Formation of an electron-temperature superlattice in the presence of a constant electric field

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(Submitted April 30, 1976)
Zh. Eksp. Teor. Fiz. **71**, 1583–1591 (October 1976).

The influence of band bending in a sample on the conditions for the formation of an electron-temperature superlattice in a semiconductor with hot electrons is studied (the Bénard problem for hot electrons in an electric field). It is assumed that the heating of the electron gas is due to intraband absorption of light. It is shown that the band bending produced by, e.g., an external voltage (as in the field effect) can lower the threshold for the appearance of the superlattice by two to three orders of magnitude. In this case the effect can be observed not only at liquid-helium but also at liquid-nitrogen temperatures. In addition, the conditions on the dependence of the energy-relaxation time and electronic thermal conductivity on the electron temperature that were imposed in the absence of a voltage become unnecessary.

PACS numbers: 64.60.Eq

1. INTRODUCTION. BASIC EQUATIONS

In a previous paper[1] the problem of the formation of an electron-temperature superlattice during spatially nonuniform heating of electrons as a result of intraband absorption of light was formulated and solved in the linear approximation. It was assumed that there was no external electric field and that, in the absence of illumination, the bands were not bent. The mechanism of the onset of instability of the one-dimensional distribution of electron temperature T was connected with the specific temperature dependence of the kinetic coefficients—the energy-relaxation time τ and electronic thermal conductivity $\varkappa$. In the presence of a voltage across the sample, however, there appears another instability mechanism, analogous to that considered in the hydrodynamic problem of Bénard.[2,3] In our problem it consists in an increase of the pressure of the electron gas near the illuminated ("lower") face of the sample under the action of the force $e\mathbf{E}$ due to the electric field (see Fig. 1).[1)] (In hydrodynamics a gravitational field appears in place of the electric field.)

In the present paper we consider the problem of the formation of an electron-temperature superlattice (the Bénard problem for hot electrons) in the presence of a given potential difference V between the illuminated and dark sides of the sample (see the figure); it is assumed that there is no through current along the z axis.

As before[1] we shall consider a material with monopolar conduction, assuming that the characteristic electron-electron collision time is appreciably longer than the momentum-relaxation time and, at the same time, much shorter than the energy-relaxation time.

The problem under consideration, while similar in essence to the hydrodynamic problem, nevertheless differs appreciably from the latter in two respects. First, the electric field is screened; in the conditions under consideration the voltage associated with it is, in practice, nonzero only in the layers of space charge near the boundaries of the sample. Secondly, as is well-known, with the above-indicated relative values of the characteristic times in the problem, instead of a system of hydrodynamic equations we must use the Poisson, continuity and energy-transport equations, supplementing them by expressions for the fluxes of energy and charge. We note that, unlike in[1], it is now no longer possible to neglect the diffusion current.

As in the absence of an electric field, in the problem under consideration there are four characteristic lengths: the screening length r_0, the "cooling length" $\lambda_0^{-1}=(2\varkappa_0\tau_0/3)^{1/2}$ (the subscript 0 here and in the following refers to the corresponding quantity in the absence of electron heating), the reciprocal γ_0^{-1} of the light-absorption coefficient,[2)] and the sample thickness l. As before,[1] we shall consider the frequently realized conditions

$$r_0\ll\lambda_0^{-1}\ll\gamma^{-1}\ll l. \tag{1}$$

In this case the absorption of light (and heating of the electrons) occurs principally in the volume of the sample (at distances greater than λ_0^{-1} from the surface), and we can let l tend to infinity.

In the following the quantities $r_0\lambda_0$ and $\gamma\lambda_0^{-1}$ will play the role of the basic small parameters. In particular, the smallness of the first of these makes it possible, firstly, to use the quasi-neutrality approximation outside the space-charge region, and, secondly, to avoid a detailed analysis of the processes in this region (just as is done in quantum mechanics when solving the Schrödinger equation with a strongly localized potential[4]).

