[1)]For simplicity we shall refer only to DD interactions, and accordingly to the dipole–dipole pool (DDP), although all the results can be easily generalized also to other types of spin–spin interactions, such as exchange interactions.

[2)]We note that this conclusion contradicts the idea of treating the subsystem $\hat{\mathscr{H}}_{CR}$ as a separate quasi-equilibrium pool.[6]

[3)]This circumstance was pointed out to us by L. L. Buishvili.

[4)]This damping, of course, is not irreversible, but here, at least in principle, experiments of a spin-echo type are possible.

[5)]At the same time, this "line" can be regarded as a result of induced two-spin transitions: in this sense it is the analog of the low-frequency "Van Vleck satellites" in magnetic-resonance spectra.

[1]N. Bloembergen, S. Shapiro, P. S. Pershan, and J. O. Artman, Phys. Rev. **114**, 445 (1959).

[2]B. N. Provotorov, Zh. Eksp. Teor. Fiz. **42**, 882 (1962) [Sov. Phys. JETP **15**, 882 (1962)].

[3]M. Goldman, Spin Temperature and Nuclear Magnetic Resonance in Solids, Oxford, 1970.

[4]J. Jeener, H. Eisendrath, and R. Van Steenwinkel, Phys. Rev. **133**, A478 (1964); J. Jeener, Advances in Magnetic Resonance, ed. J. S. Waugh, 3, New York, 1968.

[5]V. A. Atsarkin and M. I. Rodak, Usp. Fiz. Nauk 107, 3 (1972).

[6]V. A. Skrebnev, Phys. Status Solidi 60b, 51 (1973).

[7]R. L. Strombotne and E. L. Hahn, Phys. Rev. **133**, A1616 (1964).

[8]B. Sapoval and D. Lepine, J. Phys. Chem. Solids **27**, 115 (1966).

[9]M. I. Rodak, Zh. Eksp. Teor. Fiz. **61**, 832 (1971) [Sov. Phys. JETP **34**, 443 (1972)]; Fiz. Tverd. Tela (Leningrad) **15**, 404 (1973) [Sov. Phys. Solid State **15**, 290 (1973)].

[10]S. A. Al'tshuler and B. M. Kozyrev, Elektronnyĭ paramagnitnyĭ rezonans (Electron Paramagnetic Resonance), Nauka, 1972, Ch. 4. [Academic, 1964].

[11]C. P. Slichter, Princples of Magnetic Resonance, Harper, 1963.

[12]J. Jeener, R. Du Bois, and P. Broekart, Phys. Rev. **139**, A1959 (1965).

[13]H. Dubois, C. R. Acad. Sci. (Paris) **264**, 503 (1967).

[14]D. H. Zubarev, Neravnovesnaya statisticheskaya termodinamika (Nonequilibrium Statistical Thermodynamics), Nauka, 1971.

[15]L. L. Buishvili, A. A. Davituliani, and M. D. Zviadadze, Fiz. Tverd. Tela (Leningrad) 16, 1559 (1974) [Sov. Phys. Solid State 16, 1017 (1974)].

Translated by J. G. Adashko

Stratification of a heated electron-hole plasma

B. S. Kerner and V. V. Osipov

Moscow Institute of Radio, Electronics, and Automation

(Submitted April 15, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1542–1554 (October 1976)

It is found that the homogeneous density distribution of a quasineutral electron-hole plasma becomes unstable when the plasma is uniformly heated by photogeneration or in an electric field, even if the condition for ordinary superheat instability is not fulfilled and external fluxes (temperature and concentration gradients, or a current) are absent. The instability is of an aperiodic nature and arises with respect to perturbations with a characteristic wave vector $k_0=(Ll)^{-1/2}$, where L is the bipolar diffusion length and l is the hot electron cooling length. It is shown in the "hydrodynamic" approximation that at $L>l$ stratification of the plasma occurs when the hot-electron momentum is scattered by charged centers, irrespective of the mechanism of the electron energy relaxation. From an analysis of the nonlinear one-dimensional problem it follows that the stable distributions of the carrier density and of the carrier temperature are those having the form of large-amplitude oscillations with a period of the order of $(Ll)^{1/2}$. The stratification mechanism considered may be one of the causes of the "drop formation" observed experimentally in a nonequilibrium electron-hole plasma of a semiconductor.

PACS numbers: 72.30.+q, 71.80.+j

1. INTRODUCTION. STRATIFICATION MECHANISM

In homogeneous systems that are disturbed by external actions out of their thermodynamic equilibrium state, ordered inhomogeneous stationary states are frequently produced.[1] Thus, for example, superheat instability, which leads to a non-single-valued current-voltage characteristic, produces in semiconductors an inhomogeneous distribution of the electric field or of the current density.[2,3] The formation of such inhomogeneous states can be connected with the presence of not only external but also "latent" (with respect to one of the parameters) negative differential resistance.[4,5] When external conditions produce temperature gradients or carrier-density gradients, the onset or the ordered structure takes place under less stringent requirements with respect to the semiconductor parameters.[6,7] In particular, in the presence of a temperature gradient, static spatially-periodic distributions are produced both in liquids (the Benard phenomenon) and in systems of hot electrons in semiconductors.[7] In this paper we consider an instability homogeneously heated by a quasi-neutral electron-hole plasma or a weakly-ionized gas, leading to stratification even in those cases when the conditions of the superheat instability are not satisfied and there are no fluxes (gradients of the temperature and of the carrier density) as specified by the external conditions.

