Concerning the possible states of a one-dimensional Fermi system

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A one-dimensional model of a Fermi gas in which the interaction is most fully taken into account is considered. It is shown that, besides the earlier-indicated instabilities, a metal-dielectric parametric transition can occur in the system. The behavior of the correlation functions at large distances is studied, and it is shown that the long-wave gapless collective excitations make a contribution to the exponent of the power law according to which these functions decrease. The collective-excitation spectrum is investigated. A phase diagram of the states is obtained. A comparison with the results of other approaches is carried out. Ways are indicated of generalizing the obtained results to the case in which the kinetic coupling between the filaments in a quasi-one-dimensional system is taken into account.

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INTRODUCTION

Recently there has been a sharp increase in the volume of experimental investigations of quasi-one-dimensional objects, e.g., the TCNQ salts[1]; compounds of the A-15 type[2,3]; quasi-one-dimensional magnetic substances (TMMC, $CsNiF_3$, etc.)[4]; compounds with variable valency, with planisquare complexes of Ir or Pt as bases (for example, KCP)[5]; certain "secondary crystals"[6] obtained by filling under pressure with molten metal regular channels of small (5–15 Å) diameter that exist in certain natural dielectric matrices (chrysotile asbestos[7,8] and zeolites[9,10]). These investigations showed that the standard (based on the self-consistent field method (SCM)) band theory of highly anisotropic substances, in which the description is carried on in terms of single-particle excitations, is unable to account for the increased role of collective and fluctuation phenomena that is observed in quite a number of quasi-one-dimensional substances. It became clear that these effects turn out to be automatically lost when the SCM is used. On the other hand, it followed from the investigation of certain exactly soluble one-dimensional models that the role of these effects in the purely one-dimensional situation should be exceptionally great. This stimulated anew the interest of theorists in purely one-dimensional systems.

It should not, however, be forgotten that certain exact properties of one-dimensional models are peculiar only to one dimension, and surely ought not be fulfilled in really investigable quasi-one-dimensional objects, in which the one-dimensional filaments are coupled, albeit weakly. Let us recall some of such exact results.

1. A phase transition cannot occur at $T_c \neq 0$ in a one-dimensional system when the interaction range is finite; ODLRO and DLRO are destroyed. This is a consequence of the enhanced role of fluctuations in a one-dimensional system.

2. Allowance for the correlations between the current carriers can completely eliminate the single-particle states from an entire energy interval near the Fermi surface, and all the corresponding degrees of freedom will become collective degrees of freedom—this enhances the role of the collective effects in the observed phenomena.

3. The single-particle states (if such states are allowed in a one-dimensional lattice) turn out to be localized even in an arbitrarily weak random potential, i.e., the low-temperature conductivity in a one-dimensional lattice with defects cannot be metallic.

When allowance is made for even a very weak transverse coupling between the filaments, these results lose their absolute validity. A phase transition ($T_c \neq 0$) becomes possible, although the critical temperature T_c in the weak-coupling case is very low. In the region $T < T_c$ ODLRO or DLRO can exist, i.e., the correlation functions at large distances tend to a finite (albeit very small in the case of weak coupling) limiting value. The collective and fluctuation effects weaken. The conditions for localization in a random potential become more rigid.

Thus, purely one-dimensional models cannot provide a fully adequate description for real quasi-one-dimensional systems. Nevertheless, the investigation of one-dimensional models is useful for the understanding of the physics of quasi-one-dimensional systems. Indeed, there exists above T_c some temperature range (fairly wide in the case of weak coupling) in which, for the description of some properties of the quasi-one-dimensional system, the filaments can be considered to be uncoupled. Although the collective and fluctuation properties here are not as important as in the one-dimensional case, they are many times stronger than in the ordinary three-dimensional situation. It would be absurd to try to find T_c, remaining within the framework of a one-dimensional model, but the tendency towards the formation of a new phase can be followed. It is this circumstance that allows the construction of "phase diagrams" in the space of the coupling constants (see Fig. 2), as was first done in Dzyaloshinskiĭ and Larkin's paper[11] in the case of three coupling constants.

Furthermore, it should be borne in mind that the many-particle, one-dimensional models can serve as a suitable zeroth approximation in the construction of the theory of quasi-one-dimensional systems if, using some

form of perturbation theory, we are able to introduce a weak coupling between the filaments. Naturally we should in this case, treat such a zeroth approximation correctly.

Unfortunately, the one-dimensional models that are of the greatest physical interest contain not one, but several coupling constants and are not exactly soluble. The noncritical use of approximate computational methods that have proved to be quite satisfactory in three-dimensional situations have led in quite a number of cases to incorrect results (this pertains especially to investigations of the Peierls transition and one-dimensional superconductivity). Therefore, the improvement of the methods of investigation of the one-dimensional models that are not amenable to exact solution is of independent interest.

1. CHOICE OF THE MODEL

At present two groups of exactly soluble one-dimensional models of interacting Fermi particles are known. These are the case of the Fermi gas with a δ-function interaction (Gaudin and Yang[12]) and the so-called Hubbard model (Lieb and Wu[13]). The exact solutions allowed the computation of the ground-state energy, the energy of the Bose branches of the spectrum, as well as certain characteristics of the one-electron spectra. However, the exact wave functions are so complex that it is impossible to compute with their aid the correlation functions and settle the question of the nature of the ground state.

The second group of exactly soluble one-dimensional models is made up of different variants of the model, first considered by Tomonaga,[14] of a Fermi gas with long-range interaction. In this model, because of the simplicity of the interaction, it is possible to compute the correlation functions in addition to the spectrum. Subsequently, the model was extended: the spin degrees of freedom were taken into account,[5] and an additional interaction describing the backward scattering was introduced[16] (the model with two coupling constants). However, an exact solution could be found only for a particular value of one of the coupling constants.

Since it is difficult to make any judgment about the magnitude and nature of the interaction in a one-dimensional system, it is of interest to investigate one-dimensional models in which the interaction is most fully taken into account, and which contain an even larger number of coupling constants (see below). To find for these models the exact solution is a virtually hopeless problem; therefore, the question of reliable methods of investigation of these models arises in all its acuteness.

In a one-dimensional Fermi gas two tendencies are manifested clearly: the Cooper pairing of electrons with opposite momenta and the Peierls pairing of an electron and a hole with opposite momenta. This leads to a situation in which it is necessary to simultaneously allow for two scattering channels—the Peierls and Cooper channels—in the analysis of the question of the possible states of a one-dimensional system in the framework of perturbation theory.[17] Allowance for the simplest set of important diagrams, which corresponds to the choice of the dominant logarithms, amounts to the summation of parquet diagrams.[17] In the parquet approximation, a finite phase-transition temperature is obtained, which ought not be the case in a one-dimensional system.[18] It follows from this that this approximation is inadequate in the one-dimensional case.

Next to the parquet approximation, the most convenient approximation is obtained by the method of multiplicative renormalizations.[19] In this case, as was to be expected, it turns out that the transition temperature is equal to zero. However, the methods of the theory of the renormalization group are also of limited accuracy, especially in the case when we get into the strong-coupling region, as obtains in the case under discussion. Therefore, even such a general method is not always reliable for the solution of the problem of the possible states of a one-dimensional Fermi system.