We introduce the notation indicated in Table I. Then the basic equations of the problem will have the form

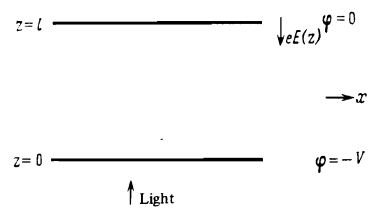

FIG. 1. Section of the sample in the (x, z) plane. The thickness of the sample along the z axis is denoted by l and the potential $\varphi(z)$ of the electric field is normalized by the condition $\varphi(l)=0$. The dimensions for the sample in the x and y directions are assumed to be arbitrarily large.

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TABLE I.

Quantity	Symbol	Unit of measurement	
		In the absence of degeneracy	For strong degeneracy
Coordinate	x, y, z	λ_0^{-1}	λ_0^{-1}
Time	t	τ_0	$\frac{2}{3}\tau_0$
Electron concentration	n	n_0	n_0
Current density	j		
Drift velocity	u	$u_0=\lambda_0^{-1}\tau_0^{-1}$	$u_0=\frac{3}{2}\lambda_0^{-1}\tau^{-1}$
Thermal conductivity	$\varkappa$	$\varkappa_0$	$\varkappa_0$
Electron energy-flux density	q		
Differential thermopower	α	$3/2e$	$1/e$
Light-energy flux	I	$\frac{3}{2}nT_0u_0$	$n_0T_0u_0$
Mean energy of the electrons (per electron)	W	T_0	T_0
Fermi level	F	T_0	T_0
Potential of electric field	φ	$3T_0/2e$	T_0/e
Field intensity	$\mathbf{E}=-\nabla\varphi$	$3T_0\lambda_0/2e$	$T_0\lambda_0/e$
Mobility	μ	$\frac{2e}{3T_0\tau_0\lambda_0^2}$	$\frac{3e}{2T_0\tau_0\lambda_0^2}$

Note: The quantities n_0 and F_0 refer (in the absence of illumination) to the region outside the layer of space charge.

$$\frac{\partial n}{\partial t}+\operatorname{div} n\mathbf{u}=0, \tag{2}$$

$$\frac{\partial (nW)}{\partial t}+\operatorname{div}\mathbf{q}-en(\mathbf{uE})+\frac{3}{2}n\tau^{-1}(T-T_0)=\gamma I, \tag{3}$$

$$-\nabla^2\varphi=4\pi\varepsilon^{-1}(n-n_0). \tag{4}$$

The factor in the last term in the left-hand side of (3) is connected with the definition we have taken for τ. We note that in Eq. (2) (as in the boundary condition (8) below) the recombination of electrons is not taken into account. This is justified in conditions when the parameters $r_0\lambda_0$ and $s\lambda_0\tau_0$ are small, where s is the rate of surface recombination.

Furthermore, let $\mu \propto T^r$. Then, for the velocity u and energy flux q we obtain, using the Einstein relation between the mobility and diffusion coefficient (throughout, only the formulas for a nondegenerate gas are written out explicitly),

$$\mathbf{u}=-\mu(\nabla\varphi+\alpha\nabla T-(T/e)\nabla\ln n), \tag{5}$$

$$\mathbf{q}=-n\varkappa\nabla T+(5/2+r)n\mathbf{u}T. \tag{6}$$

The solutions of Eqs. (2)–(4) should be bounded as $x\to\pm\infty$, $y\to\pm\infty$ and $z\to\infty$. At $z=0$ the boundary conditions have the form (cf. formulas (6) and (7) of[1])

$$u_z=0, \tag{7}$$

$$\varkappa\partial T/\partial z=\nu(T-T_0). \tag{8}$$

Here ν is the phenomenological parameter introduced earlier,[1] which describes the cooling of electrons across a contact.[3)]

One further boundary condition is shown in the figure:

$$\varphi(0)=-V. \tag{9}$$

Here, in the conditions indicated in the figure, $V>0$.

