Magneto-gravitational effect

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A magneto-gravitational effect of the following form is considered: a body rotating in a strong constant field of a magnetic dipole whose axis coincides with the axis of rotation generates electromagnetic radiation if the rotation is accompanied by a change of the gravitational potential with time. This effect is due to the fact that the equations of electrodynamics in a strong variable gravitational field have very different solutions from the nonrelativistic equations. Two cases are considered in which the gravitational field varies: *a*) an axisymmetric ellipsoid rotates around an axis that does not coincide with the symmetry axis; *b*) a triaxial ellipsoid rotates about a principal axis. In the second case, the radiation is very different from magnetic dipole radiation.

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1. INTRODUCTION

In the case of rotation of a body that has a strong meridional magnetic field, not only magnetic dipole radiation is emitted but also a further radiation due to an effect of the general theory of relativity; we call this magneto-gravitational radiation. It is similar in nature to a number of effects considered earlier by various people. Gertsenshteĭn[1] considered the generation of a gravitational wave by an electromagnetic wave propagating in a strong magnetic or electric field. Dubrovich[2] investigated the opposite effect of generation of an electromagnetic wave by a gravitational wave propagating in a constant magnetic field. Finally, Zel'dovich[3] considered a coupled electromagnetic-gravitational wave in which there is mutual generation of one type of wave by the other in the presence of a constant magnetic field.

The magneto-gravitational effect which we study here is as follows. Consider a body with a strong axisymmetric magnetic field. For simplicity, we shall consider a field of the form

$$\mathbf{H}^0(\mathbf{r})=[3\mathbf{r}(\boldsymbol{\mu}\mathbf{r})-r^2\boldsymbol{\mu}]/r^5,$$

where μ is the magnetic moment and $\mathbf{r}$ the radius vector. Suppose the body rotates about an axis that is not the axis of axial symmetry. This can occur in two cases.

a) The body has axial symmetry but the axis of symmetry does not coincide with the vector $\boldsymbol{\omega}$ of the angular velocity. Such noncoincidence of these directions can occur in a star because of nonsymmetric collapse or the influence of tidal forces in close binaries.

b) The body does not have axial symmetry; for example, it is a triaxial ellipsoid, which, as we assume for simplicity, rotates about one of the principal axes.

In these cases, the rotation of the body will be accompanied by a variation of the gravitational potential with the time, and if the relativistic parameter Φ/c^2 (Φ is the gravitational potential) and the angular velocity of rotation are sufficiently large, then, as we shall see, the variation of the gravitational potential in the magnetic field gives rise to the emission of electromagnetic waves. Such a situation may occur in neutron stars and, in particular, in pulsars. The electromagnetic radiation will consist of two parts: magnetic dipole radiation (if the magnetic moment does not coincide with the direction of the axis of rotation) and the radiation due to the magneto-gravitational effect. In what follows, we shall determine the ratio of the intensities of the two.

In this paper, we consider the simplest case when the direction of the magnetic moment and the axis of rotation coincide, and therefore, there is no magnetic dipole radiation. A tendency for this limiting case to be realized in a rotating figure does indeed exist. The meridional magnetic field is produced by circular currents. It is readily seen that the Coriolis force tends to arrange these currents in the plane perpendicular to the axis of rotation and, therefore, the magnetic moment along the direction of the axis of rotation. At the same time, the magnetic field produced by the currents themselves will not prevent their displacement since the displacement is parallel to the field. The time during which this tendency can be realized is inversely proportional to the magnetic field strength, the angular velocity ω, and the mean free time of the electrons in these currents. We shall not consider what are the conditions under which the time needed for the orientation of the currents is reasonably short but restrict ourselves to this qualitative argument, which shows that the limiting case we consider of no magnetic dipole radiation is not merely a formal model.

2. DERIVATION OF LINEARIZED EQUATIONS OF ELECTRODYNAMICS IN THE PRESENCE OF A VARIABLE MAGNETIC FIELD

Since the ratio of the gravitational radius to the radius of a star is less than unity for even a neutron star, in the solution of the problem we can consider small perturbations of a pseudo-Euclidean metric, linearizing the equations with respect to it; in addition, we can also assume that the generated electric and magnetic fields are small. The equations of electrodynamics in three-dimensional notation have the form[4]

$$\frac{\partial}{\partial x^\alpha}(\sqrt{\gamma}\,H^\alpha/\sqrt{g_{00}})=0, \tag{1}$$

$$e^{\alpha\beta\delta}\frac{\partial E_\delta}{\partial x^\beta}=-\frac{1}{c}\frac{\partial}{\partial t}(\sqrt{\gamma}\,H^\alpha/\sqrt{g_{00}}), \tag{2}$$

$$e^{\alpha\beta\delta}\frac{\partial H_\delta}{\partial x^\beta}=\frac{1}{c}\frac{\partial}{\partial t}(\sqrt{\gamma}\,E^\alpha/\sqrt{g_{00}}), \tag{3}$$

$$\frac{\partial}{\partial x^\alpha}(\sqrt{\gamma}\,E^\alpha/\sqrt{g_{00}})=0. \tag{4}$$

