Calculation of rates of formation of μ-mesic hydrogen molecules

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The rates of formation of the $pp\mu$, $pd\mu$, $pt\mu$, $dd\mu$, $dt\mu$ and $tt\mu$ mesic molecules have been calculated and also their dependence on the energy of collision of the mesic atom has been investigated. Good agreement has been obtained between the theoretical and the experimental values of the rates of formation of the $pp\mu$ and $pd\mu$ mesic molecules.

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INTRODUCTION

The process of formation of μ-mesic molecules is a part of the many-branched chain of mesic atomic processes which occur when μ^--mesons are stopped in a mixture of isotopes of hydrogen. The final stages of this chain of processes are the decay of the μ^--meson, or its capture by the proton of the nucleus of one of the hydrogen isotopes, or the reaction of the fusion of the nuclei of the mesic molecule formed.[1–3] The probabilities of the last two processes depend in an essential manner on the rates of formation of the mesic molecules. Moreover a knowledge of these rates is required for precision measurements of the weak interaction constants in the rections $\mu^- + p \to n + \tilde{\nu}_\mu$ and $\mu^- + d \to 2n + \tilde{\nu}_\mu$ involving hydrogen nuclei. The advantage of such measurements in comparison with similar measurements on complex nuclei consists of the fact that they are free of indefiniteness due to an insufficient knowledge of nuclear structure.

Until recently notable disagreements occurred between calculated[1,4,5] and measured[6–10] values of the rates of formation of the $pd\mu$ mesic molecules. In the present work these disagreements have been eliminated. Moreover, in this paper we have calculated the rates of formation of the $pp\mu$, $pt\mu$, $dd\mu$, $dt\mu$ and $tt\mu$ mesic molecules.

The mechanism of formation of μ-mesic molecules of hydrogen was discussed in detail in earlier works.[1–5] As is well known, electric dipole $E1$-transitions play a fundamental role in this case, and as a result of this the $pp\mu$, $pd\mu$ and $pt\mu$ mesic molecules which exist only in rotational states with orbital angular momenta $L = 0$ and $L = 1$ can be formed only from the s- p-and d-states describing the scattering of $p\mu$, $d\mu$ and $t\mu$ mesic atoms by the protons of the H_2 molecules. In particular, in the case of slow ($\varepsilon \lesssim 0.1$ eV) collisions transitions from the s-states of scattering to the rotational states of the mesic molecules with orbital angular momentum $L = 1$ predominate. In the presence of resonance for a d-wave (for example, in the $d\mu + p$ collisions[11]) the transition $(L = 2) \to (L = 1)$ competes with this transition. The $dd\mu$, $dt\mu$ and $tt\mu$ mesic molecules, in addition to the rotational states with $L = 0$ and $L = 1$, also exist in the states with $L = 2$ and $L = 3$ (only for $tt\mu$), and in accordance with this it is possible for them to be formed from scattering states with higher orbital angular momenta. However, in the region of collision energies ($\varepsilon \leqslant 50$ eV) of interest to us the probability of transitions from these states is small and we shall not consider them.

As is well known,[1,3,12], when mesic molecules are formed in excited states of low binding energy an appreciable contribution to the total probability can be made by monopole $E0$-transitions. The probabilities of these transitions are also calculated in the present paper.

THE CALCULATION METHOD

We follow through the scheme of making the calculations using the example of formation of $pd\mu$ mesic molecules. When μ^--mesons are stopped in a mixture of hydrogen and deuterium at first a mixture of $p\mu$ and $d\mu$ is formed, and this is followed by a rapid transition $p\mu + d \to d\mu + p$ with a rate of $\lambda_{pd} = 1.7 \times 10^{10}$ sec^{-1} (in liquid hydrogen).[1–4,13]

Subsequently in collisions of $d\mu$ mesic atoms with hydrogen molecules the following reaction takes place

$$d\mu + H_2 \to [(pd\mu)^+ pe^-]^+ + e^-. \qquad (1)$$

Thus, the $pd\mu$ mesic molecule that has been formed, i.e., the $(pd\mu)^+$ ion, becomes the nucleus of a peculiar molecular ion, while the energy given off in the formation of the $pd\mu$ molecule is carried off by the conversion electron. The system of coordinates in which the rate $\lambda_{pd\mu}$ of the formation of the $pd\mu$ mesic molecule is calculated is shown in Fig. 1.