We shall introduce dimensionless quantities, thereby expressing u, E, etc. in the units indicated in the table. We also put $T=T_0(1+\xi)$ and denote the dimensionless diffusion coefficient by $D=\frac{2}{3}$.

Keeping the previous notation for the dimensionless variables, we obtain in place of (3)–(5)

$$\frac{\partial[n(1+\xi)]}{\partial t}-\operatorname{div}(n\varkappa\nabla\xi)+\frac{5+2r}{3}\operatorname{div}[n\mathbf{u}(1+\xi)]-n(\mathbf{uE})+n\xi\tau^{-1}(\xi)=\gamma I(z). \tag{3'}$$

$$-\eta\nabla^2\varphi=n-1, \tag{4'}$$

$$\mathbf{u}=-\mu[\nabla^2\varphi+\alpha\nabla\xi+D(1+\xi)\nabla\ln n], \tag{5'}$$

$$\eta=3\varepsilon T_0\lambda_0^2/8\pi n_0e^2=3r_0^2\lambda_0^3/2. \tag{10}$$

According to (1), $\eta\ll 1$.

Equation (2) and the boundary condition (9) keep their previous form and the condition (8) is rewritten in the form

$$\varkappa\partial\xi/\partial z=\nu'\xi, \quad \nu'=\nu\varkappa_0^{-1}\lambda_0^{-1}. \tag{8'}$$

We note that in Eq. (3') $\tau(\xi)\to 1$ as $\xi\to 0$.

In the conditions under consideration the absorption coefficient γ depends on the coordinates, even if, in accordance with what was said earlier, we assume it to be independent of T. In fact, $\gamma=n(z)\sigma$, where σ is the cross-section for capture of a photon by a conduction electron. Consequently, in the volume-charge region,

$$I(z)=I_m\exp\left[-\int_0^z\sigma n(z')\,dz'\right].$$

In this region, however, the argument of the exponential function is itself small by virtue of (1), and $I(z)\approx I_m$. On the other hand, outside the space-charge region, where (in the system of units assumed above) $\gamma=\sigma$, the usual relation

$$I(z)=I_m\exp(-\gamma z). \tag{11}$$

is valid.

The situation could change only under very large band-bending, when the concentration of electrons near the surface is raised to such an extent that γ^{-1} becomes the smallest length and a transition to the case of surface absorption occurs.[1] The corresponding critical voltage is easily estimated using the expressions obtained below for the electron concentration. It is clear that it turns out to be fairly large. This possibility is not considered again in the present paper.

The subsequent course of the calculation is the same as before[1]: we find the one-dimensional static solution corresponding to the absence of free convection in the electron gas and then investigate its stability against

small three-dimensional variations of the electron temperature.

2. THE STATIC ONE-DIMENSIONAL SOLUTION

We put $\mathbf{u}=0$, $\varphi=\varphi_s(z)$, $n=n_s(z)$, $\xi=\xi_s(z)$ and confine ourselves, as before,[1] to the case of small superheating, putting $\xi_s \ll 1$ and confining ourselves to terms of first order in ξ_s.

According to (3') and (5'),

$$\varphi_s'+\alpha\xi_s'+D\frac{d\ln n_s}{dz}=0 \tag{12}$$

and

$$-\xi_s''-\frac{d\ln n_s}{dz}\xi_s'+\xi_s=\gamma I_m(z). \tag{13}$$

Taking into account the condition $\xi_s \to 0$ as $z \to \infty$, we obtain from (12)

$$n_s=\exp\left(-\frac{\varphi_s+\alpha\xi_s}{D}\right)\approx\exp\left(-\frac{\varphi_s}{D}\right) \tag{14}$$

