substrate cooled with liquid helium leads to the formation of a new modification of this metal. It was shown by electron diffraction1) that the low-temperature modification of ytterbium is an amorphous state. Amorphous ytterbium films are metastable and undergo an irreversible transition into the crystalline states (*a*–*c* transitions) when heated to a definite temperature T_{tr} and when they reach a critical thickness d_{cr} in the course of the condensation.

The value of d_{cr} obtained in preliminary investigations[6] was ~ 3000 Å. It is known, however,[1] that impurities greatly increase the stability of amorphous metallic films, increasing both their crystallization temperature and their critical thickness. The value of d_{cr} for the purest ytterbium films obtained recently is 500–1000 Å.[7] It has also been shown[7,8] that the crystallization temperature of amorphous ytterbium films increases in accordance with a hyperbolic law with decrease of their thickness. T_{tr} of subcritical thickness is approximately 14 °K. The amorphous ytterbium layers whose thickness is smaller by several percent than critical undergo an *a*–*c* transition also when a magnetic field of definite intensity is applied perpendicular to the layer.[9]

In this paper we report an investigation of the electronic properties of amorphous Yb films in the interval of thicknesses, temperatures, and fields that limit their stability.

2. PROCEDURE

Amorphous ytterbium films were obtained by condensation in ultrahigh vacuum (~ 10^{-12} mm Hg) on glass substrates cooled with liquid helium, using a procedure described earlier.[7,10] The rate of condensation was from 3 to 200 Å/min. Variation of the rate of condensation within these limits did not influence the results. The initial material was ytterbium of 99.7% purity. The impurities (in percents) were: erbium—0.02; lutecium $\leq 1\times 10^{-2}$; calcium—0.05; iron <0.01; copper $<5\times 10^{-3}$; tantalum—0.2. The ratio of the resistance at room temperature R(293 °K) to the resistance at liquid helium temperature R(4.2 °K) of the initial ytterbium annealed in vacuum was of the order of seven. The same ratio for the investigated films with thickness less than 1000 Å was on the average 2.5–3. The electric resistance and the Hall coefficients were measured by a potentiometer method using a six-probe scheme and an R-348 potentiometer. The magnetic field for the galvanomagnetic investigations was produced by a superconducting solenoid. When the films were heated, the temperature was determined with the aid of a platinum thermometer. Special experiments have shown[7] that when condensed on a glass substrate cooled with liquid helium the temperature of the ytterbium films did not exceed 5 °K. The film thickness was determined by interferometry or from the optical density.[11] The thickness was determined accurate to 15–20%. This in the main is the cause of the scatter in the determination of the resistivity ρ and of the Hall coefficient R_H for different films. The error in the determination of ρ is due, in addition, to the uncertainty in the width of the film (owing to the presence of thinner wedge-like edges).

3. EXPERIMENTAL RESULTS

3.1. Resistivity

The resistivity of both amorphous and crystalline ytterbium films depends essentially on their purity. For films of equal thickness obtained in different experiments, ρ sometimes differed by a factor of two. It is interesting that in this case none of the films had at room temperature a resistivity exceeding the value of ρ for the bulky metal, 27×10^{-6} Ω-cm. This points to a negligible contamination of the films. On the other hand, the difference in their purity is due obviously to the inhomogeneous distribution of the impurities in the initial metal. The face-centered cubic modification of ytterbium (fcc Yb) is a semimetal[12] and has a low carrier density.[13] It will be shown below that amorphous ytterbium is an even more typical semimetal. This apparently explains also the very high sensitivity of the resistivity of ytterbium to an even negligible content of impurity atoms.

The last circumstance makes it difficult to obtain plots of $\rho(d)$ for the investigated films. However, the determination of the resistivity of amorphous films of different thicknesses, obtained in a single experiment by successive evaporation (with the intermediate thicknesses calculated from the condensation time) shows that ρ decreases with decreasing thickness. This is illustrated by curve 1 of Fig. 1. Curve 2 characterizes the $\rho(d)$ dependence at 4.2 °K for crystallize films whose resistivity corresponds to curve 1 in the amorphous state.

