Effect of the nature of electron reflection on the penetration of an electromagnetic wave through a metallic plate

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The topic considered is the effect of the nature of the reflection of electrons by the surface on the passage of a doppleron wave or of a Gantmakher-Kaner component through a metallic plate in a constant magnetic field perpendicular to its surface. It is shown that the amplitude of the transmitted signal in the strong-field range is much larger for diffuse reflection than for specular. The doppleron excitation is considerably more efficient in the case of diffuse reflection. Furthermore, an important role in the amplification of the signal is played by the skin layer that is produced by the transmitted wave at the opposite side of the plate.

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It is well known[1] that the impedance of a metal under anomalous skin-effect conditions is only slightly dependent on the nature of the reflection of electrons from the surface. The presence of a constant magnetic field may in principle change the situation. Thus the behavior of the impedance of a semi-infinite metal near the threshold for a helicon[2,3] or doppleron[4,5] wave depends significantly on how the electrons are reflected. At the same time, it has been shown[6,7] that the basic features of cyclotron resonance in metals are not very sensitive to the nature of the reflection. The same is true of the passage of a helicon through a plate[8] and of a number of other phenomena.

The treatment of the electromagnetic properties of metals for diffuse reflection presents a much more complicated problem than for specular. Therefore the authors of the majority of papers on the theory of the penetration of a radiofrequency field through metals have considered only specular reflection, assuming that diffuseness will not lead to qualitative changes of the results. In the present paper it is shown that in a number of cases this assumption turns out to be incorrect. Thus when a doppleron[9] or a Gantmakher–Kaner (GK) wave[10] is excited in the plate, the transmitted signal may be much larger in the case of diffuse reflection than in the case of specular.

The first part of the article is devoted to a qualitative discussion of the role that the nature of the electron reflection plays in the penetration of an electromagnetic wave in the nonlocal mode. In the second part, the conclusions deduced from the qualitative discussion are illustrated with a simple model of a compensated metal. In this model, the electron Fermi surface is a lens formed by two paraboloidal bowls with the rims joined; the Fermi surface of the holes is a cylinder with axis parallel to the axis of the lens; the constant magnetic field H (z axis) and the normal to the plate surface are directed along the axis of symmetry of the Fermi surface.

This model permits exact solution of the problem of the field distribution in a semi-infinite metal and in a metallic plate, both for specular and for diffuse reflection of the electrons from the surface.

1. We shall consider the low-frequency, strong-magnetic-field range $\omega \ll \nu \ll \omega_c$, where ω is the frequency of the exciting field, ν is the frequency of collision of the electrons with phonons and impurities, and ω_c is the cyclotron frequency of the electrons. First we shall consider the impedance of a semi-infinite metal, and the differences in doppleron excitation between specular and diffuse reflection. Let a plane monochromatic wave with amplitude $\mathscr{E}$ be normally incident on a semi-infinite metal. This boundary condition of the problem can be written in the form

$$\left(1+\frac{1}{iq_0}\frac{d}{d\zeta}\right)E(\zeta)\Big|_{\zeta=0}=2\mathscr{E}, \tag{1}$$

where we have introduced the dimensionless quantities $\zeta=2\pi z/u$, $q_0=\omega u/2\pi c$; u is the maximum displacement of the electrons along the magnetic field during a cyclotron period,

$$u=2\pi\left(\frac{v_z}{\omega_c}\right)_{ext}=\frac{c}{eH}\left|\frac{\partial S}{\partial p_z}\right|_{ext},$$

where $S(p_z)$ is the area of the section of the Fermi surface by the plane $p_z=\text{const}$ (p_z is the component of the electron momentum along the magnetic field H). To terms of order q_0 we can write

$$E'(0)=2iq_0\mathscr{E}.$$

If there are no nonlocal effects in the conductivity, then we get both in the specular and in the diffuse case the same field distribution

$$E_0(\zeta)=2\mathscr{E}\frac{q_0}{q_s}e^{iq_s\zeta},$$

where q_s is the root of the dispersion equation and describes the skin layer. The surface impedance, which determines the energy that flows into the semi-infinite metal in unit time, has the form

$$Z_\infty = 4\pi q_0 i E_0(0)/cE_0'(0),$$
$$Z_\infty = 4\pi q_0/cq_s. \qquad (2)$$

In the presence of nonlocal effects, singularities appear in the conductivity σ as a function of the wave vector **k**; these correspond to the extreme displacements of the electrons and cause the existence of a doppleron and of a Gantmakher–Kaner wave. As a result, there occurs a redistribution of the field among three components: a "skin" component, a "doppleron" component, and a Gantmakher–Kaner component.