Apart from this, a specific property of the one-dimensional system consists in the distinct role of the collective excitations.[1)] In a model with a linear spectrum,[14] this manifests itself in the fact that all the degrees of freedom turn out to be collective. Since the spectrum in the vicinity of the Fermi surface can always be linearized, all the low-lying excitations for any other model are also collective excitations, and, apparently, what Menyhard and Solyom did in[19], in going outside the framework of the parquet approximation (as a result of which $T_c \to 0$), was equivalent to making allowance for the contribution of the collective excitations.

On account of the foregoing, the most suitable model for the investigation of the question of the possible states of a one-dimensional system is the model with a linear spectrum, where the collective excitations turn out to be the most distinct states, especially as the investigation in the present case is not confined within the framework of perturbation theory (the Cooper and Peierls anomalies depend weakly on the shape of the initial spectrum, because of their logarithmic character).

Thus, let us choose the zeroth-order Hamiltonian in the form

$$H_0 = v_F \sum_{p,s} p\,(a_{ps}^+ a_{ps} - b_{ps}^+ b_{ps}), \tag{1}$$

where $a^+(a)$ and $b^+(b)$ are respectively the operators of creation (annihilation) of an electron with $p>0$ and $p<0$. This Hamiltonian has been investigated in a number of papers.[14-16,20] It can also be written in terms of density operators:

$$H_0 \to \overline{H}_0 = \frac{2\pi v_F}{L} \sum \left[\rho_s^{(1)}(p)\rho_s^{(1)}(-p) + \rho_s^{(2)}(-p)\rho_s^{(2)}(p)\right], \tag{2}$$

where

$$\rho_s^{(1)}(p) = \sum_k^{p,s} a_{p+k,s}^+ a_{ks}, \qquad \rho_s^{(2)}(p) = \sum_k b_{p+k,s}^+ b_{ps}$$

are the operators for electrons with spin s and with p

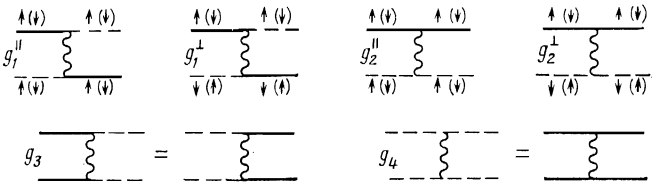

FIG. 1. The elementary processes corresponding to the various contributions to the interaction Hamiltonian (4).

>0 and $p<0$, operators which satisfy the following commutation relations:

$$[\rho_s^{(j)}(-p), \rho_{s'}^{(j')}(p')] = \delta_{jj'}\delta_{ss'}\delta_{pp'}\frac{pL}{2\pi}, \tag{3}$$

where L is the volume of the one-dimensional system, i.e., the length of the filament. This is the mathematical expression of the property that all the degrees of freedom in such a model can be referred to collective degrees of freedom. A system with such a zeroth-order Hamiltonian can be adequately described in terms of single-particle excitations (H_0) and in terms of density oscillations ($\bar{H}_0$). In the presence of interaction, the concept of a single-particle spectrum turns out, strictly speaking, to be useless, and the eigenstates will be only collective excitations.

The interaction Hamiltonian under consideration has the form

$$\begin{aligned} H_{int} = {} & \frac{1}{L}\sum_{(p,s)} \delta(p_1+p_2-p_3-p_4)\, a^+_{p_1s} b^+_{p_2s'} a_{p_3s'} b_{p_4s} (g_1^{\parallel}\delta_{ss'} + g_1^{\perp}\delta_{s-s'}) \\ & + \frac{1}{L}\sum_{(p,s)} \delta(p_1+p_2-p_3-p_4)\, a^+_{p_1s} b^+_{p_2s'} b_{p_3s'} a_{p_4s} (g_2^{\parallel}\delta_{ss'} + g_2^{\perp}\delta_{s-s'}) \\ & + \frac{g_3}{2L}\sum_{(p,s)} [\delta(p_1+p_2-p_3-p_4+4p_F)\, a^+_{p_1s} a^+_{p_2s'} b_{p_3s'} b_{p_4s} \\ & + \delta(p_1+p_2-p_3-p_4-4p_F)\, b^+_{p_1s} b^+_{p_2s'} a_{p_3s'} a_{p_4s}] \\ & + \frac{g_4}{2L}\sum_{(p,s)} \delta(p_1+p_2-p_3-p_4)\, [a^+_{p_1s} a^+_{p_2s'} a_{p_3s'} a_{p_4s} + b^+_{p_1s} b^+_{p_2s'} b_{p_3s'} b_{p_4s}]. \end{aligned} \tag{4}$$

Other methods of specifying the interaction exist, but in the final analysis they all amount to the redefinition of the coupling constants in Eqs. (6) and (7). The physical conditions imposed on the various contributions to the interaction will be formulated later. The elementary interaction processes responsible for the various contributions to (4) are shown in Fig. 1.

The interaction with the constants $g_1^{\parallel}, g_1^{\perp}$ and $g_2^{\parallel}, g_2^{\perp}$ describes backward and forward scattering, respectively. The constant g_4 corresponds to the interaction of the electrons within the subsystems with $p>0$ and $p<0$. The contribution to the interaction with the constant g_3 allows us to take the presence of the crystal lattice into account and turns out to be important with $4p_F = 2\pi/a$, where a is the lattice constant. Notice that in (4) the term with g_3 and g_4 for $s=s'$ vanishes on account of the Pauli principle. This fact will be used below. The case with $g_1^{\parallel} = g_1^{\perp} = g_3 = 0$ and $g_2^{\parallel} = g_2^{\perp} = g_4 = g$ corresponds to the Tomonaga–Luttinger (TL) model with allowance for the spin degrees of freedom.[15,16] The Hubbard model corresponds to the choice $g_1^{\parallel} = g_1^{\perp} = g_2^{\parallel} = g_2^{\perp} = g_3 = g_4 = g$. Here we have in mind some analog of this model. Let us recall that in the standard Hubbard model[13] the bare spectrum has the form $\varepsilon(k) = \frac{1}{2}w\cos ka$, where w is the band width, whereas in the present model the spectrum is linear and unbounded, though below we shall introduce a cutoff parameter that will imitate the finiteness of the band width.

2. EXACT SOLUTION OF THE MODEL FOR SOME PARTICULAR RELATION BETWEEN THE COUPLING CONSTANTS

In a recent paper, Luther and Emery[16] proposed an original solution for the one-dimensional fermion model corresponding to the following substitution in (4): $g_3 = g_4 = 0$, $g_2^{\perp} = g_2^{\parallel} = g$ and $g_1^{\parallel} = -\frac{3}{5}2\pi v_F$. We shall use their method to investigate the more representative Hamiltonian (4). In the model with a linear spectrum there exists for the fermion operators the following Bose representation[20,21]:

$$\psi_s^{1,2}(x) \leftrightarrow \bar{\psi}_s^{1,2}(s) = \frac{e^{\pm ip_F x}}{(2\pi\alpha)^{1/2}} \exp\left[\pm\frac{2\pi}{L}\sum_p \frac{e^{-\alpha|p|/2 - ipx}}{p}\rho_s^{1,2}(p)\right], \tag{5}$$

where v_F/α is the cutoff parameter; in the TL model it should be allowed at the end of the computations to tend to infinity. Following Luther and Emery, we shall assume that v_F/α is equal to the band width.

