that the "nuclear phase" Φ_L contains the contribution of the left cuts from the rescattering graphs (which can themselves lead to bound and virtual states, cf., [9]). Together with the contribution from the exchange of particles, as in the usual nuclear phase, there is also present the contribution of the Coulomb forces.

We now discuss the application of our results under conditions of phase-shift analysis as compared with the unitarization procedure in the work of Ascoli and Wyld. [4] As the authors themselves have noted, unitarization in[4] reduces to the summation of the rescattering series with the propagators in Fig. 3 being replaced by δ-functions. Then, naturally, the analytical properties of $B^J(\sigma, s)$ are violated and as a result of this singularities appear at $s = s_1, s_2$, i.e., at those threshold values of the energy s when the formation of a resonance is possible simultaneously in two subsystems (13) and (23) of three particles. Taking into account the width of the resonance, the singularities s_1 and s_2 are shifted to the second sheet but there remains a strong dependence of $B^J(\sigma, s)$ on s in the neighborhood of these points. The forcing of such an incorrect dependence on to the physical amplitude B^J, apparently, is what leads to the deterioration of the quality of the fit.

The results of our analysis show that unitarization leads (in the single-channel case) to the appearance of the additional phase Φ_r, which should not affect the procedure of phase analysis, in which the total phase $\Phi_r + \Phi_L$ is determined. At the same time the phase ν, appearing in Φ_L is determined by the real analytic solution of N/D equations taking into account arbitrary exchange mechanisms (arbitrary left cuts), and therefore must pass through $\pi/2$, if there exists a resonance in the $\rho\pi$ system.

In principle the phase Φ_r can cancel a part of the resonance dependence of Φ_L. For example, in the state $J^P = 1^+$ of the $\rho\pi$-system, where the existence of a $A1$-resonance is assumed, the phase Φ_r is positive and falls off with energy (while Φ_L increases in the resonance case). But the total change in Φ_r over the range 900 MeV $\leqslant s^{1/2} \leqslant$ 1400 MeV is according to preliminary estimates ~10° and therefore is not likely to explain the absence of resonance behavior in the total phase $\Phi_r + \Phi_L$.

The authors are grateful to I. V. Chuvilo for discussions which have stimulated the present investigation.

[1]G. Ascoli, D. V. Brockway, H. B. Crawley, L. B. Eisenstein, R. W. Hanft, M. L. Ioffredo, and U. E. Kruse, Phys. Rev. Lett. **25**, 962 (1970). G. Ascoli *et al.*, Phys. Rev. D7, 669 (1973).

[2]M. Deutchmann *et al.*, Phys. Rev. Lett. **49B**, 388 (1974).

[3]J. D. Hansen, G. T. Jones, G. Otter, and G. Rudolf, Nucl. Phys. **B81**, 403 (1974).

[4]G. Ascoli and H. W. Wyld, Phys. Rev. **D12**, 43 (1975).

[5]I. J. R. Aitcheson and R. J. Golding, Phys. Rev. Lett. **59B**, 288 (1975).

[6]Yu. A. Simonov, Preprint de Vrye Universitet, Amsterdam, 1975, Nucl. Phys. **A268**, (1976).

[7]N. I. Muskhelishvili, Singulyarnye integral'nye uravneniya (Singular Integral Equations), Nauka, 1968.

[8]G. Goebel, Phys. Rev. Lett. **13**, 149 (1964), C. Schmid, Phys. Rev. **154**, 1363 (1967), A. M. Badalyan, M. I. Polikarpov and Yu. A. Simonov, Preprint ITEP-38, 1975.

[9]Yu. A. Simonov, Zh. Eksp. Teor. Fiz. **69**, 1905 (1975) [Sov. Phys. JETP **42**, 966 (1976)].

Translated by G. Volkoff

Properties of heavy leptons and their manifestation in the $e^+e^- \to$ hadrons reaction

Ya. I. Azimov, L. L. Frankfurt, and V. A. Khoze

B. P. Konstantinov Institute of Nuclear Physics, Academy of Sciences of the USSR, Leningrad

(Submitted July 1, 1976)
Zh. Eksp. Teor. Fiz. **72**, 63–72 (January 1977)

The manifestations of heavy leptons in various characteristics of e^+e^- annihilation are examined. The properties of the heavy-lepton decays and their contributions to $R(s)$ and $\epsilon(s)$ are discussed with allowance for event-selection effects. The inclusive spectra of the secondary particles are also discussed. It is shown that the characteristic properties expected for the heavy leptons are consistent with the data on anomalous muon production.

