Forces acting on vortices moving in a pure type II superconductor

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The motion of vortices in a very pure type II superconductor is analyzed in the case $\kappa \gg 1$ on the basis of the microscopic theory of superconductivity. It is shown that, similar to He II, the motion of the vortices is determined by the balance between the Magnus force of mutual friction between the superfluid and normal components. This latter force can be expressed in terms of the transport and transverse scattering cross sections for normal excitations on the vortex.

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1. INTRODUCTION

The motion of Abrikosov vortex filaments[1] in type II superconductors is very intensively studied at the present time. The dissipation processes which take place in the motion of vortices, which lead to the existence of a finite resistance of the superconductor in the mixed state, have been studied in special detail. The microscopic theory (see the review of Ref. 2) gives in principle the possibility of calculating the diagonal components of the conductivity tensor in the superconducting alloys at any values of the magnetic field and temperature. The situation with regard to the Hall effect is otherwise—no clear understanding has been achieved here to date. According to the simple phenomenological model, the vortex has a core consisting of normal electrons, around which superconducting currents circulate. On the basis of this model, Bardeen and Stephen[3] have found that the Hall angle is determined by the relation

$$\text{tg}\,\theta_H = \omega_c \tau,$$

where $\omega_c = eh_{\text{eff}}/mc$, and h_{eff} is some effective magnetic field at the core of the vortex, which, according to Ref. 4, has the order of H_{c2}. Attempts at a microscopic consideration of the Hall effect were undertaken in Ref. 5, where the Hall effect developed as a result of the rather artificial introducting of corrections to the chemical potential, and in Ref. 6, where curvature of the Fermi surface was taken into account.

In type II superconductors, just as in the normal metal, the Hall effect can arise from curvature of the trajectories of the electrons in the magnetic field. Furthermore, another mechanism is possible, namely, the dragging of the vortices by the passing superfluid flow, which takes place in superfluid helium. In the motion of the vortices, a mean electric field

$$\mathbf{E} = \frac{1}{c}[\mathbf{B} \times \mathbf{u}], \qquad (1)$$

is induced, where $\mathbf{u}$ is the velocity of the vortex and $\mathbf{B}$ is the magnetic induction. If the vortex is partially dragged by the flow, this leads to the appearance of a component $\mathbf{E}$, perpendicular to the transport current (see Fig. 1), where

$$\text{tg}\,\theta_H = -u_{\parallel}/u_{\perp} = E_{\perp}/E_{\parallel}.$$

Thus, the experiments of Refs. 7, 8 in pure superconductors yield $\tan\theta_H = 1$.

As is well known,[9] the superfluid flow in He II acts on the vortex with the Magnus force

$$\mathbf{F}_m = \frac{2\pi}{m_{\text{He}}}[\mathbf{v}_s - \mathbf{u} \times \boldsymbol{\mu}]\rho_s,$$

where $\rho_s = m_{\text{He}} N_s$ is the density of the superfluid component, $\mathbf{v}_s$ is its velocity at large distances from the vortex, and μ is the unit vector of circulation of the velocity. On the other hand, in motion of the vortex, a frictional force develops between it and the normal component $\mathbf{F}$, which depends on the difference $\mathbf{u} - \mathbf{v}_n$ and is determined by the processes of scattering of the normal excitations by the vortex. The balance of forces $\mathbf{F}_m + \mathbf{F} = 0$ connects the velocity of the vortex $\mathbf{u}$ with the velocities of the superfluid and normal motions in the vicinity of the vortex.

It is of interest to understand the analogy of the properties of pure superconductors and He II. In considering below the pure superconductor (the required degree of purity will be established later) we impose the condition $\varkappa \gg 1$ on the Ginzburg–Landau parameter. This allows us to neglect the effect of the magnetic field at distances of the order of the coherence distance ξ from the center of the vortex, and to assume the vortices to be isolated at $H \sim H_{c1}$. In this case, the Hall effect is due to the dragging of the vortices by the superfluid current. Such a formulation of the problem preserves the analogy with He II, but for the superconductor, it has a methodological character, since in pure type II superconductors that are known at the present time, beginning with vanadium and niobium, the parameter $\varkappa \sim 1$;

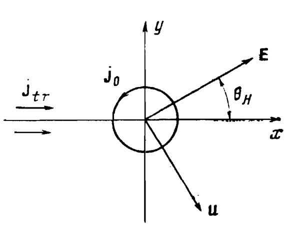

FIG. 1.

however, it allows us to trace out the features of the similarity and the difference between the superconductor and He II.

2. THE ORTHOGONALITY RELATION

The specifics of a pure superconductor in comparison with an alloy also appear in the fact that there is a rather sharp spike in the magnetic field at the center of the vortex here; in the case $\varkappa \sim 1$ it can reach values of the order of H_{c2}.[10] At $\varkappa \gg 1$, however, H_{max} is apparently much smaller than H_{c2}.[4,10] We neglect the contribution of this field. It will be shown in Appendix 2 how to take this effect into account.

The phase of the order parameter changes by 2π in going around the origin, which is placed at the center of the fixed vortex,

$$\Delta_0=|\Delta_0|e^{i\varphi}.$$

We assume that the electrons have a quadratic spectrum $p^2/2m$, i.e., we neglect the band structure of the metal and, consequently, umklapp processes. In such a situation, the electron system has Galilean invariance if the interaction with the lattice is sufficiently weak. The moving vortex shifts as a whole in the zeroth approximation in $\mathbf{u}$. By virtue of what has been said above, it is convenient to set

$$\Delta(\mathbf{r})=\Delta_0(\mathbf{r}-\mathbf{u}t)e^{2im\mathbf{u}\mathbf{r}}+\Delta_1, \tag{2}$$

$$G^{R(A)}_{\varepsilon,\varepsilon'}(\mathbf{p},\mathbf{p}')=G^{R(A)}_{(0)\tilde{\varepsilon},\tilde{\varepsilon}'}(\mathbf{p}-m\mathbf{u},\mathbf{p}'-m\mathbf{u})+G^{R(A)}_1,$$
$$F^{+R(A)}_{\varepsilon,\varepsilon'}(\mathbf{p},\mathbf{p}')=F^{+R(A)}_{(0)\tilde{\varepsilon},\tilde{\varepsilon}'}(\mathbf{p}+m\mathbf{u},\mathbf{p}'-m\mathbf{u})+F^{+R(A)}_1 \tag{3}$$