We shall consider a model of a compensated metal in which there is only one group of electrons, with maximum displacement u, and the displacement of the holes is much smaller than u, so that their contribution to the conductivity is described by the local approximation. We shall consider waves with "minus" circular polarization, in which the field rotates in the same sense as do electrons in a magnetic field. The dispersion equation for the normal modes in an infinite metal has the form

$$k^2 = \frac{4\pi i\omega}{c^2}\sigma_-(k) = \frac{4\pi\omega}{c^2}\frac{nec}{H}\left\{\left\langle\left(1-\frac{\omega}{\omega_c}+\frac{kv_z}{\omega_c}-i\frac{\nu}{\omega_c}\right)^{-1}\right\rangle-1\right\}, \qquad (3)$$

where n is the concentration of the electrons and m is their effective mass. The angular brackets signify averaging over the electron distribution; the unity in the wavy brackets corresponds to the local Hall conductivity of the holes.

This dispersion equation has two solutions k_s and k_D, of which the first represents the complex wave vector of the skin component of the field, the second the wave vector of the doppleron. In the production of the skin component of the field, all the carriers play an important role; but in the formation of the doppleron, only the electrons moving in the direction of its propagation. Hereafter we shall use the dimensionless wave vector $q = ku/2\pi$ and, accordingly, the solutions

$$q_{s,D} = k_{s,D}u/2\pi.$$

The real part of q_D is negative and in modulus less than unity.[9] Hereafter we shall be interested in the strong-field range, in which

$$|q_s| \ll 1, \quad \mathrm{Re}(1+q_D) \ll 1.$$

The external wave produces, near the surface of the metal, an electromagnetic field in which are present both components with wave vectors close to the wave vector of the doppleron q_D, and Gantmakher–Kaner waves. These components of the field also excite corresponding waves. The amplitude of the excited wave depends significantly on the nature of the reflection of electrons from the surface: the doppleron and the Gantmakher–Kaner wave are excited considerably less in the case of specular reflection than in diffuse reflection. The reason is that in the diffuse case an electron reflected from the surface loses the momentum acquired in the skin layer, whereas in the specular case it keeps it.

The electric fields comprising the skin layer with wave vectors q_D and $-q_D$ are almost equal in magnitude and opposite in direction (this is a consequence of the inequality $|q_s| \ll |q_D|$). An electron approaching the surface interacts in resonance fashion with the component $E(-q_D)$; a reflected electron, with the component $E(q_D)$. Therefore a reflected electron acquires in the skin layer a momentum almost equal in magnitude and opposite in sign to that, that it acquired during its flight to the surface. Thus when the nature of the reflection is specular, the resultant change of momentum is greatly diminished. But in the case of diffuse reflection, the electron gives up the acquired momentum to the surface, so that compensation does not occur, and the effect turns out to be much larger. The field acting on a resonance electron that has undergone diffuse reflection is

$$E_{eff}(q_D) \sim \frac{1}{q_D - q_s},$$

while the effective field in specular reflection is

$$E_{eff}^{sp}(q_D) = E_{eff}(q_D) + E_{eff}(-q_D) \sim \frac{1}{q_D-q_s} + \frac{1}{-q_D-q_s} = \frac{2q_s}{q_D^2-q_s^2},$$

that is,

$$E_{eff}^{sp}(q_D)/E_{eff}(q_D) \approx -2q_s.$$

As a result it is found that

$$\left.\frac{E_D}{E_s}\right|_{sp} \Big/ \left.\frac{E_D}{E_s}\right|_{dif} = -2q_s, \qquad (4)$$

that is, the amplitude of a doppleron (or of a Gantmakher–Kaner wave) is significantly larger in the diffuse case than in the specular.

We turn to the expression for the impedance Z_∞; this we write in the form

$$Z_\infty = \frac{4\pi q_0}{c} i \frac{E_s(0)+E_D(0)+E_{GK}(0)}{E_s'(0)+E_D'(0)+E_{GK}'(0)}, \qquad (5)$$

where the indices s and D refer to the skin and doppleron components of the field, the indices GK to the Gantmakher–Kaner component.