The gist of the considered stratification mechanism of an electron-hole plasma (weakly ionized gas) consists in the following. Assume that electron-hole pairs are homogeneously generated in a semiconductor by photoexcitation. Naturally, this holds true for samples whose thickness does not exceed α_{ph}^{-1}. The electron density is then directly proportional to the light absorption coefficient α_{ph} and to the density Φ of the incident photons. If the energy of the exciting photons exceeds the width of the forbidden band (or the ionization energy of the atoms in the gas) by a certain amount $\Delta\varepsilon$, then photon absorption will give rise to hot carriers. At sufficiently high carrier density, the frequency of the interelectron collisions is high and the electrons have time to become thermalized.[2,8,9] The effective electron temperature T_e is determined by the equation of the heat balance between the system of hot electrons and the lattice

$$W=\Delta\varepsilon\alpha_{ph}\Phi=n(T_e-T_0)/\tau_\varepsilon, \tag{1}$$

where τ_ε is the time of energy relaxation of the hot electrons, and T_0 is the lattice temperature. The distribution of T_e in space is characterized by the diffusion cooling length $l\approx(D_e\tau_\varepsilon)^{1/2}$.[2,3,8,9] At the same time, when the quasi-neutrality condition is satisfied, it can be assumed that the concentrations of the electrons and holes (or of the ions in the gas) are equal to each other $(n=p)$, and their distribution in space is characterized by the bipolar diffusion length $L=(D\tau_r)^{1/2}$, where D is the coefficient of bipolar diffusion[9–11] and τ_r is the recombination time of the non-equilibrium carriers. Since τ_r is as a rule larger than τ_ε by many orders of magnitude, the characteristic distribution length L of the carrier density can exceed by many times the characteristic distribution length l of the effective temperature. We consider under these conditions the evolution of an inhomogeneous temperature fluctuation produced in a region with characteristic dimension $\sim(Ll)^{1/2}$. Since the dimension of the region of variation of T_e greatly exceeds l, it can be assumed that the heat-balance equation (1) is locally satisfied. At the same time, the hot carriers (as a result of the heat flow) will leave this hotter region so that the concentration of the carriers in it decreases, since the length L of their distribution greatly exceeds the dimension of the region of the fluctuation of T_e. In other words, the hot electrons will be vigorously ejected from the hotter region. This in turn decreases the power given up by the system of hot electrons to the lattice. Since the power W fed to the electron system is uniform, i.e., is independent of T_e or n, the decrease of n in the region of the fluctuation of T_e can lead, as can be seen from Eq. (1), to further increase of the temperature at the place where T_e increases. As a result of this growth of the fluctuations of T_e, the sample breaks up into alternating "cold" and "hot" regions with characteristic dimension $\sim(Ll)^{1/2}$, corresponding respectively to increased and decreased density of the electron-hole plasma.

Naturally, such a stratification will take place when the decrease of the power given up by the electron system to the lattice on account of the departure of the electrons from the hotter region exceeds the increase of the power due to the increase of T_e. As a consequence, the condition for the realization of the stratification is determined by the temperature dependences of the energy and momentum relaxation times τ_ε and τ_p of the electrons. As will be shown below, in the case of the power law dependences $\tau_p\sim T_e^\alpha$ and $\tau_\varepsilon\sim T_e^s$, the necessary condition for stratification is

$$\alpha+s>0. \tag{2}$$

This condition is satisfied at sufficiently low temperatures, when the momentum of the carriers is scattered mainly by the charged centers. In this case $\alpha=\frac{3}{2}$ and $\alpha+s>0$ at any energy relaxation mechanism, since $s\geqslant-\frac{1}{2}$[2,8,9] (in semiconductors the scattering of the carrier energy, as a rule, is by acoustic phonons, and in weak ionized gases the scattering is by neutral atoms, so that more likely we have $s=-\frac{1}{2}$).

Stratification can take place also when an electron-hole plasma is heated by an electric field even under conditions of impact ionization, if the power flowing from the electric field to the electrons depends little on their concentration. The latter is realized in the case when the electron momentum is scattered by mobile holes (or by ions in gas). Then, for a quasi-neutral plasma $(n=p)$ we have $\tau_p\sim T_e^{3/2}/n$,[9–11] and consequently the conductivity σ_e does not depend on the electron concentration, and is only a function of T_e. Owing to the dependence of σ_e on T_e, the Joule power supplied to the electron system is $W=\sigma_e E^2\sim T_e^{3/2}$. As a result, the necessary condition for the stratification of an electron-hole plasma is less stringent

$$2\alpha+s>0 \tag{3}$$

and the stratification takes place at a lower value of the electron temperature T_e.

We emphasize that the stratification conditions (2) and (3) are substantially less stringent than the condition $\alpha+s>1$ of the superheat instability, which can be satisfied only in exceptional cases.[2,3,8] In addition, in contrast to superheat instability, when the stability loss is relative to the long-wave perturbations,[2,3] in this case the homogeneous temperature distribution becomes unstable with respect to perturbations with a distinct value $k_0\approx(lL)^{-1/2}$.

2. FUNDAMENTAL EQUATIONS

We consider for the sake of argument an electron-hole plasma in a semiconductor. At a sufficiently high density and relatively low temperatures, the time of the electron–electron collisions turns out to be much shorter than the characteristic time τ_ε of the electron energy relaxation, i.e., the electron gas has time to become thermalized.[8–10] As a rule, the momentum relaxation time is $\tau_p\ll\tau_\varepsilon$, and consequently the drift velocity of the carriers is small compared with the thermal velocity. As a result, the kinetic coefficients and the power given up by the electron and hole gas to the lattice is a function only of their effective temperatures T_e and T_h and of the Fermi quasi levels F_e and F_h.[8–10] Under these conditions, the "hydrodynamic" approximation is valid, in which the carrier density and their effective temperatures, when the electrons and holes are not degenerate, are determined by the following system of equations[8–10]:

$$\frac{\partial n}{\partial t}=\frac{1}{e}\operatorname{div}\mathbf{J}_e-n\nu_r+R_g, \tag{4}$$

$$\frac{3}{2}\frac{\partial(nT_e)}{\partial t}=-\operatorname{div}\mathbf{J}_{\varepsilon e}+W_e-n\frac{T_e-T_0}{\tau_\varepsilon},$$
$$\frac{3}{2}\frac{\partial(pT_h)}{\partial t}=-\operatorname{div}\mathbf{J}_{\varepsilon h}+W_h-p\frac{T_h-T_0}{\tau_\varepsilon'}; \tag{5}$$