PACS numbers: 14.60.−z, 13.65.+i

1. INTRODUCTION

According to current ideas (e.g. [1–5]) the reaction $e^+e^- \to$hadrons is a rather complicated process in the energy region $2.4 \lesssim s^{1/2} \lesssim 7.4$ GeV, there being at least two components:

a) In the region $s^{1/2} \gtrsim 2.4$ GeV there is apparently a virtually steady quark-parton component (q component) composed of the usual light quarks $q=u, d, s$. This component should correspond to a logarithmic growth of the multiplicity, to scale invariant structure functions in the inclusive hadron spectra, and to hadron jets. It is also assumed that the contributions from this component to R and ε, where

$$R=\sigma(e^+e^-\to\text{hadrons})/\sigma(e^+e^-\to\mu^+\mu^-),\quad \varepsilon=\langle E_{ch}\rangle/s^{1/2}$$

(E_{ch} is the energy of the charged particles), are almost constant ($R_q\approx 2-2.5$ and $\varepsilon_q\approx 0.6-0.66$).

b) At $s^{1/2}\gtrsim 3.6$ GeV there apparently comes into play a second component associated with the production of pairs of new heavy particles that cannot be composed of light quarks alone. In particular, the production of heavy meson pairs ($D\bar{D}$, $D\bar{D}^*$, $D^*\bar{D}^*$, etc.) is expected (e.g. [3–7]). Here the symbols D and D^* represent mesons of composition $Q\bar{q}$ or $\bar{Q}q$, where Q and $\bar{Q}$ are the quarks occurring in the ψ particle. In the accessible energy region, this component is not so smooth as the q component, but we can speak of it as a unified component if we are unable (or for some reason do not wish) to distinguish the contributions from the individual channels.

The data on anomalous μe pairs[8] indicate that the heavy component also contains heavy lepton pairs L^+L^-, with $m_L\sim 1.8$ GeV. The anomalous muon production[9] also seems to indicate the presence of such heavy leptons. A substantial number of papers (e.g., [10–14]) have already been published on the properties of μe events[8] under the assumption that they are due to leptonic decays of heavy-lepton pairs.

Even before the data of Perl *et al.*[8] were published, the production of heavy-lepton pairs was invoked to provide an explanation (even though a partial one) of the growth of R at $s^{1/2}\gtrsim 3.6$ GeV (e.g., [1,7,15]; also see[10]). Also associated with heavy leptons were the normal expectations (e.g., [3,10]) of describing the fall of $\varepsilon(s)$ with increasing s,[16,17] the appearance of hadron jets, and the behavior of the hadron inclusive spectra (see[5]). Rough estimates were made of the effect of hadronic decays of heavy leptons and heavy hadrons on such characteristics of the $e^+e^-\to$ hadrons reaction as the average charged-particle multiplicity $\langle n_{ch}\rangle$ and the average energy $\langle E_{track}\rangle$ of a charged particle (e.g., [4,5]).

In this paper we present a detailed study of the manifestations of heavy leptons in e^+e^- annihilation. With a realistic selection of events, these manifestations turn out to be not quite what is usually expected. Heavy-lepton decays, especially the contribution of the hadron continuum, are discussed in Sec. 2. It is shown that the characteristics expected for a heavy lepton with ton with $m_L\sim 1.8$ GeV agree well with the experimental data.[8,9] In Sec. 3 we discuss the contributions from production and subsequent decay of heavy leptons to the basic characteristics of the $e^+e^-\to$ hadrons reaction at $s^{1/2}\gtrsim 3.6$ GeV with allowance for event selection effects. In Sec. 4 we discuss the inclusive spectra of particles produced as a result of e^+e^- annihilation into a pair of new heavy particles (in particular, heavy leptons) with subsequent decay of the new particles. In Sec. 5 we list the qualitative characteristics of μe events resulting from L^+L^- decay.

As will be evident from an analysis of their properties, the heavy leptons and heavy hadrons differ substantially from one another. Hence it is now reasonable to speak no longer of two components in the $e^+e^-\to$ hadrons reaction, but of three components that differ in nature: the q, heavy lepton, and heavy hadron components.

2. REQUIRED PROPERTIES OF THE $L^\pm$ HEAVY LEPTONS

We shall assume that the $L^\pm$ lepton is of sequential type,[18] i.e., that it has a new lepton number and is associated with a new neutrino ν_L. This hypothesis is consistent with the properties of μe events[8] and with the apparent absence of $L^\pm$ particles in neutrino experiments.[19,20] The properties of such leptons have been discussed in a number of papers[10,18,21,22] in the context of the universal weak interaction hypothesis, both for massless ν_L neutrinos and for massive ones. Fujikawa and Kawamoto[10] also examined a more general V, A structure for the (L, ν_L) current, including the $V+A$ version which, for example, is possible in vectorlike models.[23,24]

The calculations for the decays

$$\begin{aligned} L^-\to\; &\nu_L+e^-+\bar{\nu}_e \quad (a)\\ &\nu_L+\mu^-+\bar{\nu}_\mu \quad (b)\\ &\nu_L+\pi^-(K^-) \quad (c)\\ &\nu_L+\rho^- \quad (d) \end{aligned} \tag{1}$$

are standard and reliable (e.g., [10,21,22]). For the $V\pm A$ version with $m_L=1.8$ GeV we obtain the ratios[10]

$$\Gamma(L^-\to\nu_L\pi^-):\Gamma(L^-\to\nu_L\rho^-):\Gamma_e\approx\begin{cases}0.55:1.2:1\\0.79:1.65:1\end{cases} \tag{2}$$