and so on, where $\tilde{\varepsilon}=\varepsilon-\mathbf{p}\cdot\mathbf{u}$, $\tilde{\varepsilon}'=\varepsilon-\mathbf{p}'\cdot\mathbf{u}$, the functions with the zero subscript correspond to the fixed vortex, and the corrections Δ_1, G_1, F_1 are proportional to $\mathbf{u}$. With the help of the equations for $G^{R(A)}$ and $F^{+R(A)}$, we can establish the fact that G_1, F_1^+ are expressed only in terms of Δ_1 and do not contain time derivatives of Δ_0, $\mathscr{G}_0$:

$$\mathscr{G}^{R(A)}_1(\mathbf{p},\mathbf{p}')=-\int\mathscr{G}^{R(A)}_{(0)\varepsilon}(\mathbf{p},\mathbf{p}_1)\hat{H}_1(\mathbf{k}_1)\mathscr{G}^{R(A)}_{(0)\varepsilon}(\mathbf{p}_1-\mathbf{k}_1,\mathbf{p}')\frac{d^3\mathbf{k}_1\,d^3\mathbf{p}_1}{(2\pi)^6}, \tag{4}$$

where

$$\mathscr{G}=\begin{pmatrix}G & F\\ -F^+ & \bar{G}\end{pmatrix},\quad \hat{H}_1=\begin{pmatrix}0 & -\Delta_1\\ \Delta_1^* & 0\end{pmatrix}.$$

This circumstance is violated when account is taken of the interaction with the lattice. Estimate of the corresponding term in the collision integral, which arises under the transformation (2), (3), shows that (4) takes place when $l\gg\xi$. We shall impose a much stronger condition on the path length below.

The order parameter and the current are expressed in terms of the Green's function in the following way[11]:

$$\frac{\Delta_\omega(\mathbf{k})}{|g|}=\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{d\varepsilon}{4\pi i}\Big[F^R_{\varepsilon_+,\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)\operatorname{th}\frac{\varepsilon_-}{2T}-\operatorname{th}\frac{\varepsilon_+}{2T}F^A_{\varepsilon_+,\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)+F^{(a)}_{\varepsilon_+,\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)\Big], \tag{5}$$

$$\mathbf{j}_\omega(\mathbf{k})=-\frac{2e}{m}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{d\varepsilon}{4\pi i}\mathbf{p}\Big[G^R_{\varepsilon_+,\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)\operatorname{th}\frac{\varepsilon_-}{2T}-\operatorname{th}\frac{\varepsilon_+}{2T}G^A_{\varepsilon_+,\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)+G^{(a)}_{\varepsilon_+,\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)\Big], \tag{6}$$

where $\mathbf{p}_\pm=\mathbf{p}\pm\mathbf{k}/2$, $\varepsilon_\pm=\varepsilon\pm\omega/2$. The anomalous Green's functions for the pure superconductor have the form

$$\mathscr{G}^{(a)}_{\varepsilon_+,\varepsilon_-}(\mathbf{p}_+,\mathbf{p}_-)=-\frac{\omega}{2T}\operatorname{ch}^{-2}\frac{\varepsilon}{2T}\int\mathscr{G}^R_{(0)\varepsilon}(\mathbf{p}_+,\mathbf{p}_1)\hat{H}_\omega(\mathbf{k}_1)\mathscr{G}^A_{(0)\varepsilon}(\mathbf{p}_1-\mathbf{k}_1,\mathbf{p}_-)\frac{d^3\mathbf{p}_1\,d^3\mathbf{k}_1}{(2\pi)^6},$$

where $\hat{H}_\omega$ is composed of Δ_ω and Δ_ω^* patterned after $\hat{H}_1$. This expression assumes that the normal component of the electron liquid is at rest relative to the laboratory set of coordinates (the thermostat, the role of which is played by the lattice). Interaction with the lattice comes about from the impurities and phonons. We select such a model, in which the basic mechanism is scattering from impurities, i.e., we assume that the phonon relaxation time $\tau_{ph}\sim\Theta_D^2/T^2$ is larger than the impurity relaxation time τ. The interaction with the lattice maintains the normal excitations at rest relative to the thermostat in the case of sufficiently small vortex density n_L: the small number of vortices cannot lead to motion of the entire mass of the normal component of the electron liquid. Actually, if the frictional force per unit length of the vortex on the normal component has the form $F=-D(u-v_n)$, then a force $-n_L\mathbf{F}$ acts on a unit volume of the normal component. On the other hand, the momentum transferred to the impurities by a unit volume of the normal component, moving with velocity $\mathbf{v}_n$ averaged over the volume, will be $-\mathbf{F}_{1at}\sim mN_n\mathbf{v}_n/\tau$. It is seen from the condition $F_{1at}-n_L\mathbf{F}=0$, that, in the case

$$n_L\ll mN_n/\tau D \tag{7}$$

the velocity of the normal component $v_n\ll u$.

It is important to emphasize that the free flight time τ can be so large here that the Galilean invariance (2), (3) is no longer violated, and the scattering of the excitations by the individual vortex takes place more rapidly than scattering from impurities. As we have seen, for satisfaction of the first condition, it suffices to require $\tau\gg\Delta^{-1}$. The inequality necessary for the satisfaction of the second condition can be obtained in the following way. The scattering probability by a vortex of unit length for an electron located in the volume occupied by the vortex is equal to $\tau_v^{-1}\sim\sigma v_F\xi^{-2}$, where σ is the corresponding scattering cross section, which, in accord with Eq. (22) (see below) has the order of p_F^{-1}. Requiring that this probability be larger than τ^{-1}, we obtain

$$\tau\gg E_F/\Delta^2.$$

The condition (7) for the vortex density takes the form

$n_L\xi^2 \ll E_F/\Delta^2\tau$.