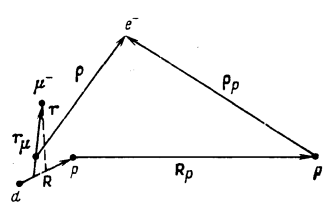

FIG. 1. R—radius-vector, joining the nuclei d and p, r_μ and r—radius-vectors of the μ^--meson with the origin at the center of mass of the nuclei d and p and at the geometrical center of the segment R respectively; ρ and ρ_p are the radii-vectors drawn from the center of mass of the system $pd\mu$ and from the second nucleus of the H_2 molecule to the point where the electron is situated; $\mathbf{R}_p$—vector joining the nuclei in the H_2 molecule.

The rate $\lambda_{pd\mu}$, reduced to the density of nuclei of liquid hydrogen $N_0 = 4.25\times10^{22}$ cm^{-3}, is equal to

$$\lambda_{pd\mu}=\sigma v N_0 \text{ sec}^{-1}, \tag{2}$$

where v is the velocity of the relative motion of the $d\mu$ mesic atom and the H_2 molecule, while σ is the total cross section for the formation of the $pd\mu$ mesic molecule. The formula for the differential cross section corresponding to the electron being emitted in the momentum range $\mathbf{q}$ to $\mathbf{q}+d\mathbf{q}$ has the form[14]

$$v\,d\sigma=\beta\frac{2\pi}{\hbar}\delta(E_f-E_i)\,dq\sum_{m_L}\Big|\int d\mathbf{R}\,d\mathbf{r}\,d\rho\,\Psi^{(f)*}(\mathbf{r},\mathbf{R}) \times\psi^{(f)*}(\rho)\hat{H}_{int}\psi^{(i)}(\rho)\,\Psi^{(i)}(\mathbf{r},\mathbf{R})\Big|^2. \tag{3}$$

Here $\psi^{(i)}(\rho)$ and $\psi^{(f)}(\rho)$ are the wave functions for the emerging electron in the initial (i) and the final (f) states, $\Psi^{(i)}(\mathbf{r},\mathbf{R})$ and $\Psi^{(f)}(\mathbf{r},\mathbf{R})$ are respectively the wave functions for the system $d\mu+p$ and for the $pd\mu$ molecule, β is the statistical factor which has to be taken into account in the case of formation of mesic molecules with identical nuclei (in the case of mesic molecules with different nuclei $\beta=1$).

Summation over mL is carried out over all the allowed values of the component m_L of the orbital angular momentum L of the mesic molecule that has been formed, $E_i=\varepsilon_I+\varepsilon$ and $E_f=\varepsilon_{vL}+q^2/2m$ are the total energies of the system $d\mu+H_2$ in the initial and the final states respectively; $\varepsilon_I=-15.44$ eV is the binding energy of the electron in the ground state of the H_2 molecule, ε_{vL} is the binding energy of the $pd\mu$ mesic molecule in the state of orbital angular momentum L and virbational quantum number v; $\mathbf{q}$ is the momentum of the conversion electron the absolute magnitude of which is equal to $q=[2m(\varepsilon+|\varepsilon_{vL}|-|\varepsilon_I|)]^{1/2}$, m is the electron mass, $\varepsilon=q^2/2m$ is the energy of relative motion of the $d\mu$ mesic atom and the H_2 molecule, M is the effective mass of the system $pd\mu$, which can be expressed in terms of the deuteron mass $M_d=M_1$, the proton mass $M_p=M_2$ and of the μ-meson mass M_μ in accordance with the formulas[15]

$$M=M_0/m^*,\quad 1/M_0=1/M_1+1/M_2,\quad 1/m^*=1/M_\mu+1/4M_0. \tag{3a}$$

Mesic molecules are formed under the action of the perturbation

$$\hat{H}_{int}=-\frac{e^2}{R_{1e}}-\frac{e^2}{R_{2e}}+\frac{e^2}{R_{\mu e}}+\frac{e^2}{\rho}, \tag{4}$$

where R_{ie} ($i=1,2,\mu$) is the distance from the nuclei M_i and the μ-meson to the electron. Taking into account the inequality $R_i\ll\rho$, where R_i are the distances of the nuclei and the meson from the centre of mass of the $pd\mu$ mesic molecule, we have in the dipole approximation

$$\hat{H}_{int}=-e\mathbf{d}\rho/\rho^3, \tag{5}$$

where the dipole moment $\mathbf{d}=\mathbf{d}(\mathbf{r},\mathbf{R})$ is expressed in terms of different coordinates (cf., Fig. 1) in the following manner:

$$\begin{aligned}\mathbf{d}&=-e(\mathbf{R}_1+\mathbf{R}_2-\mathbf{R}_\mu)=-e[\varkappa\mathbf{R}+(1+M_\mu/M_i)\mathbf{r}_\mu]\\&=-e\left[\frac{\varkappa}{2}\left(1-\frac{M_\mu}{M_i}\right)\mathbf{R}+\left(1+\frac{M_\mu}{M_i}\right)\mathbf{r}\right],\\ \varkappa&=\frac{M_2-M_1}{M_2+M_1},\quad M_i=M_1+M_2+M_\mu.\end{aligned} \tag{6}$$