Here and below the upper (lower) value is for $m_{\nu_L}=0$ ($m_{\nu_L}=0.3\,m_L$) and $\Gamma_e\equiv\Gamma(L^-\to\nu_L e^-\bar{\nu}_e)$, so that[21,22]

$$\Gamma_e=\frac{G^2m_L^5}{192\pi^3}N\left(\frac{m_{\nu_L}}{m_L}\right),$$
$$N(y)=(1-y^4)(1-8y^2+y^4)-24y^4\ln y \tag{3}$$

(all the $L^\pm$ partial widths are the same for the $V-A$ and $V+A$ versions). As is evident from (2), the ratio of the sum of the widths for decays (1) to the width Γ_e is ~ 3.8 for $m_{\nu_L}=0$ and increases with increasing m_{ν_L}, reaching ~ 4.5 at $m_{\nu_L}=0.3\,m_L$. It follows from the data of[8] that $\Gamma_{tot}/\Gamma_e\approx 6\pm 1$ for the $L^\pm$ leptons. This means that decays (1) must be very important. We note that only one charged particle (one track) is produced in each of these decays.

The calculations for the decays

$$\begin{aligned} L^-\to\; &\nu_L+A_1^- \quad (a)\\ &\nu_L+K^{*-} \quad (b) \end{aligned} \tag{4}$$

are less reliable[10,21,22]; they give[10]

$$\Gamma(L\to\nu_L A_1):\Gamma(L\to\nu_L K^*):\Gamma_e\approx\begin{cases}0.44:0.06:1\\0.46:0.08:1\end{cases}. \quad (5)$$

The greatest theoretical uncertainty arises in treating the multihadron decay

$$L^-\to\nu_L+\text{hadron continuum}. \quad (6)$$

The spectral functions

$$\sum_F \langle 0|J_\mu^{w+}(0)|F\rangle\langle F|J_\nu^w(0)|0\rangle(2\pi)^3\delta(q-p_F)$$
$$=\rho_1(q^2)(q_\mu q_\nu-q^2g_{\mu\nu})+\rho_2(q^2)q_\mu q_\nu \quad (7)$$

are introduced to treat this process.[21,22] There is no interference between V and A in these functions,[21] so that

$$\rho_i=\rho_i^V+\rho_i^A.$$

The following equations[10,21,22] are usually used in the region $q^2\geqslant 1\ (\text{GeV})^2$:

$$\rho_1^V(q^2)\approx\frac{R(q^2)}{8\pi^2},\quad \rho_2^V(q^2)\approx 0,\quad \rho_i^A\approx\rho_i^V. \quad (8)$$

The first two of Eqs. (8) follow from the conserved vector current hypothesis provided it is assumed, in accordance with the quark model, that the $I=1$ state accounts for 75% of the $e^+e^-\to$ hadrons cross section. The relation between ρ_i^A and ρ_i^V is obtained from asymptotic chiral symmetry. Then for $m_L=1.8$ GeV and $R_{eff}\approx 1.5$ we obtain[10]

$$\Gamma_{hc}/\Gamma_e\approx\begin{cases}0.97\\0.48\end{cases},\quad \Gamma_{tot}/\Gamma_e\approx\begin{cases}5.2\\5.5\end{cases}. \quad (9)$$

For $m_L=1.8$ GeV, however, the q^2 values are small: $\langle q^2\rangle\sim 1.5\ (\text{GeV})^2$. Hence we may suppose that the contribution to ρ_1^A from the five-pion state will be depressed on account of phase space considerations, while the three-pion contribution is associated mainly with A_1. Then, after separating A_1, we may assume that

$$\rho_1^A\ll\rho_1^V. \quad (10a)$$

In addition, the $\omega\pi$ state with $I=1$ may play a considerable part in the $e^+e^-\to$ hadrons reaction in this energy region (see, e.g., the discussion in[25]). If this is the case, then, neglecting the isoscalar contribution, we obtain

$$\rho_1^V(q^2)\approx R(q^2)/6\pi^2. \quad (10b)$$

In this case

$$\Gamma_{hc}/\Gamma_e\approx\begin{cases}0.65\\0.32\end{cases},\quad \Gamma_{tot}/\Gamma_e\approx\begin{cases}4.92\\5.33\end{cases}. \quad (11)$$

A comparison of (9) and (11) illustrates the magnitude of the uncertainty in the theoretical estimates; however, we feel that (11) is to be preferred.