Substituting expressions (2) and (3) in (5) and (6), and limiting ourselves to terms of first order in $\mathbf{u}$, we obtain a set of linear equations for Δ_1 and j_1. Since the initial equations (5), (6) possess translational invariance we can write down for the obtained linear set of equations sort of a orthogonality relation in analogy to what was done in Ref. 12. Its derivation is given in Appendix 1. The result is of the form

$$\frac{\pi}{e}[\mathbf{n}_H, \mathbf{d}](\mathbf{j}_{tr} - N_s e\mathbf{u}) = L^{-1}\int_{V_{R_1}} d^3\mathbf{r}_1 \int \frac{d\varepsilon}{4\pi i}\,\mathrm{Sp}\{\hat{H}_d \mathscr{G}_1^{(a)}(\mathbf{r}_1, \mathbf{r}_1')\}_{\mathbf{r}_1'=\mathbf{r}_1} \quad (8)$$

(the frequency indices on the $\mathscr{G}$ functions are omitted here and below). Integration is carried out over the volume of a cylinder of radius $R_1 \gg \xi$ with center at the origin. N_s is the density of "superconducting" electrons at large distances from the center of the filament: $N_s = N - N_n$, where

$$N_n = \int_{\Delta_\infty}^{\infty} \frac{\varepsilon}{(\varepsilon^2 - \Delta_\infty^2)^{1/2}}\,\mathrm{ch}^{-2}\frac{\varepsilon}{2T}\frac{d\varepsilon}{2T},$$

L is the length of the vortex, $\mathbf{n}_H$ the unit vector in the direction of the magnetic field, $\mathbf{d}$ is an arbitrary constant vector and the operator $\hat{H}_d$ consists of $\Delta_d = \mathbf{d}\nabla\Delta_0$, $\Delta_d^* = \mathbf{d}\nabla\Delta_0^*$: $\hat{H}_d = \mathbf{d}\cdot\nabla H_0$. We have

$$\mathscr{G}_1^{(a)}(\mathbf{r}, \mathbf{r}') = -\frac{1}{2T}\mathrm{ch}^{-2}\frac{\varepsilon}{2T}\int d\mathbf{S}_2\Big\{\mathscr{G}^R(1,2)\Big[\frac{\mathbf{p}_{2'}}{2m}(\mathbf{u}\nabla_{2'})\mathscr{G}^A(2',1')\Big]_{2'=2}$$
$$+\Big[\frac{\mathbf{p}_{2'}^+}{2m}\mathscr{G}^R(1,2')\Big]_{2'=2}(\mathbf{u}\nabla_2)\mathscr{G}^A(2,1')\Big\}, \quad (9)$$

where $\mathbf{p}_{2'} = i\nabla_{2'}$, $\mathbf{p}_{2'}^+ = i\nabla_{2'}$. The integration over the $\mathbf{r}_2$ coordinate is carried out over the surface of a cylinder of radius $R_2 \gg R_1$.

We transform the volume integral over $d^3\mathbf{r}_1$ in (8) to an integral over a surface of radius R_1. For this purpose, as in Appendix 1, we use the identity

$$\hat{H}_d = \mathbf{d}\nabla\mathscr{G}_{(0)}^{-1R(A)}$$

as well as the equation of motion

$$\mathscr{G}^{-1}\mathscr{G}^{R(A)}(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}').$$

As a result, we obtain

$$\int_{V_{R_1}} \mathrm{Sp}\{\hat{H}_d\mathscr{G}_1^{(a)}\}d^3\mathbf{r}_1 = -\frac{i}{2T}\mathrm{ch}^{-2}\frac{\varepsilon}{2T}\iint d(S_{R_1})_i\, d(S_{R_2})_k$$
$$\times\mathrm{Sp}\Big\{\frac{(p_2)_k}{2m}(\mathbf{u}\nabla_2)\mathscr{G}^A(2,1)\frac{(p_1)_i}{2m}(\mathbf{d}\nabla_1)\mathscr{G}^R(1,2)$$
$$+(\mathbf{u}\nabla_2)\frac{(p_1^+)_i}{2m}\mathscr{G}^A(2,1)(\mathbf{d}\nabla_1)\frac{(p_2^+)_k}{2m}\mathscr{G}^R(1,2) \quad (10)$$
$$+\Big[\frac{(p_2)_k(p_{1'}^+)_i}{(2m)^2}(\mathbf{u}\nabla_2)\mathscr{G}^A(2,1')\Big]_{1'=1}(\mathbf{d}\nabla_1)\mathscr{G}^R(1,2)$$
$$+(\mathbf{u}\nabla_2)\mathscr{G}^A(2,1)\Big[\frac{(p_{1'})_i(p_2^+)_k}{(2m)^2}(\mathbf{d}\nabla_1)\mathscr{G}^R(1',2)\Big]_{1'=1}\Big\}.$$

The derivation of (8), (10), together with the use of the equation of motion, is actually equivalent to the derivation, given by Iordanskiĭ,[13] for the force of mutual friction in He II.

3. BALANCE OF FORCES ACTING ON THE VORTEX

Writing $\mathbf{j}_{tr} = N_s e\mathbf{v}_s$, we can represent the left side of (8) in the form $\mathbf{F}_m\cdot\mathbf{d}$, where the Magnus force (or the Lorentz force) acting on the vortex from the transport current is

$$\mathbf{F}_m = \pi N_s[\mathbf{v}_s - \mathbf{u}\times\mathbf{n}_H] = \frac{1}{c}N_s e[\mathbf{v}_s - \mathbf{u}\times\Phi_0]$$

($\Phi_0 = \mathbf{n}_H\pi c/e$ is the magnetic flux quantum). Thus, there is a quantity on the right side of (8) which has the meaning of a force (with opposite sign) acting on the vortex from the normal excitations. In He II, as is well known,[13,14] we can express this force in terms of the transverse and transport scattering cross sections of excitations by the vortex. We shall show that this is also the case for the pure superconductor.