The wave function for the initial state of the electron in the hydrogen molecule was chosen by us in the form proposed by Heitler and London (in atomic units $e=\hbar=m=1$):

$$\psi^{(i)}(\rho)=\left[\frac{Z_0^3}{2\pi(1+\Delta)}\right]^{1/2}[\exp(-Z_0\rho)+\exp(-Z_0|\rho-\mathbf{R}_p|)], \tag{7}$$

where Z_0 is the effective charge of the proton in the hydrogen molecule, while R_p is the equilibrium distance between the protons. In accordance with[16] the following values of these quantities have been adopted:

$$Z_0=1.19;\quad R_p=1.40;\quad \Delta=0.677. \tag{8}$$

The wave function $\psi^{(f)}(\rho)$ for the electron with momentum $\mathbf{q}$ in the Coulomb field of the charge Z^* normalized to $\delta(\mathbf{q}-\mathbf{q}')$ has the well known form[14]:

$$\psi^{(f)}(\rho)=\frac{1}{4\pi q}\sum_{L=0}^{\infty}i^L(2L+1)e^{-i\sigma_L}R_{qL}(\rho)P_L(\cos\theta_{q\rho}), \tag{9}$$

where $\theta_{q\rho}$ is the angle between the vectors $\mathbf{q}$ and ρ, while the Coulomb function is defined by the relations

$$R_{qL}(\rho)=C_{qL}\frac{Z^{*L}}{(2L+1)!}(2q\rho)^L e^{-iq\rho}F(L+1+i\eta,2L+2;2iq\rho)\underset{\rho\to\infty}{\longrightarrow}\left(\frac{2}{\pi}\right)^{1/2}\frac{1}{\rho} \times\sin\left(q\rho+\eta\ln2q\rho-\frac{\pi L}{2}+\sigma_L\right). \tag{10}$$

Here $\delta_L=\arg\Gamma(L+1-i\eta)$ is the Coulomb phase, $\eta=Z^*/q$, Z^* is the effective charge of the system $[(pd\mu)^+pe^-]$.

Separating out the dimensional factors in all the expressions containing the coordinates ρ and the momenta $\mathbf{q}$ of the electron and carrying out an integration over them we obtain the following expression for the rate of formation of mesic molecules by means of the conversion $E1$-transition[1)]

$$\lambda(E1)=\beta N_0\frac{8\pi}{3}\frac{me^4}{\hbar^3a_0^2}\frac{Z_0^3}{1+\Delta}\frac{1}{q}|I(q)|^2 \times\sum_{m_L}\Big|\int d\mathbf{R}\,d\mathbf{r}\,\Psi^{(f)*}(\mathbf{r},\mathbf{R})\,\mathbf{d}(\mathbf{r},\mathbf{R})\,\Psi^{(i)}(\mathbf{r},\mathbf{R})\Big|^2. \tag{11}$$

Here $a_0=\hbar^2/me^2$ and the following notation has been introduced:

$$I(q)=I_1(q)+I_2(q),\quad I_1(q)=\int_0^\infty d\rho R_{qL}(\rho)e^{-Z_0\rho}, \tag{12}$$

$$I_2(q)=\tfrac{1}{2}\int_0^\infty d\rho R_{qL}(\rho)\int_{-1}^{1}d\cos\theta_{\rho R_p}\exp(-Z_0|\rho-\mathbf{R}_p|)$$

and all quantities are expressed in terms of atomic units. The explicit form of the integrals $I_1(q)$ and $I_2(q)$ and other details of calculations are given in[17].

The wave functions representing the motion of the μ^--meson and of the nuclei have been calculated in the two-level approximation of the perturbed-stationary-states method[13]

$$\Psi^{(i,f)}(\mathbf{r},\mathbf{R})=\varphi_1(\mathbf{r};R)\psi_1^{(i,f)}(\mathbf{R})+\varphi_2(\mathbf{r};R)\psi_2^{(i,f)}(\mathbf{R}), \tag{13}$$

where

$$\varphi_{1,2}(\mathbf{r};R)=2^{-1/2}[\varphi_g(\mathbf{r};R)\mp\varphi_u(\mathbf{r};R)], \tag{14a}$$

while $\varphi_g(\mathbf{r};R)$ and $\phi_u(\mathbf{r};R)$ are the even and odd solutions of the two-center problem[18] normalized by the condition

$$\int d\mathbf{r}\varphi_g^2(\mathbf{r};R)=\int d\mathbf{r}\varphi_u^2(\mathbf{r};R)=1. \tag{14b}$$

They represent the motion of the μ^--meson in the field of two fixed nuclei (M_1 and M_2) in two lowest energy states.