(allowance for the term with ξ_s here only gives corrections that are unimportant for the following). Substituting the expression (14) into the Poisson equation we obtain the standard self-consistent problem, the solution of which is well-known. In particular,

$$\varphi_s'=\left\{2D\eta^{-1}\left[\exp\left(-\frac{\varphi_s}{D}\right)+\frac{\varphi_s}{D}-1\right)\right]\right\}^{1/2}. \tag{15}$$

Using formulas (14) and (15), it is not difficult to solve Eq. (13) explicitly for the cases of weak and strong fields, corresponding, respectively, to the conditions $eV \ll T_0$ and $eV \gg T_0$ (in the usual units). There is, however, no necessity for this, since it is clear beforehand that, by virtue of (1), no appreciable heating can occur in the volume-charge region. For this reason it is sufficient to confine ourselves to examining (13) outside the volume-charge region (replacing the quantity ν' in formula (8') by some effective value of it where necessary; it will be seen from the following that the exact value of ν' is not important for us, so long as it is not anomalously large).

In other words, for the function $\xi_s(z)$ we can use the solution found earlier.[1] In particular, in the most interesting region ($z \gg 1$) $\xi_s \approx I_m \exp(-\gamma z)$.

3. DEVIATION FROM THE STATIC SOLUTION

We put, as in[1],

$$\xi=\xi_s+\delta\xi,\quad \varphi=\varphi_s+\delta\varphi,\quad n=n_s+\delta n, \tag{16}$$

in which $\delta\xi$, $\delta\varphi$, δn and $\mathbf{u}$ are proportional to $\exp(i\mathbf{k}\cdot\mathbf{r}+st)$ and the coefficients of proportionality are, respectively, $f_1(z)$, $f_2(z)$, $f_3(z)$ and $\mathbf{f}(z)$. Here $\mathbf{r}=\{x,y\}$, $\mathbf{k}=\{k_x,k_y\}$, $\mathbf{f}=\{f_\perp,f_z\}$; the components of the two-dimensional vector $\mathbf{k}$ should be real.

It follows from Poisson's equation (4) that the function f_3 is small in the parameter η compared with f_1 and f_2. Next, linearizing the formulas for the velocity in $\delta\xi$, $\delta\varphi$ and δn and discarding small quantities, we obtain

$$f_z=-\mu_s(\Phi'-f_1\varphi_s'),\quad \mathbf{f}_\perp=-i\mathbf{k}\mu_s\Phi, \tag{17}$$

$$\Phi=f_2+\alpha_s f_1+Df_3 n_s^{-1}. \tag{18}$$

In this case the continuity equation (2) takes the form

$$\Phi''-Q_1\Phi'-k^2\Phi=Q_2(z), \tag{19}$$

$$Q_1=-\frac{n_s'}{n_s},\quad Q_2=f_1'\varphi_s'+f_1(\varphi_s''-D^{-1}\varphi_s'^2). \tag{20}$$

According to (7) and (17), at $z=0$ we have

$$\Phi=f_1\varphi_s'. \tag{21}$$

Finally, we turn to the energy-transport equation. Here it is sufficient to confine ourselves to the part of the sample lying outside the space-charge region. As before,[1] we introduce the notation

$$\dot\tau=\left(\frac{d\ln\tau}{d\ln T}\right)_0,\quad \dot\varkappa=\left(\frac{d\ln\varkappa}{d\ln T}\right)_0,\quad \dot\nu'=\left(\frac{d\ln\nu'}{d\ln T}\right)_0,$$

we then obtain

$$f_1''-[k^2+s+1-(2\dot\tau+\dot\varkappa+s\dot\varkappa)\xi_s']f_1=\tfrac{2}{3}F\xi_s'f_z. \tag{22}$$

After linearization the boundary condition (8') takes the form (to within terms of order η)

$$f_1'(0)=\nu_e'f_1(0), \tag{23}$$

$$\nu_e'=\{\nu'[1+\dot\nu'\xi_s(0)]-\dot\varkappa\xi_s'(0)\}[1+\dot\varkappa\xi_s(0)]^{-1}. \tag{24}$$

Equation (22) differs in its right-hand side from the corresponding equation of the previous paper.[1] Such terms also appeared in[1], but were discarded on account of the smallness of the parameter γ. However, in the presence of a voltage across the sample these terms may turn out not to be small. Moreover, in conditions when the parameter η is small, they alone describe the effect of an external electric field on the function f_1 and, thereby, on the condition for the appearance of a temperature superlattice.