We make use of the well-known (see, for examples, Refs. 4, 15, 16) expansion of equilibrium Green's functions in the eigenfunctions of the Bogolyubov equations:

$$\mathscr{G}_\varepsilon^{R(A)}(\mathbf{r}_1, \mathbf{r}_2) = -\sum_n \frac{\hat{U}_n(\mathbf{r}_1)\hat{U}_n^+(\mathbf{r}_2)}{\varepsilon - E_n \pm i\delta}, \quad (11)$$

where the summation is carried out over all states, both with positive and with negative energies, $\delta \to +0$,

$$\hat{U}_n = \begin{pmatrix} u_n \\ -v_n \end{pmatrix}, \quad \hat{U}_n^+ = (u_n^*, v_n^*).$$

The functions $\hat{U}$ satisfy the equations

$$E\sigma_z\hat{U} = \Big[\frac{(-i\nabla)^2}{2m} - E_F + \hat{H}\Big]\hat{U}. \quad (12)$$

In the substitution of (11) in (10), we note that if one of the states belongs to the discrete spectrum $|E| < \Delta_\infty$,[17] then the surface integral vanishes for this state because of the exponential decay of $\hat{U}_n$ at $\rho \gg \xi_0$. Thus, only sums over the continuous spectrum enter into the expression (10). Expressing the functions $\hat{U}$ at $\rho \gg \xi$ in terms of the phases of the scattering, we can write down for the force of mutual friction a formula which contains the scattering cross section.

The solution (12) for each E in the continuous spectrum is doubly degenerate. The corresponding linearly independent function is

$$\hat{U}_+ e^{i(\nu\pm 1/2)\varphi + ikz}w_+(\rho), \quad \hat{U}_- = e^{i(\nu\pm 1/2)\varphi + ikz}\hat{w}_-(\rho). \quad (13)$$

The upper sign applies to u, the lower for v and ν takes on half-integer values. The asymptotic form of the expression $\hat{w}(\rho)$ is given in the work of Cleary[15] (see also Ref. 16):

$$\hat{w}_+(\rho) = \Big(\frac{2}{\pi q_+\rho}\Big)^{1/2}\hat{\beta}_+\cos\chi\cos\Big(q_+\rho - \frac{\pi}{2}|\nu| - \frac{\pi}{4} + \delta_1\Big)$$
$$+\Big(\frac{2}{\pi q_-\rho}\Big)^{1/2}\hat{\beta}_-\sin\chi\cos\Big(q_-\rho - \frac{\pi}{2}|\nu| - \frac{\pi}{4} - \delta_1\Big), \quad (14)$$

$$\hat{w}_-(\rho)=-\left(\frac{2}{\pi q_+\rho}\right)^{1/2}\beta_+\sin\chi\cos\left(q_+\rho-\frac{\pi}{2}|\nu|-\frac{\pi}{4}+\delta_2\right)$$
$$+\left(\frac{2}{\pi q_-\rho}\right)^{1/2}\beta_-\cos\chi\cos\left(q_-\rho-\frac{\pi}{2}|\nu|-\frac{\pi}{4}-\delta_2\right), \qquad (15)$$

where

$$q_\pm^2=p_F^2-k^2\pm 2m(E^2-\Delta_\infty^2)^{1/2},$$
$$\beta_\pm=2^{-1/2}\left\{\begin{matrix}[1\pm(E^2-\Delta_\infty^2)^{1/2}/E]^{1/2}\\ -[1\mp(E^2-\Delta_\infty^2)^{1/2}/E]^{1/2}\end{matrix}\right\}.$$

The functions $\hat{U}$ with real $\hat{w}(\rho)$ guarantee the absence of a radial component of the current for the equilibrium vortex. The quantity χ describes the scattering with change of the particle-hole channel.

The part of the sum (11) corresponding to the continuous spectrum is of the form

$$\mathcal{G}_\varepsilon^{R(A)}(\mathbf{r}_1,\mathbf{r}_2)=-\sum_\nu\int\frac{dk\,dE}{(2\pi)^2}\frac{mE}{(E^2-\Delta_\infty^2)^{1/2}}\frac{\hat{U}_+(1)\hat{U}_+^+(2)+\hat{U}_-(1)\hat{U}_-^+(2)}{\varepsilon-E\pm i\delta}. \qquad (16)$$

Substituting (16) in (10), with account of (13)–(15), and after cumbersome calculations, we obtain

$$\frac{\pi}{e}[\mathbf{n}_H\times\mathbf{d}](\mathbf{j}_{tr}-N_s e\mathbf{u})$$
$$=\int\limits_{|\varepsilon|>\Delta_\infty}\frac{d\varepsilon}{4\pi T}\mathrm{ch}^{-2}\frac{\varepsilon}{2T}\int\limits^{p_F}\frac{dk}{2\pi}q^3\{\sigma_{tr}(\mathbf{ud})+\sigma_\perp[\mathbf{n}_H\times\mathbf{d}]\mathbf{u}\}. \qquad (17)$$

Here we have used the following notation:

$$q^2=p_F^2-k^2,\quad \sigma_{tr}={}^1/_2(\sigma_{tr}^{(1)}+\sigma_{tr}^{(2)}),\quad \sigma_\perp={}^1/_2(\sigma_\perp^{(1)}+\sigma_\perp^{(2)}).$$

The cross sections in (17) are expressed in terms of the amplitude of the scattering of excitations by the vortex, which are identical with those obtained in the work of Cleary[15]:

$$f_1(\varphi)=\left(\frac{1}{2\pi q}\right)^{1/2}\sum_\nu e^{i(\nu+1/2)\varphi}\{\cos^2\chi(e^{2i\tilde{\delta}_1}-1)+\sin^2\chi(e^{2i\tilde{\delta}_2}-1)\},$$
$$f_2(\varphi)=\left(\frac{1}{2\pi q}\right)^{1/2}\sum_\nu e^{i(\nu+1/2)\varphi}\{\sin^2\chi(e^{2i\tilde{\delta}_1}-1)+\cos^2\chi(e^{2i\tilde{\delta}_2}-1)\},$$
$$g(\varphi)=\left(\frac{1}{2\pi q}\right)^{1/2}\sum_\nu e^{i(\nu+1/2)\varphi}\sin\chi\cos\chi(e^{2i\delta_1}-e^{2i\delta_2}),$$

where

$$\delta_{1,2}(\varepsilon,k,\nu)=\delta_{1,2}(\varepsilon,k,\nu)+{}^1/_4\pi\,\mathrm{sign}\,\nu,$$
$$\tilde{\delta}_{1,2}(\varepsilon,k,\nu)=\delta_{1,2}(\varepsilon,k,\nu)-{}^1/_4\pi\,\mathrm{sign}\,\nu. \qquad (18)$$

