emitted in the direction of **R**: an elliptically polarized wave with ellipticity that depends on the angle ϑ, and a plane polarized wave with polarization parallel to the z axis. The magnetic field of the wave is determined in the same way as in the preceding case, and the Poynting vector is equal to

$$S = \frac{cn}{8\pi} \left(\frac{2\omega F}{cR}\right)^2 \sin^2\vartheta \{\cos^2\vartheta + (\cos^2\vartheta + {}^2/_3 \sin^2\vartheta)^2$$
$$+ [(\cos^2\vartheta + {}^2/_3 \sin^2\vartheta)^2 - \cos^2\vartheta]\cos 4\omega(t - R/c)\}.$$

In this case, the magneto-gravitational radiation is very different from the magnetic dipole radiation as regards the frequency of the emitted waves, the angular distribution, the intensity of the radiation, and its dependence on the angular velocity of rotation.

This effect is important in the case of rapid rotation and in the presence of strong magnetic and gravitational fields. In accordance with the classical theory, rapid rotation leads to a departure from spherical shape of the rotating body and to its replacement by a Maclaurin ellipsoid of revolution and a triaxial Jacobi ellipsoid. It is this second case that we consider in model b). At even higher angular velocities, one can have more complicated equilibrium figures. [8] However, small deviations from these classical figures are unstable. In[9], Tsygan made the assumption that for sufficiently large velocities the deviations increase and may become

stable. If this is true (though it should be noted that it has not yet been proved), the rotation of such a body, in which the radius vector of its surface as a function of the direction is characterized by the spherical function $Y_{lm}(\vartheta, \varphi)$, must lead to an increase in the frequency of the radiation in model b) in proportion to the quantum number m and of the intensity of the electromagnetic radiation by a factor m^8.

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Optical activity of heavy-metal vapors—a manifestation of the weak interaction of electrons and nucleons

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The feasibility of detecting parity nonconservation in atomic transitions by observing the rotation of the plane of polarization of light in heavy-metal vapors is discussed. The angle of rotation of the plane of polarization when the vapor temperature is 1200°C is $\sim 10^{-5}$ rad/m in thallium and lead and 10^{-7}–10^{-6} rad/m for various transitions in bismuth.

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1. INTRODUCTION

It has been noted recently that there is a fairly real possibility of detecting the weak interaction of electrons with nucleons by observing parity-nonconservation effects in atomic transitions. The first to draw attention to these effects was Zel'dovich as long ago as 1959, [1] and since then they have been discussed repeatedly by theorists[2–5] (cf. also[6,7]). An extremely important step was taken by Bouchiat and Bouchiat who pointed out in their note[3] that parity-nonconservation effects are enhanced in heavy atoms to the extent that their observa-

tion in induced doubly-forbidden $M1$ transitions lies on the borders of the possible.

It was recently pointed out that it is feasible to detect parity nonconservation in atomic transitions by the rotation of the plane of polarization of light[1] in heavy-metal vapors[8–10] (see also the note[11], in which the analogous effect in the radio-frequency region is discussed). In the present paper we consider the question of near which transitions and in which chemical elements we must look for optical activity. We then calculate the effect in the elements that appear to be the most suit-

able: thallium, lead and bismuth. The original results were obtained by us in the summer of 1975 and are contained in the review[12]. In the autumn of the same year we became acquainted with the note[13], which gave results of a numerical calculation of the degree of circular polarization in bismuth that differed substantially from ours. In view of this, together with V. V. Flambaum we carried out careful numerical calculations, which led to a certain improvement in the accuracy of the original results. However, an appreciable discrepancy with the work of[13] remained. A discussion of the accuracy of our calculations and a comparison with the article[13] (and also with the recent paper[14]) are contained in Sec. 6 of our article. In the last section we discuss the present-day experimental possibilities and indicate the limits on external magnetic fields, which imitate the effect.

The calculations performed, while rather cumbersome and laborious, are entirely worthwhile, since searches for the optical activity under discussion are already being carried out by several experimental groups at the present time.[15,16,43]

2. IN WHICH ATOMS AND TRANSITIONS IS IT PROFITABLE TO LOOK FOR OPTICAL ACTIVITY?

We write the refractive index for right- and left-polarized quanta near a resonance at frequency ω_0 in the form

$$n = 1 - \frac{2\pi N \overline{|M_\pm|^2}}{\hbar} \left\langle \frac{1}{\Omega + i\Gamma/2} \right\rangle \qquad \Omega = \omega - \omega_0 - \frac{v}{c}\omega_0. \qquad (2.1)$$

Here N is the density of atoms of the medium, and Γ is the width of the excited level. For dipole transitions the operator $\hat{M}_\pm$ is equal to the corresponding projection of the dipole moment. The bar on top denotes summation of the square of the matrix element over the final polarizations and averaging over the initial polarizations of the atoms. The angular brackets denote averaging over v, the projection of the velocity of the atom on the direction of the light ray.

If parity is not conserved the matrix elements of $\hat{M}_\pm$ are not equal to each other and can be represented in the form

$$M_\pm = M \pm F M_1 = M(1 \pm P/2), \qquad (2.2)$$

where F is a dimensionless small parameter, M_1 is the admixed matrix element of the wrong parity, and P is the degree of circular polarization of the radiation, related as follows to the probabilities $w_\pm$ of emission of right- and left-polarized quanta:

$$P = \frac{w_+ - w_-}{w_+ + w_-} = \frac{|M_+|^2 - |M_-|^2}{|M_+|^2 + |M_-|^2} = 2F\frac{M_1}{M}. \qquad (2.3)$$

The plane of polarization of the light is rotated over a length l through the angle

$$\psi = \frac{\omega}{2c} l \operatorname{Re}(n_+ - n_-) = -\frac{2\pi N\omega}{\hbar c} lF(M \cdot M_1 + MM_1\cdot) \left\langle \frac{\Omega}{\Omega^2 + \Gamma^2/4} \right\rangle$$

$$= -\frac{2\pi N\omega}{\hbar c} l\overline{|M|^2} P \left\langle \frac{\Omega}{\Omega^2 + \Gamma^2/4} \right\rangle. \qquad (2.4)$$

In addition, the absorption coefficients for right- and left-polarized quanta are also found to be different:

$$\alpha_\pm = 2\frac{\omega}{c}\operatorname{Im} n_\pm = \frac{4\pi N\omega}{\hbar c}\overline{(|M|^2 \pm F\overline{MM_1})} \left\langle \frac{\Gamma/2}{\Omega^2 + \Gamma^2/4} \right\rangle$$

$$= \frac{4\pi N\omega}{\hbar c}\overline{|M|^2}(1 \pm P) \left\langle \frac{\Gamma/2}{\Omega^2 + \Gamma^2/4} \right\rangle. \qquad (2.5)$$