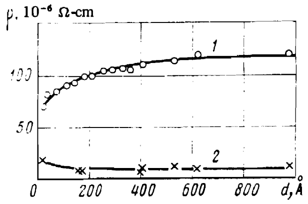

FIG. 1. Dependence of the resistivity of amorphous (curve 1) and crystalline (curve 2) ytterbium films on their thickness at T = 4.2 °K.

At room temperature, the resistivity of crystalline ytterbium films of thickness less than 600 Å is as a rule $(15-20)\times 10^{-6}$ Ω-cm, which is appreciably lower than the value for the bulky metal. This contradicts the data obtained by others,[14] who observed an increase of ρ with decreasing thickness of the crystalline ytterbium films condensed in a vacuum ~ 10^{-6} Torr. However, as shown recently,[15] thin ytterbium films obtained in such a vacuum contain a large number of other phases (YbH_2, Yb_2O_3). That the films investigated in[14] were considerably contaminated is indicated by the high value of ρ for thick ytterbium layers (~ 40×10^{-6} Ω-cm). The resistivity of thick (1000–18 000 Å) annealed ytterbium films investigated in the present study was (25–27) $\times 10^{-6}$ Ω-cm, which agrees with the value of ρ of the bulky metal.

It should be noted that when discussing the behavior of crystalline ytterbium films we have in mind its fcc phase. An x-ray analysis of thick ytterbium layers (~ 2 μ) condensed at low temperatures of the substrate and annealed to ~ 460 °K has revealed that they contain only the fcc phase at room temperature. Plots of the temperature dependence of the resistivity of such layers in the interval 1.5–460 °K [6] are fully reversible and contain no anomalies whatever to indicate the presence of phase transition from the fcc modification of ytterbium into the hcp modification.[16,17] The form of the $R(T)$ dependence of thin annealed ytterbium films is the same as that of thick films. Thus, in the temperature region 1.5–460 °K, no signs of the hexagonal modification of Yb have been observed in the investigated film. It is possible that this is due to the presence of internal stresses in the films.

A characteristic feature of amorphous ytterbium films is the negative temperature coefficient of resistivity (TCR). Its absolute value depends on the purity of the films—as a rule, at equal film thicknesses, large resistivities go with large negative values of the TCR. An investigation of amorphous films of different thicknesses, obtained in the course of a single experiment by successive additional evaporation, has shown that the absolute value of their TCR is smaller the thicker the film (Fig. 2). In films of below-critical thickness, no changes of the resistivity with decreasing temperature were observed in fact. The character of the variation of the resistivity of amorphous ytterbium changes with decreasing temperature. From the temperature of the start of crystallization down to T = 4–5 °K the plot

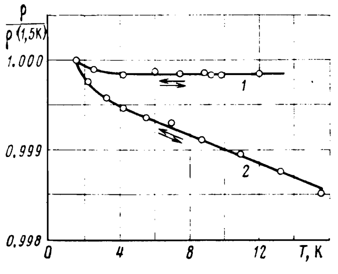

FIG. 2. Temperature dependence of the resistivity of amorphous ytterbium films of thickness 150 Å (curve 1) and 30 Å (curve 2); $\rho(1.5\,°\mathrm{K})$—resistivity at $T = 1.5\,°\mathrm{K}$.

of $\rho(T)$ is a straight line. At lower temperatures it is satisfactorily described by an exponential law. The activation energies calculated in the latter case amount to 10^{-6}–10^{-8} eV (depending on the layer thickness and its purity). After going over to the crystalline state, all the films thicker than 20 Å have positive TCR.