We have

$$\sigma_{tr}^{(1),(2)}=\int\limits_{-\pi}^{\pi}[|f_{1,2}(\varphi)|^2+|g(\varphi)|^2](1-\cos\varphi)d\varphi,$$
$$\sigma_\perp^{(1),(2)}=\int\limits_{-\pi}^{\pi}[|f_{1,2}(\varphi)|^2+|g(\varphi)|^2]\sin\varphi\,d\varphi.$$

The difference from Ref. 15 lies in the fact that the term designated by σ_s in Ref. 15 and having a very large value does not appear in these formulas.

The force acting on the moving vortex from the normal excitations is, according to (17), equal to

$$\mathbf{F}=-\int\limits_{|\varepsilon|>\Delta_\infty}\frac{d\varepsilon}{4\pi T}\mathrm{ch}^{-2}\frac{\varepsilon}{2T}\int\limits_0^{p_F}\frac{dk}{2\pi}q^3\{\sigma_{tr}\mathbf{u}+\sigma_\perp[\mathbf{u}\times\mathbf{n}_H]\}. \qquad (19)$$

4. SCATTERING OF THE EXCITATIONS BY THE VORTEX

The phases of the scattering $\delta_{1,2}$ change by a quantity of the order of unity in a change of the impact parameter $b=\nu/q$ by a quantity of the order of ξ. Therefore,

$$\delta(\nu+1)-\delta(\nu)\sim(q\xi)^{-1}\ll 1 \qquad (20)$$

at all ν except $\nu=-\frac{1}{2}$. As follows from Appendix 2, the difference $\delta(\frac{1}{2})-\delta(-\frac{1}{2})\sim 1$. The quantity $\chi=\pi/4$.[15] Using (20), we can simplify the formulas for the cross sections. We begin with the transverse cross section. Separating the terms with $\nu'=-\nu=\pm\frac{1}{2}$ from the sum over the azimuthal numbers, and expanding in terms of the small differences (20), we find

$$\sigma_\perp=\frac{1}{q}\sum_{\nu\neq-1/2}[\delta_1(\nu)+\delta_2(\nu)-\delta_1(\nu+1)-\delta_2(\nu+1)]+\frac{1}{q}\sin\alpha\cos\gamma,$$

where

$$\alpha=\delta_1(1/2)+\delta_2(1/2)-\delta_1(-1/2)-\delta_2(-1/2),$$
$$\gamma=\delta_1(1/2)-\delta_2(1/2)+\delta_1(-1/2)-\delta_2(-1/2).$$

Carrying out the summation, we obtain

$$\sigma_\perp=-\frac{1}{q}[\delta_1(+\infty)+\delta_2(+\infty)-\delta_1(-\infty)-\delta_2(-\infty)]+\frac{\alpha}{q}+\frac{1}{q}\sin\alpha\cos\gamma. \qquad (21)$$

The first term in this formula is the analog of the ordinary quasiclassical formula for the transverse cross section

$$\sigma_\perp=\int\limits_{-\infty}^{\infty}(\delta p_\perp)\frac{db}{p}=\frac{\Delta S(+\infty)-\Delta S(-\infty)}{p},$$

where $\Delta S(b)$ is the change in the classical action in passing along the trajectory. The phases $\delta_{1,2}$ at large b are calculated in Appendix 2:

$$\delta_{1,2}=-\frac{\pi}{4}\frac{\varepsilon}{(\varepsilon^2-\Delta_\infty^2)^{1/2}}\mathrm{sign}\,\nu.$$

The first term in (21) therefore yields

$$\sigma_\perp'=\frac{\pi}{q}\frac{\varepsilon}{(\varepsilon^2-\Delta_\infty^2)^{1/2}},$$

which corresponds accurately to the cross section obtained for He II: $\sigma_\perp=2\pi/m_{\mathrm{He}}v_g$,[13,14] where v_g is the group velocity of the excitations. The second and third terms in (21) contain the phase jump α. It is connected with the term $\frac{1}{4}\pi$ sign ν in (18) and is due to the fact that, in contrast with He II, "the superfluids" in the superconductor are not the quasiparticles (electrons) themselves, but their bound pairs: while, the phase of the order parameter changes by 2π in going around the origin of the coordinates, the phase of the single-electron wave function changes by π.

It was shown in Appendix 2 that $\alpha=-\pi$; therefore,

$$\sigma_\perp=-\frac{\pi}{q}\left[1-\frac{\varepsilon}{(\varepsilon^2-\Delta_\infty^2)^{1/2}}\right]. \quad (22)$$

Similar simplifications can be obtained also for the transport cross section. We have

$$\sigma_{tr}=\frac{1}{q}\sum_{\nu=-1/2}\{\sin^2[\delta_1(\nu)-\delta_1(\nu+1)]+\sin^2[\delta_2(\nu)-\delta_2(\nu+1)]\}$$

$$+\frac{1}{q}\left\{\cos^2\left[\delta_1\left(\frac{1}{2}\right)-\delta_2\left(-\frac{1}{2}\right)\right]+\cos^2\left[\delta_1\left(-\frac{1}{2}\right)-\delta_2\left(\frac{1}{2}\right)\right]\right\}. \quad (23)$$

The terms entering into the summation over ν are of the order of $(q\xi)^{-2}$. Since the number of terms of the summation $\sim q\xi$, this part of the cross section is of the order of $1/q^2\xi$. The second part of the cross section (23) requires a more detailed consideration. It is shown in Appendix 2 that $\delta_1(\pm\frac{1}{2})-\delta_2(\mp\frac{1}{2})=+\pi/2$ in the quasiclassical approximation.[1)] Therefore, this part of the cross section vanishes in the quasiclassical approximation. The difference in the phase differences from $\pm\pi/2$ can appear in the next approximation in the quasiclassicality parameter $(p_F\xi)^{-1}$. Therefore, the second term in the cross section (23) does not exceed the first term in order of magnitude. Thus, $\sigma_{tr}\sim 1/q^2\xi$. The calculation of σ_{tr} requires a calculation of the phase; this is possible only with the help of numerical methods, which go beyond the scope of the present work.