Therefore, the polarization of the light changes from linear to elliptical. The ratio of the small semi-axis of the ellipse to the large semi-axis is

$$\chi = \frac{\omega}{2c} l \operatorname{Im}(n_+ - n_-) = l(\alpha_+ - \alpha_-)/4. \qquad (2.6)$$

We emphasize that the quantities ψ and χ characterizing the parity-nonconservation effects in the given case are, unlike the degree of circular polarization P, proportional not to the ratio but to the product of the principal and admixed matrix elements. Therefore, unlike in the experiment proposed by Bouchiat and Bouchiat[3] to observe the circular polarization, it is not profitable to look for optical activity in the vicinity of a strongly forbidden $M1$ transition. Also unfavorable is the situation when the principal transition is allowed. For allowed transitions the absorption coefficient α is very large and, since the path length l cannot appreciably exceed α^{-1}, the obtainable rotation angle ψ and ellipticity χ turn out to be extremely small.

It is natural, therefore, to turn to the case in which the principal transition is an $M1$ transition and the admixture is $E1$. As is well-known, an $M1$ transition occurs without additional suppression only between terms belonging to the same electron configuration. To observe the small optical-activity effects it is also highly desirable that this transition from the ground state be in or near the visible part of the spectrum. Such a situation obtains in the heavy elements. Finally, the substance should have appreciable vapor pressure at a reasonable temperature. If, somewhat arbitrarily, we regard a pressure of ~ 10 mm as "appreciable" and a temperature of ~ 1200 °C as "reasonable," the range of suitable elements is narrowed to tellurium, iodine, europium, thallium, lead, bismuth and polonium.

Of all the elements listed, tellurium has the lowest nuclear charge ($Z = 52$) and, therefore, we should expect the smallest effect in it. This shortcoming is not compensated by any advantages. In tellurium the transitions under discussion lie in the infrared region ($\lambda = 21,048;\ 9,471$ Å). Moreover, in the vapor of this element there are incomparably more Te_2 molecules than free tellurium atoms. Absorption of light on account of the molecular component in tellurium vapor may turn out to be an additional complication.[2] Analogous considerations also apply to iodine ($Z = 53$, $\lambda = 13,152$ Å).

With regard to europium, the energies of the states belonging to the same electronic configuration as its ground state are not yet known.

Finally, polonium ($Z = 84$, $\lambda = 4,613;\ 5,941$ Å) pos-

sesses high radioactivity, which makes it very difficult to work with. This circumstance is made all the more annoying by the fact that, for polonium, the problem of a sufficiently powerful monochromatic light source is evidently solved (see below), since the second of the lines indicated above almost coincides with one of the lines of the helium-neon laser.

Thus, at the present time, the most suitable elements for the experiments under discussion are thallium (Z = 81, λ = 12, 833 Å), lead (Z = 82, λ = 12, 789 Å) and bismuth (Z = 83, λ = 8, 757; 6, 477; 4, 616; 3, 015 Å).

We shall discuss now the requirements to be imposed on the stability of the frequency and on the linewidth of the light source. We shall be interested in the situation in which the upper-level width Γ (arising from the resonant transfer of excitation in collisions with atoms in the ground state) is much smaller than the Doppler broadening $\Delta_D = \omega_0(2kT/Mc^2)^{1/2}$ (M is the mass of the atom). In order that ψ, the angle of rotation of the plane of polarization, be not too small, the detuning $\Delta = \omega - \omega_0$ should be comparable with Δ_D. But since ψ is an odd function of the detuning, the stability of the frequency and the linewidth of the source should also be at least comparable with the Doppler broadening, which in our case amounts to about $10^{-6} \omega_0$. This quantity is smaller than the hyperfine splitting, so that a transition will occur only between particular components of the hyperfine structure of the levels.

We note that the rotation angle ψ can be represented in the following form:

$$\psi = \alpha l P g / 2 f,$$
$$\alpha = \frac{1}{2}(\alpha_+ + \alpha_-) \approx \alpha_\pm, \quad g + i f = \left\langle \frac{\Delta_D}{\Omega + i\Gamma/2} \right\rangle. \quad (2.7)$$

Since the quantity αl clearly cannot be much greater than unity, the angle ψ differs from the degree of circular polarization P by only a numerical factor g/f. But the principal advantage of the experiment proposed is that it is incomparably easier, evidently, to measure small angles of rotation of the plane of polarization than to measure a small degree of circular polarization. An additional advantage for thallium and lead is that the factor g/f for them can amount to 50–70 in realizable conditions (see below).

We turn now to the calculation of the optical activity of vapors of thallium, lead and bismuth.

3. THALLIUM

The ground state of thallium is $6s^2 6p_{1/2}$. It is elementary to find the amplitude of the $6p_{1/2}$–$6p_{3/2}$ transition of interest to us. Its square, averaged over the polarizations of the lower state and summed over the polarizations of the upper state, is equal to

$$\overline{|M|^2} = 2\mu^2/9 = 0.222\mu^2, \quad (3.1)$$

where μ is the Bohr magneton.

In the situation of interest to us, when the hyperfine structure of the line is resolved, it is well-known that the probability is a maximum for the transition between the states with the largest values of the total angular momentum F of the atom. The optical activity is also found to be at its greatest at the same time, and, therefore, we shall confine ourselves to treating just this transition. For thallium, for which the nuclear spin $i = \frac{1}{2}$ for both the stable isotopes Tl²⁰³ and Tl²⁰⁵, we are concerned with the transition $F = 1 \rightarrow F' = 2$. The result of summing the square of the amplitude over the projections of F and F' and dividing by the total number of initial states, i.e., the quantity by which we must replace $|M|^2$ in formula (2.1), is written in the form

$$|\overline{M}|^2 = {}^5/_8 \overline{|M|^2} = 5\mu^2/36 = 0.139\mu^2. \quad (3.2)$$

We note that the above transition in thallium can also occur as an electric-quadrupole transition, and this leads to additional absorption of light. A numerical calculation gives the following value of the matrix element of the square of the valence-electron radius: $\langle P_{3/2} | r^2 | P_{1/2} \rangle = 19a^2$ (a is the Bohr radius). From this it is not difficult to obtain that the quadrupole absorption in thallium amounts to only 6%.