The derivatives of these components are connected with them by the relations

$$E_s'(0) \sim q_s E_s(0), \quad E_D'(0) \sim E_D(0), \quad E_{GK}'(0) \sim E_{GK}(0).$$

The fields $E_D(0)$ and $E_{GK}(0)$ are considerably smaller than the electric field $E_s(0)$ of the skin component, in both the specular and the diffuse cases. Therefore, as follows from (4) and (5), in specular reflection the impedance is determined, as usual, by the skin layer: $Z_\infty = 4\pi q_0/cq_s$. In the diffuse case, however, the numerator in the expression for the impedance is determined by the skin field $E_s(0)$, the denominator by the sum of the last two terms $E_D'(0) + E_{GK}'(0)$; that is, by the field of the doppleron and the Gantmakher–Kaner component. Therefore the impedance no longer determines the effective depth of penetration of the field, as is customarily supposed. Thus the impedance of a metal with diffuse reflection of the electrons differs in an important way from the impedance with specular reflection.

2. We shall now consider the propagation of an electromagnetic field in a plate of thickness d, on which an external wave is incident from the left. We shall suppose the plate so thick that near the left surface one may neglect the signal reflected from the right surface and traveling back. In other words, we shall take into account the effect of the right-boundary reflection only on the transmitted signal. In order to simplify the formulas, we shall suppose that the thickness of the skin layer is much smaller than the extinction distance of the doppleron, and that only the doppleron and the Gantmakher–Kaner wave reach the right boundary; that is, that the following conditions are satisfied:

$$\operatorname{Im} q_s L \gg \operatorname{Im} q_D L > 1, \quad L = 2\pi d/u. \tag{6}$$

In diffuse reflection, an electron loses information about the field. Therefore for the reflected electrons (and it is they that carry the field back into the depth of the metal), the field produced at the right boundary by the arriving doppleron (or GK wave) may be treated as an external source. Consequently, these electrons produce at the right boundary a field distribution similar to the distribution at the left boundary. Under these conditions the field inside the plate is

$$E(\zeta) = E_0(\zeta) + AE_0(L-\zeta),$$

where $E_0(\zeta)$ is the field in a semi-infinite metal. The constant A is determined from the boundary condition at the right surface,

$$\left(1 - \frac{1}{iq_0}\frac{d}{d\zeta}\right)E(\zeta)\Big|_{\zeta=L} = 0, \tag{7}$$

which states that no external wave is incident on the plate from the right. From this condition follows

$$E_0'(L) - AE_0'(0) = 0.$$

As a result we get

$$E(\zeta) = E_0(\zeta) + E_0'(L)E_0(L-\zeta)/E_0'(0),$$

or, by the definition of the impedance of a semi-infinite metal,

$$E(\zeta) = E_0(\zeta) - i\frac{cZ_\infty}{4\pi q_0}\frac{E_0'(L)}{E_0(0)}E_0(L-\zeta). \tag{8}$$

The field at the right boundary of the metal has the value

$$E(L) = E_0(L)\left(1 - i\frac{cZ_\infty}{4\pi q_0}\frac{E_0'(L)}{E_0(L)}\right). \tag{9}$$

The second term is almost entirely determined by the field of the skin layer formed as a result of the reflection. This term is always much larger than the first. Consequently, in experiments on passage of an electromagnetic field through a metal, what is measured is not the signal of the transmitted doppleron, but the field of the skin layer excited by this doppleron and oscillating in phase with it.

In the specular case, electrons bring up the field of the doppleron, and the same electrons take exactly the same field back into the depth of the metal. The magnetic fields of the arriving and reflected dopplerons compensate each other at the surface $\zeta = L$ to terms of order q_0; that is, no skin layer at all is formed on reflection, but the electric fields of the two dopplerons are so combined that

$$E(L) = 2E_0(L). \tag{10}$$

3. We turn now to consideration of the impedance of a plate under antisymmetric excitation, as occurs when a specimen is placed in the coil of an oscillatory circuit. Here what is usually measured is the energy that flows into the specimen in unit time,

$$W = \left\langle\!\left\langle \mathbf{n}\frac{c}{4\pi}\{[\mathbf{E}_a\mathbf{H}_a]|_{\zeta=0} - [\mathbf{E}_a\mathbf{H}_a]|_{\zeta=L}\}\right\rangle\!\right\rangle = \frac{c}{8\pi}\operatorname{Re}(-i)E_{a-}(0)H_{a-}^*(0),$$

where $\langle\langle\ldots\rangle\rangle$ means a time average.