3.2. Magnetoresistance

The resistivity of crystalline ytterbium films at a temperature 4.2 °K increases in a magnetic field perpendicular to the film like H^n (where $n \approx 2$). Such a variation of ρ was observed in the field interval from zero to 80 kOe. Amorphous films behave differently. In fields up to several kOe, the dependence of the magnetoresistance on the field is close to quadratic (Fig. 3). Further increase of the field (at any rate up to 80 kOe), however, leads to a slowing down of the increase in the resistivity—it changes in fields stronger than 15–20 kOe like H^n (where $n \approx \frac{1}{3}$). The quadratic section on the plot of the magnetoresistance against the field increases rapidly with increasing temperature, and at $T \approx 10$ °K a quadratic dependence is observed up to $H \approx 14$ kOe (Fig. 3, curve 2).

Amorphous ytterbium films are characterized by a strong dependence of the magnetoresistance on the film thickness (Fig. 4). It has turned out here that in contrast to the resistivity, the magnetoresistance of amorphous films is much less sensitive to their purity. The points on Fig. 4 (curve 1) correspond to samples whose resistivity differed sometimes by a factor of two (at close thicknesses). Naturally, the entire dependence $\Delta\rho/\rho = f(H)$ coincides also for different films of equal thickness. Figure 4 (curve 2) illustrates the change of the magnetoresistance with thickness in a field 4.5 kOe. At $H \leqslant 4.5$ kOe, a quadratic dependence of the magnetoresistance on the field is observed for amorphous ytterbium films in the entire investigated thickness interval.

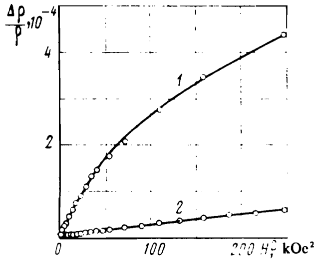

FIG. 3. Change of the resistivity of ytterbium film ~110 Å thick in a magnetic field at the following temperatures: 1—4.2 °K, 2—10.3 °K; $\Delta\rho = \rho_H - \rho$, ρ_H is the resistivity in a magnetic field H, ρ is the resistivity in the absence of a magnetic field.

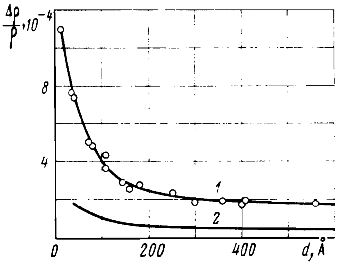

FIG. 4. Dependence of the magnetoresistance of amorphous ytterbium films on their thickness at $T = 4.2$ °K; curve 1—$H \approx 16$ kOe, 2—$H \approx 4.5$ kOe.

The magnetoresistance of the crystalline ytterbium films is so sensitive to their purity and degree of annealing, that the scatter of the data for different samples does not make it possible to reveal the form of the dependence of the magnetoresistance on the thickness. It is clear only that the magnetoresistance decreases with decreasing thickness of the crystalline films. Thus, whereas for films of thickness ~1000 Å the value of $\Delta\rho/\rho$ is $(8-10)\times 10^{-3}$ (at $H \approx 16$ kOe), the films of thickness ~20 Å have a value of $\Delta\rho/\rho$ that differs little from the value in the amorphous state (~1×10^{-3}). At low temperatures (< 12 °K) the magnetoresistance of annealed ytterbium films remains practically unchanged with decreasing temperature (Fig. 5, curve 1). Raising the temperature above 12 °K leads to a decrease of the magnetoresistance. The temperature dependence of the magnetoresistance of thick ytterbium films, just as that of a bulky sample, is less strongly pronounced. Thus, for an ytterbium layer ~1.8 μ thick the value of $\Delta\rho/\rho$ in a field ~16 kOe decreases to approximately one-half when the temperature is increased from 1.7 to 25 °K.