The wave functions $\psi_j^{(i)}(\mathbf{R})$ and $\psi_j^{(f)}(\mathbf{R})$ ($j=1,2$), representing the relative motion of the nuclei in the initial and final states have the form

$$\psi_j^{(i)}(\mathbf{R})=\sum_{L=0}^{\infty}i^L(2L+1)\frac{1}{kR}\chi_j^{(i)}(R)P_L(\cos\theta_{kR}), \tag{15}$$

where $k^2=2M\varepsilon$, θ_{kR} is the angle between the vectors k and R

$$\psi_j^{(f)}(\mathbf{R})=\frac{1}{R}\chi_j^{(f)}(R)Y_{Lm_L}(\Theta,\Phi). \tag{16}$$

The functions $\chi_j^{(i)}(R)$ and $\chi_j^{(f)}(R)$ are solutions of the system of equations

$$\begin{aligned}&\frac{d^2\chi_1}{dR^2}+\left[2M\varepsilon-\frac{L(L+1)}{R^2}\right]\chi_1=V_{11}\chi_1+V_{12}\chi_2+2Q_{12}\frac{d\chi_2}{dR},\\&\frac{d^2\chi_2}{dR^2}+\left[2M(\varepsilon-\Delta E)-\frac{L(L+1)}{R^2}\right]\chi_2=V_{21}\chi_1+V_{22}\chi_2+2Q_{21}\frac{d\chi_1}{dR}\end{aligned} \tag{17}$$

with the following boundary conditions: for $R=0$

$$\chi_j^{(i)}(0)=\chi_j^{(f)}(0)=0, \tag{18a}$$

for $R\to\infty$ for the initial state ($0<\varepsilon<\Delta E$)

$$\chi_1^{(i)}(R)\sim\sin\left(kR-\frac{L\pi}{2}+\delta_L\right),\quad \chi_2^{(i)}(R)\sim\exp\{-[2M(\Delta E-\varepsilon)]^{1/2}R\}, \tag{18b}$$

for $R\to\infty$ for the final state ($\varepsilon=\varepsilon_{vL}<0$)

$$\begin{aligned}&\chi_1^{(f)}(R)\sim\exp\{-[-2M\varepsilon_{vL}]^{1/2}R\},\\&\chi_2^{(f)}(R)\sim\exp\{-[2M(\Delta E-\varepsilon_{vL})]^{1/2}R\}.\end{aligned} \tag{18c}$$

In terms of these definitions the energy of collision ε and the binding energy ε_{vL} are computed with respect to the ground energy level of the isolated mesic atom with the nucleus M_1 of the heavy isotope. In the adiabatic approximation the isotopic difference of the levels of the $d\mu$ and $p\mu$ mesic atoms is equal to $\Delta E=-\varkappa/2M$. The effective potentials $V_{ij}\equiv V_{ij}(R)$ and $Q_{ij}=Q_{ij}(R)$, which are expressed in terms of the energies $E_g(R)$ and $E_u(R)$ of the two-center problem and the matrix elements of the operators $\nabla_{\mathbf{R}}$ and $\Delta_{\mathbf{R}}$ with respect to the wave functions $\varphi_g(\mathbf{r};R)$ and $\varphi_u(\mathbf{r};R)$, have been calculated in[19] In order to solve equations (17) with the boundary conditions (18c) an algorithm has been chosen which utilizes the continuous analogue of the Newton method[20] which guarantees that the accuracy in calculating the functions is $\sim10^{-4}$.

The boundary conditions (18b) correspond to the two-channel scattering problem taking into account the closed channel. In order to solve it an effective algorithm has been developed[11,21] based on the method of phase functions, which also guarantees an accuracy not worse than $\sim10^{-3}$.

The functions $\chi_j^{(f)}(R)$ are normalized by the condition

$$\int_0^{\infty}\{[\chi_1^{(f)}(R)]^2+[\chi_2^{(f)}(R)]^2\}dR=1, \tag{19}$$

while the function $\chi_1^{(i)}(R)$ is normalized to unit amplitude (18b) for $R\to\infty$.