We shall define the impedance of a plate under antisymmetric excitation by the expression

$$W = \left(\frac{c}{4\pi}\right)^2\frac{1}{4}\operatorname{Re} Z_{a-}H_{a-}(0)H_{a-}^*(0), \quad (|H_{a-}(0)| \approx 2|\mathscr{E}|).$$

Then

$$Z_{a-} = \frac{8\pi q_0}{c}\frac{iE_a(0)}{E_a'(0)} = \frac{8\pi q_0}{c}i\frac{E(0)-E(L)}{E'(0)-E'(L)},$$

where $E(\zeta)$ is the field under one-sided excitation. Since $E'(L)/E'(0)$ is a small quantity of order q_0, the impedance of the plate has the form

$$Z = 2Z_\infty(1 - E(L)/E(0)). \tag{11}$$

In the specular case we get for a thick plate

$$Z^{sp} = 2Z_\infty^{sp}(1 - 2E_0(L)/E_0(0)). \tag{12}$$

Analogously, in the diffuse case

$$Z^d = 2Z_\infty^d\left(1 + i\frac{cZ_\infty^d}{4\pi q_0}\frac{E_0'(L)}{E_0(0)}\right). \tag{13}$$

Therefore in order to calculate the impedance of the plate in the strong-field range, it is sufficient to know the impedance of the semi-infinite metal and the asymptotic expression for the ratio $E_0(L)/E_0(0)$. We note that in this case the observed oscillations of absorption are due to the electric field of the skin layer excited by the doppleron, which is much larger than the electric field of the doppleron itself.

4. We shall now apply these relations to the model of a compensated metal described at the beginning of the article. The appropriate dispersion equation for "minus" polarization, in dimensionless variables, has the form (see[5])

$$q^2 = \xi\left(\frac{1-i\gamma}{(1-i\gamma)^2 - q^2} - 1\right),$$

where

$$\xi=\omega\omega_p^2u^2/4\pi^2\omega_c c^2,\quad \omega_p^2=4\pi ne^2/m,\quad \gamma=\nu/\omega_c.$$

In a strong field, i.e., when $\xi \ll 1$, it has two roots: the skin solution $q_s=(i\gamma\xi)^{1/2}$ and the doppleron solution $q_D=-1+\xi/2+i\gamma$. In this model the nonlocal conductivity as a function of the complex variable q has no branch point; therefore the Gantmakher–Kaner component is absent. The inequality mentioned above, $\mathrm{Im}\,q_sL \gg \mathrm{Im}\,q_DL$, is equivalent to the condition

$$\gamma\ll\xi. \tag{14}$$

We consider first a semi-infinite metal. The field distribution in the specular case is determined by the expression[11]

$$E_0^{sp}(\zeta)=E'(0)\frac{(-1)}{\pi}\int_{-\infty}^{\infty}dq\,e^{iq\zeta}\left[q^2-\xi\left(\frac{1-i\gamma}{(1-i\gamma)^2-q^2}-1\right)\right]^{-1}.$$

On evaluating the integral and substituting $E'(0)$, we find

$$E_0^{sp}(\zeta)=\frac{2q_0\mathscr{E}}{q_D^2-q_s^2}\left\{\frac{q_D^2-1}{q_D}e^{iq_D\zeta}-\frac{q_s^2-1}{q_s}e^{iq_s\zeta}\right\}, \tag{15}$$

and by use of the inequalities $|1+q_D|\ll 1$ and $|q_s| \ll |q_D|$ we get

$$E_0^{sp}(\zeta)=2q_0\mathscr{E}\left[\frac{1}{q_s}\exp(iq_s\zeta)+\xi\exp(iq_D\zeta)\right].$$

The impedance of the semi-infinite metal in this case is

$$Z_\infty^{sp}=\frac{4\pi q_0}{c}\frac{1+q_Dq_s}{q_Dq_s(q_D+q_s)}\approx\frac{4\pi q_0}{cq_s}=\frac{4\pi\omega_c}{c\omega_p}\left(-\frac{i\omega}{\nu}\right)^{1/2}\sim H\left(\frac{\omega}{\nu}\right)^{1/2}. \tag{16}$$