The $\Delta\rho/\rho = f(T)$ dependence of amorphous ytterbium films manifests itself very strongly, especially when the temperature is lower than 5 °K (Fig. 5, curve 2). The growth of the magnetoresistance with decreasing temperature is larger the thinner the film.

3.3. The Hall effect

The Hall effect was investigated for amorphous and crystalline ytterbium films in fields up to 40 kOe. In this field interval, the Hall voltage increases linearly with increasing fields. In other words, up to 40 kOe the Hall constant does not depend on the magnetic field for either amorphous or crystalline samples. In the temperature region where amorphous ytterbium films exist ($0 < T < T_{\mathrm{tr}}$), their Hall coefficients do not depend on the temperature (Fig. 6, curve 1). In the a–c transition (in the course of heating) accompanied by a jumplike irreversible decrease of the resistance, a jumplike change takes place also in the Hall constant (dashed in Fig. 6). As seen from Fig. 6, curve 2, a weak change

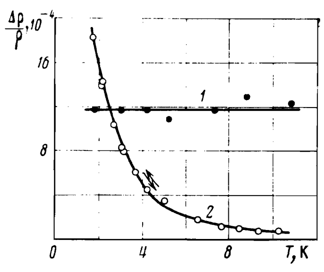

FIG. 5. Temperature dependence of the magnetoresistance for a crystalline (curve 1) and amorphous (curve 2) ytterbium film of thickness 110 Å; $H \approx 16$ kOe.

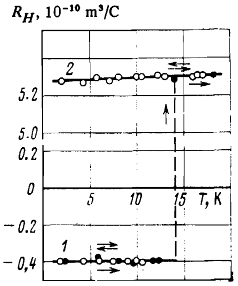

FIG. 6. Variation of the Hall coefficient with temperature for an ytterbium film 150 Å thick; 1—amorphous state, 2—crystalline state; ●—heating, ○—cooling.

of the Hall coefficient is observed in crystalline ytterbium films in the considered temperature interval. The behavior of the Hall coefficient is approximately the same at low temperatures also in the case of the initial bulky Yb. However, further increase of the temperature (approximately from 10–15 to 80 °K) leads to an appreciable decrease of the Hall coefficient, just as in the case of the hcp modification of Yb.[18]

Figure 7b shows the dependence of the Hall coefficient on the thickness of amorphous ytterbium films. Just as the magnetoresistance, the Hall coefficients of such films are much less senstive to their purity than the resistivity. The obtained variation of the Hall coefficients with thickness is due, in our opinion, to the change in the coordination structure of the amorphous ytterbium films. Such a change was observed, for example, by electron diffraction in amorphous bismuth films.[19]

For crystalline ytterbium films, the Hall coefficient increases smoothly with increasing thickness (Fig. 7a). In this case the plot of $R_H(d)$ is a curve with saturation. The limit to which the curve tends is ~ 13×10^{-10} m³/C (for an ytterbium layer 1.8 μ thick we have $R_H = 12.6\times10^{-10}$ m³/C at $T = 4.2$ °K).

4. DISCUSSION OF RESULTS

4.1. Crystalline films

As shown by investigations of the magnetoresistance of fcc single crystals of ytterbium in strong fields,[20] this metal is compensated, i.e., in terms of the two-band model[21–23] it has equal numbers of mobile electrons and holes. It appears that this applies to a full extent also to the crystalline ytterbium films investigated in the present study. The experimental determination of the values of the electric conductivity, magnetoresistance, and Hall coefficient makes it possible to estimate for such films the number of carriers and their mobilities. To this end one solves the system of equations[24,25]

$$\sigma=\rho^{-1}=en(\mu_e+\mu_h),\quad R_x=\frac{1}{en}\frac{\mu_h-\mu_e}{\mu_h+\mu_e},\quad \frac{\Delta\rho}{\rho B^2}=\mu_e\mu_h. \tag{1}$$

Here σ is the conductivity, $n = n_e = n_h$ is the number of electrons or holes per unit volume; μ_e and μ_h are respectively the electron and hole mobilities; B is the magnetic induction.