Separating out the dimensional factors, carrying out the integration over the coordinates r and R and the summation over the components m_L of the final state we arrive at the final expression for $\lambda(E1)$:

$$\lambda(E1)=\beta\frac{32\pi^2}{3}(N_0a_0^3)\left(\frac{m}{m^*}\right)^5\frac{Z_0^3}{1+\Delta}\frac{1}{q}|I(q)|^2|\langle\mathbf{d}\rangle|^2\frac{me^4}{\hbar^3}\ \text{sec}^{-1}. \tag{20}$$

Here m^* is the effective mass defined by formulas (3a), while the value of $\langle\mathbf{d}\rangle$ for the transitions $(L=0)^{(i)}\to(L=1)^{(f)}$ in units of $e=\hbar=m^*=1$ is equal to

$$\begin{aligned}|\langle\mathbf{d}\rangle|^2&=\frac{1}{4\pi}\sum_{m_L}\left|\int d\mathbf{R}\,d\mathbf{r}\,\Psi^{(f)*}(\mathbf{r},\mathbf{R})\,d(\mathbf{r},\mathbf{R})\,\Psi^{(i)}(\mathbf{r},\mathbf{R})\right|^2\\&=\left[\frac{\varkappa}{2}\left(1-\frac{M_\mu}{M_t}\right)J_1+\left(1+\frac{M_\mu}{M_t}\right)J_2\right]^2,\end{aligned} \tag{21}$$

where

$$J_1=\frac{1}{k}\int_0^{\infty}(\chi_1^{(i)}\chi_1^{(f)}+\chi_2^{(i)}\chi_2^{(f)})R\,dR,\quad J_2=\frac{1}{k}\int_0^{\infty}(\chi_2^{(i)}\chi_2^{(f)}-\chi_1^{(i)}\chi_1^{(f)})D_{gu}(R)\,dR, \tag{22}$$

$$D_{gu}(R)=\frac{\mathbf{R}}{R}\int d\mathbf{r}\,\varphi_g(\mathbf{r};R)\mathbf{r}\varphi_u(\mathbf{r};R).$$

All the formulas reamin valid for the transitions $(L=1)^{(i)}\to(L=0)^{(f)}$. For the transitions $(L=2)^{(i)}\to(L=1)^{(f)}$ expression (21) should be multiplied by an additional factor 2.

In calculating the rates of formation of mesic molecules with identical nuclei the formulas quoted above must be somewhat altered. In this case $\varkappa=0$, $\chi_1^{(f)}(R)=\chi_2^{(f)}(R)=2^{-1/2}\chi_g^{(f)}(R)$ and in accordance with this

$$\Psi^{(f)}(\mathbf{r},\mathbf{R})=\varphi_g(\mathbf{r};R)\frac{1}{R}\chi_g^{(f)}(R)Y_{Lm_L}(\Theta,\Phi). \tag{23a}$$

Moreover, the functions $\Psi^{(i,f)}(\mathbf{r},\mathbf{R})$ have a definite symmetry with respect to the operation of interchange of nuclei. Under such an operation the wave functions acquire a factor[14] $\gamma=(-1)^{L+l+2I_1-I}$, where L and l are the orbital angular momenta of the relative motion of the nuclei and of the meson respectively, I_1 and I are respectively the spin of each of the nuclei and the total spin of both nuclei, with $\gamma=-1$ for the $pp\mu$ and $tt\mu$ mesic molecules with fermion-nuclei, and $\gamma=+1$ for the $dd\mu$ mesic molecule with boson-nuclei. Since the functions $\varphi_g(\mathbf{r};R)$ and $\varphi_u(\mathbf{r};R)$ correspond to the values $l=0$ and $l=1$[18] we conclude from this that rotational states of mesic molecules with odd values of orbital angular momenta $L=1,3,\ldots$ are possible only when the total spin of the nuclei is $I=1$. Since the values of γ and I are conserved, the dipole transitions from the states

TABLE I. Rates of formation of the $pp\mu$ and $pd\mu$ mesic molecules.

Source	$\lambda_{pp\mu}$, 10^6 sec^{-1}	$\lambda_{pp\mu}$, 10^6 sec^{-1}
Dzhelepov *et al.* (1962)	1.5 ± 0.6	...
Bleser *et al.* (1963)	1.89 ± 0.20	5.8 ± 0.3
Conforto *et al.* (1964)	2.55 ± 0.18	6.82 ± 0.25
Bodyashov *et al.* (1968)	2.74 ± 0.25	...
Bystritskiĭ *et al.* (1975)	2.34 ± 0.17	5.53 ± 0.16
Zel'dovich and Gershteĭn (1960)	2.6	1.3
	3.	
Cohen *et al.* (1960	3.9	3.0
Present work for $\varepsilon = 0.04$ eV	2.20	5.91