Here, the Greek indices take three values: 1, 2, 3; $e^{\alpha\beta\delta}$ is the completely antisymmetric unit tensor, γ is the determinant of the three-dimensional metric tensor $\gamma_{\alpha\beta}=-g_{\alpha\beta}$ (this equation is true only for $g_{0\alpha}=0$, and the metric of the post-Newtonian approximation, which we shall use in what follows, satisfies this condition). All raising and lowering of indices is done by means of the three-dimensional metric $\gamma_{\alpha\beta}$. In the approximation of a weak gravitational field, i.e., a metric that is nearly pseudo-Euclidean,[4]

$$g_{00}=1+2\Phi/c^2,\quad \gamma_{11}=\gamma_{22}=\gamma_{33}=1-2\Phi/c^2,$$
$$\sqrt{\gamma}=1-3\Phi/c^2,\quad \gamma^{11}=\gamma^{22}=\gamma^{33}=1+2\Phi/c^2.$$

Since Eqs. (1)–(4) contain both covariant and contravariant components of the electric and magnetic field vectors, we must make more precise what we understand by **E** and **H**. In accordance with the meaning of the problem under consideration, a strong electric field $\mathbf{E}_0$ independent of the gravitation is absent; therefore, because **E** is small, it is immaterial whether we identify it with E^α or E_α in the linear approximation. It is convenient to represent the field **H** in the form of the sum $\mathbf{H}^0+\mathbf{H}^1$, where $\mathbf{H}^0$ has the classical value already given in the introduction. In order to satisfy Eq. (4) in the stationary case, we must identify $\mathbf{H}^0$ with the covariant components of H^0_α. Thus, we finally obtain a system of equations linearized with respect to the gravitational field and the generated electromagnetic field, and this has the form

$$\operatorname{div}\mathbf{H}=2c^{-2}\mathbf{H}^0\nabla\Phi, \tag{5}$$

$$\operatorname{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}+\frac{2}{c^3}\mathbf{H}^0\frac{\partial\Phi}{\partial t}, \tag{6}$$

$$\operatorname{div}\mathbf{E}=0, \tag{7}$$

$$\operatorname{rot}\mathbf{H}=\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}. \tag{8}$$

Here, by **H** we understand the magnetic field generated in the effect we are considering.

To solve these equations by analogy with ordinary electrodynamics, we introduce pseudovector and pseudoscalar potentials $\tilde{\mathbf{A}}$ and Ψ in accordance with the definitions (with allowance for (8))

$$\mathbf{E}=\operatorname{rot}\tilde{\mathbf{A}},\quad \mathbf{H}=\frac{1}{c}\frac{\partial\tilde{\mathbf{A}}}{\partial t}+\nabla\Psi.$$

Substituting into (5) and (6), we obtain

$$\Delta\Psi+\frac{1}{c}\frac{\partial}{\partial t}\operatorname{div}\tilde{\mathbf{A}}=\frac{2}{c^2}\mathbf{H}^0\nabla\Phi,$$

$$\operatorname{grad}\operatorname{div}\tilde{\mathbf{A}}-\Delta\tilde{\mathbf{A}}=-\frac{1}{c^2}\frac{\partial^2\tilde{\mathbf{A}}}{\partial t^2}-\frac{1}{c}\nabla\frac{\partial\Psi}{\partial t}+\frac{2}{c^3}\mathbf{H}^0\frac{\partial\Phi}{\partial t}.$$

Imposing on the pseudopotentials an additional condition (analog of the Lorentz condition in ordinary electrodynamics)

$$\operatorname{div}\tilde{\mathbf{A}}+\frac{1}{c}\frac{\partial\Psi}{\partial t}=0, \tag{9}$$

we obtain for $\tilde{\mathbf{A}}$ and Ψ the wave equations

$$\Box\Psi=\frac{2}{c^2}\mathbf{H}^0\nabla\Phi,\quad \Box\tilde{\mathbf{A}}=-\frac{2}{c^3}\mathbf{H}^0\frac{\partial\Phi}{\partial t}. \tag{10}$$