Let us use (5) to rewrite H_{int} in terms of the density operators. As in[16], the total Hamiltonian can then be represented in the form of a sum of two Hamiltonians describing the spin excitations (H_1) and the density oscillations (H_2):

$$H_0 + H_{int} \to H_1 + H_2,$$

$$\begin{aligned} H_1 = {} & \frac{2\pi v_F}{L}\sum_k [u_1(\sigma_1(k)\sigma_1(-k) + \sigma_2(-k)\sigma_2(k)) + \gamma_1\sigma_1(k)\sigma_2(-k)] \\ & + 2\gamma_1 w\int\frac{dx}{2\pi\alpha}\operatorname{ch}\left\{\frac{2\pi}{L}\sqrt{2}\sum_k \frac{e^{-\alpha|k|/2 - ikx}}{k}[\sigma_1(k) + \sigma_2(k)]\right\}, \end{aligned} \tag{6}$$

$$\begin{aligned} H_2 = {} & \frac{2\pi v_F}{L}\sum_k [u_2(\rho_1(k)\rho_1(-k) + \rho_2(-k)\rho_2(k)) - \gamma_2\rho_1(k)\rho_2(-k)] \\ & + 2\gamma_3 w\int\frac{dx}{2\pi\alpha}\operatorname{ch}\left\{\frac{2\pi}{L}\sqrt{2}\sum_k \frac{e^{-\alpha|k|/2 - ikx}}{k}[\rho_1(k) + \rho_2(k)]\right\}, \end{aligned} \tag{7}$$

where

$$\rho_{1,2}(k) = \frac{1}{\sqrt{2}}\sum_s \rho_s^{1,2}(k)$$

are the density operators for the particles with $p>0$ and $p<0$, operators which satisfy the following commutation relations:

$$\begin{aligned} & [\rho_1(-k), \rho_1(k')] = [\rho_2(k), \rho_2(-k')] = \delta_{kk'}kL/2\pi, \quad [\rho_1(k), \rho_2(k')] = 0; \\ & \sigma_{1,2}(k) = \frac{1}{\sqrt{2}}\sum_s s\rho_s^{1,2}(k) \end{aligned} \tag{8}$$

are the spin-density operators, satisfying similar commutation relations, for the subsystems with $p>0$ and $p<0$. The operators $\rho_{1,2}(k)$ and $\sigma_{1,2}(k)$ commute with one another; therefore, $[H_1, H_2] = 0$. In (6) and (7) we have introduced the dimensionless coupling constants

$$\gamma_{1,2} = \frac{g_1^{\parallel} - g_2^{\parallel} \pm g_2^{\perp}}{2\pi v_F}, \quad \gamma_3 = \frac{g_3}{2\pi v_F}, \quad \gamma_4 = \frac{g_1^{\perp}}{2\pi v_F}, \quad u_{1,2} = 1 \pm \frac{g_4}{2\pi v_F}. \tag{9}$$

TABLE I. Values of the exponents of the power-law singularities of the correlation functions at $T=0$, $q=0$ or $q=2k_F$, and $\Omega \sim 0$ ($\mathrm{Im}N(2k_F, \Omega) \sim \Omega^{\nu}$) and the values for the spin susceptibility χ_0 and the compressibility, χ_T, of the system.

	$\gamma_1 \geqslant \lvert\gamma_4\rvert$ $\gamma_2 \geqslant \lvert\gamma_3\rvert$	$\gamma_1 \geqslant \lvert\gamma_4\rvert$ $\gamma_2 < \lvert\gamma_3\rvert$	$\gamma_1 \leqslant \lvert\gamma_4\rvert$ $\gamma_2 < \lvert\gamma_3\rvert$	$\gamma_1 \leqslant \lvert\gamma_4\rvert$ $\gamma_2 \geqslant \lvert\gamma_3\rvert$
$N(2k_F\Omega)$	$-2+\mu_1+\mu_2$	$-2+\mu_1$	-2	$-2+\mu_2$
$\chi(2k_F\Omega)$	$-2+\mu_1^{-1}+\mu_2$	$-2+\mu_1^{-1}$	—	—
$P_s(0,\Omega)$	$-2+\mu_1+\mu_2^{-1}$	—	—	$-2+\mu_2^{-1}$
$P_t(0,\Omega)$	$-2+\mu_1^{-1}+\mu_2^{-1}$	—		
χ_0	$\frac{2}{\pi v_F}\frac{1}{u_1-\bar\gamma_1}$	$\frac{1}{\pi v_F}\frac{1}{u_1-\bar\gamma_1}$	$\frac{1}{\pi v_1}\left(\frac{2\pi\Delta_1}{kT}\right)^{1/2}\exp\left[-\frac{\Delta_1}{kT}\right]$	
χ_T	$\frac{\pi}{2v_F}\frac{1}{k_F^2}\frac{1}{u_2-\bar\gamma_2}$	$\frac{\pi}{v_2}\frac{1}{(2k_F)^2}\left(\frac{2\pi\Delta_2}{kT}\right)^{1/2}\exp\left[-\frac{\Delta_2}{kT}\right]$	$\frac{\pi}{2v_F}\frac{1}{(k_F)^2}\frac{1}{u_2-\bar\gamma_2}$	

Here $\mu_1^{\pm}=(u_1+\bar\gamma_1)/(u_1-\bar\gamma_1)$, $\mu_2^{\pm}=(u_2+\bar\gamma_2)/(u_2-\bar\gamma_2)$.

When $\gamma_2=-\frac{3}{5}u_2$, the Hamiltonian H_2 can be diagonalized exactly. For this purpose, it is necessary to perform a canonical transformation with $\exp[iS]$, where

$$S=\frac{\pi \ln 2}{L}\sum_k \frac{\rho_1(k)\rho_2(k)}{k}$$

after which, having been rewritten in the Fermi representation, H_2 assumes the form

$$H_2=v_2\sum_p p(a_p^+a_p+b_p^+b_p)+\gamma_3 w\sum_p (a_p^+b_{p-2p_F}+b_{p-2p_F}^+a_p), \qquad (10)$$

where $v_2=\frac{4}{5}v_F u_2$. The spectrum of (10) is known:

$$\varepsilon^{(\pm)}(p)=v_F p_F \mp \mathrm{sign}\,(p\pm p_F)\{[v_2(p\pm p_F)]^2+\Delta_2^2\}^{1/2}, \qquad (11)$$

where $\Delta_2=\gamma_3 w$ has the meaning of a gap in the density-oscillation spectrum. For $\gamma_1=-\frac{3}{5}u_1$, the spectrum of the operator H_1 can also be found exactly. It has the form of (11), with v_2 replaced by $v_1=\frac{4}{5}v_F u_1$ and Δ_2 by $\Delta_1=\gamma_4 w$, i.e., there is a gap in the spin-excitation spectrum. This last result coincides with Luther and Emery's results[16] if we set in (4) $g_3=g_4=0$, $g_2^{\perp}=g_2^{\parallel}=g$.