At a temperature of 1200 °C the vapor pressure of thallium is 100 mm.[17] We select the detuning Δ in such a way that in these conditions the absorption coefficient $\alpha = 1$ m⁻¹. If, for definiteness, we take it that the thallium–thallium scattering cross-section σ, leading to the broadening of the line, is 0.5×10^{-14} cm² (according to the experimental data of[18], $\sigma < 10^{-14}$ cm²), this value of α is attained with $\Delta = 5.3\Delta_D$.

We pass now to the calculation of the circular polarization that is induced by the weak, parity-nonconserving interaction of the electrons with the nucleons. Assuming infinitely heavy nucleons and a point nucleus, we write the Hamiltonian of the P-odd interaction of a relativistic electron with the nucleus in the form

$$H = -\frac{G\hbar^3}{\sqrt{2}\,c} Zq\delta(\mathbf{r})\gamma_5. \quad (3.3)$$

Here $G = 10^{-5}/m_p^2$ is the Fermi weak-interaction constant and m_p is the proton mass. The quantity q is related in the following way to the coupling constants G_p and G_n of the electron axial current with the proton and neutron vector currents:

$$GZq = ZG_p + (A - Z)G_n, \quad (3.4)$$

where Z and A are the charge and atomic number of the nucleus. The value of q depends on the choice of model; in the final analysis, it is this quantity which should be found in the experiment under discussion. For definiteness we shall use Weinberg's model[19] in the calculation. In this,

$$q = 1 - A/2Z - 2\sin^2\theta, \quad (3.5)$$

where the mixing angle θ is a free parameter. An analysis, in the framework of this model, of the neutrino experiment to study neutral currents[20,21] gives $\sin^2\theta$ = 0.32. In this case, for thallium, lead and bismuth, $q \approx -0.9$.

Because the interaction (3.3) is local, i.e., because of the presence of the δ-function in it, it leads to mixing of one-electron states $s_{1/2}$ and $p_{1/2}$ only. The corresponding matrix element is equal to[3)]

$$\langle s_{1/2}|H|p_{1/2}\rangle = i\frac{Gm^2\alpha^2 Z^3 Rq}{\pi\sqrt{2}}(v_s v_{p_{1/2}})^{-3/2}\frac{me^4}{2\hbar^2}. \qquad (3.6)$$

Here m is the electron mass and α is the fine-structure constant. We eliminate the divergence that arises in the calculation of the matrix element of $\delta(\mathbf{r})$ with relativistic wavefunctions by introducing a finite nuclear radius $r_0 = 1.2\times10^{-13}A^{1/3}$ cm. In this case the relativistic enhancement factor R is equal to

$$R = 4(2Zr_0/a)^{2\gamma-2}[\Gamma(2\gamma+1)]^{-2}, \quad \gamma = (1-Z^2\alpha^2)^{1/2}. \qquad (3.7)$$

It can be shown that more accurate allowance for the finite size of the nucleus has little effect on the quantity R. This enhancement factor increases rapidly at large Z and amounts to 8.5 for thallium.

The effective principal quantum numbers ν of the s and p states of the valence electron are determined from the spectrum of thallium[23]: $\nu_{6p_{1/2}} = 1.49$; $\nu_{7s} = 2.19$; $\nu_{8s} = 3.22$; $\nu_{9s} = 4.24$; $\nu_{10s} = 5.26$.

A simple calculation leads to the following result for the ground-state wavefunction containing an admixture of $ns_{1/2}$ states:

$$|6p_{1/2}\rangle + i\cdot10^{-10}[1.10|7s_{1/2}\rangle + 0.42|8s_{1/2}\rangle + 0.25|9s_{1/2}\rangle + 0.14|10s_{1/2}\rangle + \ldots]. \qquad (3.8)$$

In order to determine the contribution of these states to the circular polarization it remains to find the amplitudes of the E1 transitions $ns_{1/2}-6p_{3/2}$. Their moduli can be determined from the experimental data on the oscillator strengths in thallium.[24,25] Similar values are also given by a numerical calculation, from which the signs of these amplitudes can also be determined.[26] The radial integrals found in this way

$$\rho_n = \int_0^\infty dr r^3 R_{ns} R_{6p_{1/2}}$$

are $\rho_7 = 2.82a$, $\rho_8 = 0.66a$, $\rho_9 = 0.36a$ and $\rho_{10} = 0.28a$.

By an elementary calculation we find that the contribution of the indicated states to the circular polarization of the radiation in the $6p_{1/2}-6p_{3/2}$ transition amounts to 1.9×10^{-7}. A numerical calculation shows that the higher states of the discrete spectrum can be neglected, but the contribution of the continuous spectrum is found to be equal to 0.5×10^{-7}. Thus, the total contribution of excitations of the $6p$ electron to the circular polarization is $P' = 2.4\times10^{-7}$.

A large contribution to the effect is given by the states that belong to the configuration $6s6p^2$, which arise on excitation of a $6s$ electron. All these, apart from one, possess positive energy and thus are resonances in the continuous spectrum.

The wavefunctions of these states are calculated in the intermediate-coupling approximation. The matrix elements of the spin-orbit interaction and of the non-

spherical Coulomb interaction between the electrons for the configuration sp^2 are given, in particular, in the book by Condon and Shortley.[27] Solving the corresponding secular equations and then comparing with the experimentally known spectrum of these states[23,28] leads to the following values of the Slater integrals $F^2(1,1)$ and $G^1(0,1)$ and of the spin-orbit interaction parameter ζ (the notation from[27] is used):

$$\tfrac{1}{5}F^2(1.1) = 2100 \text{ cm}^{-1}, \quad \tfrac{1}{3}G^1(0.1) = 8800 \text{ cm}^{-1}, \quad \zeta = 5500 \text{ cm}^{-1}. \qquad (3.9)$$