4. CONDITION FOR FORMATION OF A TEMPERATURE SUPERLATTICE

As can be seen from (20), the terms with Q_1 and Q_2 in (19) can differ appreciably from zero only in the space-charge region. Outside this region, Eq. (19) coincides, naturally, with the corresponding equation of the paper.[1] The essential point, however, is that now, unlike in[1], the function $\Phi(z)$ no longer vanishes as $z \to 0$ but satisfies the condition (21).

In a weak field Eq. (19) can be solved exactly without difficulty in the entire range of variation of z. It is simpler, however, to use the same device as in the solution of the Schrödinger equation with a "strong point-interaction".[4] Namely, by virtue of the small size of the screening length ($\eta \ll 1$), we shall regard the whole space-charge region as the plane $z=0$ and subject the solution of Eq. (19) for $Q_1=Q_2=0$ to the boundary condition (21) directly. In this way we obtain

$$\Phi'(z)=f_1(0)\varphi_s'(0)\exp(-kz). \tag{25}$$

Returning to the formulas (17) we see that, in con-

trast to a system without an external field, the velocity u is now already nonzero in the approximation in which the above equalities are valid. As can be seen from the boundary condition (21) and formula (5), this is connected with the change in the diffusion coefficient on heating of the electron gas, i.e., in the final analysis, with the increase in the pressure of the electron gas in the space-charge region. (The other factor arising from the immediate change in the electron concentration in this region does not play a role in the conditions under consideration, by virtue of the small size of the parameter η.) In other words, an imbalance of the drift, thermoelectric and diffusion currents arising on heating, and leads to a finite value of the drift velocity.

It is sufficient to solve Eq. (22) just in the region $z \gg 1$, in which the heating principally occurs (cf.[1]).[4] In this case it is convenient to put

$$f_1(z)=f_1(0)\psi(z), \tag{26}$$

where ψ is a new unknown function. In fact, in conditions of heating, according to (23) $f_1(0)\neq 0$. Using now the equalities (15), (17) and (25), we obtain

$$\psi''-\left[\frac{\gamma^2 p^2}{4}-q^2\exp(-\gamma z)\right]\psi=b\exp[-(k+\gamma)z], \tag{27}$$

$$b=a\gamma^2 I_m,\quad p=2\gamma^{-1}(1+s+k^2)^{1/2},\quad q^2=(2\dot{\tau}+\dot{\varkappa}+s\dot{\varkappa})\gamma I_m, \tag{28a}$$

$$a=\frac{2^{1/2}|F_0|}{(5+2r)T_0 r_0\lambda_0}\left[\exp\left(\frac{eV}{T_0}\right)-\frac{eV}{T_0}-1\right]^{1/2}. \tag{29a}$$

Equation (27) is also valid in conditions of strong degeneracy of the electron gas, if we put

$$p=2\gamma^{-1}(1+cs+k^2)^{1/2},\quad q^2=(2\dot{\tau}+\dot{\varkappa}+cs\dot{\varkappa})\gamma I_m, \tag{28b}$$

$$c=\pi^2/2F_0,\quad a=\frac{\pi^{1/2}}{9}(r+1)\frac{T_0^2}{F_0^2}r_0^{-1}\lambda_0^{-1}\left[\frac{6}{5}\frac{(1+eV/F_0)^{5/2}-5eV/2F_0-1}{1+eV/F_0}\right]^{1/2} \tag{29b}$$

In formulas (29a) and (29b), by F_0, V and r_0 we now mean quantities with the usual dimensions, and in (29b) the screening length for the degenerate gas appears. We note that in the absence of degeneracy $F_0<0$.