Calculation shows that with increasing thickness of the crystalline ytterbium films, from 20 to 1000 Å, the mobility of the electrons increases from 2×10^{-2} to 4.5×10^{-2} m²/V-sec, while that of the holes increases from 2×10^{-2} to 5.5×10^{-2} m²/V-sec. For thick ytterbium layers, the difference between the electron and hole mobilities ($\Delta\mu = \mu_h - \mu_e$) increases. The value of $\Delta\mu$ for a layer of thickness 1.8 μ is ~ 3×10^{-2} m²/V-sec, and for the thinnest films (d = 20–30 Å) the mobilities of the electrons and holes practically coincide. The density of the mobile electrons (holes) in crystalline ytterbium films 20–1000 Å thick varies in the range $(0.6-1.2)\times10^{21}$ cm^{-3}; the number of carriers tends to increase with decreasing film thickness. The calculations were performed for $T = 4.2$ °K. The data obtained on the number of carriers are in satisfactory agreement with the results for bulky fcc Yb[13] ($n \approx 0.4\times10^{21}$ cm^{-3}).

Knowing the number and mobility of the carriers in the Yb crystalline films, we can estimate also their average mean free path l.[25] In the case $n_e = n_h = n$, the expression for l takes the form (in the c.m.s.)

$$\bar{l}=3.23\cdot10^{-34}\frac{n^{1/3}(\mu_e+\mu_h)}{2e}. \tag{2}$$

Calculation shows that the mean free path of the carriers increases with increasing thickness (in the interval 20–1000 Å) from 400 to 1500 Å (at 4.2 °K).

As noted above, crystalline ytterbium films have an anomalously low resistivity at room temperature. The galvanomagnetic properties of these films at T ~ 300 °K were not investigated in the present study. It is therefore impossible to interpret this result unambiguously. However, judging from the electronic properties of crystalline ytterbium films at low temperatures, it can be assumed that the number of carriers increases in thin films (compared with the bulky metals) also at room temperature.

4.2. Amorphous films

From general considerations, on the basis of only the experimental data, one can advance certain hypotheses concerning the carriers in amorphous ytterbrium films. Thus, the appreciable increase of the magnetoresistance with decreasing thickness of the amorphous films (Fig. 4) or else with decreasing temperature (Fig. 5, curve 2) indicate an increase of the carrier mobility μ. The relatively weak change of the electric conductivity of such films, compared with the increase in the magnetoresistance, may be due to the compensating decrease of the number of carriers. Qualitatively, the same results are obtained by calculation with

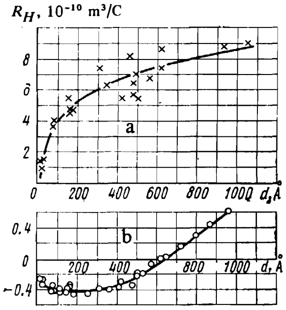

FIG. 7. Dependence of the Hall coefficient of the thickness of crystalline (Fig. a) and amorphous (b) ytterbium films.

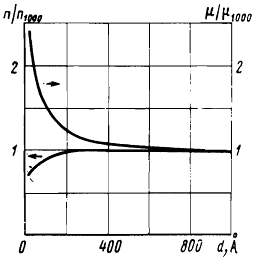

FIG. 8. Dependence of the number and mobility of the electrons (holes) on the thickness of amorphous ytterbium films; $n_{1000} = 1.85 \times 10^{20}$ cm^{-3} and $\mu_{1000} = 1.4 \times 10^{-2}$ m^2/V-sec are the concentration and mobility of the electrons (holes) in amorphous films ~1000 Å thick.