We note that the value $\Gamma_{tot}/\Gamma_e\sim 5$ looks very natural from the quark point of view, since L decay can give rise to two lepton systems, $e\nu_e$ and $\mu\nu_\mu$, and (including color) three light-quark systems.[5,22]

Let us consider the quantity ε, the fraction of the energy carried off in L decay by charged particles. For decays (1) and (4), ε can be calculated in a standard manner (e.g. [5,10]) and always turns out to be smaller than ε_q; thus, the neutrino carries off a considerable part of the energy, which increases with increasing m_{ν_L}. The result for decay (6) is substantially model dependent. For the usual assumptions (8) one obtains $\varepsilon_{hc}\approx 0.44$ for $m_{\nu_L}=0$ and $\varepsilon_{hc}\approx 0.38$ for $m_{\nu_L}=0.3\ m_L$.[10] If the $\omega\pi$ system plays the principal part in decay (6), however, we have $\varepsilon_{hc}\approx 0.5-0.55$ for $m_{\nu_L}=0$.

The most characteristic property of the leptons with $m_L\sim 1.8$ GeV is that they usually produce just one charged particle (one track) when they decay. One can obtain the following estimates, which depend on the assumptions made concerning decay (6):

$$\Gamma(\geqslant 3)/\Gamma(1)=\begin{cases}0.16\text{–}0.22\\0.1\text{–}0.12\end{cases}, \quad (12)$$

$$\delta_1=\Gamma(1)/\Gamma_{tot}\approx\begin{cases}0.84\\0.9\end{cases}, \quad (13)$$

$$\Gamma_e/\Gamma(1)\leqslant\begin{cases}0.25\\0.21\end{cases}. \quad (14)$$

Here $\Gamma(n)$ is the total width for decay with the production of n charged particles. The value of δ_1 turns out to be practically the same for both the assumptions concerning decay (6) under discussion, and the inequality (14) does not depend on them at all.

Relations (12) and (14) are in good agreement with the experimental data,[9] which give

$$\left.\frac{\Gamma(\geqslant 3)}{\Gamma(1)}\right|_{exp}<\frac{1}{3},\quad \left.\frac{\Gamma_e}{\Gamma(1)}\right|_{exp}<\frac{1}{3}. \quad (15)$$

This a weighty argument in favor of the assertion that anomalous muon production[9] is indeed due to heavy leptons, and not to hadrons, for in the latter case one would naturally expect $\Gamma(\geqslant 3)/\Gamma(1)>1$ (see, e.g.[5-7])1).

3. THE HEAVY-LEPTON CONTRIBUTION TO THE OBSERVED PROPERTIES OF THE $e^+e^-\to$ HADRONS PROCESS

The behavior of $\varepsilon(s)$, which has been investigated experimentally for events with $n_{ch}\geqslant 3$,[17] is determined by the formulas[4,5,10]

$$\varepsilon(s)=\varepsilon_h(s)+\sum_i(\varepsilon_{L_i}-\varepsilon_h(s))\bar R_{L_i}(s)/\bar R(s),$$
$$\bar R(s)=\bar R_h(s)+\sum_i\bar R_{L_i}(s),\quad \bar R_{L_i}(s)={}^1/_2 v_i(3-v_i^2)\bar B_{L_i}, \quad (16)$$

where the summation is taken over all possible heavy-lepton pairs, $v_i^2=1-4m_{L_i}^2/s$, $\bar R_h$ and ε_h are the contributions from purely hadronic events with $n_{ch}\geqslant 3$, $\bar R_{L_i}$ and ε_{L_i} are the contributions from events involving L^+L^--pair production and having $n_{ch}\geqslant 3$, and $\bar B_{L_i}$ is the fraction of such events in L^+L^- production.

Since $\varepsilon_{L_t} < \varepsilon_h$, the quantity $\varepsilon(s)$ begins to fall at the heavy-lepton-production threshold. Under the usual assumptions, $\varepsilon_L \approx 0.39$ for heavy leptons with $m_L \approx 1.8$ GeV. This enabled Fujikawa and Kawamoto[10] to describe the experimental behavior of $\varepsilon(s)$ for $s^{1/2} \lesssim 6$ GeV, using the contribution from one L^+L^- pair with $\bar{B}_L \sim 1$. Depending on the properties of decay (6), ε_L may be larger, and this reduces the expected rate of fall of $\varepsilon(s)$. Even more important, according to (13) selecting events with $n_{ch} \geq 3$ gives $\bar{B}_L \lesssim 1 - \delta_1^2 \lesssim 0.2 - 0.3$. Therefore, a single heavy-lepton pair cannot account for the behavior of $\varepsilon(s)$ even in the region $s^{1/2} \leq 6$ GeV.

The heavy-lepton contribution to $R_{exp}(s)$ is given by the same formula as R_L, but with a different weight B_L corresponding to different recording conditions.[16] The limitations $\theta_{copl} > 20°$ on the track coplanarity angle θ_{copl} reduces the relative number of $n_{ch} = 2$ events. As $s^{1/2}$ increases, the contribution of two-track events from L^+L^--pair decay falls. Rough estimates show that we may expect this contribution to decrease by ~30% at $m_L \approx 1.8$ GeV and $s^{1/2} \approx 7.4$ GeV. If we then exclude ee and $e\mu$ events, we shall have $B_L(s^{1/2} \approx 7.4 \text{ GeV}) \lesssim 0.75$. This is considerably lower than the value $B_L \approx 1$ usually assumed (e.g. [10]). According to the same estimates the relative number of two-track events is lower by a factor of ~2 at $s^{1/2} \approx 11$ GeV. Under realistic conditions with $1 - v \ll 1$ it falls by $\sim (1 - v)/\sin^2\theta_{min}$, where θ_{min} is the minimum value of θ_{copl}. Of course these estimates are merely illustrative. The actual situation depends on a number of other conditions—in particular, on the procedure employed to deal with rejected events.