3. STATE DIAGRAM

The fact that the equivalent Hamiltonian in the Boson representation can be written in the form of the sum of H_1 and H_2, which are such that $[H_1, H_2]=0$, while the bare coupling constants $g_1^{\parallel}$, $g_1^{\perp}$, $g_2^{\parallel}$, $g_2^{\perp}$, and g_3 collect in definite combinations (see (6) and (7)), is an expression of a definite symmetry of the system. This can be verified with the aid of first-order perturbation theory in the four-fermion interaction described by the formula (4).

As can be seen from (6) and (7), the constants γ_1 and γ_2 can be referred to a long-range interaction, γ_1 describing its antisymmetric—with respect to spin—part and γ_2 its symmetric part. Generally speaking, a long-range interaction presupposes the existence of a cutoff parameter $\varkappa$, such that $\varkappa \ll p_F$. However, in the present model $\varkappa$ enters only as a scale for the momentum $p/\varkappa$ and energy $\varepsilon/v_F\varkappa$. For example, for the imaginary part of the susceptibility $N(Q,\omega)$ ($Q=q-2p_F$), which describes the tendency of the system towards a Peierls doubling (see (13)) in the spin-zero variant of the TL model,[2)] Luther and Peschel[20] have obtained the expression:

$$\mathrm{Im}\,N(Q,\omega)=\begin{cases} 0, & |\omega|<cQ, \\ \dfrac{1}{\pi^2 v_F}\Gamma^2(\nu)\sin^2(\pi\nu)\left[\dfrac{4}{(\omega/\bar\omega)^2-(Q/\varkappa)^2}\right]^{\nu}, & |\omega|>cQ, \end{cases} \qquad (12)$$

$$\bar\omega=c\varkappa,\quad c=v_F[1-(2\gamma)^2]^{1/2},\quad \gamma=g/2\pi v_F,$$
$$\nu=1-[(1-2\gamma)/(1+2\gamma)]^{1/2},$$

$\Gamma(\nu)$ is a gamma function of ν. A similar expression is obtained for $\mathrm{Im}\,P(q,\omega)$, which describes the tendency towards Cooper pairing, by making the substitution $\gamma \to -\gamma$.[20] Thus, allowance for $\varkappa$ has no effect on the form of the exponents ν, and since we are interested only in these exponents (see Table I), we have not explicitly included $\varkappa$ in the Hamiltonian.

A surprising circumstance, peculiar to the model, is the fact that the interaction defined by the constant g_4—an interaction which we also interpret as a long-range interaction—amounts to a trivial renormalization of the sound speed, $v_F \to v_F u_2$, and the velocity of the spin excitations, which is a consequence of the model under consideration and certain assumptions concerning the form of the corresponding contribution in (4).

Luther and Emery assumed that, for their model ($g_3=g_4=0$, $g_2^{\parallel}=g_2^{\perp}=g$), the value $\gamma_1=-\frac{3}{5}$ is, from the standpoint of the renormalization group, a fixed point for an entire range, $-\frac{3}{5}\leqslant\gamma_1<0$, of bare γ_1 values. Using arguments based on the theory of multiplicative renormalizations, they proposed a state diagram that was corrected in a subsequent paper by Lee.[16] Lee and Chui[22] succeeded to some extent in justifying Luther and Emery's assumptions, drawing attention to the connection between the system described by (6) and the problem of the two-dimensional Coulomb gas studied earlier by Kosterlitz.[23] Their main result consists in the following: a gap exists in the spin-excitation spectrum (i.e., in the spectrum of the operator H_1) only when $\gamma_1<|\gamma_4|$. This assertion can also be fully referred to the operators, H_1 and H_2, being investigated in the present paper: $\Delta_1\neq 0$, if $\gamma_1<|\gamma_4|$ and $\Delta_2\neq 0$ if $\gamma_2<|\gamma_3|$.

The remaining region of γ_1 values ($\gamma_1\geqslant|\gamma_4|$) for H_1 can be readily studied, since it corresponds to a "zero-charge" situation. The renormalized coupling constants for the spin excitations of characteristic frequency Ω and momentum q possess the following property: $\gamma_1(z)\to\bar\gamma_1=(\gamma_1^2-\gamma_4^2)^{1/2}$ and $\gamma_4(z)\to\bar\gamma_4=0$, for $z\to 0$, where $z=\max[T,\Omega,v_F,u_2q]$ (T is the temperature). Thus, as a result of the renormalization, H_1 goes over, as $z\to 0$, into the TL Hamiltonian $\bar H_1$ with $\bar\gamma_1=(\gamma_1^2-\gamma_4^2)^{1/2}$, which has been well studied.[20]

Carrying out similar investigations for H_2 in the region $\gamma_2\geqslant|\gamma_3|$, we find that the effective Hamiltonian describing the interaction of acoustic phonons of frequencies lower than Ω and momenta smaller than q has the form of the TL Hamiltonian $\bar H_2$ with $\bar\gamma_2=(\gamma_2^2-\gamma_3^2)^{1/2}$ in the limit as $z\to 0$.

The symmetry of the state can be established by investigating the behavior of the various correlation func-

tions at large times and distances. The correlator $N(x, t)$, which is equal to

$$N(x,t)=-i0(t)\langle[U(x,t),U^{+}(0,0)]\rangle, \tag{13}$$

where

$$U(x,t)=\sum_{s}[\psi_{1s}^{+}(x,t)\psi_{2s}(x,t)+\psi_{2s}^{+}(x,t)\psi_{1s}(x,t)],$$

characterizes the tendency of the system towards the doubling of the period. Similarly, $\chi(x, t)$, defined as an advanced Green function of the operator $\psi_{1\uparrow}^{+}(x)\psi_{2\downarrow}(x)+\psi_{2\uparrow}^{+}(x)\psi_{1\downarrow}(x)$, and the functions P_s and P_t, defined as advanced Green functions respectively of the operators $\psi_{1\downarrow}^{+}(x)\psi_{2\uparrow}^{+}(x)$ and $\psi_{1\uparrow}^{+}(x)\psi_{2\downarrow}^{+}(x)$, describe the tendency of the system towards antiferromagnetic ordering (χ) and towards singlet (P_s) and triplet (P_t) pairing.

The correlators defined above can be computed in the (k, ω)-representation. For $z \ll \Delta_1, \Delta_2$ the dominant contribution to the correlators is made by the long-wave, gapless excitations, which, as has been indicated, are effectively described by a TL-type Hamiltonian. For $z \gg \Delta_1$, we neglect in H_1 the term with γ_4, as a result of which H_1 in this region of variables turns out to be equivalent (with allowance for the correct cutoff) to the TL Hamiltonian.

In the table we give the exponents of the power behavior that is manifested by the spectral representations of the above-defined correlation functions for $z \to 0$. Also given in the table are the values for the spin susceptibility χ_0 and the compressibility $\varkappa_T = \partial n/\partial \mu$ (n is the density and μ is the chemical potential). We have constructed on the basis of the table a state diagram for $T=0$.

In that region of values of the coupling constants where H_1 and H_2 assume the form of the TL Hamiltonian, i.e., for $\gamma_1 \geq |\gamma_4|$ and $\gamma_2 \geq |\gamma_3|$, it is necessary to impose such additional limitations on the coupling constants that $\chi_0 > 0$ and $\varkappa_T > 0$. This is a condition of stability of the system against spontaneous decay for H_2 and a transition into the ferromagnetic state for H_1. From the table we obtain, for $\gamma_1 \geq |\gamma_4|$, $\bar{\gamma}_1 > u_1$ and for $\gamma_2 \geq |\gamma_3|$, $\bar{\gamma}_2 > u_2$. Hence we obtain the right and upper boundaries in the diagram.