The expansions of the exact wavefunctions in the Russell–Saunders wavefunctions have the following appearance:

$$|^2S_{1/2}\rangle' = 0.444|^2S_{1/2}\rangle + 0.894|^2P_{1/2}\rangle - 0.056|^4P_{1/2}\rangle, \quad 76\,804 \text{ cm}^{-1},$$
$$|^2P_{1/2}\rangle' = 0.846|^2S_{1/2}\rangle - 0.439|^2P_{1/2}\rangle - 0.302|^4P_{1/2}\rangle, \quad 67\,150 \text{ cm}^{-1},$$
$$|^4P_{1/2}\rangle' = 0.295|^2S_{1/2}\rangle - 0.087|^2P_{1/2}\rangle + 0.952|^4P_{1/2}\rangle, \quad 45\,220 \text{ cm}^{-1},$$
$$|^2P_{3/2}\rangle' = 0.977|^2P_{3/2}\rangle - 0.075|^4P_{3/2}\rangle + 0.201|^2D_{3/2}\rangle, \quad 81\,120 \text{ cm}^{-1},$$

$$|^4P_{3/2}\rangle' = 0.055|^2P_{3/2}\rangle + 0.993|^4P_{3/2}\rangle + 0.103|^2D_{3/2}\rangle, \quad 49\,826 \text{ cm}^{-1},$$
$$|^2D_{3/2}\rangle' = -0.207|^2P_{3/2}\rangle - 0.089|^4P_{3/2}\rangle + 0.974|^2D_{3/2}\rangle, \quad 62\,000 \text{ cm}^{-1},$$
$$|^4P_{5/2}\rangle' = 0.933|^4P_{5/2}\rangle - 0.359|^2D_{5/2}\rangle, \quad 53\,050 \text{ cm}^{-1},$$
$$|^2D_{5/2}\rangle' = 0.359|^4P_{5/2}\rangle + 0.933|^2D_{5/2}\rangle, \quad 64\,740 \text{ cm}^{-1}. \qquad (3.10)$$

Alongside the wavefunction we indicate the energy of the corresponding term, reckoned from the ground state. The positions of the terms $^2P'_{3/2}$ and $^2D'_{3/2}$ are not known experimentally, and calculated values of the energies are given for these. The center of the $6s6p^2$ band (i.e., the position of the terms under consideration when $F^2(1,1) = G^1(0,1) = \zeta = 0$) is $E = 71,500$ cm^{-1}.

The effective principal quantum numbers of a $6p_{1/2}$ electron in the configuration $6s6p^2$ $(\nu'_{6p_{1/2}})$ and of a $6s$ electron in the configuration $6s^26p$ (ν_{6s}), which are necessary for the calculation of the mixing, can be determined by regarding the corresponding electron as added to the TlIII ion in the $6s6p$ state. Using the positions of the centers of the corresponding bands (the intermediate-coupling approximation for the configuration sp is considered in[27]) and the magnitude of the fine-splitting parameter ζ, we find $\nu'_{6p_{1/2}} = 1.49$ and $\nu_{6s} = 1.00$. The fact that ν for a $6p$ electron is the same in the $6s^26p$ (see above) and $6s6p^2$ states is entirely natural, since the screening properties of the ns and np electrons are, crudely speaking, the same.

The radial integral ρ_6, which determines the amplitudes of the E1 transitions $6s^26p-6s6p^2$, is found from the experimental data on the photo-ionization of thallium in the neighborhood of the resonance level $^2D'_{3/2}$.[29,30 4)] It is equal to $\rho_6 = -1.8a$. We note that the numerical calculation of[26] leads to an appreciably larger value of $|\rho_6|$ $(\rho_6 = -2.9a)$. However, since it also overestimates ν_{6s} (1.08 instead of the experimental 1.00), and the quantity $|\rho_6|$ increases rapidly with ν_{6s}, we prefer to rely upon the experimental data, taking from the numerical calculation only the sign of the matrix element.

The subsequent calculation of the contribution of the $6s6p^2$ states to the circular polarization does not contain difficulties of principle but is extremely cumbersome if one proceeds directly. Therefore, we shall apply a comparatively compact method, using the formalism of second quantization. We introduce creation and annihilation operators a^+, a, b^+, b, c^+, c for the $6s_{1/2}$, $6p_{1/2}$

and $6p_{3/2}$ electrons. The mixing operator and the operator for the $E1$ transition with $\Delta J_z = 1$ have the following appearance:

$$\hat{H} = \langle s_{\frac{1}{2}} | H | p_{\frac{1}{2}} \rangle [a_\uparrow^+ b_\uparrow + a_\downarrow^+ b_\downarrow - b_\uparrow^+ a_\uparrow - b_\downarrow^+ a_\downarrow], \quad (3.11)$$

$$\hat{D}_+ = -\frac{ie}{\sqrt{3}} \Big[\rho \Big(c_{\frac{1}{2}}^+ a_\uparrow + \frac{1}{\sqrt{3}} c_{\frac{1}{2}}^+ a_\downarrow - a_\uparrow^+ c_{-\frac{1}{2}} - \frac{1}{\sqrt{3}} a_\uparrow^+ c_{-\frac{1}{2}} \Big)$$

$$+ \bar{\rho}(^2/_3)^{\frac{1}{2}} (b_\uparrow^+ a_\downarrow + a_\uparrow^+ b_\downarrow) \Big]. \quad (3.12)$$

The contribution of the $6s6p^2$ states to the admixture matrix element of the $E1$ transition can be written, obviously, as follows:

$$\sum_n \left\{ \frac{\langle 6s^2 6p_{\frac{1}{2}} | \hat{H} | n \rangle \langle n | \hat{D}_+ | 6s^2 6p_{\frac{1}{2}} \rangle}{E(6s^2 6p_{\frac{1}{2}}) - E_n} + \frac{\langle 6s^2 6p_{\frac{1}{2}} | \hat{D}_+ | n \rangle \langle n | \hat{H} | 6s^2 6p_{\frac{1}{2}} \rangle}{-E_n} \right\}. \quad (3.13)$$

Since the spacing between the terms of this configuration is much smaller than the average spacing between the $6s6p^2$ and $6s^2 6p$ configurations we can take the energy denominators outside the summation, having replaced E_n by a certain average value. Then, if we make use of the completeness condition, the expression (3.13) is brought to the form

$$\frac{\langle 6s^2 6p_{\frac{1}{2}} | \hat{H}\hat{D}_+ | 6s^2 6p_{\frac{1}{2}} \rangle}{E(6s^2 6p_{\frac{1}{2}}) - \bar{E}_{\frac{1}{2}}} + \frac{\langle 6s^2 6p_{\frac{1}{2}} | \hat{D}_+ \hat{H} | 6s^2 6p_{\frac{1}{2}} \rangle}{-\bar{E}_{\frac{1}{2}}}. \quad (3.14)$$

The states of $6s^2 6p$ are written, obviously, as follows (we confine ourselves to the values $J_z = +\frac{1}{2}$ and $J_z = +\frac{3}{2}$):

$$|6s^2 6p_{\frac{1}{2}}, J_z = \frac{1}{2}\rangle = b_\uparrow^+ a_\uparrow^+ a_\downarrow^+ |0\rangle, \quad |6s^2 6p_{\frac{3}{2}}, J_z = \frac{3}{2}\rangle = c_{\frac{3}{2}}^+ a_\uparrow^+ a_\downarrow^+ |0\rangle.$$

$$(3.15)$$

The calculation of the amplitude (3.14) is carried out trivially using (3.11), (3.12) and (3.15) and leads to the result

$$ie\rho_6 \frac{1}{\sqrt{2}} \langle 6s_{\frac{1}{2}} | H | 6p_{\frac{1}{2}} \rangle \frac{1}{E(6s^2 6p_{\frac{1}{2}}) - \bar{E}_{\frac{1}{2}}}. \quad (3.16)$$