$$\frac{\partial\rho}{\partial t}=\operatorname{div}\mathbf{J}=\operatorname{div}(\mathbf{J}_e+\mathbf{J}_h),$$
$$\varepsilon\operatorname{div}\mathbf{E}=4\pi\rho=4\pi e(p-n), \tag{6}$$

where R_g and $\nu_r=\tau_r^{-1}$ are the generation rate and the frequency of the recombination of the non-equilibrium carriers; p and n are the hole and electron densities; $W_{e,h}=\alpha_{\rm ph}\Phi\Delta\varepsilon_{e,h}+\mathbf{J}_{e,h}\cdot\mathbf{E}$ is the power delivered to the electron (hole) system when the carriers are heated by light or by an electric field. We shall assume a dependence of the momentum relaxation time on the energy in the form $\tau_p\sim T^\alpha_{e,h}$. In this case the densities $\mathbf{J}_e$ and $\mathbf{J}_h$ of the electron and hole components of the current density $\mathbf{J}$ and the electron energy flux density $\mathbf{J}_{\varepsilon e}$ and of the hole energy $\mathbf{J}_{\varepsilon h}$ are respectively equal to

$$\mathbf{J}_e=\sigma_e\mathbf{E}+eD_e\nabla n+(1+\alpha)\mu_e n\nabla T_e, \tag{7}$$
$$\mathbf{J}_h=\sigma_h\mathbf{E}-eD_h\nabla p-(1+\alpha')\mu_h p\nabla T_h, \tag{8}$$
$$\mathbf{J}_{\varepsilon e}=-\frac{1}{e}\left(\frac{5}{2}+\alpha\right)T_e(\mathbf{J}_e+\mu_e n\nabla T_e), \tag{9}$$
$$\mathbf{J}_{\varepsilon h}=\frac{1}{e}\left(\frac{5}{2}+\alpha'\right)T_h(\mathbf{J}_h-\mu_h p\nabla T_h), \tag{10}$$

where $\sigma_e=e\mu_e n$, $\sigma_h=e\mu_h p$ and $D_e=\mu_e T_e/e$, $D_h=\mu_h T_h/e$ are the electric conductivities and the diffusion coefficients of the electrons and holes, respectively.

The characteristic dimension $k_0^{-1}\approx(lL)^{1/2}$ of the resultant inhomogeneity of the plasma density and of the temperature greatly exceed the Debye screening lengths even at relatively small concentrations of the carriers ($n>10^{12}$–10^{13} cm^{-3}). In addition, the characteristic growth times of the fluctuations turn out to be much less than the Maxwellian relaxation time τ_M. In other words, even at relatively low density of the electron-hole plasma the quasi-neutrality conditions $\rho\ll en$, $\rho\ll ep$, and $\omega t_M\ll 1$ are satisfied; these conditions, as follows from (6), allow us to assume[8,9] that the electron and hole densities are practically equal ($n=p$), and

$$\operatorname{div}\mathbf{J}=\operatorname{div}(\mathbf{J}_e+\mathbf{J}_h)=0. \tag{11}$$

We shall consider below the conditions for the stratification of a plasma that is uniformly heated in the absence of external fluxes, i.e., when the temperature and carrier-density gradients on the lateral surfaces of the sample are equal to zero; these conditions follow from the vanishing of the electron and hole components of the heat flux and of the current at the boundaries.

3. STABILITY OF HOMOGENEOUS DISTRIBUTION OF THE PLASMA DENSITY WHEN UNIFORMLY HEATED DURING THE PHOTOGENERATION PROCESS

The effective hole mass m_h^* (or the mass of an ion in a gas) in many superconductors is much larger than the effective electron mass m_e^*, so that we first consider homogeneous heating of an electron-hole plasma in the photogeneration process, when $m_h^*\gg m_e^*$. The hole energy relaxation time τ_ε' is in this case much shorter than the electron energy relaxation time τ_ε,[8–10] and in accordance with (5) the temperature T_h of the hole gas should differ much less from the lattice temperature T_0 than does T_e, so that T_h can be assumed equal to T_0. We note that heating of the hole gas, as will be shown below, leads to a stratification under less stringent requirements with respect to the semiconductor parameters.

Inasmuch as, by definition, the current at the sample boundaries is equal to zero, it follows from (11) that we have $\mathbf{J}=0$ in the entire sample. This makes it possible to determine from (7) and (8), and also from the quasineutrality condition ($n=p$), the value of the electric field

$$\mathbf{E}=-\frac{e(D_e-D_h)\nabla n+(1+\alpha)\mu_e n\nabla T_e}{\sigma_e+\sigma_h}. \tag{12}$$

From (7) and (12) it follows that

$$\mathbf{J}_e=eD\nabla n+(1+\alpha)\mu n\nabla T_e, \tag{13}$$

where $D=(\sigma_e D_h+\sigma_h D_e)/(\sigma_e+\sigma_h)$ is the bipolar diffusion coefficient,[9–11] and $\mu=\mu_e\mu_h/(\mu_e+\mu_h)$. Substituting (13) in (4), (9), and (5), we write down the equations of the continuity of the current and of the energy balance of the electrons in the form

$$\frac{\partial n}{\partial t}=\nabla\left[D\nabla n+\frac{1}{e}(1+\alpha)\mu n\nabla T_e\right]-\frac{n}{\tau_r}+\alpha_{\rm ph}\Phi, \tag{14}$$

$$\frac{3}{2}\frac{\partial(nT_e)}{\partial t}=\left(\frac{5}{2}+\alpha\right)\nabla\left\{DT_e\nabla n+\frac{1}{e}T_e\mu_e n\left[1+(1+\alpha)\frac{\mu}{\mu_e}\right]\nabla T_e\right\}+\mathbf{J}_e\mathbf{E}+\alpha_{\rm ph}\Phi\Delta\varepsilon_e-\frac{n(T_e-T_0)}{\tau_\varepsilon}, \tag{15}$$