4. COMPARISON WITH RESULTS OBTAINED FOR OTHER ONE-DIMENSIONAL MODELS

The existence of dynamic instabilities in a one-dimensional metal has been repeatedly discussed. Instability with respect to ionic displacement and phonon-mode softening was considered by Peierls and Fröhlich[24]; antiferromagnetic instability, by Overhauser.[25] Bychkov, Gor'kov, and Dzyaloshinskiĭ[17] have shown that the Peierls instability is always accompanied by the Cooper instability and that by themselves the Fröhlich and Overhauser formulas are incorrect. They correspond to the ladder approximation, which is inapplicable in the one-dimensional case. Subsequently, Dzyaloshinskiĭ and Larkin[11] proved that the antiferromagnetic instability can set in only in the case when in the one-dimensional case the transfer processes are important.

However, even the parquet approximation[11,17] is of very limited accuracy in the one-dimensional case. In their paper,[19] Menyhard and Solyom succeeded in going outside the framework of the parquet approximation. Subsequently, the question of the possible states in a one-dimensional system was discussed in a number of papers[26,27] with the aid of the method proposed by them.[19] We shall carry out a comparison with Kimura's results obtained in[27], where the model with $g_4 = 0$, $g_1^{\parallel} = g_1^{\perp} = g_1$, $g_2^{\parallel} = g_2^{\perp} = g_2$, and $g_3 \neq 0$ is investigated in the framework of the renormalization-group method.

For $|\gamma_4| \leq \gamma_1 < (u_1^2 + \gamma_4^2)^{1/2}$ and $|\gamma_3| \leq \gamma_2 < (u_2^2 + \gamma_3^2)^{1/2}$ we have a "zero-charge" situation, i.e., $\gamma_3(z), \gamma_4(z) \to 0$ for $z \to 0$. The behavior of the correlation functions in this limit is determined by both the spin and acoustic excitations. For $T \to 0$ the system exhibits a tendency towards triplet pairing (the region I_2 in Fig. 2), which can be accompanied by antiferromagnetic ordering (the region I_1) or singlet pairing (the region I_3). The choice of the coupling constants by Kimura[27] corresponds in our model to the line $\gamma_1 = |\gamma_4|$ in the region I. The lower boundary of the region I will correspond to the equation $\gamma_1 = \gamma_4 = 0$. The curves OB and OC merge and become the straight line OE. Thus, in this limiting case it turns out, in accordance with Kimuara's results,[27] that only simultaneous triplet and singlet pairing can occur in the region I. It can be seen from the diagram that in the general case, when $\gamma_1 \neq \gamma_3$, the region I breaks up into three subregions with different symmetries. Furthermore, the presence of a strong potential attraction ($\gamma_2 \gg |\gamma_3|$) facilitates singlet pairing (the region I_3), while the presence of a strong spin interaction ($\gamma_1 \gg |\gamma_4|$) leads to antiferromagnetic ordering (the region I_1).

The results obtained by Kimura[27] for the exponents of the power-law behavior of the correlation functions are, as can be seen from our solution, valid only for small values of the coupling constants, when the expansion

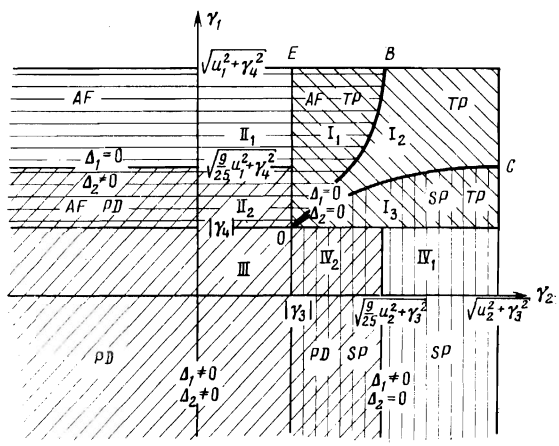

FIG. 2. The state phase diagram. PD denotes the presence in the system of a Peierls distortion with a doubling of the period; AF, antiferromagnetic ordering; SP, singlet Cooper pairing; TP, triplet pairing. The curves OB and OC are given respectively by the equations $\mu_1 + \mu_2^{-1} = 2$ and $\mu_1^{-1} + \mu_2 = 2$.

$$\mu_2=\left(\frac{1+\bar{\gamma}_2}{1-\bar{\gamma}_2}\right)^{1/2}\approx 1+\bar{\gamma}_2=1+[(g_1-2g_2)^2-g_3^2]^{1/2}. \quad (14)$$

is possible.

The spin susceptibility χ_0 turns out to be different from zero in the region I. For $\bar{\gamma}_1 \to u_1$, $\chi_0 \to \infty$, and there is a buildup of spin-density oscillations in the system. The system can go over from the paramagnetic state (the region I_2) or the antiferromagnetic state (the region I_1) into the ferromagnetic state, but this transition requires a more thorough investigation. A similar instability of the system exists at $\bar{\gamma}_2 = u_2$.

The region II: $|\gamma_3| \leq \gamma_2 < (u_2^2+\gamma_3^2)^{1/2}$ and $\gamma_1 < |\gamma_4|$. A gap Δ_2 appears in the spectrum of the electron-density oscillations, and the entire low-lying excitation branch is exhausted by spin excitations, which make the dominant contribution to the exponent of the power-law singularity of the correlation functions for $z \to 0$. Such a situation occurs in the Hubbard model with repulsion in the case of a half-filled band. The ground state turns out to be a dielectric state with antiferromagnetic ordering (the region II_1), which can be accompanied by a doubling of the period (the region II_2). The magnitude of the gap in the exact solution of the Hubbard model[13] in the strong-coupling limit is equal to γw, which coincides with Δ_2 in (11). Thus, we have both qualitative and quantitative coincidence with the known exact solutions[13] in the region II. In the weak-coupling limit ($\gamma \ll 1$) we can propose for the determination of the gap in the spectrum of H_2 a self-consistent procedure similar to the one worked out earlier by us,[28] in accordance with which we obtain for Δ_2 the expression

$$\Delta_2 = w|\gamma|^{1/2}\exp[-1/|\gamma|], \quad (15)$$

which coincides up to a pre-exponential numerical factor with the exact value of Δ_2 obtained in the Hubbard model.[13]

Kimura's calculations,[27] which pertain to the $\gamma_1=\gamma_4$ case, predict in the region II the simultaneous appearance of antiferromagnetic ordering and a Peierls doubling of the period, which agrees with our result if we set $\gamma_1=\gamma_4$. However, the values for the exponents of the power-law behavior of the correlation functions turn out to be different. Kimura found $N(2k_F,\Omega)\sim\Omega^{-3/2}$, while we found $N(2k_f,\Omega)\sim\Omega^{-1}$. This is connected with the fact that in the second-order renormalization-group method, which was used by Kimura,[27] and which correctly indicates a strong coupling in the region II, it is not possible to say anything about the existence and magnitude of the gap Δ_2 in the excitation spectrum and to find out the anomalies in the behavior of the admittances that are connected with the presence of this gap.