The second term in (3.14) vanishes. In other words, the states of $6s6p^2$ with total angular momentum $\frac{1}{2}$ do not give a contribution to the circular polarization, if we neglect their splitting. Using angular-momentum coupling in the jj-scheme, we can convince ourselves of this without second quantization, and even with almost no calculations. Taking for $\bar{E}_{3/2}$ the arithmetic-mean value 64,350 cm^{-1}, we find the contribution of the $6s6p^2$ states to the circular polarization: $P'' = 1.7 \times 10^{-7}$. The contribution of excitations of an electron from deeper shells amounts to less than one per cent, according to numerical calculations.

Thus, the total circular polarization of the radiation in the $6p_{1/2}$–$6p_{3/2}$ transition in thallium is equal to

$$P_{Tl} = P' + P'' = 4.1 \cdot 10^{-7}. \quad (3.17)$$

Using formula (2.7) it is now not difficult to obtain that, for a detuning $\Delta = 5.3\Delta_D$, the rotation of the plane of polarization in thallium is

$$\psi/l = 1.04 \cdot 10^{-5} \text{ rad/m}. \quad (3.18)$$

4. LEAD

The ground state and first excited state of lead belong to the configuration $6s^2 6p^2$ and possess total angular momenta 0 and 1, respectively. The calculation of the p^2 configuration in the intermediate-coupling approximation is well-known, [27,31-34] and we shall not dwell on it.

The square of the amplitude of the $^3P_0'$–$^3P_1$ transition of interest to us, summed over the polarizations of the upper level, is equal to

$$|\bar{M}|^2 = 0.572 \, \mu^2. \quad (4.1)$$

We shall confine ourselves to examining an even isotope of lead, e.g., Pb^{208}, in which, because of the absence of hyperfine structure, the rotation of the plane of polarization will be greatest.

The vapor pressure of lead at temperature 1200 °C is 17 mm. [17] We do not know of experimental data on collisional line-broadening for lead. If, for definiteness, we assume that the lead–lead scattering cross-section giving rise to the line-broadening is, as in thallium, equal to 0.5×10^{-14} cm^2, the absorption coefficient α is 1 m^{-1} when the detuning $\Delta = 2.6\Delta_D$.

The calculation of the admixture of states belonging to the $6p7s$ and $6p8s$ configurations does not give rise to difficulties, since they are all well described by the jj-coupling approximation. The necessary effective principal quantum numbers are as follows: $\nu_{6p_{1/2}} = 1.35$; $\nu_{7s} = 2.11$; $\nu_{8s} = 3.13$. When these states are admixed, the wavefunctions of interest to us are the following:

$$|6p^2, {}^3P_0\rangle' + i \cdot 10^{-10} [-1.52 |6p7s, {}^3P_0\rangle - 0.61 |6p8s, {}^3P_0\rangle],$$
$$|6p^2, {}^3P_1\rangle + i \cdot 10^{-10} [0.93 |6p7s, {}^1P_1\rangle + 0.39 |6p8s, {}^1P_1\rangle]. \quad (4.2)$$

The radial integrals needed for the calculation are found numerically[26]: $\rho_7 = 2.6a$; $\rho_8 = 0.9a$. These values agree well with the experimental data on the oscillator strengths.[35]

The total contribution from the admixture of $6p7s$ and $6p8s$ states to the circular polarization of the radiation in the $6p^2, {}^3P_0'$–$6p^2, {}^3P_1$ transition is equal to 1.3×10^{-7}. According to numerical calculations the contribution of higher excitations of a $6p$ electron is equal to 0.5×10^{-7} (and is due, principally, to the continuous spectrum). Thus, the total contribution of excitations of a $6p$ electron to the circular polarization is $P' = 1.8 \times 10^{-7}$.

We turn now to the estimate of the contribution of the states belonging to the $6s6p^3$ configuration, which have not been observed experimentally at all. We shall take the effective quantum number of a $6p$ electron in this configuration to be the same as in the $6s^2 6p^2$ configuration ($\nu_{6p_{1/2}}' = 1.35$), just as is the case in thallium. We find ν_{6s} for a $6s$ electron in the $6s^2 6p^2$ configuration by regarding it as added to the PbII ion in the state $6s6p^2$. The center of the $6s6p^2$ band of the PbII ion is $E_0 = 93,550$ cm^{-1} (measured from the ground state of the ion). Then $\nu_{6s} = 0.89$.

For the corresponding radial integral a numerical calculation gives the value $-2.6a$, thus overestimating

ν_{6s} (0.98 instead of the experimental 0.89). Inasmuch as the analogous calculation in thallium overestimated the quantity $|\rho_6|$ by a factor of 1.6 compared with the experimental value, it seems reasonable to us to reduce the calculated value correspondingly in this case also. We shall take $\rho_6/a = -2.6/1.6 = -1.6$.

Now, using the second-quantization formalism, just as was done for thallium, it is not difficult to show that the contribution of the $6s6p^3$ states to the circular polarization amounts to $P'' = 1.3 \times 10^{-7}$.

Thus, the total circular polarization in the $6p^2$, $^3P_0'$–$6p^2$, $^3P_1$ transition in lead is equal to

$$P_{\text{Pb}} = P' + P'' = 3.1 \cdot 10^{-7}. \qquad (4.3)$$

With detuning $\Delta = 2.6\Delta_D$, the rotation of the plane of polarization in an even isotope of lead is

$$\psi/l = 1.15 \cdot 10^{-5} \text{ rad/m}. \qquad (4.4)$$

5. BISMUTH

The ground state and first excited states of bismuth belong to the configuration $6s^2 6p^3$. Their scheme is presented in Fig. 1. From the ground state an $M1$ transition is possible to each of the four excited states. The calculation of the configuration p^3 in the intermediate-coupling approximation is also well known. [27,31,32,36]

For a transition between given components of the hyperfine structure, the square of the amplitude, summed over the projections of F and F' and divided by the total number of initial states, has the form

$$|\tilde{M}|^2 = \frac{(2F+1)(2F'+1)}{2i+1} \left\{ \begin{matrix} i & J & F \\ 1 & F' & J' \end{matrix} \right\}^2 \overline{|M|^2}. \qquad (5.1)$$