The average charged-particle multiplicity in the production of L^+L^- pairs with $m_L \approx 1.8$ GeV is $\langle n_{ch}\rangle_L \approx 2.5 - 3$; this is appreciably lower than the experimental value $\langle n_{ch}\rangle \sim 4 - 5$ found for the $e^+e^- \to$ hadrons reaction at $s^{1/2} \gtrsim 3.6$ GeV.[17] Hence the heavy-lepton contribution retards the growth of both $\langle n_{ch}\rangle$ and the average charged-particle energy $\langle E_{track}\rangle$ in a certain energy region above the threshold.[4,5] To obtain quantitative agreement with experiment,[17] however, it will evidently also be necessary to take the production of heavy-hadron pairs into account in order to obtain an average heavy-charged-particle multiplicity of ~4.[4] Because of the low value of $\langle n_{ch}\rangle_L$ it is evidently best to select events with $n_{ch} = 2$ for investigating the properties of the heavy leptons. For example, it would be interesting to find threshold behavior at $s^{1/2} \sim 3.6$ GeV in these events. One may also expect to find $\varepsilon \sim 0.35 - 0.4$ for these events.

Another characteristic property of the heavy leptons is the low kaon yield in their decay, which is due to the limited phase space available for the hadronic system. Simple estimates (e.g., [5]) of the relative number δ_K of the decays that include kaons among the decay products give values of the order of a few percent. At the same time, if the new D mesons are charmed they should have a substantial δ_K value. Selection of events containing kaons may therefore facilitate the isolation of the D-meson component in e^+e^- annihilation.

In principle, sufficiently energetic heavy leptons lead to the production of "hadron jets" (but with constant multiplicity within the jet). The angular distribution of the "jet axis" and leading hadrons associated with this mechanism has the form $\sim (1 + \cos^2\theta)$. Jets have been studied experimentally, however, in events with $n_{ch} \geq 3$[17] where the heavy-lepton contribution contains the small weight factor $\bar{B}_L$ and cannot be predominant.

We note that it is important to take the conditions of measurement into account in experimental tests of theoretical sum rules relating inclusive hadron spectra to the quantities $R(s)$, $\varepsilon(s)$, and $\langle n_{ch}\rangle$, since these quantities have been measured under different conditions.

4. INCLUSIVE SPECTRA IN HEAVY-PARTICLE-PAIR PRODUCTION

Let us consider the reaction

$$e^+e^- \to M\bar{M} \tag{17}$$

with subsequent decay of the heavy particles M and $\bar{M}$. The contribution to R from this process is

$$R_M = v\rho(s), \tag{18}$$

where v is the velocity of the M particle in the c.m. system and $\rho(s)$ depends on the properties of these heavy particles. If the width of the M particle is small so that interference between the products of the M and $\bar{M}$ decays need not be taken into account, then, neglecting spin effects, the inclusive spectrum of a particle h produced as a result of decays in reaction (17) has the form[5]

$$\frac{d\sigma^h}{d\omega}\frac{1}{\sigma(e^+e^-\to\mu^+\mu^-)} = \rho(s)\left[f_M{}^h(\omega,s) + f_{\bar{M}}{}^h(\omega,s)\right], \tag{19}$$

where $\omega = 2E_h/s^{1/2}$. The term f_M^h is related to h production in M decay and is given by

$$f_M{}^h(\omega,s) = \frac{1}{2}\int\limits_{y^-(\omega,s)}^{y_{max}(\omega,s)} dy\, G_M{}^h(y), \tag{20}$$

where the function $G_M^h(y)$ is determined by the energy spectrum of the particle h in the rest system of the particle M. The integration limits are

$$y_{max}(\omega,s) = \min\{y_{max}, y^+(\omega,s)\},$$
$$y^\pm(\omega,s) = \frac{s}{4m_M{}^2}\left[\omega \pm v\left(\omega^2 - \frac{4m_h{}^2}{s}\right)^{1/2}\right]. \tag{21}$$

Here $y_{max} = E^*_{max}/m_M$, where E^*_{max} is the maximum energy in the rest system of the M particle that an h particle can acquire in the decay

$$M \to h + \ldots \tag{22}$$

The normalization condition for $G_M^h(y)$ is

$$\int\limits_{m_h/m_M}^{y_{max}} dy\, G_M{}^h(y)\left(y^2 - \frac{m_h{}^2}{m_M{}^2}\right)^{1/2} = \langle n_h\rangle_M, \tag{23}$$

where $\langle n_h\rangle_M$ is the average h multiplicity in M decay. Similar reactions obtain for the function $f_{\bar{M}}^h(\omega, s)$ for h production in $\bar{M}$ decay.