In the region III ($\gamma_1 < |\gamma_4|$ and $\gamma_2 < |\gamma_3|$) there are no gapless excitations, and the system is a Peierls dielectric at $T=0$. This region has been studied only by Kimura,[27] but no results pertaining to the gaps Δ_1 and Δ_2 are contained in this paper.[27] As in the region II, in the region III the isothermal compressibility $\varkappa_T = \Delta n/\Delta\mu \to 0$ as $T \to 0$, which indicates that the chemical potential, μ, as a function of n has a discontinuity at $T=0$, as it should be in the Hubbard model.[13]

For $|\gamma_3| \leq \gamma_2 < (u_2^2+\gamma_3^2)^{1/2}$ and $\gamma_1 < |\gamma_4|$ (the region IV) the triplet excitations are separated by a gap ($\Delta_1 \neq 0$). At $T=0$ singlet Cooper pairing is possible (the region IV_1), and may also be accompanied by a Peierls doubling of the period (the region IV_2). For the region IV Kimura found that the Peierls doubling and the Cooper pairing always appear simultaneously, i.e., he missed the region IV_1. This difference is again due to the fact that the method used by him[27] does not allow the discovery of, and the allowance for, the presence of the gap Δ_1 in the spectrum of the operator H_1.

In the model considered by Kimura[27] it was assumed that $g_4=0$. According to Solyom,[29] allowance for g_4 in the case when $g_1^{\parallel}=g_1^{\perp}$ and $g_2^{\parallel}=g_2^{\perp}$ in the framework of the renormalization-group method significantly alters the results obtained by Kimura in[27], where $g_4=0$. Thus, Solyom[29] asserts that if $g_3=0$, then the situation turns out to be a "zero-charge" situation in the entire region of values of g_1 ($g_1(z)\to 0$), while g_2 and g_4 for $z\to 0$ do not have a universal value. In the present paper we have obtained the result that the influence of g_4 is insignificant. This quantity enters only in the definition of the bare velocity of sound, $v_F u_2$, and the velocity of the spin excitations, $v_F u_1$, and $u_1 = 1 - g_4/2\pi v_F$ and $u_2 = 1 + g_4/2\pi v_F$ participate in the subsequent calculations as dimensionless scaling factors attached to the $\{\gamma\}$. It is clear that in this case the direct expansion in a perturbation-theory series in g_4 is not correct. Nevertheless, it seems to us that the second order in the renormalization-group method is sufficient for the correct allowance for the role of the collective excitations in the $g_4 \neq 0$ case as well, but the calculation should be self-consistent, and for this purpose, for $g_4 \neq 0$, it is necessary to additionally take into account the presence of a third channel, of exchange type relative to the Peierls channel. This third channel is directly connected with the collective modes.

All the above-presented assertions pertaining to the behavior of the correlation functions and the results presented in the table were obtained for $T=0$. It is not difficult to generalize them to the $T \gg \Delta_1$ or $T \gg \Delta_2$ case. Contributions to the formation of the singularities of the correlation functions for $\Omega \to 0$ and $q=0$ or $q=2k_F$ are made in this case by both the spin and the acoustic excitations, which, as has already been indicated above, can again be described by the TL Hamiltonian. As a result, the expressions for the critical indices assume the same form as in the TL model generalized to the case in which allowance is made for spin.[15]

5. ROLE OF THE TRANSVERSE COUPLING BETWEEN THE FILAMENTS

Let us make a few comments about the generalization of the obtained results to the case of a quasi-one-dimensional system in which there is coupling between the filaments. In a quasi-one-dimensional system, besides the kinetic transverse coupling, there can exist in the transverse direction an additional interaction in which a potential part and an interaction connected with electron

transitions from filament to filament can be distinguished. It is clear that a potential interaction between the filaments will not change the main results. The critical indices in the table should be understood as certain quantities averaged over the transverse momenta. Thus, in the case of the Coulomb interaction we have for the correlator $N(2k_F, \Omega)$ in the $\Omega \to 0$ and $T=0$ limit the relation

$$N(2k_F, \Omega) \sim (\Omega/\omega_p)^{-\lambda}, \quad \lambda = (\omega_p/\varepsilon_F)^2, \tag{16}$$

where $\omega_p^2 = 4\pi e^2 n/m$ is the plasma frequency and ε_F is the Fermi energy.

The effect of the interaction associated with the electron transitions from one filament to another has been investigated by Gor'kov and Dzyaloshinskiĭ[30] in the framework of the parquet approximation, the degree of reliability of which has as yet not been completely elucidated in such problems (in the purely one-dimensional case it is inadequate). In the subsequent discussion we shall drop such type of interaction, assuming that the entire interaction is concentrated on the filaments, and study the effect of only the kinetic coupling, assuming it to be weak and characterizing it by the quantity $W_\perp$ ($W_\perp$ is the width of the electronic band in the transverse direction; $W_\perp \ll W_\parallel$). Here we can clearly expect a phase-transition temperature different from zero.

According to the standard approach (allowance for $W_\perp$ by the mean-field method), the connection between $W_\perp$ and the critical temperature, T_c, of the phase transition into a state with a given critical mode is given by the equation

$$\left(\frac{W}{W_\parallel}\right)^2 \left(\frac{T_c}{W}\right)^{\nu(T_c)} \sim \left(\frac{W_\perp}{W_\parallel}\right)^2, \tag{17}$$

where $\nu(T)$ is the exponent of the power-law singularity ($\nu < 0$ and can be taken from the table) for the correlator characterizing the corresponding mode and W is the energy scale on which the interaction is defined ($W \leq W_\parallel$). In the presence of two interacting critical modes the state with the mode corresponding to the higher T_c is realized (see, for example,[31]). However, for a more thorough consideration of the question of the T_c and the possible states, let us carry out a microscopic analysis. We shall proceed here from the fact that the present description and, in particular, the renormalization-group equations are valid for $T \gg W_\perp$. In the region $T \sim W_\perp$, the system acquires three-dimensional features. With allowance for the fact that, besides the Cooper instability, there occurs in the system instability with respect to the doubling of the period not only along, but also in the directions perpendicular to, the filaments (following Efetov and Larkin,[32] we shall call this instability the anti-Peierls instability), the behavior of the system at $T \ll W_\perp$ will be determined by the parquet equations, which are valid in this case.

According to this scheme, the following picture will obtain in the region I in the phase diagram. When $W_\perp \ll T \ll W$, the description in terms of collective excitations is valid; as the temperature approaches $W_\perp$, the fluctuations in the system will intensify, and beginning from $T \sim W_\perp$ the single-particle states will be re-established. Subsequently, the growth of the fluctuations ceases, and the system remains a normal metal right up to $T=0$.