Note. The calculated values correspond to $E1$-transitions from the state $L = 0$ of the continuous spectrum into the states $v = 0$, $L = 0$ of the $pp\mu$ ($\varepsilon_{vL} = 101.7$ eV) and $pd\mu$ ($\varepsilon_{vL} = 90.1$ eV)[22] mesic molecules. For these calculations the following values for the mass were adopted $M_\mu = 206.769$, $M_p = 1836.109$, $M_d = 3670.398$.[23] For the collision energy $\varepsilon = 0.04$ eV the quantities $q^{-1}|I(q)|^2$ and $|\langle \mathbf{d} \rangle|$ for the $pp\mu$ and $pd\mu$ mesic molecules are respectively equal to: 0.136 and 15.0; 0.140 and 21.7. The quantity $\lambda_{pp\mu}$ from reference[5] has been corrected in accordance with the remarks of Zel'dovich and Gershteĭn.[1]

$L = 0, 2, \ldots$ of the continuous spectrum are possible only from the state $\varphi_u(\mathbf{r}, R)$ describing the meson motion. In accordance with this the total wave function for the initial state has the form

$$\Psi^{(i)}(\mathbf{r}, \mathbf{R}) = \varphi_u(\mathbf{r}; R) \sum_{L=0} i^L (2L+1) \frac{1}{kR} \chi_u^{(i)}(R) P_L(\cos\theta_{kR}),$$

$$\chi_u^{(i)}(R) \underset{R\to\infty}{\longrightarrow} \sin(kR - L\pi/2 + \delta_u). \qquad (23b)$$

The formula for $\langle \mathbf{d} \rangle$ in this case is altered in the following manner:

$$|\langle \mathbf{d} \rangle| = \left(1 + \frac{M_\mu}{M_t}\right) \frac{1}{k} \int_0^\infty \chi_g^{(f)} \chi_u^{(i)} D_{gu}(R)\, dR. \qquad (24)$$

The statistical weight of the states $\beta = (2I+1)/(2I_1+1)^2$ with the total nuclear spin $I = 1$ is equal to $\beta = 3/4$ in the case of the $pp\mu$ and $tt\mu$ mesic molecules, $\beta = 1/3$ in the case of $dd\mu$.

The electric monopole $E0$-transition involving conversion of the electron of the hydrogen molecule can in principle give an appreciable contribution to the total rate of formation of mesic molecules in transitions from the s-state of the continuous spectrum into the rotational states of the mesic molecules with $L = 0$ with a low binding energy[12] (for example, $dd\mu$, $dt\mu$, $tt\mu$). In this case for the $tt\mu$ mesic molecule only the singlet state of the spins of the nuclei $I = 0$ is possible and correspondingly $\beta = 1/4$, while for $dd\mu$ only the states $I = 0, 2$ ($\beta = 2/3$) are possible.

The rate of the $E0$-transition is determined as before by formula (3) with, however, the one difference that the perturbation operator (4) should be taken into account only in the region $R_i \leq \rho \ll 1$ (cf., Fig. 1), i.e., within the boundaries of the mesic molecule. In this region the electron functions (7) and (9) can be replaced by their values for $\rho = 0$. Carrying out the integration in (3) over the coordinates ρ and over the momenta $\mathbf{q}$ and separating out the dimensional factors we obtain

$$\lambda(E0) = \beta \frac{8\pi^2}{9} (N_0 a_0^3) \left(\frac{m}{m^*}\right)^7 \frac{Z_0^3}{1+\Delta} \frac{1 + e^{-Z_0 R_p}}{1 - e^{-2\pi\eta}} |\langle Q_0 \rangle|^2 \frac{me^4}{\hbar^3} \text{ sec}^{-1}, \qquad (25)$$

where the matrix element

$$\langle Q_0 \rangle = \pi^{-1/2} \int d\mathbf{R}\, d\mathbf{r}\, \Psi^{(f)*}(\mathbf{r}, \mathbf{R}) (R_1^2 + R_2^2 - R_\mu^2) \Psi^{(i)}(\mathbf{r}, \mathbf{R}) \qquad (26)$$

is evaluated in units of $a_{m*} = \hbar^2/m^* e^2$.

DISCUSSION OF RESULTS

In Table I are given the measured rates of formation of the $pp\mu$ and $pd\mu$ mesic molecules and also the results calculated by different authors. It can be easily seen that in both cases the results of calculations of the present paper are in good agreement with the whole set of experimental data. The reason for the deviations from the earlier calculations is not always clear, since for such conclusions it is necessary to know the details of the numerical calculations which, as a rule, are not communicated. In the present paper the functions $\Psi^{(i,f)}(\mathbf{r}, \mathbf{R})$ and the corresponding integrals involving them have been evaluated with an accuracy $\sim 10^{-3}$.