where $\mathbf{J}_e$ and $\mathbf{E}$ are determined respectively by the expressions (13) and (12). We shall henceforth assume that both the energy relaxation time and, for the sake of generality, the recombination time depend on the electron energy in power law fashion: $\tau_\varepsilon=\tau_\varepsilon^0(T_e/T_0)^s$, $\tau_r=\tau_r^0(T_e/T_0)^r$. Linearizing Eqs. (12)–(15) with respect to permutations in the form $\delta T_e=(\delta T_e)_0\exp(\gamma t)\cos(\mathbf{kr})$, where in accordance with the boundary conditions $k^2=(\pi n/l_x)^2+(\pi m/l_y)^2+(\pi p/l_z)^2$ (l_x, l_y, l_z are the linear dimensions of the sample), we obtain the dispersion equation

$$\gamma^2\frac{3}{2}\tau_\varepsilon\tau_r+\gamma\left\{k^2\left[\tau_\varepsilon L^2\left(\frac{3}{2}+(1+\alpha)^2\frac{T_e}{T_e+T_0}\right)+\tau_r l^2\right]+\frac{3}{2}\tau_\varepsilon(1+r)+\tau_r\left(1-s+\frac{T_0}{T_e}s\right)\right\}+k^4l^2L^2+k^2\left\{l^2\left[1+\frac{rD}{D_e}+(1+\alpha)\frac{\mu}{\mu_e}\right]-L^2\frac{T_0}{T_e+T_0}\left[\frac{T_e}{T_0}(\alpha+s)-2-\alpha-\frac{T_0}{T_e}s\right]\right\}+1+r-s+\frac{T_0}{T_e}(s-r)=0, \tag{16}$$

where $L=(D\tau_r)^{1/2}$ is the bipolar diffusion length and $l=[(\frac{5}{2}+\alpha)D_e\tau_\varepsilon]^{1/2}\approx v_T(\tau_\varepsilon\tau_p)^{1/2}$ is the diffusion length of the cooling (of the energy range) of the hot electrons.[2] The instability condition ($\gamma>0$) is satisfied when the free term of the quadratic equation (16) relative to γ becomes less than zero, since the remaining coefficients of this equation at $s<1$ are positive. It follows therefore that the instability of the homogeneous distribution can set in relative to the long-wave perturbations ($k=0$) at

$$T_e > T_0(s-r)/(s-r-1), \quad \text{when} \quad r < s-1. \tag{17}$$

If the condition (17) is not satisfied, then the plasma becomes stratified relative to

$$k_0 = (lL)^{-1/2}\left[1+r-s+\frac{T_0}{T_e}(s-r)\right]^{1/4} \tag{18}$$

at

$$\frac{T_e}{T_e+T_0}\left[\frac{T_e}{T_0}(\alpha+s)-2-\alpha-\frac{T_0}{T_e}s\right] > 2\frac{l}{L}\left[1+r-s+\frac{T_0}{T_e}(s-r)\right]^{1/2} + \left(\frac{l}{L}\right)^2\left[1+\frac{rD}{D_e}+(1+\alpha)\frac{\mu}{\mu_e}\right]. \tag{19}$$

To satisfy the condition (17) it is necessary that the recombination time decrease when the carriers become heated, since $s \leqslant \frac{1}{2}$.[7,8] In principle, in semiconductors such a situation is possible.[2,3,12,13] As follows from the stationary equations (14) and (15), $\Delta\varepsilon_e$ is a multiple-valued function of T_e, and consequently, at the temperature T_e defined by (17), a jumplike decrease will take place in the carrier density, with a corresponding decrease in the electron temperature. In other words, a jumplike negative photoconductivity of the hysteresis type should be observed.

As a rule, however,[13] the recombination time increases with increasing temperature, and consequently the condition (17) is not satisfied. In the analysis that follows we shall therefore assume that $r \geqslant 0$. We emphasize that the stratification condition (19) can be satisfied only if $l < L$, so that the dependence of τ_r on T_e has practically no effect on the condition for the realization of the instability (19).

In a number of semiconductors the effective masses of the electron and of the hole are of the same order, so that it is expedient to analyze the stratification condition of an electron-hole plasma with $m_e^* = m_h^*$. In this case, more accurately at $n=p$, $\tau_p=\tau_p'$, $\tau_\varepsilon=\tau_\varepsilon'$ and $D_e=D_h$, the Dember field (12) is assumed equal to zero,[11] and T_e equals T_h. From the system (4)–(10) we have in place of (14) and (15)

$$\frac{\partial n}{\partial t} = \nabla\left[D_e\nabla n+\frac{1}{e}(1+\alpha)\mu_e n\nabla T_e\right]-\frac{n}{\tau_r}+\alpha_{\text{ph}}\Phi, \tag{20}$$

$$3\frac{\partial(nT_e)}{\partial t} = (5+2\alpha)\nabla\left[D_eT_e\nabla n+\frac{1}{e}(2+\alpha)\mu_e nT_e\nabla T_e\right]+\alpha_{\text{ph}}\Phi\Delta\varepsilon-2n\frac{T_e-T_0}{\tau_\varepsilon} \tag{21}$$

Linearizing Eqs. (20) and (21) with respect to the inhomogeneous perturbations, we find that the stratification of the plasma with identical carrier masses proceeds with respect to

$$k_0 = (lL_e)^{-1/2}\left[1+r-s+\frac{T_0}{T_e}(s-r)\right]^{1/4}, \tag{22}$$

when the condition

$$\alpha+s-\frac{T_0}{T_e}(1+\alpha+s) > 2\frac{l}{L_e}\left[1+r-s+\frac{T_0}{T_e}(s-r)\right]^{1/2}+\left(\frac{l}{L_e}\right)^2(2+\alpha+r) \tag{23}$$

is satisfied, where $L_e=(D_e\tau_r)^{1/2}$ is the diffusion length of the electrons (holes).