When allowance is made for a weak transverse kinetic coupling in the region II, we should distinguish two cases: $W_\perp \ll \Delta_2$ and $W_\perp \gg \Delta_2$. If $W_\perp \gg \Delta_2$, the phase transition occurs at a finite temperature. Qualitatively, the picture of the transition is as follows. When T approaches a value of the order of $W_\perp$ from above, the fluctuations in the system intensify, this buildup occurring in accordance with the laws of one-dimensional systems. In the $T \sim W_\perp$ region, the fluctuations, which at $T \gg W_\perp$ had a purely longitudinal character, begin to spread in the transverse direction, acquiring three-dimensional features. Their subsequent development can be described by parquet equations with allowance for the anti-Peierls feature. Following the above-presented arguments, we have for the invariant charge $g(t)$[19] in the case when $-\gamma_2 = |\gamma_3| = \gamma \ll 1$ the following equations:

for $t < \eta (T \gg W_\perp)$

$$\frac{dg}{dt} = -2g^2(g+1) \tag{18}$$

and for $t > \eta (T \ll W_\perp)$

$$\frac{dg}{dt} = -2g^2, \tag{19}$$

where

$$t = \ln(W/T), \quad \eta = \ln(W/W_\perp), \quad g(0) = -\gamma,$$

from which we can obtain the following estimates for T_c: for $T_0 \ll W_\perp \ll W$

$$T_c = AT_0 (W_\perp/W)^\gamma, \tag{20}$$

and for $\gamma^{1/2} T_0 \ll W_\perp \ll T_0/\gamma^{1/2}$

$$T_c = BT_0 (W_\perp/T_0)^{-1/\ln|\gamma|}, \tag{21}$$

where A, B are numbers ($A \sim 1$ and $B < 1$), $T_0 = We^{-1/2\gamma}$ is the critical temperature computed by the mean-field method for an isolated filament, while T_c corresponds to simultaneous anti-Peierls and antiferromagnetic instabilities.

However, in order to more rigorously solve the problem of just which state is realized below T_c, a more careful matching in the region $T \sim W_\perp$ is required. For $W_\perp \ll \Delta_2$ intense longitudinal fluctuations have time to develop in the system ($\gamma^{1/2} T_0$ coincides with the value of Δ_2), and it is extremely difficult to investigate this region of $W_\perp$ values (Eqs. (18) and (19) lose their meaning in this case, since $g(\eta) \sim 1$). We shall, for definiteness, assume that, for $W_\perp \ll \Delta_2$, the results obtained for one filament are valid, i.e., the system is a Mott dielectric.

Similarly, for the region IV in the phase diagram, if we take into account the anti-Peierls instability, then

when the kinetic coupling $W_\perp$ is of small magnitude, i.e., for $W_\perp \ll \Delta_1$, the quasi-one-dimensional system will not undergo any transition right up to $T=0$. For $W_\perp \gg \Delta_1$ a transition into the superconducting state, accompanied by the anti-Peierls doubling of the period, takes place. The estimates for T_c are the same as in (20).

In the region III (Fig. 2), on going from a one-dimensional to a quasi-one-dimensional system, either a state of the purely anti-Peierls type, or a state of the mixed type, in which the dielectric (anti-Peierls) phase coexists with antiferromagnetic ordering or with Cooper pairing, can be realized, depending on the relation between $W_\perp$, Δ_1, and Δ_2.

A more thorough analysis is carried out in the present authors' paper.[34]

6. CONCLUSION

In a one-dimensional Fermi system four different types of interaction should be distinguished. Let us split the potential interaction between the subsystems with $p>0$ and $p<0$ into its symmetric and antisymmetric—with respect to spin—parts. The first of them (the corresponding coupling constant is denoted by γ_2) determines, as in the usual Landau theory of the Fermi liquid, the spectrum of the density oscillations, while the antisymmetric—with respect to spin—part of the interaction (the constant γ_1) enters into the determination of the spectrum of the spin waves.

In the interaction that is accompanied by electron transitions between the subsystems, we can distinguish a part that can be called an exchange interaction. In the process of such an interaction (constant γ_4) the subsystems with $p>0$ and $p<0$ exchange only spin, the number of electrons in the subsystems remaining unchanged in the process. The fourth type of interaction is represented by U-processes. In this case only the number of electrons in the subsystems changes, the total spin (more exactly, the z component of the total spin) of the subsystems remaining unchanged (the corresponding coupling constant is designated as γ_3).

In the absence of interactions giving rise to electron transitions between the subsystems ($\gamma_3=\gamma_4=0$), the model under consideration is a generalization of the TL model to the case in which allowance is made for spin. The eigenstates of such a system are only collective excitations; strictly speaking, the concept of single-particle excitations itself turns out to be inapplicable. The correlation functions at large distances and times decrease in a power-law fashion. The contribution to the corresponding exponent of the power-law behavior is made by the long-wave parts of the collective-excitation spectra. The phase-transition temperature for the system is equal to zero.

There is a difference between the results obtained in the present paper and the results of other papers.[26,27] The coupling constants for the acoustic excitations (γ_2) and the spin excitations (γ_1) in the present model have been uncoupled, which in turn is due to a more general choice of the initial interaction Hamiltonian and leads to a situation in which there appear in the region I in the phase diagram (Fig. 2), besides the subregion I_3, the subregions I_1 and I_2.

The exact computation of the critical indices carried out in the present paper (see also[14]) indicates that some functional dependence is lost in the corresponding computations by the renormalization-group method. The expressions obtained for the critical indices by Solyom[26] and Kimura[27] are only the first terms of the expansions of the exact expressions (see (14)). This fact should evidently be linked with the fact that, in using the renormalization-group method, it is necessary to directly take account of the presence of a third channel, (of exchange type relative to the Peierls channel), which directly determines the collective-excitation spectrum, although it has no logarithmic singularities in first-order perturbation theory.

The presence in the system of interaction connected with electron transitions between the subsystems with $p>0$ and $p<0$ can significantly change the properties of the system. Thus, in the case when $\gamma_3\neq 0$ (the band is half filled), as γ_2 is varied, there occurs at the point $\gamma_2=|\gamma_3|$ a first-order parametric transition, since there is a jump in the compressibility at the transition point. This transition appears to correspond to the distinctive Mott metal-dielectric transition in a one-dimensional system. Notice that it has a purely correlative character, and is not connected with a change in the symmetry of the system as a result of the presence of some order. This in part explains the unsuccessful attempts to discover this transition in the framework of perturbation theory.

The appearance of the dielectric gap (the region II, $\gamma_2<|\gamma_3|$)[3)] inhibits the transition into some superconducting state at $T=0$, and facilitates the transition into a state with an antiferromagnetic order (the region II_1), or into a mixed state (the region II_2), in which a Peierls doubling of the period occurs along with the antiferromagnetic ordering. For $\gamma_2\geq|\gamma_3|$, the interaction defined by the constant γ_3 turns out to be effectively screened, and the system can be described as before by the TL Hamiltonian, though with modified coupling constants.

The presence of exchange interaction between the subsystems ($\gamma_4\neq 0$) can lead to a situation in which the ground state turns out to be a singlet state and the triplet excitations are separated from this state by a gap (the region IV). Such a state corresponds to a singlet magnetic substance. The spin susceptibility behaves in an activating manner. The correlation functions that are not directly connected with the triplet excitations decrease at $T\ll\Delta_1$ and at large distances according to a power law, the contribution to the exponent of the power-law behavior being made by the long-wave acoustic excitations. The assertion that only the long-wave gapless part of the collective-excitation spectrum makes a contribution to the exponent of the power-law decrease of the correlation functions was put forward as a hypothesis in Efetov and Larkin's paper[32]; their analysis pertains in the present model to the region IV and the coupling-constant values: $\gamma_3=0$, $u_2+\gamma_2=1$, and $\gamma_4\to-\infty$. In the region IV we can also expect a deviation, due to an in-

crease in the density of states near the gap, from the normal temperature dependence of the observable characteristics.