formulas (1) using the quadratic section of the $\Delta\rho/\rho = f(H)$ curves (Fig. 3). It is natural to assume that the equality $n_e = n_h = n$ is valid also for amorphous ytterbium. The functions $n(d)$ and $\mu(d)$ calculated (from the average values of ρ—Fig. 1, curve 1, of $\Delta\rho\rho$—Fig. 4, curve 2, and of R_H—Fig. 7b) are shown in Fig. 8. The calculation of μ and n for different temperatures shows that μ increases and n decreases with decreasing film temperature. The most significant change takes place when the temperature drops below 5 °K. According to Fig. 8, in amorphous Yb films of thickness ~ 1000 Å, the concentration of the mobile electrons (holes) amounts to ~ 10^{-2} per atom (at $T = 4.2$ °K). This is approximately 3–6 times less than in crystalline films. The low concentration of mobile charges in ytterbium films is qualitatively confirmed by the strong influence of the impurities on their electric conductivity. It should be noted that the obtained difference between the mobilities of the electrons and holes in amorphous ytterbium films is very small (tenths of one percent), so that the dependence of the mobilities of the electrons and holes on the thickness is represented in Fig. 8 by a single $\mu(d)$ curve. For films with negative R_H (Fig. 7b), the mobility of the electrons exceeds the mobility of the holes and, conversely, films in which $R_H > 0$ are characterized by a larger hole mobility.

Table I lists the carrier mean free paths in amorphous ytterbium films as calculated from formula (2).

The large value of the carrier mean free path in the amorphous metal should apparently cause no surprise. Thus, in many liquid metals (Cu, Au, Ag) the value of $\bar{l}$ reaches several dozen Å, while for Na and K we have $\bar{l} \approx 150$ Å.[26] The structure of amorphous metals and alloys is as a rule similar to that of a liquid,[27,28] but the degree of order in them is somewhat greater than in melts. It appears that a carrier mean free path on the order of several interatomic distances is typical also of other amorphous metals.

As is well known,[24,25] in galvanomagnetic investigation magnetic fields are regarded as "weak" if the inequality $\mu B \ll 1$ is satisfied, and for "strong" fields we have $\mu B \gg 1$. It is easy to calculate that "medium" fields ($\mu B \sim 1$) for amorphous ytterbium films are of the order of 600 kOe. Thus, the galvanomagnetic properties of amorphous ytterbium films were investigated in weak fields. According to the theoretical concepts and the experimental data,[24,25] a quadratic field dependence of the metal resistance should be observed in weak fields for all metals. At the present time it is impossible to offer an unambiguous explanation of the anomalous behavior of the function $\Delta\rho/\rho = f(H)$ of amorphous ytterbium (Fig. 3). The deviations from the quadratic variation of the magnetoresistance in weak fields may be due to the influence of the scattering of the conduction electrons by local magnetic moments caused by the small admixture of Fe atoms[29] (as already noted, the original ytterbium contained < 0.01% Fe). In crystalline films, the action of this scattering mechanism is apparently negligible against a background of the large intrinsic magnetoresistance. We note that for crystalline ytterbium films 20–30 Å thick the $\Delta\rho/\rho = f(H)$ dependence is the same as for amorphous films. Calculation of the number and mobility of the carriers in very thin crystalline films, based on the quadratic segment of the $\Delta\rho/\rho = f(H)$ curve, gives reasonable values corresponding to thick crystalline layers of Yb. This indicates that Eq. (1) yields for n and μ of the amorphous ytterbium film correct values if the quadratic segment of the $\Delta\rho/\rho = f(H)$ plot is used.

However, one cannot exclude the possibility of a change of valence upon "amorphization" of the ytterbium, as is the case, for example, when fcc Yb goes over into the hcp modification.[18] In this case a change can take place in the relative number of electrons and holes, and the calculation by Eq. (1) is no longer valid. Nonetheless, the use of the general formulas for the two-band model[24,25] gives practically the same carrier densities as were obtained under the assumption $n_e = n_h$, if it is assumed that the electron and hole mobilities in amorphous ytterbium are approximately equal. The last assumption is apparently correct, judging from the decrease in the difference between μ_e and μ_h with decreasing mean free path (film thickness) in the crystalline Yb.