The principal source of error in the quantities quoted above is associated with an insufficent knowledge of the wave function for the conversion electron in the field of the molecular ion $[(pd\mu)^+ pe^-]$. In our calculations this complex was replaced by a Coulomb center with an effective charge $Z^* = 1$. Variation of the value of Z^* within the range $1 \leq Z^* \leq Z_0$ leads to an increase of the rates $\lambda_{pd\mu}$ and $\lambda_{pp\mu}$ to the values respectively of 6.14 $\times 10^6$ and 2.26 $\times 10^6$ sec^{-1}. From this one can conclude that the error due to an insufficient knowledge of $\psi^{(f)}(\rho)$ amounts to no more than 5–10% of the quoted value.

In Table II the results of our calculations are compared with the results of the earlier calculations of the rates of formation of the $pt\mu$, $dd\mu$, $dt\mu$ and $tt\mu$ mesic molecules. As should have been expected, on the basis

TABLE II. Rate of formation of mesic molecules λ, 10^6 sec^{-1}.

Type of transition	Source	$pt\mu$	$dd\mu$	$dt\mu$	$tt\mu$
$E1$	Zel'dovich and Gershteĭn (1960)	0.4	0.01	0.002	0.65
	Cohen *et al.* (1960)	...	0.034	...	...
	Present work at $\varepsilon = 0.04$ eV	6.49	0.013	$5 \cdot 10^{-4}$	2.96
$E0$	Zel'dovich (1954)	...	0.03	0.03	...
	Present work at $\varepsilon = 0.04$ eV	...	0.029	0.033	0.002

Note. The values of $\lambda(E1)$ calculated in the present work correspond to transitions from the s-wave of the continuous spectrum into the states ($v = 0$, $L = 1$) of the mesic molecules whose binding energies are equal to[22] $\varepsilon_{01} = 92.2$, 224.1 and 229.7 eV respectively for the $pt\mu$, $dd\mu$ and $dt\mu$ mesic molecules; the quoted value of $\lambda_{tt\mu}$ corresponds to the transition into the state ($v = 1$, $L = 1$) with an energy $\varepsilon_{11} = 43.1$ eV; the rate of transition into the state ($v = 0$, $L = 1$), $\varepsilon_{01} = 287.9$ eV is equal to $\lambda_{tt\mu} = 2.7 \times 10^3$ sec^{-1}. The values of $\lambda(E0)$ correspond to transitions into the states ($v = 1$, $L = 0$) of the $dd\mu$, $dt\mu$ and $tt\mu$ mesic molecules whose energies are respectively equal to $\varepsilon_{10} = 33.2$ eV; $\varepsilon_{10} = 31.8$ eV and $\varepsilon_{10} = 81.8$ eV.[22]

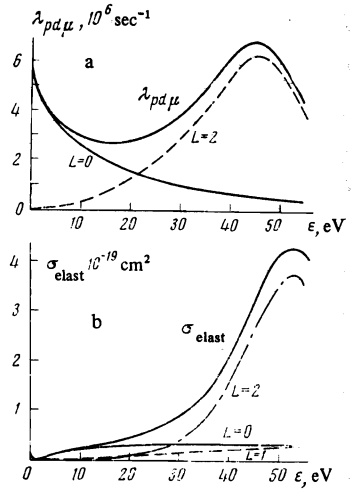

FIG. 2. a—Dependence of the rates of formation of the $pd\mu$ mesic molecule in the transitions $(L=0)^{(i)} \to (L=1)^{(f)}$, $(L=2)^{(i)} \to (L=1)^{(f)}$ and of the total rate $\lambda_{pd\mu}$ on the collision energy ε; b—dependence on ε of the partial and total cross section σ for the elastic scattering $d\mu + p$.

of earlier estimates the rate of $E0$-transitions is small, though it does exceed the rate of $E1$-transitions in the formation of the $dd\mu$ and $dt\mu$ molecules.[2)] Our attention is drawn to the significant discrepancies between the earlier and the newly calculated values of λ_{ptu} and λ_{ttu}.