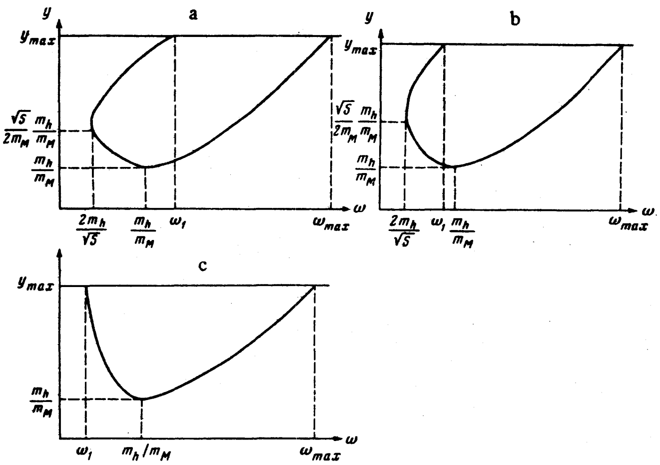

FIG. 1. Kinematic region in the y, ω plane: a) $s < s_1$, b) $s_1 < s < s_2$, c) $s_2 < s$. $\omega_{max}(s) = y_{max} + v(y^2_{max} - m_h^2/m_M^2)^{1/2}$ for all three cases; $\omega_{min} = 2m_h/s^{1/2}$ for cases a) and b), and $\omega_{min} = \omega_1(s)$ for case c).

It is convenient to depict the range of the y integration in (20) on the (y, ω) plane. We define ω_1 by the formula

$$\omega_1(s) = y_{max} - v(y^2_{max} - m_h^2/m_M^2)^{1/2}$$

and then introduce s_1 and s_2 by the equations $\omega_1(s_1) = m_h/m_M$ and $s_2 = m_M^2(2y_{max}m_M/m_h)^2$. The plots on the (y, ω) plane in the regions $s < s_1$, $s_1 < s < s_2$, and $s > s_2$ differ from one another (Fig. 1, a–c). As s increases with ω held constant, $y^-(\omega, s)$ decreases, approaching a finite limit, while $y^+(\omega, s)$ increases ($\sim s$ as $s \to \infty$). Hence $f(\omega, s)$ increases monotonically with s for any constant value of ω because of the increase in the integration range in (20). This increase can be rapid only when $\omega < \omega_1$; when $\omega > \omega_1$, f approaches the scaling-invariant regime.

Let us consider the ω dependence of f at $s = \text{const}$. If $s < s_1$ (Fig. 1a), then when $\omega > \omega_1$, f decreases with increasing ω at least as $\omega_{max} - \omega$. If $G(y) \sim (y_{max} - y)^n$, as in multihadronic decays, then $f \sim (\omega_{max} - \omega)^{n+1}$. When $\omega < m_h/m_M$, f decreases with decreasing ω. In this case the point $\bar{\omega}$ at which f is maximum lies between m_h/m_M and ω_1, its exact position depending on the properties of G. The faster G falls as $y \to y_{max}$, the farther $\bar{\omega}$ lies from ω_1. The fall of G as $y \to y_{min} = m_h/m_M$, on the other hand, brings $\bar{\omega}$ closer to ω_1.

Whenever $s > s_1$, we have $\bar{\omega} = m_h/m_M$ (Figs. 1b and 1c). When $s_2 > s > s_1$, there still remains a region $\omega < \omega_1$ in which the s dependence of f differs sharply from scaling

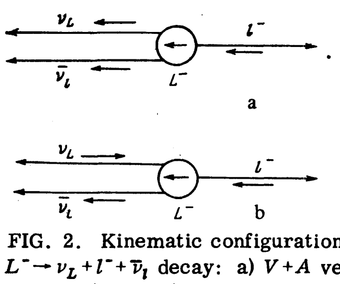

FIG. 2. Kinematic configurations for maximum energy E_l in $L^- \to \nu_L + l^- + \bar{\nu}_l$ decay: a) $V+A$ version (forbidden), b) $V-A$ version (allowed). The arrows indicate the spin directions.

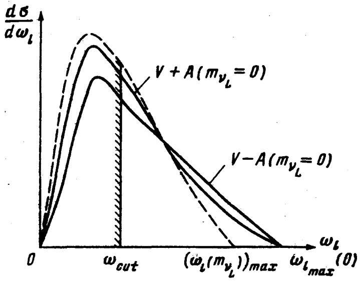

FIG. 3. Trend of the energy spectrum $d\sigma/d\omega_l$ in decay of the heavy lepton L; ω_{cut} is associated with the experimental momentum cutoff.

invariant behavior; this region shrinks with increasing s and no longer exists when $s > s_2$.

Let us consider separately the case of two-particle decay in which $G \sim \delta(y - y_0)$. Then $f = \text{const}$ in the interval

$$y_0 - v(y_0^2 - m_h^2/m_M^2)^{1/2} \leq \omega \leq y_0 + v(y_0^2 - m_h^2/m_M^2)^{1/2}.$$

and $f = 0$ outside this interval.