Substituting (22) in (19), we find

$$\mathbf{F}=-\eta\mathbf{u}-\pi\left[N_n-2f_0\left(\frac{\Delta_\infty}{T}\right)N\right][\mathbf{u}\times\mathbf{n}_H],$$

where $f_0(z)=(e^z+1)^{-1}$ is the equilibrium Fermi function. The coefficient of viscosity is

$$\eta=\int_{|\varepsilon|>\Delta_\infty}\frac{d\varepsilon}{4\pi T}\mathrm{ch}^{-2}\frac{\varepsilon}{2T}\int_0^{p_F}\frac{dk}{2\pi}q^3\sigma_{tr}.$$

At temperatures $T\sim\Delta$, we have, in order of magnitude, $\eta\sim mp_F\Delta$. At $T\ll\Delta$, the viscosity $\eta\propto e^{-\Delta/T}$. We find from (17)

$$\mathbf{j}_{tr}=N\,\mathrm{th}\left(\frac{\Delta_\infty}{2T}\right)e\mathbf{u}+\frac{e\eta}{\pi}[\mathbf{n}_H\times\mathbf{u}].$$

Thus, the effective conductivity $\sigma_f=\eta c^2/\Phi_0B$. At $T\sim\Delta$,

$$\sigma_f\sim\frac{Ne^2\Delta^{-1}}{m}\frac{H_{c2}}{B}. \quad (24)$$

The Hall angle is close to $\pi/2$:

$$\mathrm{tg}\,\theta_H=\pi N\,\mathrm{th}\left(\frac{\Delta_\infty}{2T}\right)\Big/\eta\gg 1.$$

Such a situation is analogous to that which occurs in a pure superconductor at $x\sim\tau\Delta^2/E_F\gg 1$ at low temperatures.[19] In the latter case, however, the conductivity is determined by the interaction of electrons, found in bound states at the core of the vortex, with impurities. This mechanism makes the contribution

$$\sigma_f\sim\frac{Ne^2\Delta^{-1}}{m}\frac{H_{c2}}{B}\frac{E_F^2}{\Delta x}$$

to the conductivity in the case $x\gg 1$ and $T\ll\Delta$. The contribution is greater than the quantity σ_f from (24). Therefore, we can expect that even at $T\sim\Delta$ the energy dissipations will be determined not by the scattering of the normal excitations, but by electrons in bound states in the core of the vortex.

In conclusion, the authors express their deep gratitude to L. P. Gor'kov for constant attention to the work, and also to Yu. M. Gal'perin, S. V. Iordanskiĭ, A. I. Larkin, Yu. N. Ovchinnikov and E. B. Sonin for useful discussions.

APPENDIX 1: DERIVATION OF THE ORTHOGONALITY RELATION (8)

For the derivation of (8), it is convenient to transform to a gauge in which Δ is real. In place of (2) and (3), we have

$$|\Delta|=|\Delta_0(\mathbf{r}-\mathbf{u}t)|+\Delta_1,\quad \mathbf{Q}=-\frac{c}{2e}\nabla\chi=\mathbf{Q}_0-\frac{mc}{e}\mathbf{u}+\mathbf{Q}_1 \quad (1.1)$$

(here χ is the phase of the order parameter),

$$\mathcal{G}^{(R,A)}_{\varepsilon,\varepsilon'}(\mathbf{p},\mathbf{p}')=\mathcal{G}^{(R,A)}_{(0)\varepsilon,\varepsilon'}(\mathbf{p},\mathbf{p}')-\mathcal{G}^{(R,A)}_1. \quad (1.2)$$

Substituting (1.1) and (1.2) in the equation for Δ and the current, we obtain a set of linear equations in Δ_1 and $\mathbf{Q}_1$. Using the translational invariance of the initial equations (5) and (6), we write down the orthogonality condition for them[12] (below, we transform with respect to $\mathbf{p}_+-\mathbf{p}_-=\mathbf{k}$ in the coordinate representation):

$$\frac{1}{c}\int_{V_{R_1}}[(\mathbf{j}_1+\mathbf{j}_n-Ne\mathbf{u})\mathbf{Q}_d-\mathbf{Q}_1\mathbf{j}_d]d^3\mathbf{r}$$

$$=\int_{V_{R_1}}d^3\mathbf{r}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{d\varepsilon}{4\pi i}\{\mathrm{Sp}[\hat{H}_d\mathcal{G}^{(0)}]-\mathrm{Sp}[\hat{H}'\mathcal{G}']\}, \quad (1.3)$$

where

$$\hat{H}_d=-\frac{e}{2mc}(\hat{\mathbf{p}}\mathbf{Q}_d+\mathbf{Q}_d\hat{\mathbf{p}})\sigma_z+\hat{H}_d',\quad \hat{H}_d'=\begin{pmatrix}0 & -\Delta_d\\ \Delta_d & 0\end{pmatrix},$$

$\mathbf{Q}_d=(\mathbf{d}\cdot\nabla)\mathbf{Q}_0$, $\Delta_d=\mathbf{d}\cdot\nabla\Delta_0$, $\mathbf{d}$ is an arbitrary constant vector, and we have omitted the modulus symbol for Δ. The function $\mathcal{G}'$ is equal to

$$\mathcal{G}'(\mathbf{p}_+,\mathbf{p}_-)=-\frac{1}{2T}\mathrm{ch}^{-2}\frac{\varepsilon}{2T}[\mathcal{G}^R_{(0)\varepsilon}(\mathbf{p}_+,\mathbf{p}_-)(\mathbf{p}_-\mathbf{u})-(\mathbf{p}_+\mathbf{u})\mathcal{G}^A_{(0)\varepsilon}(\mathbf{p}_+,\mathbf{p}_-)],$$

and

$$\mathbf{j}_n=-\frac{e}{m}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{d\varepsilon}{4\pi i}\mathbf{p}\,\mathrm{Sp}[\sigma_z\mathcal{G}'(\mathbf{p}_+,\mathbf{p}_-)].$$