From a comparison of (19) and (23) it is seen that the necessary condition for their realization is $\alpha+s>0$ (2). To realize the instabilities (19) or (23) it is necessary also to have $L/l \approx (m_e^*\tau_r/m_h^*\tau_\varepsilon)^{1/2} > 1$ or $L_e/l \approx (\tau_r/\tau_\varepsilon)^{1/2} > 1$, respectively. These inequalities are satisfied because $\tau_r \gg \tau_\varepsilon$. However, it must be borne in mind here that the larger the ratio of τ_r to τ_ε, the larger the energy excess $\Delta\varepsilon$ needed to heat the plasma to the critical temperature, which is determined respectively from (19) and (23). In fact it follows from (14), (15), or (20), (21) that for the homogeneous and stationary case

$$T_e-T_0=\Delta\varepsilon_e\frac{\tau_\varepsilon}{\tau_r} \quad \text{or} \quad T_e-T_0=\frac{\Delta\varepsilon}{2}\frac{\tau_\varepsilon}{\tau_r} \tag{24}$$

for strongly differing and equal masses of the electron and hole, respectively. In semiconductors, when the excess photon energy $\Delta\varepsilon$ over the width of the forbidden band is increased the absorption coefficient increases sharply, so that $\Delta\varepsilon$ cannot be too large for homogeneous photogeneration of carriers. Thus, the stratification of an electron-hole plasma is easiest to realize in non-straight-band semiconductors with close carrier effective masses at sufficiently low temperatures and when τ_r and τ_ε differ by not more than two orders of magnitude.

4. STABILITY OF HOMOGENEOUS DISTRIBUTION OF THE DENSITY OF A PLASMA HEATED IN AN ELECTRIC FIELD

In a strong electric field, generation of nonequilibrium electron-hole or gas-discharge plasma can be due not only to photoionization, but also to impact ionization of the carriers. To ascertain the influence of the impact ionization on the conditions for the realization of plasma stratification, we write down the carrier generation in the form $R_g=\alpha_{\text{ph}}\Phi+n\nu_i(T_e)$, where $\nu_i(T_e)$ is the impact-ionization frequency.[14] We note that the recombination frequency ν_r depends in principle on T_e and on $n(\nu_r \sim nT_e^{-r})$, but this circumstance, as will be shown below, has in fact no effect on the instability condition. When the carriers are heated in an electric field the power delivered to the electron system is equal to $W=\mathbf{J}_e\cdot\mathbf{E}$. Let the current flow in the direction of the z axis and consequently let the power released in the stationary case and at homogeneous distribution be $W=\sigma_e E_z^2$. In this case, σ_e, and consequently also W, are generally speaking functions of T_e and n. When $\sigma_e \sim n$, no plasma (current) stratification is realized. We shall therefore consider henceforth the case when the relaxation of the carrier momentum is determined principally by the electron-hole (electron-ion) collisions, and then under quasi-neutrality conditions ($n=p$) we have $\tau_p \sim T_e^{3/2}/n$.[2,8,9]

In the presence of an external electric field E_z, the considered system becomes essentially inhomogeneous, and expressions (7)–(10) acquire components proportional to E_z. As the result, the equations of the continuity of the current and of the energy bounds of the electrons at $m_h^* \gg m_e^*$ take the form

$$\frac{\partial n}{\partial t}=\nabla_\perp\left[D\nabla_\perp n+\frac{1}{e}(1+\alpha)\mu n\nabla_\perp T_e\right]+\frac{\partial}{\partial z}\left[\mu_e nE_z+D_e\frac{\partial n}{\partial z}+\frac{1}{e}(1+\alpha)\mu_e n\frac{\partial T_e}{\partial z}\right]-n\nu_r(n,T_e)+R_g(n,T_e), \qquad (25)$$

$$\frac{3}{2}\frac{\partial(nT_e)}{\partial t}=\left(\frac{5}{2}+\alpha\right)\nabla_\perp\left\{DT_e\nabla_\perp n+\frac{1}{e}\mu_e nT_e\left[1+(1+\alpha)\frac{\mu}{\mu_e}\right]\nabla_\perp T_e\right\}+\left(\frac{5}{2}+\alpha\right)\frac{\partial}{\partial z}\left[\mu_e nT_eE_z+D_eT_e\frac{\partial n}{\partial z}+\frac{1}{e}(2+\alpha)\mu_e nT_e\frac{\partial T_e}{\partial z}\right]$$
$$+\mathbf{J}_{e\perp}\mathbf{E}_\perp+\sigma_e E_z^2+eD_eE_z\frac{\partial n}{\partial z}+(1+\alpha)\mu_e nE_z\frac{\partial T_e}{\partial z}-n\frac{n(T_e-T_0)}{\tau_\varepsilon}, \qquad (26)$$

where the transverse components of the current $\mathbf{J}_{e\perp}$ and of the field $\mathbf{E}_\perp$ are determined by Eqs. (7), (8), and (11). It is seen from (25) and (26) that the dispersion equation is in general difficult to analyze. We note, however, that the largest growth rate is possessed by perturbations with $k_z=0$. In fact, let us consider for simplicity a plasma with $m_e^*=m_h^*$. In this case it follows from (7), (8), and (11) that the field increment δE_z at $k_z\neq 0$ is connected with the increment of the electron temperature by the relation $\delta E_z=-\alpha E_z\delta T_e/T_e$. This means that in the region where the electron temperature increases a decrease takes place in the field strength E_z, and hence in the power W, and this leads to a more stringent condition on the plasma stratification with respect to longitudinal perturbations with $k_z\neq 0$.

Linearizing Eqs. (25) and (26) with respect to transverse perturbations ($k_\perp^2=k_x^2+k_y^2$, $k_z=0$), which do not change the total current in the external circuit, and consequently also E_z, we obtain the dispersion equation

$$\gamma^2\frac{3}{2}\tau_\varepsilon\tau_r+\gamma\left\{k_\perp^2\left[\tau_\varepsilon L^2\left(\frac{3}{2}+(1+\alpha)^2\frac{T_e}{T_e+T_0}\right)+\tau_r l^2\right]+\frac{3}{2}\tau_\varepsilon\left(1+r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}\right)+\tau_r\left[1-\alpha-s+\frac{T_0}{T_e}(\alpha+s)\right]\right\}+k_\perp^4 l^2L^2$$
$$+k_\perp^2\left\{l^2\left[1+\left(r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}\right)\frac{D}{D_e}+(1+\alpha)\frac{\mu}{\mu_e}\right]-L^2\frac{T_0}{T_e+T_0}\times\left[\frac{T_e}{T_0}(2\alpha+s)-2-\alpha-\frac{T_0}{T_e}(\alpha+s)\right]\right\}+1+r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}-\alpha-s$$
$$-\frac{T_0}{T_e}\left(r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}-\alpha-s\right)=0, \qquad (27)$$