Note that the right-hand sides of these equations satisfy a relation analogous to the continuity equation:

$$-\operatorname{div}\left(\mathbf{H}^0\frac{\partial\Phi}{\partial t}\right)+\frac{\partial}{\partial t}(\mathbf{H}^0\nabla\Phi)=0. \tag{11}$$

It automatically follows from $\operatorname{div}\mathbf{H}^0=\partial\mathbf{H}^0/\partial t=0$. It is precisely because the inhomogeneities of Eqs. (10) are related by (11) that we can assert that the retarded potentials obtained by integrating the wave equations satisfy the condition (9) (cf[5]).

3. GRAVITATIONAL POTENTIAL OF A HOMOGENEOUS ELLIPSOID SLIGHTLY DIFFERENT FROM A SPHERE

As is well known (see[6]), the potential is

$$\Phi(x,y,z)=\frac{3GM}{4}\int\limits_\lambda^\infty\left[1-\frac{x^2}{a^2+s}-\frac{y^2}{b^2+s}-\frac{z^2}{c^2+s}\right]\frac{ds}{\Delta(s)}$$
$$\Delta(s)=[(a^2+s)(b^2+s)(c^2+s)]^{1/2},$$

where x, y, z are the coordinates in the system of the principal axes of the ellipsoid; a, b, c are the corresponding semiaxes; λ is the positive root of the equation

$$\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}+\frac{z^2}{c^2+\lambda}=1.$$

Finally, M is the mass of the ellipsoid and G is the gravitational constant.

In what follows, we shall assume that the ellipsoid differs little from a sphere:

$$a^2=R_0^2(1+\alpha),\quad b^2=R_0^2(1+\beta),\quad c^2=R_0^2(1+\gamma),$$

where α, β, and γ are smaller than unity in absolute magnitude. In the linear approximation with respect to them, the potential is

$$\Phi=\frac{3GM}{4}\left[\frac{4}{3r}+\frac{2R_0^2}{5r^5}(\alpha x^2+\beta y^2+\gamma z^2)-\frac{2R_0^2}{15r^3}(\alpha+\beta+\gamma)\right],$$
$$r=(x^2+y^2+z^2)^{1/2}.$$

In case a) (see the introduction), i.e., for an ellipsoid of revolution, we set $\alpha=\beta$. For the transition to the coordinate system of an external observer, in which the ellipsoid rotates around an axis $\boldsymbol{\omega}$ that does not coincide with the symmetry axis, we must make two rotations of the coordinate system: through the angle θ_ω between $\boldsymbol{\omega}$ and the symmetry axis, and through the angle ωt, which we shall measure from the plane passing through the symmetry axis and $\boldsymbol{\omega}$. Since Φ is a three-dimensional scalar, only the coordinates in its arguments will change under these transformations. Omitting the terms that do not depend on the time, we obtain for the potential of the rotating ellipsoid in the linear approximation with respect to the "deformation" parameters α, β, γ the expression

$$\Phi=\frac{3}{10}GM\frac{R_0^2}{r^5}\Big[(\alpha\cos^2\theta_\omega+\gamma\sin^2\theta_\omega-\alpha)\frac{x^2-y^2}{2}\cos 2\omega t$$

$$+(\alpha\cos^2\theta_\omega+\gamma\sin^2\theta_\omega-\alpha)xy\sin 2\omega t+(\alpha-\gamma)\sin 2\theta_\omega(xz\cos\omega t+yz\sin\omega t)\Big].$$

Here, x, y, and z are the coordinates of the point at which the field is detected in the system of the external observer; the z axis of this coordinate system is along ω.

The exact theory of rotation of such a body about an axis that does not lie along the symmetry axis[7] leads to the conclusion that the rotation axis itself precesses around the symmetry axis with angular velocity proportional to the oblateness. Therefore, the influence of the precession on the variation of the gravitational potential is proportional to the square of the oblateness. Since we restrict ourselves to the approximation linear in the oblateness, we can ignore the precession of the axis of rotation.