As has already been noted earlier, the formation of the $pd\mu$ and $pt\mu$ mesic molecules is preceded by a stage of isotropic exchange $p\mu + d \to d\mu + p$ (or $p\mu + t \to t\mu + p$), as a result of which $d\mu$ and $t\mu$ mesic atoms are formed with an initial energy respectively of 48.3 and 49.5 eV. Moreover, it is known[11] that in the d-wave for the elastic scattering processes $d\mu + p \to d\mu + p$ and $t\mu + p \to t\mu + p$ resonances occur which are a manifestation of the quasistationary states in the systems $d\mu + p$ and $t\mu + p$ with energies respectively of 55.6 and 44.9 eV.[24] As a result of this, the contribution of the

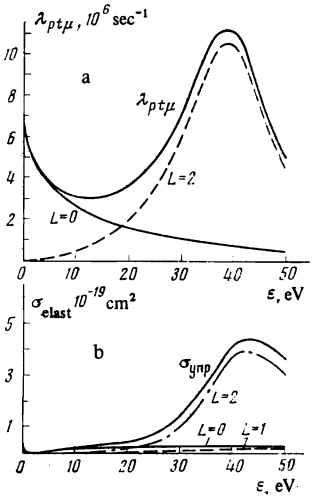

FIG. 3. a—Dependence of the rates of formation of the $pt\mu$ mesic molecule on the collision energy ε; b—dependence, on ε of the partial and total cross section for the elastic scattering $t\mu + p$.

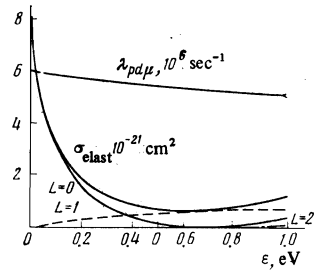

FIG. 4. The partial and the total cross sections for the elastic scattering $d\mu + p$ and of the rate $\lambda_{pd\mu}$ at low collision energies ε (the minimum in σ_{elast} is at $\varepsilon \approx 0.6$ eV).

transition $(L=2)^{(i)} \to (L=1)^{(f)}$, which is usually suppressed, increases considerably in this region of energy and exceeds by almost an order of magnitude the contribution from the transition $(L=0)^{(i)} \to (L=1)^{(f)}$. In Figs. 2 a, b and 3 a, b we have presented: a—the rates $\lambda_{pd\mu}$ and $\lambda_{pt\mu}$ and b—the cross sections σ_{elast} for the elastic scattering $d\mu + p$ and $t\mu + p$ as a function of the energy of the collision ε.

It is obvious that with the experimental values one should compare the values of $\lambda_{pd\mu}$ and $\lambda_{pt\mu}$, averaged over the whole energy interval $0.04 \leq \varepsilon \leq 45$ eV weighted with $\rho(\varepsilon)$, proportional to the time that the system finds itself in the state of energy ε. The existence of deep minima in the cross sections for the elastic scattering $d\mu + p$ and $t\mu + p$ respectively at $\varepsilon \approx 0.6$ eV and $\varepsilon \approx 1.6$ eV[3)] (Figs. 4 and 5) leads to the results that the average rates of formation of mesic molecules must be approximately equal to the average values of $\lambda_{pd\mu}$ and $\lambda_{pt\mu}$ for $\varepsilon \lesssim 1$ eV.

CONCLUSION

The accuracy of calculating the rates of formation of mesic molecules achieved in the present work at present completely satisfies the requirements and the possibilities of experiment. If necessary, the accuracy of the calculations can be increased by going outside the framework of the two-level approximation of the three-body problem, i.e., by including in the expansion (13) higher states of the two-center problem.[26] For this purpose it will also be necessary to obtain the wave functions of the electron in the field of the molecular ion H_2^+, and this in itself is quite a complicated, and until now an unsolved problem of atomic physics.

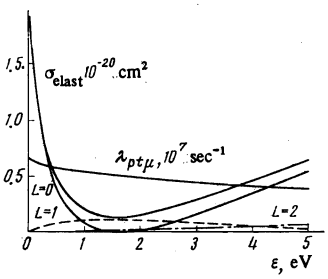

FIG. 5. The partial and the total cross sections for the elastic scattering $t\mu + p$ and the rate $\lambda_{pt\mu}$ at low collision energies ε (the minimum of σ_{elast} is at $\varepsilon \approx 1.6$ eV).

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[1)]In deriving formula (11) it was taken into account that conversion of either one of the two electrons of the H_2 molecule is equally probable.

[2)]We note that due to the existence of a weakly bound state ($L=1$, $v=1$) of the $dd\mu$ mesic molecule[22] the determining mechanism for its formation is, apparently, not the conversion $E1$-transition, but a transition with excitation of vibrational levels of the system $[(dd\mu)^* d]2e$.[27]

[3)]These minima are the effects of the well known Ramsauer–Townsend effect[5,11] and explain the large ranges for $d\mu$-atoms observed already in the first experiments of Alvarez *et al.*[25]

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