from which it follows that the homogeneous distribution of the plasma becomes unstable with respect to

$$k_{\perp 0}=(lL)^{-1/2}\left[1+r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}-\alpha-s-\frac{T_0}{T_e}\left(r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}-\alpha-s\right)\right]^{1/4}, \qquad (28)$$

when

$$\frac{T_0}{T_e+T_0}\left[\frac{T_e}{T_0}(2\alpha+s)-2-\alpha-\frac{T_0}{T_e}(\alpha+s)\right]>2\frac{l}{L}\left[1+r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}-\alpha-s-\frac{T_0}{T_e}\left(r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}-\alpha-s\right)\right]^{1/2}+\left(\frac{l}{L}\right)^2\left[1+\left(r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}\right)\frac{D}{D_e}+(1+\alpha)\frac{\mu}{\mu_e}\right]. \qquad (29)$$

In an electron-hole plasma with $m_h^*=m_e^*$ stratification sets in with respect to

$$k_{\perp 0}=(lL_e)^{-1/2}\left[1+r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}-\alpha-s-\frac{T_0}{T_e}\left(r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}-\alpha-s\right)\right]^{1/4} \qquad (30)$$

at an electron-hole-gas temperature determined from the condition

$$2\alpha+s-\frac{T_0}{T_e}(1+2\alpha+s)>2\frac{l}{L_e}\left[1+r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}-\alpha-s-\frac{T_0}{T_e}\left(r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}-\alpha-s\right)\right]^{1/2}+\left(\frac{l}{L_e}\right)^2\left(r+\frac{T_e}{\nu_i}\frac{\partial\nu_i}{\partial T_e}+2+\alpha\right). \qquad (31)$$

As seen from (29) and (31), when the plasma is heated in an electric field, a necessary stratification condition is $2\alpha+s>0$ (3). Since it was assumed in the analysis of the instability that the carrier momentum relaxation takes place in general via electron-hole (electron-ion) collisions ($\alpha=\frac{3}{2}$), it follows that the plasma stratification conditions (29) and (31) are satisfied here for any energy relaxation mechanism when $\tau_r\gg\tau_\varepsilon$ ($L\gg l$). The presence of impact ionization has in this case a relatively weak effect on the satisfaction of the instability conditions.

5. INHOMOGENEOUS STATIONARY STATES

Let us analyze the stationary states that can arise as a result of instability of the homogeneous distribution of the electron-hole plasma density when the carriers are heated by light. We assume for the sake of argument that the electrons and holes have identical masses, and that their momentum and energy are scattered by the charged impurities and phonons, respectively. In this case ($\alpha=\frac{3}{2}$, $s=-\frac{1}{2}$, $r=0$) Eqs. (20) and (21), which describe the distributions of the carrier densities and of their temperature, can be written in the form

$$\frac{\partial}{\partial t}(\eta\Theta^{-5/2})=\Delta\eta-\eta\Theta^{-5/2}+d^{-1}, \qquad (32)$$

$$\beta^2\frac{3}{8}\frac{\partial}{\partial t}(\eta\Theta^{-3/2})=\beta^2\Delta(\eta\Theta)-\eta\frac{\Theta-1}{\Theta^2}+1, \qquad (33)$$

where

$$\eta=\frac{n}{n_0}\left(\frac{T_e}{T_0}\right)^{5/2},\quad \Theta=\frac{T_e}{T_0},\quad n_0=\frac{\alpha_\Phi\Phi\Delta\varepsilon\tau_\varepsilon^0}{2T_0},$$
$$d=\frac{\Delta\varepsilon\tau_\varepsilon^0}{2T_0\tau_r},\quad \beta^2=\left(\frac{l_0}{L_0}\right)^2=\frac{4\tau_\varepsilon^0}{\tau_r}.$$

In these equations, the length is measured in units of $L_0=[D_e(T_0)\tau_r]^{1/2}$, and the length in units of τ_r. The boundary conditions on the lateral surfaces of the sample consist in the vanishing of the normal components of $\nabla\eta$ and $\nabla\Theta$. Let us study first the stationary solutions whose characteristic dimension is much greater than β^2. We can assume here the connection between η and Θ to be local, i.e., we can neglect in (33) the term $\beta^2\Delta(\eta\Theta)$:

$$\eta=[\Theta(\eta)]^2/[\Theta(\eta)-1]\qquad (n=n_0T_0^2/T_e(T_e-T_0)). \qquad (34)$$

Then Eq. (32) is transformed into

$$\Delta\eta+dU/d\eta=0,\quad U=\int(d^{-1}-\eta[\Theta(\eta)]^{-5/2})\,d\eta. \qquad (35)$$

Let us investigate the one-dimensional solutions of Eq. (35). In the one-dimensional case it takes the form of the equation of motion of a particle in a field with a potential $U(\eta)$. As seen from (35) and (34), the condition for the extremum of the function $U(\eta)$

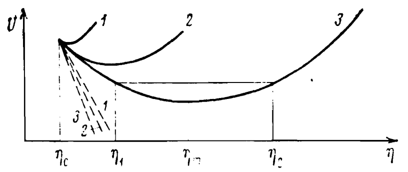

FIG. 1. Form of the potential $U(\eta)$ at different heating levels of an electron-hole plasma (an increasing number of the curve corresponds to a larger value of d, i.e., to a larger heat rise). The dashed lines show the branches of $U(\eta)$ corresponding to weak heating ($T_e - T_0 < T_0$).