The qualitative properties of f described above enable one to explain the s dependence of the inclusive spectra of charged hadrons in the $e^+e^- \to$ hadrons process by the production and subsequent decay of heavy particles. [5]

Let us consider a few specific cases.

1) Let M be a lepton L. For $L \to \nu_L \pi$ decay we obtain $y_0 = (1 - m^2_{\nu_L}/m_L^2)/2$. One can evaluate m_L and m_{ν_L} by investigating the upper limit ω_{max} of the spectrum and its energy dependence.

2) For $L^- \to \nu_L l^- \bar{\nu}_l$ decay with $m_{\nu_L} = 0$ we have

$$y_{max} = \tfrac{1}{2}(1 + m_l^2/m_L^2). \tag{24}$$

The position of $\bar{\omega}$ remains fixed at the point $\bar{\omega} = m_l/m_L$ provided $s^{1/2}/m_L \geq (m_L/m_l)^{1/2} + (m_l/m_L)^{1/2}$. For muons this means $s^{1/2} \gtrsim 7.9$ GeV if $m_L \sim 1.8$ GeV. For electrons $\bar{\omega} \approx \omega_1$ for all reasonable s values. [13] Under the experimental conditions of [8] there is an effective cutoff, $\omega > \omega_{cut} = 1.3/(s(\text{GeV}^2))^{1/2}$, and the peak of the spectrum is accessible to observation only near the L^+L^--production threshold ($v \lesssim 0.3$ for $m_L \sim 1.8$ GeV).

If $m_{\nu_L} \neq 0$, $\bar{\omega}$ will be smaller because y_{max} will be smaller. Passing from the $V-A$ version to the $V+A$ version has the same effect, but for a different reason. In this case $G(y) \to 0$ as $y \to y_{max}$. In fact, the equality $y = y_{max}$ is reached when both neutrinos are emitted in the same direction in the rest system of the L lepton while the light lepton (μ or e) is emitted in the opposite direction. In the $V+A$ version the sum of the spin projections along the momentum $\mathbf{p}_l$ of the light lepton would be $\pm 3/2$ in this case (Fig. 2). Owing to the suppression of the high energies, the spectra have the form sketched in Fig. 3. In particular, the average energy of the charged lepton is lower in the $V+A$ version than in the $V-A$ version:

$$\langle E_l \rangle_{V-A} > \langle E_l \rangle_{V+A}. \tag{25}$$

3) Kaon production in D-meson decay should give rise to structures in the inclusive kaon spectrum, which shift with energy. If $m_D \sim 2$ GeV, the peak of the inclusive kaon spectrum from D decay remains fixed at the point $\bar{\omega} = m_K/m_D \sim 1/4$, which corresponds to the momentum $\bar{p}_K = m_K(s/4m_D^2 - 1)^{1/2}$, whenever $s^{1/2} \gtrsim 5$ GeV. The discovery of such structures might facilitate the study of D mesons.

5. QUALITATIVE CHARACTERISTICS OF ANOMALOUS μe EVENTS

Let us discuss events of the type $e^+ + e^- \to e^\pm + \mu^\pm$ + missing energy under the assumption that they are due to production of L^+L^- pairs and their subsequent decays via processes (1) (a) and (b).

Such processes have already been discussed in detail in a number of papers (e.g., [10–14]). Our results are in qualitative agreement with the results of those papers, so here we shall merely list the most characteristic features of the processes.

1. The spectra of the charged leptons $l^\pm(e^\pm, \mu^\pm)$ in the variable $\omega_l = 2p_l/s^{1/2}$ have the form sketched in Fig. 3 (see[10,12–14] and the discussion in Sec. 4 of the present paper). The steeper fall of the spectra as $\omega_l \to (\omega_l)_{max}$ in the $V+A$ case is actually due to inequality (25).

2. In the case of a $V+A$ current or a $V-A$ current with a massive ν_L the quantity $\langle p_\perp^l \rangle$ is smaller than in the case of a $V-A$ current with $m_{\nu_L} = 0$ ($p_\perp^l$ is the component of the $l^\pm$ momentum perpendicular to the $L^\pm$ particle). This property, like (25), is associated with the decrease of the average $l^\pm$ energy in the $L^\pm$ rest system. It leads to greater collinearity (and coplanarity) in the emission of the electron and muon.

3. If the L^+ and L^- decay products are simultaneously detected, spin effects associated with parity violation in the decays will also appear. These effects are also more important in the $V+A$ version than in the $V-A$ version.[10,14] As numerical calculations show (e.g., [10,14]), however, they are usually not very significant.