According to (27) and (24), for $z=0$ we have

$$\psi(0)=1,\quad \psi'(0)=\nu_e'. \tag{30}$$

Equation (27) is easily solved in Bessel functions (cf.[1]). In order to trace how one passes to the limit of the regime without an external field, we first consider the case of very small voltages, putting

$$b/\gamma\equiv a\gamma I_{cr}\ll 1. \tag{31}$$

In this case the right-hand side of (31) can be regarded as a perturbation. We put $\psi=\psi_0+\psi_1+\ldots$ and $p=p_0+p_1+\ldots$, where ψ_0 and p_0 correspond to $b=0$ (i.e., to the case $V=0$).[1]

According to[1] and (30),

$$\psi_0=I_{p_0}^{-1}(2q\gamma^{-1})I_{p_0}\left[2q\gamma^{-1}\exp\left(-\frac{\gamma z}{2}\right)\right],\quad p_0=\frac{2q}{\gamma}(1-\rho)^{-1}. \tag{32}$$

Here $\rho=0(1/p_0)$ and I_{p_0} is a Bessel function. In the standard way we easily find

$$p_1^2=32qb/\gamma^2(3\gamma+4k). \tag{33}$$

Taking (28a) or (28b) into account we now obtain

$$s+k^2=[I_m/I_{cr}-1]^{1/2}. \tag{34}$$

Here the critical value of the light-energy flux at the boundary of the sample is (in the usual units)

$$I_{kp}=\frac{3nT_0}{2\gamma\tau_0(2\dot{\tau}+\dot{\varkappa}+{}^8/_3 a)}. \tag{35}$$

For $a=0$ formula (35) is transformed into the expression obtained earlier.[1] It can be seen that a voltage of the right sign (see the figure) does indeed lower the threshold value I_{cr}. Moreover, the condition $2\dot{\tau}+\dot{\varkappa}>0$, although desirable, now becomes unnecessary: it is necessary only that the denominator in the expression (35) be positive. We emphasize, however, that formula (35) is valid only so long as $a\lesssim 1$. We also note that, as can be seen from formulas (29a), (29b), this case is evidently conveniently realized in conditions of strong degeneracy, while in a nondegenerate gas the quantity a already becomes much greater than unity when $eV\approx T_0$. For this reason, in the absence of degeneracy it makes sense to consider also the other limiting case, putting

$$a\gg\gamma^2,\quad \gamma I_m\ll 1. \tag{36}$$

Here it is convenient to represent the solution of Eq. (27) in the form of an expansion in powers of $q\exp(-\gamma z/2)$. Taking (30) into account we obtain, to within terms of higher order of smallness,

$$\psi=\frac{b\exp[-(k+\gamma)z]}{1+s-2k\gamma}+\left(1-\frac{b}{1+s-2k\gamma}\right)\exp\left(-\frac{\gamma p}{2}z\right)$$

and

$$\frac{b}{1+s-2k\gamma}[(1+s+k^2)^{1/2}-(k+\gamma)]=(1+s+k^2)^{1/2}+\nu_e'. \tag{37}$$

Putting $k=0$ in formula (37), we obtain

$$s=-1+\left[\frac{\nu_e'}{2}+\left(\frac{\nu_e'^2}{4}+b\right)^{1/2}\right]^2. \tag{38}$$

On the other hand, for $s=0$ and $k\neq 0$, but $k\ll 1$ (near threshold), we find

$$k=[b-(1+\nu_e')]/b. \tag{39}$$

We see that, again, one and the same critical value of the light-energy flux at the surface of the sample corresponds to both the onset of instability of the one-dimensional distribution of electron temperature and the appearance of the temperature superlattice. This value is determined by the condition $b=1+\nu_e'$, i.e., (in ordinary units),

$$I_{cr}=\frac{1+\nu_e'}{a\gamma^2}\frac{3nT_0\lambda_0}{2\tau_0}. \tag{40}$$

According to (36) and (24), in the given case $\nu_e'\approx\nu'$. It can be seen that the exact value of ν' plays no role, so long as it is small compared with unity. Such can be

the case if, e.g., the sample borders on a vacuum at $z = 0$.