$$[\Theta(\eta_m)-1][\Theta(\eta_m)]^{1/2}=d \tag{36}$$

has at all values $d>0$ a single root corresponding to the minimum of the potential. The form of $U(\eta)$ at different values of d is shown in Fig. 1, from which it follows that $\eta(x)$, and consequently also $\Theta(x)$, have solutions only in the form of oscillations. The number of half-oscillations p is determined by the relation

$$l_x\sqrt{2}=p\int_{\eta_1}^{\eta_2}\frac{d\eta}{(U(\eta_1)-U(\eta))^{1/2}}, \tag{37}$$

and their characteristic lengths (half-period) is of the order of $L_c=\pi[U''(\eta_m)]^{-1/2}$ and increases as the plasma becomes hotter ($d=1.6$, $L_c\approx 0.68$; $d=3.5$, $L_c\approx 8.8$). As follows from the figure, the potential $U(\eta)$ has a boundary point $\eta_0=4$ ($T_e=2T_0$) that corresponds to the instability condition (23) at $l^2\ll L_e^2$. As a consequence, during each stage of plasma heating there exists a boundary condition with the maximum amplitude of the oscillations of $\eta(x)$ and $\Theta(x)$, bounded from below by the solution $\eta_0=4$.

We emphasize that the system (32) and (33) has at the considered boundary conditions no inhomogeneous stationary solutions corresponding to $\eta(x)$ and $\Theta(x)$ with strongly differing characteristic lines. In particular, it is easy to verify that at constant η there exists a single solution, $\Theta=\text{const}$. In the case of a smooth function $\eta(x)$, the equation for $\Theta(x)$ also reduces to the equation of motion of a particle in a certain potential, but in a shape of a "hump" rather than a "well." Consequently there are no inhomogeneous solutions $\Theta(x)$ satisfying the condition $\Theta'_x=0$ at $x=0$ and $l_x=0$ in this case.

Let us examine the stability of the one-dimensional solutions (the density layers of the plasma) of Eq. (35). Linearizing (32), (34), and the boundary conditions for $\eta(x)$ relative to the inhomogeneous perturbations $\delta\eta(x)e^{\nu t}$, we obtain

$$\hat{H}\delta\eta=[-d^2/dx^2-V(x)]\delta\eta=\gamma V(x)\delta\eta, \tag{38}$$

$$d\,\delta\eta/dx|_{x=0,\,l_x}=0, \tag{39}$$

where

$$V(x)=\left.\frac{d^2U}{d\eta^2}\right|_{\eta(x)}=\frac{3\Theta(x)-1}{2[\Theta(x)]^{1/2}[\Theta(x)-2]}, \tag{40}$$

with $\Theta>2$ ($\eta>4$) for all the solutions of (35), so that $V(x)>0$. Thus, the stability problem reduces to the Sturm–Liouville problem. To obtain the concrete form of $V(x)$ in (38) it is necessary to substitute the solution of (35), which describes that $\eta(x)$ distribution whose stability is under investigation.[2,3,15] The solution is unstable if at least one of the eigenvalues γ of Eq. (38) turns out to be larger than zero.

Differentiating (35) with respect to x we obtain for the one-dimensional case

$$\hat{H}\,d\eta/dx=0.$$

It follows therefore that $d\eta/dx$ is an eigenfunction of the operator $\hat{H}$, corresponding to $\gamma=0$, but satisfying boundary conditions other than (39). Since $d\eta/dx$ vanishes at the limits of the interval $(0, l_x)$, it follows from the Sturm theorem that the eigenfunction $\delta\eta_0(x)$ of the operator $\hat{H}$, corresponding to $\gamma=0$ and satisfying the conditions (39) has one more zero than the function $d\eta/dx$. Consequently, the longer-wavelength perturbations $\delta\eta(x)$ have eigenvalues $\gamma<0$, while the short-wavelength perturbations have $\gamma>0$. Thus, as expected from the linear theory, the long-wave solution $\eta(x)$ with period $L_c\gg k_0^{-1}$ turn out to be unstable with respect to the short-wavelength perturbations of $\delta\eta(x)$. The reason for this instability lies in the fact that the ejection of the electrons from the region in which T_e rises is more intense the smaller the characteristic dimension of the region where the temperature is increased. In other words, the system tends to break up into layers with ever shorter wavelengths. However, at large values of k the local connection between η and Θ (34) is violated, i.e., diffusion heat flows assume an important role and tend to equalize the temperature distribution. This, on the one hand, limits the period of the oscillations to a value on the order of k_0^{-1}, and on the other hand, as follows from the linear theory, leads to stability of solutions with an oscillation period smaller than $(lL_e)^{1/2}$. We note that with decreasing plasma heat rise $d^2U/d\eta^2$ at the minimum point, $U(\eta)$ increases sharply (see the figure), so that the period of the corresponding solutions $\eta(x)$ of Eq. (35) decreases. Since the perturbation $\delta\eta_0(x)$, which corresponds to the solution of Eq. (38) at $\gamma=0$, has the same number of zeroes as $\eta(x)$, it follows therefore that all the $\delta\eta(x)$ that are solutions of (38) oscillate strongly at $\gamma>0$. One should therefore expect the oscillations of $\eta(x)$ with a period comparable with $(lL_e)^{1/2}$ to be stable in the case of a small heat rise ($T_e\approx 2T_0$).

To investigate the stable solutions corresponding to a strongly heated plasma, it is necessary to consider the complete system of equations (32) and (33). An analysis of this system shows that solutions that oscillate with a period $(lL_e)^{1/2}$ but with small amplitude also turn out to be unstable, at least with respect to the longer-wavelength perturbations. In fact, at small oscillation amplitudes, by averaging the equation for the fluctuations over a length exceeding $(lL_e)^{1/2}$, we arrive at an investigation of a linear problem, from which it follows that in the case of a strong heat rise we have $\gamma>0$ in a wide range of k, including $k<k_0$. Thus, we can conclude that the only stable oscillating solutions are those with large amplitude and with a period on the order of $(lL_e)^{1/2}$.

6. CERTAIN REMARKS

In the narrow-band semiconductors of the type CdHgTe, PbSnTe and InSb, on account of thermal generation, the intrinsic carrier density can be high enough even at relatively low temperatures. An electron-hole plasma of the required concentration can also be produced by injection from a $p-n$ junction. In these cases, the stratification of the electron-hole gas considered in the present paper will take place in the absence of photogeneration of impact ionization, when the carriers are heated either by light absorbed by the free carriers or by an electric field.