Using an appropriate coordinate transformation, in case b) as well we can also readily obtain an expression for the gravitational potential in the system of the external observer:

$$\Phi=\frac{3}{10}GM\frac{R_0^2}{r^5}(\alpha-\beta)\left[\frac{x^2-y^2}{2}\cos 2\omega t+xy\sin 2\omega t\right].$$

4. MAGNETO-GRAVITATIONAL RADIATION IN MODELS a) AND b)

Substituting the expression for the gravitational potential of the corresponding model into (10), we obtain an expression for $\tilde{\mathbf{A}}$ in the form of a retarded potential defined at the point $\mathbf{R}$ at time t:

$$\tilde{\mathbf{A}}(\mathbf{R},t)=\frac{1}{2\pi c^3}\int\frac{\mathbf{H}^0(\mathbf{r})}{|\mathbf{R}-\mathbf{r}|}\frac{\partial}{\partial t}\Phi\left(\mathbf{r},t-\frac{|\mathbf{R}-\mathbf{r}|}{c}\right)d^3r.$$

In case a), we can ignore retardation. Then, integrating over r from R_0 to ∞ (which is justified because of the rapid decrease of the integrand), we obtain

$$\tilde{A}_x^{(1)}=-\frac{B}{R}\sin\omega\left(t-\frac{R}{c}\right),\quad \tilde{A}_y^{(1)}=\frac{B}{R}\cos\omega\left(t-\frac{R}{c}\right),\quad \tilde{A}_z^{(1)}=0,$$

$$B=\frac{(\alpha-\gamma)}{25}\frac{GM}{c^2R_0}\frac{\mu\omega}{c}\sin 2\theta_\omega.$$

The electric field is $\mathbf{E}=\operatorname{curl}\tilde{\mathbf{A}}$, so that

$$E_x^{(1)}=-\frac{B\omega}{Rc}\cos\vartheta\sin\omega\left(t-\frac{R}{c}\right),\quad E_y^{(1)}=\frac{B\omega}{Rc}\cos\vartheta\cos\omega\left(t-\frac{R}{c}\right),$$

$$E_z^{(1)}=\frac{B\omega}{Rc}\sin\vartheta\sin\omega\left(t-\frac{R}{c}\right).$$

Here, the x axis is chosen such that the direction to the observer $\mathbf{R}$ lies in the xz plane and forms an angle ϑ with the z axis.

Thus, in the direction $\mathbf{R}$ there propagate two waves of frequency ω; one of them is circularly polarized and the other is plane polarized and the direction of polarization is parallel to the axis of rotation. The angular distributions of these two fields are different. To determine the magnetic fields of the two waves, we can use Eq. (6). We note that the term associated with the variability of the gravitational potential produces a negligibly small contribution at great distances, and we can therefore use the ordinary Maxwell equation. This leads to the following expression for the Poynting vector:

$$S_{\rm mg}^{(1)}=\frac{c\mathbf{R}}{8\pi R}\left(\frac{B\omega}{Rc}\right)^2[1+\cos^2\vartheta-\sin^2\vartheta\cos 2\omega t].$$

Let us compare this expression with the Poynting vector for the magnetic dipole radiation of a system in which the magnetic moment $\boldsymbol{\mu}$ in the expression for $\mathbf{H}^0$ is inclined at angle θ to the axis of rotation:

$$S_{\rm md}=\frac{c\mathbf{R}}{8\pi R}\left(\frac{\mu\sin\theta\omega^2}{Rc^2}\right)^2[1+\cos^2\vartheta-\sin^2\vartheta\cos 2\omega t].$$

We arrive at the conclusion that the angular distributions of the radiation are the same and the ratio of the intensities is

$$\frac{S_{\rm mg}^{(1)}}{S_{\rm md}}=\left[\frac{(\alpha-\gamma)\sin 2\theta_\omega}{25\sin\theta}\frac{GM}{c^2R_0}\right]^2.$$

Thus, in this case the magneto-gravitational effect can be interpreted as changing the magnetic moment. In accordance with what we have said above, the magneto-gravitational effect is the more important compared with the magnetic dipole effect the smaller is the angle between the magnetic moment and the axis of rotation and the closer is the radius of the body to its gravitational radius.

We now turn to case b). Substituting the previously calculated potential of the gravitational field into the expression for the retarded potential for $\tilde{\mathbf{A}}$, we make in the argument of the gravitational potential the expansion

$$|\mathbf{R}-\mathbf{r}|=R-\mathbf{rn},\quad \mathbf{n}=\mathbf{R}/R$$

and ignore r in the denominator of the integrand. Expanding now $\Phi(t-R/c+\mathbf{r}\cdot\mathbf{n}/c)$ in a series in powers of $\mathbf{r}\cdot\mathbf{n}/c$ to $(\mathbf{r}\cdot\mathbf{n}/c)^2$ and integrating over r from R_0 to ∞, we obtain

$$\tilde{A}_x^{(2)}=\frac{F}{R}\cos\vartheta\sin\vartheta\sin 2\omega\left(t-\frac{R}{c}\right),$$