4. Properties 1 and 2 show that with the cutoffs that obtain under the experimental conditions of[8] ($\theta_{copl} > 20°$, $p_l > 0.65$ GeV/c) there is a greater loss of $e\mu$ events for the $V+A$ version or the $V-A$ version with $m_{\nu_L} \neq 0$ than for the $V-A$ version with $m_{\nu_L} = 0$. In principle this might tend to increase the value of Γ_e/Γ_{tot} extracted from the experimental data.[8]

A clear confirmation of the $V+A$ version of heavy lepton decay might prove to be a weighty argument in favor of vectorlike theories of the weak interactions.[23,24] In such models ν_L might be identified with $(\nu_e)_R$ or $(\nu_\mu)_R$.

In some vectorlike models (e.g., [24]), two charged heavy leptons with masses lying close together might in principle exist. The available data[8] do not entirely rule out this possibility. This might be verified by investigating not only μe events, but also other events involving L^+L^- decay, e.g., μh and hh events, where h represents a charged hadron.

The authors thank J. D. Bjorken, V. N. Gribov, and B. L. Ioffe for valuable discussions.

1)On completing this work we learned that Snow[26] had reached the same conclusion.

1J. D. Bjorken, Invited paper presented at the 1973 International Symposium on Electron and Photon Interactions at High Energies, Bonn, 1973; SLAC-PUB-1318, 1973.

2J. D. Bjorken and B. L. Ioffe, Usp. Fiz. Nauk 116, 115 (1975) [Sov. Phys. Usp. 18, 361 (1975)].

3H. Harari, Rapporteur talk at the 1975 Lepton Photon Symposium, Stanford; Lecture notes at the SLAC Summer Institute on Particle Physics, 1975.

4L. L. Frankfurt and V. A. Khoze, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 69 (1976) [JETP Lett. **23**, 61 (1976)]; Yad. Fiz. **23**, 926 (1976) [Sov. J. Nucl. Phys. **23**, (1976]; preprint LIYaF (Leningrad Inst. Nucl. Phys.) No. 207, 1975.

5Ya. I. Azimov, L. L. Frankfurt, and V. A. Khoze, Lektsii na XI Zimneĭ shkole LIYaF po fizike yadra i élementarnykh chastits, preprint LIYaF No. 220–222, 1976 (Lectures at the eleventh Winter School of the Leningrad Inst. Nucl Phys. on nuclear and elementary particle physics, Leningrad Inst. Nucl. Phys. preprint No. 220–222, 1976).

6V. I. Zakharov, B. L. Ioffe, and L. B. Okun', Usp. Fiz. Nauk. 117, 228 (1975) [Sov. Phys. Usp. 18, 757 (1975)].

7L. L. Frankfurt and V. A. Khoze, Materialy X Zimneĭ shkoly LIYaF po fizike yadra i élementarnykh chastits (Materials of the Leningrad Inst. Nucl. Phys. tenth Winter School on nuclear and elementary particle physics), Part II, 1975, p. 196.

8M. L. Perl *et. al.*, Phys. Rev. Lett. 35, 1489 (1975); Preprint SLAC-PUB-1592, 1975; SLAC-PUB-1664, 1975.

9M. Cavalli-Sforza *et al.*, Preprint SLAC-PUB-1685, 1975; Phys. Rev. Lett. **36**, 558 (1976).

10Kazuo Fujikawa and Noburu Kawamoto, Phys. Rev. Lett. **35**, 1560 (1975); Preprint DESY-76/01, 1976.

11Kazuo Fujikawa and Noburu Kawamoto, Preprint DESY-75/52, 1975.

12So-Young Pi and A. I. Sanda, Phys. Rev. Lett. **36**, 1 (1976).

13A. V. Berkov, E. D. Zhizhin, and Yu. P. Nikitin, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 408 (1976) [JETP Lett. **23**, 367 (1976)].

14Soo Yong Park and A. Yildiz, Preprint Lyman Lab. 10/75, 1975.

15J. J. Sakurai, Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies, Cornell, 1971.

16J. E. Augustin *et al.*, Phys. Rev. Lett. 34, 764 (1975); G. J. Feldman and M. L. Perl, Phys. Rep. **19C**, 233 (1975).

17R. F. Schwitters, Invited Talk at the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, 1975.

18M. L. Perl and R. Rapidis, SLAC rep., SLAC-PUB-1496, 1974.

19B. C. Barish, CALT-68052, 1975; C. Albright, FNAL preprint, 1975.

20V. V. Lapin and V. N. Folomeshkin, Preprint IFVE OTF 75-66, 1975.

21Yung-Si Tsai, Phys. Rev. **D4**, 2821 (1971).

22J. D. Bjorken and C. H. Llewellyn Smith, Phys. Rev. **D7**, 887, 1973.

23A. De Rújula, Howard Georgi, and S. L. Glashow, Phys. Rev. **D12**, 3589 (1975).

24H. Fritzsch, M. Gell-Mann, and P. Minkowski, Phys. Lett. **59B**, 256 (1975); H. Fritzsch, Preprint CALT-68-524, 1975.

25F. B. Close, Invited talk at the International Symposium on Storage Ring Physics, Flaine, 1976; G. Parrour, Preprint LAL 1285, 1976.

26George A. Snow, Phys. Rev. Lett. **36**, 766 (1976).

Translated by E. Brunner