The field distribution in the diffuse case is determined by the equation

$$\frac{d^2E_0^d}{d\zeta^2}=-\xi\int_0^\infty d\eta\,E_0^d(\eta)\frac{1}{2\pi}\int_{-\infty}^{\infty}dq\,e^{iq(\zeta-\eta)}\left(\frac{1-i\gamma}{(1-i\gamma)^2-q^2}-1\right)$$

with the boundary condition (1). A similar equation was solved earlier.[12,13] Its solution has the form

$$E_0^d(\zeta)=2q_0\mathscr{E}\frac{(1+q_D)\exp(iq_D\zeta)-(1+q_s)\exp(iq_s\zeta)}{q_D(1+q_D)-q_s(1+q_s)}. \tag{17}$$

By use of the inequality $\gamma\ll\xi\ll 1$, we get the field distribution

$$E_0^d(\zeta)=2q_0\mathscr{E}\left(\frac{2}{\xi}\exp(iq_s\zeta)-\exp(iq_D\zeta)\right)$$

and the impedance

$$Z_\infty^d=\frac{4\pi q_0}{c}\frac{1}{1+q_D+q_s}\approx\frac{8\pi q_0}{c\xi}=\frac{16\pi^2\omega_c}{\omega_p^2u}\sim H^2. \tag{18}$$

Thus the impedance of a semi-infinite metal in the diffuse case is in fact significantly different from the impedance in the specular case.

5. We shall now consider the field in a thick plate. In accordance with the considerations presented above, the field distribution under one-sided excitation in the specular case has the form

$$E^{sp}(\zeta)=2q_0\mathscr{E}\left\{\frac{1}{q_s}e^{iq_s\zeta}+\xi\left(e^{iq_D\zeta}+e^{iq_D(2L-\zeta)}\right)\right\}, \tag{19}$$

and in the diffuse case

$$E^d(\zeta)=2q_0\mathscr{E}\left\{\frac{2}{\xi}e^{iq_s\zeta}-e^{iq_D\zeta}-e^{iq_DL}\left[e^{iq_D(L-\zeta)}-\frac{2}{\xi}e^{iq_s(L-\zeta)}\right]\right\}. \tag{20}$$

The impedance of the plate under antisymmetric excitation is determined by the expressions

$$Z^{sp}=\frac{8\pi q_0}{c}\frac{1}{q_s}(1-2q_s\xi e^{iq_DL}), \tag{21}$$

$$Z^d=\frac{8\pi q_0}{c}\frac{2}{\xi}(1-e^{iq_DL}), \tag{22}$$

whence it is evident that the amplitude of the doppleron oscillations of impedance with diffuse reflection is different by the factor

$$\Delta Z^d/\Delta Z^{sp}\sim 1/\xi^2\gg 1.$$

Our model enables us to obtain exact expressions for the field in a plate with an arbitrary degree of specularity p. For this purpose it is necessary to solve the kinetic equation for the distribution function of the electrons, with allowance for the nature of their reflection from the surfaces of the plate, and by means of this solution to calculate the current density that enters Maxwell's equations. As a result, the following integro-differential equation is obtained:

$$\left(\frac{d^2}{d\zeta^2}-\xi\right)E(\zeta)=-i\xi\int_0^L d\eta\,E(\eta)\left\{\frac{1}{2}\exp[-(i+\gamma)|\zeta-\eta|]-\frac{p\exp[-(i+\gamma)L]}{1-p^2\exp[-2(i+\gamma)L]}\left[\cos(1-i\gamma)(\zeta+\eta-L)+p\exp[-(i+\gamma)L]\cos(1-i\gamma)(\zeta-\eta)\right]\right\}. \tag{23}$$

The Fourier transform of the kernel of this equation is a rational function. Therefore it reduces to a differential equation with constant coefficients. The corresponding characteristic equation is identical with the dispersion equation and consequently has the two roots q_s and q_D. As a result, Eq. (23) is easily solved. Under one-sided excitation of the plate, the solution of (23) with the boundary conditions (1) and (7) has a very cumbersome form; in this case we shall give only the approximate expressions for a thick plate in the limits of purely specular ($p=1$) and purely diffuse ($p=0$) reflection. For $p=1$, the field distribution has the form

$$E^{sp}(\zeta)=\frac{2q_0\mathscr{E}}{q_D^2-q_s^2}\left\{\frac{q_D^2-1}{q_D}\left[\exp(iq_D\zeta)+\exp[iq_D(2L-\zeta)]\right]-\frac{q_s^2-1}{q_s}\exp[iq_s\zeta]\right\}, \tag{24}$$

and the impedance of the plate under antisymmetric excitation is

$$Z^{sp}=\frac{8\pi q_0}{c}\frac{1+q_sq_D}{q_sq_D(q_s+q_D)}\left[1-\frac{2q_s(1-q_D^2)}{(1+q_sq_D)(q_s-q_D)q_D}\exp(iq_DL)\right]. \tag{25}$$