At any rate, the results cannot be regarded as quantitative. This is only a very rough approximation, which makes it possible to explain qualitatively the experimentally observed results.

In addition, it must be emphasized that if the carrier mean free paths in the amorphous and crystalline ytterbium films are appreciable in comparison with the film thickness, the dependence of the electronic parameters of these films on their thickness and temperature is apparently distorted. All these dependences are the results of superposition of size effects[30] on the change in the number and mobility of the carriers. The latter can be due to a considerable degree to the possible changes of the coordination structure of the amorphous metal as the condensed layer becomes thicker.[19]

It is difficult to say anything definite as yet concerning the causes of the negative TCR of amorphous Yb films. According to modern notions, the resistance of amor-

TABLE I.

d, Å	T, K	$\bar{l}$, Å
40	4.2	300
300	4.2	200
700	4.2	180
1000	4.2	180
150	1.75	500
	4.2	200
	10.5	60

phous or a liquid metal is determined principally by the scattering of the carriers as a result of the disordered arrangement of the atoms.[31] It is believed[19,26] that the temperature-induced change of the resistance of amorphous metals is determined mainly by the temperature-induced structure changes (both irreversible and reversible). It appears that these changes lead also to the existence of a negative temperature coefficient of resistivity in amorphous ytterbium films. It is interesting that the increase of the negative TRC with decreasing film thickness is accompanied by a decrease of the resistivity.

Attention is called to the fact that the TRC of molten Yb is also negative,[32] and its resistivity $\sim 107\times10^{-6}$ Ω-cm[32]) is close to the value of ρ of thick amorphous films.

The electronic properties of amorphous metals are frequently analogous to the electronic properties of the same metals in the crystalline state under pressure.[33] By now, the behavior of Yb with increasing pressure has been thoroughly investigated.[13,34–36] It has been shown that with increasing pressure the magnetoresistance decreases, the Hall constant drops to zero and reverses sign, the resistivity decreases, and the TRC reverses sign. A comparison of our present data with the results of an investigation of crystalline ytterbium under pressure shows that the electronic properties (ρ, R_H, the sign of TRC) of the amorphous modification of ytterbium and of its fcc phase under pressure ($p \approx 20$ kbar at $T = 293$ °K[34,35] or $p \approx 13$ kbar at $T = 4.2$ °K[34,36]) are close to each other. Calculation[13] shows that with increasing pressure a decrease takes place in the number of carriers in the fcc Yb (by approximately five times at $T = 2$ °K and $p = 13$ kbar). As shown above, the numbers of the carriers in the amorphous and crystalline ytterbium films are in approximately the same ratio. The reversal of the sign of the Hall coefficient (from positive to negative) both in crystalline fcc Yb under pressure and in amorphous films with decreasing thickness, is due to the increase of the electron mobility relative to the hole mobility. The latter is obviously connected with the more appreciable decrease of the effective mass of the d electrons in comparison with the small change in the effective mass of the sp holes.[37] Thus, just as for other metals,[33] electronic states of the amorphous modification of ytterbium and of its fcc phase under pressure are identical. It is assumed[34,35] that the shift of the 5d band relative to the 5p6s bands decreases the overlap of the bands on the Fermi surface in the fcc phase of Yb under pressure. A result of such a shift is a smooth transition from semimetallic to semiconducting properties. A similar shift of the bands takes place apparently in amorphous ytterbium. This assumption seems plausible, since according to Gubanov's theory[4] the loss of the long-range order in the metal should lead to a certain broadening of the allowed energy bands.