The curly brackets here denote a $6j$-symbol. The nuclear spin of bismuth is $i = \frac{9}{2}$. As before, we shall be interested in transitions between states with the maximum values of F and F'. For these transitions the quantities $|\tilde{M}|^2$ are[5]

$$|\tilde{M}_1|^2 = \frac{91}{400}\overline{|M_1|^2} = 0.056\mu^2, \quad F = F' = 6,$$

$$|\tilde{M}_2|^2 = \frac{1}{4}\overline{|M_2|^2} = 0.007\mu^2, \quad F = 6, \quad F' = 7,$$

$$|\tilde{M}_3|^2 = \frac{13}{40}\overline{|M_3|^2} = 0.010\mu^2, \quad F = 6, \quad F' = 5, \qquad (5.2)$$

$$|\tilde{M}_4|^2 = \frac{91}{400}\overline{|M_4|^2} = 0.0007\mu^2, \quad F = F' = 6.$$

The small value of the amplitude of the last transition arises from the mutual cancellation of comparatively large terms, each of which is not very accurat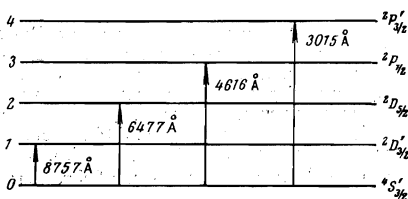ely calculated. Therefore, it is most likely that all the results for the fourth transition are true only in order of magnitude.

All the transitions under consideration in bismuth can also occur as electric-quadrupole transitions. Using the calculated value $\langle r^2 \rangle = 11a^2$, we can show that the quadrupole absorption is negligibly small in all transitions except the second, in which, for $F = 6$ and $F' = 7$, it amounts to 20% of the magnetic-dipole absorption.

According to the handbook[17], the partial vapor pressure of atomic bismuth at temperature 1200 °C is 23 mm. (According to the same data, the partial pressure of Bi_2 and more complicated molecules is equal to 24 mm at this temperature.) If, being guided by the experimental data on thallium, [18] we take for bismuth $\sigma \lesssim 10^{-14}$ cm^2, the absorption coefficient $\alpha = 1$ m^{-1} at detunings $\Delta_1 = 2.0\Delta_D$, $\Delta_2 = 1.4\Delta_D$, $\Delta_3 = 1.5\Delta_D$. But for the fourth transition, even for $\Delta = 0$ the absorption coefficient is 0.5 m^{-1}, so that here it is advantageous to take $\Delta = 0.9\Delta_D$; at this point the rotation angle ψ_4 is a maximum.

We proceed to the calculation of the circular polarization. The closest levels of opposite parity belong to the configuration $6p^2 7s$. They can be calculated in the intermediate-coupling approximation (cf., e.g., [37]). A simpler method of calculation, leading to similar results, is based on the assumption that the $7s$ electron is added to the $6p^2$ configuration of the BiII ion without changing the state of the latter. The effective quantum numbers of the $6p$ and $7s$ electrons are $\nu_{6p1/2} = 1.23$, $\nu_{7s} = 2.00$. With admixture of $6p^2 7s$ states, the wavefunctions of the $6p^3$ configuration are as follows:

$$|^4S_{3/2}'\rangle - i \cdot 10^{-10}[1.04|^4P_{3/2}'\rangle + 1.00|^2P_{3/2}'\rangle - 0.09|^2D_{3/2}'\rangle],$$

$$|^2D_{3/2}'\rangle - i \cdot 10^{-10}[0.42|^4P_{3/2}'\rangle + 0.22|^2P_{3/2}'\rangle + 0.90|^2D_{3/2}'\rangle],$$

$$|^2D_{5/2}\rangle + i \cdot 10^{-10}[0.33|^4P_{5/2}'\rangle + 0.94|^2D_{5/2}'\rangle]. \qquad (5.3)$$

$$|^2P_{3/2}\rangle - i \cdot 10^{-10}[0.85|^4P_{3/2}'\rangle - 0.95|^2S_{3/2}'\rangle],$$

$$|^2P_{1/2}'\rangle + i \cdot 10^{-10}[0.63|^4P_{1/2}'\rangle + 0.50|^2P_{1/2}'\rangle - 0.13|^2D_{1/2}'\rangle].$$

In bismuth, unlike thallium and lead, to calculate the effect it is necessary to know not only the radial integral

$$\rho_7 = \int_0^\infty dr\, r^3 R_{7s} R_{6p\,1/2}$$

but also the quantity

$$\bar{\rho}_7 = \int_0^\infty dr\, r^3 R_{7s} R_{6p1/2}.$$

A numerical calculation[26] leads to the values $\rho_7 = 2.2a$, $\bar{\rho}_7 = 1.5a$, which agree with the experimental data on the lifetimes of the excited states of bismuth. [38,39]

The contribution of the admixture of $6p^2 7s$ states to the circular polarization of the radiation for the $M1$ transitions being discussed is as follows (in the order of the positions of the levels): 1.1×10^{-7}; 1.0×10^{-7}; 3.1×10^{-7}; 4.2×10^{-7}. For the contribution of higher excitations of a $6p$ electron, including the continuous spec-

FIG. 1.

trum, a numerical calculation gives 0.7×10^{-7}; 0.8×10^{-7}; 1.3×10^{-7}; 2.1×10^{-7}.

Thus, the total contribution of excitations of a $6p$ electron to the circular polarization is

$$P_1' = 1.8 \cdot 10^{-7}, \quad P_2' = 1.8 \cdot 10^{-7}, \quad P_3' = 4.4 \cdot 10^{-7}, \quad P_4' = 6.3 \cdot 10^{-7}.$$

The states corresponding to excitation of a $6s$ electron and belonging to the configuration $6s6p^4$, like the analogous states in lead, have never been observed in bismuth. However, it turns out to be possible to take them into account. For a $6p$ electron in this configuration we shall take the same value of ν as in the $6s^2 6p^3$ configuration: $\nu'_{6p_{1/2}} = 1.23$. The experimental data on the spectrum of BiII are insufficient for an unambiguous determination of ν_{6s}. Therefore, we shall use the calculated value 0.9, incorporating the same correction -0.09 as in lead: $\nu_{6s} = 0.81$. Finally, we estimate ρ_6 by incorporating the correction factor 0.62 into the calculated value $-2.41a$ (as in thallium and lead): $\rho_6 = -1.5a$. Now it is already straightforward to calculate the contribution of the states of $6s6p^4$. For the different transitions it is equal to

$$P_1'' = 1.5 \cdot 10^{-7}, \quad P_2'' = 2.4 \cdot 10^{-7}, \quad P_3'' = 3.1 \cdot 10^{-7}, \quad P_4'' = 5.3 \cdot 10^{-7}.$$

The final results for the circular polarization of the radiation in the transitions discussed and for the optical activity of bismuth vapor are presented in Table I. In it we also give the analogous data (see above) for thallium and lead.