An interesting state is realized in the region III; here there are no gapless excitations, and for $T \ll \min[\Delta_1, \Delta_2]$ the disruptive influence of the long-wave collective excitations will be considerably weakened. At $T=0$ there exists in the system long-range order corresponding to the state of a Peierls dielectric.

The transverse kinetic coupling can significantly alter the results obtained for a one-dimensional system. If we take the anti-Peierls instability into account, then for $W_\perp \gg \Delta_1, \Delta_2$ the state diagram obtained in Dzyaloshinskiĭ and Larkin's paper[11] in the framework of the parquet approximation is qualitatively re-established. In this case T_c can be a complicated function of $W_\perp$ (see (20) and (21)). Apparently, there also exists a region of sufficiently small values of $W_\perp$ ($W_\perp \ll \Delta_1$ and $W_\perp \ll \Delta_2$) where the results obtained for a one-dimensional system do not qualitatively change. This is in regard to the states of the Mott (the region II) and Peierls (the region III) dielectrics and the state of the singlet magnetic substance (the region IV).

Although the entire analysis pertained to, and is valid for, a model with a linear spectrum with allowance for the cutoff suggested by Luther and Emery,[17] we hope that the obtained results (the table and the phase diagram in Fig. 2) and the assertions concerning the spectrum and the role of the collective excitations in a one-dimensional system are of sufficiently general character, since of real importance in the present problem (in the problem of the behavior of the correlation functions at large distances) is the shape of the spectrum near the Fermi surface, the linear character of which is always assumed, and the proposed interaction Hamiltonian is representative enough for it to describe all the known physical situations.

Recently, Emery, Luther, and Peschel[33] made allowance for U processes, using a scheme similar to ours (see Sec. 2); the correlation functions were, however, not investigated by these authors and, consequently, the question of the possible states of a one-dimensional system was not considered.

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[1)]This can already be seen from the fact that the velocity of sound in a one-dimensional Fermi system coincides with v_F, whereas in two- and three-dimensional ideal Fermi gases it turns out to be less than v_F, being equal respectively to $v_F/\sqrt{2}$ and $v_F/\sqrt{3}$, which leads to sound attenuation in these systems.

[2)]In the spin-zero variant of the TL model, the two diagrams with $g_2^{\parallel}$ and $g_2^{\perp}$ (Fig. 1) do not differ from each other, and only one of them needs to be retained; therefore, the factor of two connected with spin drops out in the computation involving the intermediate states, and this alters the results drastically. The square-root singularity[15] in the single-particle Green function is replaced by a pole singularity[20]:

$$G(p,\omega)=\frac{(p/\varkappa)^{\nu_2}}{[(\omega-v_F p)(\omega-cp)]^{\nu_2}} \to G(p,\omega)=\frac{(p/\varkappa)^{\nu_2}}{\omega-v_F p}.$$

The expressions for the correlation functions also change.

[3)]The interaction constants $\{\gamma\}$ are defined in this way: the positive values of γ_2 correspond to attraction of electrons with opposite momenta, the negative values, to repulsion; the positive values of γ_1 correspond to ferromagnetic coupling between the electrons, the negative values, to antiferromagnetic coupling; the interaction defined by the constants γ_3 and γ_4 is, on account of the indicated off-diagonal nature of these constants, determined by the absolute values $|\gamma_3|$ and $|\gamma_4|$.

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An investigation of the dispersion law for carriers in bismuth doped with acceptor-type impurities

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The Shubnikov–de Haas effect has been investigated in single-crystal bismuth samples doped with Sn and Pb in fields of up to 60 kOe at liquid-helium temperatures. An increase in the cyclotron masses $m_{cT}(\epsilon_{FT})$ of holes at T with increase in the volume of the Fermi surface was found. In the framework of the Kane two-band model, a gap of $\epsilon_{gT} = (200 \pm 40)$ meV was found at T. The energy dependence of the low cyclotron masses of carriers $m_{cL}(\epsilon_{FL})$ at L on lowering the Fermi level (**H** parallel to the binary axis) was found to be linear. The parameters of the Abrikosov dispersion law for carriers at the point L were determined as $\nu_\chi = 0.99 \times 10^8$ cm/s and $\nu_y = 0.74 \times 10^8$ cm/s. It was shown that the gap ϵ_{gL} at L lies in the interval $0 < |\epsilon_{gL}| < 15$ meV. It was also shown that the band overlap in the acceptor-doped bismuth samples investigated did not depend on the impurity concentration. It was found that the parameter $\epsilon_{0v} + \epsilon_{gL}/2 = (46 \pm 2)$ meV.

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1. INTRODUCTION

The energy spectrum of bismuth has been investigated in a large number of experimental and theoretical papers.[1–4] However, right up to the present time models describing the spectrum of carriers at the points L and T in the reduced Brillouin zone, as well as certain important parameters of the spectrum have not been given unique values, owing to the complexity of the band structure of bismuth and the strong anisotropy of its properties. There are considerable discrepancies in the estimates of the degree of deviation of the hole spectrum from parabolic.[5–8] At the same time the values of the gap parameter of the electron spectrum at L found in various papers do not agree not only in magnitude, but even in sign.[4,7–16]

The introduction of acceptor-type impurities (Pb, Sn) into the bismuth lattice allows the Fermi level to be displaced within wide limits. This opens up the possibility in principle of deriving the energy dependence of the basic parameters of the carrier spectrum at the overlapping extrema of the conduction band L_s and the valence band T_{45}^-,[3] which is essential for checking the theoretical models describing the laws of carrier dispersion at the L- and T-extrema.[1–4,17] No less interesting is the problem of finding new extrema in the valence band of Bi on suitably lowering the Fermi level.

At acceptor impurity concentrations below 0.1 at.% the lattice parameters of Bi do not vary markedly,[3] and the effect of the acceptors is apparently only to make the electron and hole concentrations unequal and to change the electron and hole mobilities.

Theoretical estimates indicate that localization of carriers at impurity centers in Bi has an extremely low probability, owing to the strong screening of the impurity potential by free carriers.[19] It should be noted that even when an impurity level is formed, it inevitably falls within the spectrum of allowed states, owing to the overlap of the valence band with the conduction band, which must lead to delocalization. At the present time there is no direct evidence of the existence of local or quasi-local impurity levels in the spectrum of impurity-doped bismuth.

The investigation of the Fermi surface in impurity-doped bismuth by quantum-oscillation effects is made very difficult because the relaxation times τ of the electrons and holes, which reach $\sim 10^{-9}$ s in pure single crystals of bismuth at liquid-helium temperatures,[20,22] drop rapidly with increase in the impurity concentration. At the same time the dominant scattering mechanism at liquid-helium temperatures becomes scattering by ionized impurities,[5,23–26] which leads to a marked dependence of τ on energy.

Decrease in relaxation time with increase in impurity concentration by several orders does not impose severe restrictions on the investigation of galvanomagnetic and thermomagnetic effects in Bi, but this involves its own inherent difficulties. These are primarily associated with the ambiguity in the interpretation of the experimental data, as a result of the obvious arbitrariness in the choice of a model for the band structure. Additional