In many semiconductors, at low temperatures and high excitation levels, electron-hole drops are produced and are interpreted as a result of exciton interaction.[16] As follows from our present results, a phenomenon similar to "drop formation" can occur also without allowance for the interaction between excitons. Indeed, in germanium the concentration of the free electrons and holes becomes equalized already at $T_0 = 6$ °K with the concentrations of the excitons, so that the conditions of the stratification considered here can be satisfied also at lower temperatures. Thus, for example, in[17] is presented experimental proof that in samples of pure germanium subjected to strong optical surface excitation the electron-hole plasma becomes stratified into a "plasma vapor" and a "plasma liquid" both at $T_0 = 4.2$ °K and at $T_0 = 1.8$ °K.

Murashev, Ivanov, and Shotov[18] have studied experimentally the spatial distribution of the recombination radiation produced in pure germanium by injection of carriers from contacts. The investigations show that the non-equilibrium carriers are strongly heated in the electric field, and a spatially-periodic distribution of both the hot-carrier temperature and of the position of the maximum of the spectral exciton-emission line are observed up to $T_0 = 77$ °K. These effects can also be connected with the stratification mechanism considered here. Under the experimental conditions, the principal mechanism of carrier momentum relaxation is most likely electron-hole scattering, i.e., the stratification condition (3) is satisfied. In addition, the dimension of the hot-carrier temperature oscillations observed in[18] is of the same order as the period of the stratification of the electron-hole plasma considered in the present paper, which amounts to several units of $10^{-3}-10^{-2}$ cm at the typical parameters of germanium.[16]

Under the experimental conditions, the electron-hole gas can turn out to be degenerate. In this case the necessary stratification conditions (2) and (3) are also satisfied, inasmuch as for strongly degenerate electrons we have $\alpha = 0$ and $s = 1$ regardless of the mechanisms of their energy and momentum relaxation.[7]

In a just-published paper[19] reporting an experimental study of the kinetics of formation of electron-hole drops in germanium, it was noted that the radii of all the drops are practically identical, and their size does not depend on the excitation level, increasing somewhat with rising temperature and tending to 10^{-3} cm. One cannot exclude the possibility that these effects are also connected with the here-considered stratification of an electron-hole plasma as its temperature rises.

It is our pleasant duty to thank L. V. Keldysh for a discussion of the results.

[1] J. Prigogine and G. Nicolis, Quart. Rev. Biophys. **4**, (2/3), 107 (1971).

[2] A. P. Volkov and Sh. M. Kogan, Usp. Fiz. Nauk **96**, 633 (1968) [Sov. Phys. Usp. **11**, 881 (1969)].

[3] V. L. Bonch-Bruevich, I. P. Zvyagin, and A. G. Mironov, Domennaya élektricheskaya neuštoǐchivost' v poluprovodnikakh (Domain Electric Instability in Semiconductors), Nauka, 1972.

[4] B. S. Kerner and V. V. Osipov, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 122 (1973) [JETP Lett. **18**, 70 (1973)].

[5] V. V. Osipov, Abstract of Doctoral Dissertation, Leningrad Polytech. Inst., 1975.

[6] L. E. Gurevich and I. V. Ioffe, Zh. Eksp. Teor. Fiz. **61**, 1133 (1971); **62**, 1531 (1972) [Sov. Phys. JETP **34**, 605 (1972); **35**, 803 (1972)].

[7] V. L. Bonch-Bruevich, Zh. Eksp. Teor. Fiz. **67**, 2204 (1974) [Sov. Phys. JETP **40**, 1093 (1975)].

[8] F. G. Bass and Yu. G. Gurevich, Groyachie élektrony i sil'nye élektromagnitnye volny v plazme poluprovodnikov i gazovogo razryada (Hot Electrons and Strong Electromagnetic Waves in the Plamsa of Semiconductors or of gas Discharges), Nauka, 1975.

[9] Kinetic Processes in Gases and Plasmas, A. R. Hochstim, ed., Academic, 1969.

[10] É. D. Lozanksiǐ and O. B. Firsov, Teoriya iskry (Theory of Sparks), Atomizdat, 1975.

[11] R. A. Smith, ed. Semiconductors, Academic, 1963.

[12] V. L. Bonch-Bruevich, Fiz. Tverd. Tela (Leningrad) **6**, 2037 (1964) [Sov. Phys. Solid State **6**, 1607 (1965)]; V. L. Bonch-Bruevich and S. G. Kalashnikov, Fiz. Tverd. Tela (Leningrad) **7**, 750 (1965) [Sov. Phys. Solid State **7**, 599 (1965)].

[13] V. L. Bonch-Bruevich and E. G. Landsberg, Phys. Status Solidi **29**, 9 (1968).

[14] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **48**, 1692 (1965) [Sov. Phys. JETP **21**, 1135 (1965)].

[15] G. F. Bass, V. S. Bochkov, and Yu. G. Gurevich, Zh. Eksp. Teor. Fiz. **58**, 1814 (1970) [Sov. Phys. JETP **31**, 972 (1970)].

[16] In: Éksitony v poluprovodnikakh (Excitons in Semiconductors), ed. B. M. Vul, Nauka, 1971; L. V. Keldysh, Usp. Fiz. Nauk **100**, 513 (1970) [Sov. Phys. Usp. **13**, 291 (1970)].

[17] B. M. Ashkinadze and I. M. Fishman, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 336 (1975) [JETP Lett. **22**, 153 (1975)].

[18] M. S. Murashev, V. N. Ivanov, and A. P. Shotov, Zh. Eksp. Teor. Fiz. **70**, 1009 (1976) [Sov. Phys. JETP **43**, 526 (1976)].

[19] V. S. Bagaev, N. V. Zamkovets, L. V. Keldysh, N. N. Sibel'din, and V. A. Tsvetkov, Zh. Eksp. Teor. Fiz. **70**, 1501 (1976) [Sov. Phys. JETP **43**, 783 (1976)].

Translated by J. G. Adashko