Integration in (1.3) is carried out over the volume of a cylinder of radius $R_1\gg\xi$. In the left side of (1.3) we have separated out a surface integral. Since, at large distances from the vortex,

$$\mathbf{j}_n=N_ne\mathbf{u},\quad \mathbf{j}_d=\frac{N_se}{2m}\nabla\chi_d,\quad \mathbf{j}_{1\infty}=\frac{N_se}{2m}(\nabla\chi_1+2m\mathbf{u}),$$

the linear part takes the form[12]

$$\frac{L\pi}{e}[\mathbf{n}_H\times\mathbf{d}](\mathbf{j}_{1\infty}-N_s e\mathbf{u})+\int\limits_{V_{R_1}}\frac{\chi_d}{2e}\operatorname{div}\mathbf{j}_n d^3\mathbf{r}+\int\limits_{V_{R_1}}[\operatorname{div}\mathbf{j}-\mathbf{u}\nabla Ne]\frac{\chi_d}{2e}d^3\mathbf{r}.$$

The last term in this expression vanishes by virtue of the condition of continuity. Integrating by parts on the right side of (1.3), we obtain

$$\frac{\pi}{e}[\mathbf{n}_H\times\mathbf{d}](\mathbf{j}_{1\infty}-N_s e\mathbf{u})=L^{-1}\int\limits_{V_{R_1}}d^3\mathbf{r}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{d\varepsilon}{4\pi i}$$
$$\times\left\{\operatorname{Sp}[\hat{H}_d'(\mathcal{G}^{(a)}-\mathcal{G}')]-\frac{\chi_d}{2m}\operatorname{div}[\mathbf{p}(G^{(a)}-G'-\bar{G}^{(a)}+\bar{G}')]\right\}. \qquad (1.4)$$

The anomalous Green's function for a pure superconductor[11] in the coordinate representation is

$$\mathcal{G}^{(a)}_{\varepsilon_+,\varepsilon_-}(\mathbf{r},\mathbf{r}')=\frac{i}{2T}\operatorname{ch}^{-2}\frac{\varepsilon}{2T}\int\mathcal{G}_\varepsilon^R(\mathbf{r},\mathbf{r}_2)\hat{H}_u(\mathbf{r}_2)\mathcal{G}_\varepsilon^A(\mathbf{r}_2,\mathbf{r}')d^3\mathbf{r}_2, \qquad (1.5)$$

where $\hat{\hat{H}}_u$ is obtained from $\hat{\hat{H}}_d$ by the substitution of $(\mathbf{u}\cdot\nabla)\mathbf{Q}_0$ and $(\mathbf{u}\cdot\nabla)\Delta_0$ for $\mathbf{Q}_d$, Δ_d. The integral over $d^3\mathbf{r}_2$ in (1.5) converges at distances of the order of ξ; therefore, in place of integration over the entire space, we can integrate over the volume of a cylinder of radius $R_2\gg R_1\gg\xi$. Making use of the fact that

$$\hat{H}_u=(\mathbf{u}\nabla)\hat{H}_0=\mathbf{u}\nabla\mathcal{G}_{(0)}^{-1},$$

and the equation of motion

$$\mathcal{G}_{(0)}^{-1}\mathcal{G}_{(0)}^{R(A)}(\mathbf{r}_1,\mathbf{r}_2)=\delta(\mathbf{r}_1-\mathbf{r}_2),$$

we separate from (1.5) an integral over the surface of a cylinder of radius R_2:

$$\mathcal{G}^{(a)}(\mathbf{r},\mathbf{r}')=\mathcal{G}_1^{(a)}+\mathcal{G}'(\mathbf{r},\mathbf{r}')\theta(R_2-|\rho|)\theta(R_2-|\rho'|), \qquad (1.6)$$

where $\mathcal{G}_1^{(a)}$ is given by the formula (9), and

$$\theta(x)=\begin{cases}1, & x\geqslant 0\\ 0, & x<0\end{cases}.$$

With the help of (1.6), after some transformations, we can obtain the following for the term in curly brackets in (1.4):

$$\{(\Delta_d+i\chi_d\Delta_0)F_1^{+(a)}(\mathbf{r},\mathbf{r})+(\Delta_d-i\chi_d\Delta_0)F_1^{(a)}(\mathbf{r},\mathbf{r})\}.$$

It follows from (9) that $e^{-i\varphi}F_1^{+(a)}$ and $e^{i\varphi}F_1^{(a)}$ in the gauge with real Δ are equal to $F_1^{+(a)}$ and $F_1^{(a)}$ in the gauge $\Delta=|\Delta|e^{i\varphi}$. Thus we arrive at the expression (8).

APPENDIX 2: PHASES OF THE SCATTERING

We first note that the spike in the magnetic field at the center of the vortex, about which we spoke in Sec. 2, is taken into account by replacing $-i\nabla$ in (12) by $-i\nabla\mp e\mathbf{A}/c$, where $\mathbf{A}$ is the corresponding vector potential. The relation (17) remains unchanged in this case.