$$\tilde{A}_y^{(2)}=-\frac{F}{R}\cos\vartheta\sin\vartheta\cos 2\omega\left(t-\frac{R}{c}\right),$$

$$\tilde{A}_z^{(2)}=-\frac{2F}{3R}\sin^2\vartheta\sin 2\omega\left(t-\frac{R}{c}\right),$$

$$F=\frac{3GM}{175c^2R_0}\frac{(2\omega)^3R_0^2\mu}{c^3}(\alpha-\beta).$$

Here, as before, the radius vector of the point of observation $\mathbf{R}$ forms an angle ϑ with the axis of rotation and lies in the xz plane. The components of the electric field vector are

$$E_x^{(2)}=\frac{2\omega F}{cR}\sin\vartheta\cos^2\vartheta\sin 2\omega\left(t-\frac{R}{c}\right),$$

$$E_y^{(2)}=-\frac{2\omega F}{cR}\sin\vartheta\left(\cos^2\vartheta+\frac{2}{3}\sin^2\vartheta\right)\cos 2\omega\left(t-\frac{R}{c}\right),$$

$$E_z^{(2)}=-\frac{2\omega F}{cR}\cos\vartheta\sin^2\vartheta\sin 2\omega\left(t-\frac{R}{c}\right).$$

Therefore, in this case two waves of frequency 2ω are emitted in the direction of **R**: an elliptically polarized wave with ellipticity that depends on the angle ϑ, and a plane polarized wave with polarization parallel to the z axis. The magnetic field of the wave is determined in the same way as in the preceding case, and the Poynting vector is equal to

$$S=\frac{cn}{8\pi}\left(\frac{2\omega F}{cR}\right)^2\sin^2\vartheta\{\cos^2\vartheta+(\cos^2\vartheta+{}^2/_3\sin^2\vartheta)^2$$
$$+[(\cos^2\vartheta+{}^2/_3\sin^2\vartheta)^2-\cos^2\vartheta]\cos 4\omega(t-R/c)\}.$$

In this case, the magneto-gravitational radiation is very different from the magnetic dipole radiation as regards the frequency of the emitted waves, the angular distribution, the intensity of the radiation, and its dependence on the angular velocity of rotation.

This effect is important in the case of rapid rotation and in the presence of strong magnetic and gravitational fields. In accordance with the classical theory, rapid rotation leads to a departure from spherical shape of the rotating body and to its replacement by a Maclaurin ellipsoid of revolution and a triaxial Jacobi ellipsoid. It is this second case that we consider in model b). At even higher angular velocities, one can have more complicated equilibrium figures.[8] However, small deviations from these classical figures are unstable. In[9], Tsygan made the assumption that for sufficiently large velocities the deviations increase and may become stable. If this is true (though it should be noted that it has not yet been proved), the rotation of such a body, in which the radius vector of its surface as a function of the direction is characterized by the spherical function $Y_{lm}(\vartheta,\varphi)$, must lead to an increase in the frequency of the radiation in model b) in proportion to the quantum number m and of the intensity of the electromagnetic radiation by a factor m^8.

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Optical activity of heavy-metal vapors—a manifestation of the weak interaction of electrons and nucleons

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The feasibility of detecting parity nonconservation in atomic transitions by observing the rotation of the plane of polarization of light in heavy-metal vapors is discussed. The angle of rotation of the plane of polarization when the vapor temperature is 1200°C is $\sim 10^{-5}$ rad/m in thallium and lead and 10^{-7}–10^{-6} rad/m for various transitions in bismuth.

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1. INTRODUCTION

It has been noted recently that there is a fairly real possibility of detecting the weak interaction of electrons with nucleons by observing parity-nonconservation effects in atomic transitions. The first to draw attention to these effects was Zel'dovich as long ago as 1959,[1] and since then they have been discussed repeatedly by theorists[2–5] (cf. also[6,7]). An extremely important step was taken by Bouchiat and Bouchiat who pointed out in their note[3] that parity-nonconservation effects are enhanced in heavy atoms to the extent that their observation in induced doubly-forbidden $M1$ transitions lies on the borders of the possible.

It was recently pointed out that it is feasible to detect parity nonconservation in atomic transitions by the rotation of the plane of polarization of light[1)] in heavy-metal vapors[8–10] (see also the note[11], in which the analogous effect in the radio-frequency region is discussed). In the present paper we consider the question of near which transitions and in which chemical elements we must look for optical activity. We then calculate the effect in the elements that appear to be the most suit-