We note in conclusion that some of the features observed in the behavior of amorphous ytterbium films in the present study, such as: 1) the increase of the conductivity with decreasing thickness, 2) the negative temperature coefficient of resistivity, 3) the deviation of the function $\Delta\rho/\rho = f(H)$ from quadratic, a deviation that increases with decreasing temperature, and 4) the sharp decrease of the magnetoresistance with decreasing thickness and temperature, were all observed at low temperatures and for films (but not crystalline) of another typical semimetal, bismuth.[38] It appears that this common behavior is due to the similarity of the electronic spectra and of the singularities of the scattering of the carriers in the thin-film state of these semimetals.

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Distribution of “hot” electrons in a metal at low temperatures

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The electron distribution function at $T=0$ is found for a thin metal plate in a “heating” stationary and uniform electric field (T is the lattice temperature) in the case in which the “temperature” approximations (Kaganov et al., 1956, 1967) are not satisfied.

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1. Attention was called in the work of one of the authors[1] to the importance of the study of the effects of heating of an electron gas as applied to the analysis of the resistive behavior of thin superconducting films, in which such a heating is first revealed by the hysteresis of the critical current. The temperature dependence of this hysteresis was calculated in Ref. 1 in the approximation of two different temperatures for the electrons and the lattice[2] and it turned out to be in good agreement (for films of In and Sn) with experiment.[3]

As is well known, the electron-temperature approximation allows us to “bypass” the investigation of the kinetic equation for the electron distribution function in a strong electric field, and is correct when the criterion of thermalization of the electron gas is satisfied: $\nu_{ep}^{\varepsilon} \ll \nu_{ee}$. Here ν_{ep}^{ε} is the frequency of the energy relaxation of the electrons from phonons and ν_{ee} is the frequency of interelectron collisions. Taking into account the well-known estimates

$$\nu_{ee} \sim \frac{\varepsilon_F}{\hbar}\left(\frac{T_e}{\varepsilon_F}\right)^2, \quad \nu_{ep}^{\varepsilon} \sim \frac{\Theta_D}{\hbar}\left(\frac{T_e}{\Theta_D}\right)^3$$

(here T_e is the characteristic energy of the electrons, ε_F is the Fermi energy, Θ_D is the Debye temperature, $T_e \ll \Theta_D$), it is easy to see that a rough estimate for the electron temperature T_e, below which thermalization is guaranteed as a result of the interelectron collisions, is given by the inequality $T_e \ll \Theta_D^2/\varepsilon_F$. For such metals as Sn, In, Pb, Nb, the right side of this inequality is smaller than or of the order of 1 °K. At the same time, the electron temperature for these superconductors, as follows from the results given in Ref. 1, even with a current j_{c2}, is only slightly below critical (apart from the dependence on the lattice temperature), i.e., $T_e \gtrsim 3$ °K and the thermalization criterion given above is obviously not satisfied. Moreover, what has been pointed out above means that precisely the opposite situation is of practical interest for the metals mentioned above. In such a case, the role of interelectron collisions in the formation of the distribution function of hot electrons is of little importance.

In this connection, the problem arises as to the finding of the electron distribution function in “heating” electric field in the absence of interelectron collisions generally. In the case of nondegenerate semiconductors, the answer to a similar question has already been obtained by Davydov.[4] By a significant simplifying circumstance, used in Ref. 4, account was taken of the known[5] ineffectiveness of the energy relaxation of the electrons from acoustic phonons in semiconductors. Therefore, expanding the desired distribution function in the collision integral of electrons with phonons in a series in the small ratio of the energies of the phonon and electron, we can replace the corresponding integral operator by a differential one. In metals, however, at $T_e \ll \Theta_D$, the energy relaxation is effective and a similar simplification is impossible.[1)] Moreover, in contrast with the nondegenerate semiconductors, the collision integral of electrons with phonons in a metal contains a nonlinearity in the distribution function, associated with the necessity of taking into account the Pauli principle. The difficulties mentioned (absence of a small parameter and the essential nonlinearity of the problem, see Eq. (6) below) provide, in our opinion, the reason for the