According to Ref. 15, we write out

$$\hat{w}(\rho)=(p\rho)^{-1/2}\{\hat{g}(\rho)e^{iS-i\pi/4}+\hat{g}^*(\rho)e^{-iS+i\pi/4}\}, \qquad (2.1)$$

where

$$p=(q^2-\nu^2/\rho^2)^{1/2},\quad S=\int\limits_b^\rho p\,d\rho,\quad \hat{g}=\begin{pmatrix}g_u\\-g_v\end{pmatrix},$$

and $b=\nu/q$. The regularity of $\hat{w}(\rho)$ as $\rho\to 0$ requires $\hat{g}(\rho)$ to be real at the turning point $\rho=b$ in the case $b\gtrsim\xi$. We get for $\hat{g}$,

$$\begin{aligned}\left(\varepsilon-\frac{\nu}{2m\rho^2}\right)g_u+\frac{ip}{m}\frac{\partial g_u}{\partial\rho}&=|\Delta|g_v,\\ \left(\varepsilon-\frac{\nu}{2m\rho^2}\right)g_v-\frac{ip}{m}\frac{\partial g_v}{\partial\rho}&=|\Delta|g_u.\end{aligned} \qquad (2.2)$$

Equations (2.2) have two linearly independent solutions:

$$\hat{g}^{(1)}=\begin{pmatrix}a^{(1)}\\-a^{*(1)}\end{pmatrix},\quad \hat{g}^{(2)}=\begin{pmatrix}a^{(2)}\\a^{*(2)}\end{pmatrix}.$$

Let $a=a_r+ia_i$. Then

$$\begin{aligned}\left(\varepsilon-\frac{\nu}{2m\rho^2}\mp\Delta\right)a_r^{(1),(2)}&=\frac{p}{m}\frac{\partial a_i^{(1),(2)}}{\partial\rho},\\ \left(\varepsilon-\frac{\nu}{2m\rho^2}\pm\Delta\right)a_i^{(1),(2)}&=-\frac{p}{m}\frac{\partial a_r^{(1),(2)}}{\partial\rho}\end{aligned} \qquad (2.3)$$

(the modulus symbol on Δ is omitted). We get

$$a_i^{(1)}=\sin\zeta_1,\quad a_r^{(1)}=\lambda_1\cos\zeta_1,$$
$$a_i^{(2)}=\lambda_2\sin\zeta_2,\quad a_r^{(2)}=\cos\zeta_2.$$

Here,

$$\begin{aligned}g_u^{(1)}&\propto(\lambda_1+1)e^{i\zeta_1}+(\lambda_1-1)e^{-i\zeta_1},\\ g_v^{(1)}&\propto(\lambda_1-1)e^{i\zeta_1}+(\lambda_1+1)^{-i\zeta_1}\end{aligned} \qquad (2.4)$$

$$\begin{aligned}g_u^{(2)}&\propto(1+\lambda_2)e^{i\zeta_2}+(1-\lambda_2)e^{-i\zeta_2},\\ g_v^{(2)}&\propto-(1-\lambda_2)e^{i\zeta_2}-(1+\lambda_2)e^{-i\zeta_2}.\end{aligned} \qquad (2.4')$$

In accord with (2.1) and (14), (15), as $\rho\to\infty$,

$$\zeta_{1,2}=\frac{m}{q}(\varepsilon^2-\Delta_\infty^2)^{1/2}\rho+\delta_{1,2}.$$

It follows from (2.3) that as $\rho\to\infty$, we have

$$\lambda_{1,2}=\left(\frac{\varepsilon+\Delta_\infty}{\varepsilon-\Delta_\infty}\right)^{1/2}$$

which corresponds to $\chi=\pi/4$.[15]

At $b\gg\xi$ we can assume $\Delta=\text{const}$ and obtain from (2.3)

$$\delta_{1,2}=-\frac{\pi}{4}\frac{\varepsilon}{(\varepsilon^2-\Delta_\infty^2)^{1/2}}\operatorname{sign}\nu.$$

At small impact parameters ($\nu\sim 1$), the quasiclassical equation (2.3) is applicable only at large distances from the turning point $b\sim q^{-1}$. The boundary conditions for g are obtained with the aid of matching with the exact solutions of (12) at small distances (cf. Ref. 17). In the region $\rho\ll\xi$ in (12), we can omit Δ. We find

$$\hat{w}_\pm=\pm A_\pm\begin{pmatrix}1\\0\end{pmatrix}J_{|\nu+1/2|}(q\rho)+A_\mp\begin{pmatrix}0\\-1\end{pmatrix}J_{|\nu-1/2|}(q\rho).$$

Comparing this with (2.4), (2.4′), we see that the quasiclassical equations (2.3) must be solved with the boundary conditions

$$\operatorname{tg}[\zeta_1(\rho=0)] = -\lambda_1(\rho=0)\operatorname{sign}\nu,$$
$$\operatorname{ctg}[\zeta_2(\rho=0)] = -\lambda_2(\rho=0)\operatorname{sign}\nu \qquad (2.5)$$

In Eqs. (2.3), we can omit the term with ν and set $p = q$. For $\lambda_{1,2}$ and $\zeta_{1,2}$, we have

$$(\varepsilon-\Delta)\lambda_1 = \frac{g}{m}\frac{\partial\zeta_1}{\partial\rho},$$
$$\varepsilon+\Delta = -\frac{q}{m}\frac{\partial\lambda_1}{\partial\rho}\operatorname{ctg}\zeta_1 + \frac{q}{m}\lambda_1\frac{\partial\zeta_1}{\partial\rho}, \qquad (2.6)$$
$$(\varepsilon-\Delta)\lambda_2 = \frac{q}{m}\frac{\partial\zeta_2}{\partial\rho},$$
$$\varepsilon+\Delta = \frac{q}{m}\frac{\partial\lambda_2}{\partial\rho}\operatorname{tg}\zeta_2 + \frac{q}{m}\lambda_2\frac{\partial\zeta_2}{\partial\rho}. \qquad (2.6')$$

It follows from (2.6), (2.6′) and the boundary conditions (2.5) that at $\nu \sim 1$,

$$\lambda_2(\rho,-\nu) = \lambda_1(\rho,\nu),$$
$$\zeta_2(\rho,-\nu) = \zeta_1(\rho,\nu) + \frac{\pi}{2}\operatorname{sign}\nu.$$

Therefore the phases of $\delta_1(\nu)$ and $\delta_2(-\nu)$ are connected by the relation

$$\delta_1(\nu) - \delta_2(-\nu) = -\frac{\pi}{2}\operatorname{sign}\nu.$$

The phase jump is

$$\alpha = \delta_1(^1/_2) - \delta_1(-^1/_2) + \delta_2(^1/_2) - \delta_2(-^1/_2) = -\pi.$$

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