The substantially smaller angles of rotation in bismuth vapor are due to the lower vapor pressure (as compared with thallium), the smaller transition amplitudes (as compared with lead), and, finally, the larger spin of the bismuth nucleus, which leads to the complicated hyperfine structure.

6. DISCUSSION OF THE ACCURACY OF THE CALCULATION. COMPARISON WITH OTHER RESULTS

The results of different calculations of the effect differ appreciably from each other. For convenience of comparison, they are brought together in Table II.

The appreciable difference between the results of the present work and our original results arises, primarily, for the following reasons. First, the numerical calculations have made it possible to take into account the contribution of high excitations of a $6p$ electron including the continuous spectrum. Secondly, they have shown that for the effective principal quantum number of a $6p_{1/2}$ electron in formula (3.6) we must use $\nu_{6p_{1/2}}$ directly,

TABLE I.

Atom	J	F	J'	F'	λ, Å	$\lvert \bar{M} \rvert^2/\mu^2$	$P \cdot 10^7$	$\sigma \cdot 10^{14}$, cm^2	Δ/Δ_D	$(\psi/l) \cdot 10^7$, rad/m
Tl	$1/2$	1	$3/2$	2	12832.8	0.139	4.1	0.5	5.3	104
Pb^{208}	0	0	1	1	12788.93	0.572	3.1	0.5	2.6	115
Bi	$3/2$	6	$3/2$	6	8757.45	0.056	3.3	$\leqslant 1$	2.0	26
			$5/2$	7	6477.23	0.007	4.2	$\leqslant 1$	1.4	6
			$1/2$	5	4616.39	0.010	7.5	$\leqslant 1$	1.5	15
			$3/2$	6	3015.22	0.0007	11.6	$\leqslant 1$	0.9	2

TABLE II. Results of the calculations of the circular polarization $P \cdot 10^7$ in various papers.

Atom	This paper	Our original result[12]	Our original result[13]*	[14]*
Tl	4.1	3.1		−5.1
Pb	3.1	2.1		
Bi	3.3	2.0	7.0	6.9
	4.2	2.3	9.2	
	7.5	4.5	−12.4	
	11.6	7	19.2	

*Quantities equal to $P/2$ were used in the papers[13,14].

and not the value of ν_{6p} obtained after averaging over the fine structure, as was done previously.

The numerical solution of the Dirac equation, by means of which our results were obtained, was carried out with the two-parameter potential

$$V(r) = -\frac{2}{r} \left\{ \frac{Z-1}{H(e^{r/d}-1)+1} + 1 \right\}, \tag{6.1}$$

proposed in[41] (Z is the nuclear charge). Starting from the requirement that we have the best fit for the terms of the ground configuration $6s^2 6p^k$, including the fine structure, and also for the configuration $6s^2 6p^{k-1} ns$, the following parameter values were chosen:

$$H_{\text{Tl}} = 15.041; \quad H_{\text{Pb}} = 14.045; \quad H_{\text{Bi}} = 12.049; \quad d_{\text{Tl}} = 1.580; \quad d_{\text{Pb}} = 1.538; \quad d_{\text{Bi}} = 1.401. \tag{6.2}$$

With these values, as already noted above, the calculated values of the radial integrals for the excitations of a $6p$ electron agree well with the available experimental data. This means that the valence-electron functions obtained behave correctly at large distances. The behavior of the wavefunctions at short distances has been checked by a calculation[41] of the hyperfine structure of thallium, lead and bismuth with allowance for mixing of configurations. These calculations also lead to good agreement with experiment. Somewhat less reliable is the calculation of the contribution to the effect from the $6s6p^{k+1}$ configuration, inasmuch as we have chosen the values of the radial integrals ρ_6 on the basis of a single experiment on thallium, [29,30] and the numerical calculations are not in agreement with this experiment. From our point of view, the question of the magnitude of these integrals can only be resolved convincingly by experiment, e.g., by an exact measurement of the polarizability of the atoms of thallium, lead and bismuth (cf. the calculation in[26]) or by a direct determination of the oscillator strengths. We note that if the values obtained by the numerical calculation for the radial integrals turned out to be correct, our predictions for the effect would increase by 30–40%.

An indubitable source of error in our calculation (and, incidentally, in the calculations of[13,14]) is the use of the single-particle approximation. However, as estimates based on a calculation of the hyperfine structure show,[41] allowance for mixing of configurations can hardly change the answer by more than 15–20%.

As regards the discrepancies between our results and the results of[13,14], we find it difficult to point to their origin because of the absence of a sufficiently detailed account of the numerical calculations in the indicated articles. We note only that the procedure used in these papers assumes the same effective potential for the electrons both in the normal configurations ($6s^2 6p^k$ and $6s^2 6p^{k-1} ns$) and in the configuration $6s 6p^{k+1}$. However, the possibility of describing these states by means of a single effective potential is by no means obvious. In any case, using the potential (6.1) (the same form of potential was taken in[13]) we have not been able to fit simultaneously all the energy levels, including the fine structure, needed for the calculation. In conclusion, we remark that we have no doubts about the correctness of our results for the signs of the effect (see Table II) for all the transitions.

7. EXPERIMENTAL POSSIBILITIES. BOUNDS ON STRAY EXTERNAL MAGNETIC FIELDS

Experiments with bismuth are already being performed at the present time.[15,16,43] In view of this we should like to draw attention to the fact that, under reasonable conditions, the angle of rotation in thallium and lead vapors can be substantially greater than in bismuth, despite the fact that the degrees of circular polarization for these three atoms are close (see Table I). The corresponding transitions lie in a range that can be covered by tunable parametric generators.

Moreover, unlike bismuth, thallium and lead have stable isotopes. By comparing the effect in different isotopes (the corresponding difference is small $\sim 1/Z$), one could in principle determine not only the quantity q, which is the result of averaging the weak interaction over all the nucleons of the nucleus, but also the neutron and proton weak-interaction constants G_n and G_p themselves (cf. (3.4)). Obviously, the inaccuracy of the atomic calculations does not impede the extraction of such information from experiments with isotopes of one element.