We note that the right-hand side of (40) does not contain $\dot{\tau}$ or $\dot{\varkappa}$ and the sign of these quantities plays no role here (by direct calculation it is easy to convice oneself that the same formula (40) is also obtained—in the conditions (36)—for $q^2 < 0$). The meaning of this result is clear: in so strong a field the second of the mechanisms indicated in Sec. 1 for the instability of the static one-dimensional distribution dominates.

Comparing formulas (35) (for $a = 0$) and (40), we see that in a sufficiently strong electric field the critical value of the light-energy flux at the boundary of the sample differs from the corresponding value in the absence of a field by the factor

$$(\lambda_0/a\gamma)(1+\nu_e')(2\dot{\tau}+\dot{\varkappa}). \tag{41}$$

(For $2\dot{\tau}+\dot{\varkappa} < 0$ this comparison is completely meaningless, since then the instability does not arise in the absence of a field.) According to (36) this factor is much smaller than unity.

In the absence of degeneracy the value of a can turn out to be fairly large. Correspondingly, for the formation of a temperature superlattice the fairly stringent conditions that were obtained earlier[1] become unnecessary and we may now think not only of InSb but also of other materials, including germanium and, perhaps, silicon. Thus, for $n_0 = 10^{16}$ cm^{-3}, $r = \frac{3}{2}$ (momentum scattering by a charged impurity), $T_0 = 75$ K and $V = 0.33$ V,[5)] we obtain $a \approx 6 \times 10^{11}$. For the estimate we put $\mu_0 = 10^4$ cm^2/V·sec, $\tau_0 = 10^{-10}$ sec, $\gamma = 10$ cm^{-1} (the latter value is obtained by extrapolation of the data in Fig. 3.41 of the book by Pankove[5] to the case when the wavelength of the heating light is 1.4×10^{-2} cm). It is easy to convince oneself that all the inequalities used above are satisfied here, and for the critical value of the light-energy flux we obtain $I_{cr} = 1.4 \times 10^{-5}(1+\nu)$W/cm^2. The value of ν' can, apparently, be made sufficiently small (cf. formulas (8) and (7) from[1]). Thus, by applying a voltage across the sample we can lower I_{cr} by approximately three orders of magnitude as compared with the result of[1].

[1)] For convenience we consider the motion of particles with positive charge e.

[2)] In the following we shall assume that γ does not depend on T (for which there are well-known reasons[1]). Accordingly, the subscript 0 of γ can be dropped.

[3)] As a result of an error by the author in the proof-reading the quantity ν appeared with the wrong sign in formulas (7), (7′), (7″) and (36) of[1]. The results of the paper, however, were obtained using the correct form of these formulas.

[4)] We note that, in this case, the problems (unimportant in the conditions under consideration) of precisely where we must impose the boundary condition (8) and of whether we can confine ourselves to the local form of this condition disappear. In fact, the electrons participating in the transport of energy across the boundary of the sample (at $z = 0$) are principally those which are moving in the direction of the boundary and are already at a distance from it of the order of the momentum mean free path.

[5)] This is, apparently, the maximum voltage (eV less than the half-width of the forbidden band) that can be applied to a germanium plate without producing unacceptably large bending of the bands. The author is grateful to V. B. Sandomirskiĭ for a discussion of this point.

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Translated by P. J. Shepherd