In the case of diffuse reflection, the field and the impedance are given by the expressions

$$E^d(\zeta)=2q_0\mathscr{E}\left\{\frac{(1+q_D)\exp(iq_D\zeta)-(1+q_s)\exp(iq_s\zeta)}{(1+q_D+q_s)(q_D-q_s)}\right.$$
$$+\frac{(1+q_s-q_D)(1+q_D)(q_D+q_s)}{(1-q_D)(1+q_D+q_s)^2(q_D-q_s)^2}\exp(iq_DL)\Big[(1+q_D)\exp[iq_D(L-\zeta)]$$
$$\left.-\frac{2q_D(1+q_s)}{(q_D+q_s)(1-q_D+q_s)}\exp[iq_s(L-\zeta)]\Big]\right\}, \quad (26)$$

$$Z^d=\frac{8\pi q_0}{c(1+q_D+q_s)}\left[1+\frac{2q_D(1+q_D)(q_D+q_s)}{(1-q_D)(1+q_D+q_s)(q_D-q_s)}\exp(iq_DL)\right]. \quad (27)$$

In the range of moderately strong fields, where the inequalities

$$\gamma\ll\xi\ll1, \quad (28)$$

are satisfied, (24) and (25) reduce to (19) and (21), (26) and (27) to (20) and (22); that is, the results of the qualitative reasoning presented earlier are confirmed by the rigorous calculation.

6. We shall give also an exact expression for the impedance of a plate of arbitrary thickness in the case of antisymmetric excitation. For an arbitrary value of p, the impedance Z is determined by the expressions

$$Z=\frac{8\pi q_0}{c}\{q_s[(1-i\gamma)^2-q_D^2]T_D^-T_s^+-q_D[(1-i\gamma)^2-q_s^2]T_D^+T_s^-$$
$$-\lambda(1-i\gamma)(q_s^2-q_D^2)T_D^-T_s^-\}\{q_Dq_s(q_s^2-q_D^2)T_D^+T_s^+$$
$$-\lambda(1-i\gamma)\{q_D[(1-i\gamma)^2-q_D^2]T_D^+T_s^--q_s[(1-i\gamma)^2-q_s^2]T_D^-T_s^+\}\}^{-1}, \quad (29)$$

$$T_{D,s}^{\pm}=1\pm\exp(iq_{D,s}L), \quad \lambda=(1-p)/(1+p). \quad (30)$$

In the case of a thick plate, when $T_s^{\pm}\to1$, while $T_D^{\pm}$ differs from unity by a small oscillatory quantity, the expression (29) for the impedance in the field range (28) takes the form

$$Z\approx\frac{8\pi q_0}{c}\left[\frac{1+\lambda}{\lambda\xi+q_s}-2\xi\left(\frac{\lambda-q_s}{\lambda\xi+q_s}\right)^2\exp(iq_DL)\right]. \quad (31)$$

From (31) it follows that for values of $\lambda>|q_s|/\xi$, the dependence of the smooth and oscillatory parts of the impedance on H is the same as with purely diffuse reflection. In the range $\lambda\leq|q_s|$, the amplitude of the oscillations of the impedance decreases to values characteristic of purely specular reflection. The range $|q_s|<\lambda<|q_s|/\xi$ is transitional.

Thus for a simple model with a paraboloidal Fermi surface, we have demonstrated the correctness of the conclusion reached by qualitative reasoning, that there is a significant increase of the amplitude of doppleron oscillations of the impedance of a plate with diffuse reflection of the electrons, as compared with the amplitude with specular reflection. Here it should be mentioned that the dependence of the amplitudes themselves on the magnetic field is sensitive to the model; that is, to the form of the singularity of the nonlocal conductivity in the neighborhood of the Doppler-shifted cyclotron resonance.

In the model treated, the Gantmakher–Kaner effect is absent. It is obvious, however, that the mechanism described above for amplification of the oscillations when there is diffuse reflection may occur also in relation to the Gantmakher–Kanrr oscillations.

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