Finally, by comparing the effect in transitions between different hyperfine components (these differences are also relatively small: $\sim 1/Z$, cf.[11,12]), one could in principle determine also the coupling constants of the electron vector current with the proton and neutron axial currents by using, respectively, thallium and an odd isotope of lead.

We now turn our attention to the rather complicated problem of how to eliminate a stray external magnetic field, which, as is well known, also leads to rotation of the plane of polarization of light. There exist several mechanisms by which a magnetic field induces optical activity. We shall discuss the two most dangerous.

First, an external magnetic field leads to mixing of different hyperfine states. Owing to this effect, the quantities $|\tilde{M}_\pm|^2$ acquire the following factors:

Thallium:

$$1 \mp g \mu H / 4 \Delta E, \quad \Delta E = 0.018 \text{ cm}^{-1}, \quad g = \text{⅓}. \tag{7.1}$$

The field sufficient to imitate the effect is found from the condition $g \mu H / 4 \Delta E = P$ and is $H_1 = 0.5 \times 10^{-3}$ G.

Bismuth:

$$
\begin{aligned}
&1) \quad 1 \pm \text{³/₄} (g_0 / \Delta E_0 - g_1 / \Delta E_1) \mu H, \quad H_1 = 0.8 \cdot 10^{-3} \text{ G}; \\
&2) \quad 1 \mp 9 g_2 \mu H / 14 \Delta E_2 \mp 0.34 g_0 \mu H / \Delta E_0, \quad H_1 = 1.6 \cdot 10^{-3} \text{ G}; \\
&3) \quad 1 \pm 3 g_0 \mu H / 4 \Delta E_0, \quad H_1 = 1.3 \cdot 10^{-3} \text{ G}; \\
&4) \quad 1 \pm \text{³/₄} (g_0 / \Delta E_0 - g_1 / \Delta E_1) \mu H, \quad H_1 = 1.2 \cdot 10^{-3} \text{ G}; \\
&\Delta E_0 = -0.10 \text{ cm}^{-1}; \quad \Delta E_1 = -0.26 \text{ cm}^{-1}; \quad \Delta E_2 = 0.59 \text{ cm}^{-1}; \\
&\Delta E_4 = 0.12 \text{ cm}^{-1}; \quad g_0 = 1.63; \quad g_1 = 1.24; \quad g_2 = 1.20; \quad g_4 = 1.26.
\end{aligned}
\tag{7.2}
$$

In these formulas the quantities ΔE_i are the hyperfine-structure intervals between the components being mixed, and their values are taken from[42,36]; i is the label of the electronic term in bismuth (see Fig. 1); g_i is the gyromagnetic ratio for this term. In the second transition in bismuth the electric-quadrupole contribution has been taken into account. The point is not only that it is appreciable in itself (20%), but that in it the magnetic-field correction is enhanced in comparison with the same correction in the $M1$ transition, inasmuch as $|\Delta E_0| \ll |\Delta E_2|$. In an even isotope of lead there is no hyperfine structure, and mixing of fine-structure components leads to weak restrictions and imitates the effect only at fields of the order of several gauss.

Secondly, a mechanism that leads to more stringent restrictions on the magnetic field is the difference in the resonance frequencies for right- and left-polarized quanta that arises because of the Zeeman splitting of the lines by a longitudinal field. The extra factors appearing in $|\tilde{M}_\pm|^2$ on account of the Zeeman effect are as follows.

Thallium:

$$1 \pm g \mu H \varepsilon / \Delta_D, \quad \varepsilon = \varepsilon(\Delta) = \Delta_D \partial \ln g(\Delta) / \partial \Delta. \tag{7.3}$$

The function $g(\Delta)$ is defined in formula (2.7). The field sufficient to imitate the effect is $H_2 = 5.9 \times 10^{-5}/\varepsilon$ G.

Lead:

$$1 \pm g \mu H \varepsilon / \Delta_D, \quad g = \text{³/₂}, \quad H_2 = 4.0 \cdot 10^{-5}/\varepsilon \text{ G}. \tag{7.4}$$

Bismuth:

$$
\begin{aligned}
&1) \quad 1 \pm \frac{1}{8}(g_0 + g_1) \frac{\mu H}{\Delta_D} \varepsilon, \quad && H_2 = 28 \cdot 10^{-5}/\varepsilon \text{ G}, \\
&2) \quad 1 \pm \left(\frac{10}{7} g_2 - \frac{3}{4} g_0 \right) \frac{\mu H}{\Delta_D} \varepsilon, \quad && H_2 = 33 \cdot 10^{-5}/\varepsilon \text{ G}, \\
&3) \quad 1 \pm \left(\frac{7}{8} g_0 - \frac{1}{4} g_3 \right) \frac{\mu H}{\Delta_D} \varepsilon, \quad && H_2 = 32 \cdot 10^{-5}/\varepsilon \text{ G}, \quad g_3 = 0.67, \\
&4) \quad 1 \pm \frac{1}{8}(g_0 + g_4) \frac{\mu H}{\Delta_D} \varepsilon, \quad && H_2 = 270 \cdot 10^{-5}/\varepsilon \text{ G}.
\end{aligned}
\tag{7.5}
$$

We note that, unlike the rotation that arises because of parity nonconservation, the rotation of the plane of polarization in a magnetic field as a result of the second mechanism is an even function of the detuning Δ. This fact can be used effectively in efforts to control this background field.

Flambaum, with whom we carried out the numerical calculations. We are sincerely grateful to W. K. Martin and E. Schreider for information on the spectra of the rare-earth elements. E. Schreider also drew our attention to the theoretical possibility of using polonium in the experiments discussed.

1)The fact that parity nonconservation leads to the appearance of optical activity was first noted by Zel'dovich.[1]

2)It should be noted that searches for parity nonconservation in the electronic spectra of molecules containing heavy atoms are also of great interest. However, reliable estimation of the effect here is extremely difficult. Therefore, we do not consider the question of parity nonconservation in molecular transitions.

3)The derivation of formula (3.6) is described in[22,12], and in the second of these papers a comparatively simple qualitative explanation of the rapid growth of the mixing with nuclear charge Z is also given.

4)We have not been able to establish correspondence between the values given in[30] for the oscillator strength f and the radial integral. We shall use the value $f = 0.39$, which certainly corresponds to the cross-section data given in[29] when the correction indicated in[30] is made for the error in the determination of the density.

5)We note that, at least in the Russian translation of the article[31] (we have not been able to acquaint ourselves with the original), the lower halves of certain columns (for $\chi > 1$) in the table for the total strengths of the transitions between terms of the $6p^3$ configuration are interchanged, making this table useless for calculating the transitions of interest to us.

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