Anomalous absorption of electromagnetic waves in a randomly inhomogeneous collisionless magnetoactive plasma

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The mechanism whereby normal waves lose energy in a magnetoactive collisionless plasma is considered; this mechanism is connected with excitation of normal scattered waves under conditions when their refractive indices can be infinite. Excitation of quasilongitudinal waves near the resonance directions is effected by the scattering of the waves from the inhomogeneities of the plasma electron density. The case $u=\omega_H^2/\omega^2<1$, when the extraordinary wave is resonant, is analyzed in the greater detail. This case is of importance for wave propagation in the ionosphere and can be invoked to explain the anomalous absorption of ordinary waves in the ionosphere regions adjacent to the reflection point. A comparative analysis is presented of absorption due to different types of inhomogeneities (isotropic small-scale inhomogeneities and large-scale inhomogeneities elongated along the magnetic field). It is shown that at propagation angles χ close to zero, the integral absorption due to different types of inhomogeneities is the same, whereas as $\chi\sim\pi/2$, only small-scale inhomogeneities are significant. The considered mechanism of wave absorption in a collisionless plasma with fluctuations of the electron density can be used to heat a laboratory plasma by a high-frequency electromagnetic field.

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1. INTRODUCTION

It is well known that under certain conditions even weakly turbulent perturbations of the electron density can strongly influence the propagation and radiation processes in a magnetoactive plasma. The interaction produced by the plasma inhomogeneity between different modes leads to mutual transformation of the energies of the normal waves. We are interested here in wave interaction in a magnetoactive collisionless plasma under resonance conditions, when the refractive index $n_{1,2}^2(\vartheta)$ of one of the normal waves can become infinite. For the case $u=\omega_H^2/\omega^2\ll 1$ ($\omega_H=|e|H/mc$ is the electron gyrofrequency and ω is the cyclic frequency of the wave), which we shall consider, the refractive index of the extraordinary wave $n_1(\vartheta)$ can become infinite on a conical surface $\vartheta=\vartheta_r$ defined by the resonance condition

$$N(\vartheta)=\varepsilon\sin^2\vartheta+\eta\cos^2\vartheta,$$

where ε and η are the components of the dielectric tensor of the unperturbed magnetoactive plasma. In a reference frame with the z axis directed along the magnetic field **H**, this tensor is given by

$$\varepsilon_{np}=\begin{Vmatrix}\varepsilon & -ig & 0\\ ig & \varepsilon & 0\\ 0 & 0 & \eta\end{Vmatrix}.$$

(A harmonic time dependence in the form $e^{i\omega t}$ is assumed.)

$$\varepsilon=1-v/(1-u),\ g=vu^{1/2}/(1-u),\ \eta=1-v,$$
$$v=\omega_0^2/\omega^2,\quad \omega_0^2=4\pi e^2\langle N\rangle/m$$

is the square of the Langmuir frequency of the unperturbed plasma with a concentration equal to the average value of the electron density in the turbulent plasma. The electron density in the turbulent plasma $N(\mathbf{r})=\langle N\rangle+\Delta N(\mathbf{r})$ is assumed to be a random statistically homogeneous function of the coordinates. $n_1^2(\vartheta)$ is large near the resonance directions and the scattering into extraordinary waves greatly exceeds the scattering into ordinary waves. We note that near $\vartheta=\vartheta_r$ the field of the extraordinary wave has a quasi-electrostatic character and can be calculated under certain conditions within the framework of the quasi-static approximation. [1]

The present article can be regarded as a continuation of an earlier study, [2] in which the effective dielectric constant of a collisionless inhomogeneous magnetoactive plasma at the plasma-resonance frequencies was calculated in the quasi-electrostatic approximation. The need for returning to the resonant-absorption problem is dictated by the many published reports of investigations of the modification of the ionosphere plasma by powerful electromagnetic radiation. [3–5] The results of these studies offer evidence of a strong influence of the modified plasma region on the propagation of waves whose reflection level is located in the modified region. A number of workers[6,7] attribute the experimentally observed anomalous absorption of the ordinary wave to scattering of the latter by electron-density inhomogeneities that are elongated along the magnetic field and are the results of the action of the high-power radiation.

The most complete information on wave propagation can be obtained from the general formulas of the electrodynamics of a randomly inhomogeneous magnetoactive plasma. [2,8,9] Knowing the effective dielectric constant at the plasma-resonance frequencies, we can calculate the absorption of normal waves propagating at an arbitrary angle to the magnetic field. It is important that even in the case of small-scale inhomogeneities in the plasma ($k_0l\ll 1$, $k_{1,2}l\ll 1$, $k_{1,2}=k_0n_{1,2}$ are the wave numbers of the extraordinary and ordinary waves, $k_0=\omega/c$, l is the correlation radius of the electron-density fluctuations $\Delta N(\mathbf{r})$), the quasi-electrostatic approximation must be used in intervals of v that are adjacent to the boundaries of the resonance region ($\varepsilon=0$, $v=1-u$; $\eta=0$, $v=1$). To calculate the absorption in these cases it is necessary to use the exact formulas of the wave theory.

The importance of the mechanism that governs the absorption of electromagnetic wave energy by a collisionless magnetoactive plasma through excitation of quasi-electrostatic oscillations near the resonant surfaces is not restricted to radiowave propagation in the ionosphere. Resonant absorption of the energy of an electromagnetic field can be effectively used in plasma heating. However, it is precisely the ionosphere which provides an interesting example of "locking" the energy within a sufficiently narrow layer, followed by conversion of this energy into heat. Figure 1 shows a plot of the refractive indices of the extraordinary (n_n^2) and ordinary (n_2^2) waves against the parameter v at a fixed value of the propagation angle ϑ (which is not exactly equal to 0 or $\pi/2$). When ϑ is varied in the interval (0, $\pi/2$) the resonant value

$$v_\infty=\frac{1-u}{1-u\cos^2\vartheta}$$

($\lim n_1^2(\vartheta)=\infty$ at $v\to v_\infty$, $v-v_\infty>0$) lies in the interval ($1-u$, 1) and moves from the point $v=1-u$ at $\vartheta=\pi/2$ to the point $v=1$ at $\vartheta=0$. In a statistically inhomogeneous plasma (we assume for simplicity that the average electron density $\langle N\rangle$ depends only on z) the extraordinary scattered waves (and scattering into extraordinary waves is predominant, owing to resonance), turn out to be "trapped" in the region (v_∞, $1+u^{1/2}$), and their energy goes entirely to heating of the plasma. The integral absorption, connected with the statistical transformation in the ionosphere, will be considered in Sec. 5. We shall discuss first the expression for the effective dielectric tensor of a magnetoactive plasma, on the use of which is based the most general method of calculating the absorption due to statistical wave transformation. The definition of $\varepsilon_{np}^{\text{eff}}(\omega,\mathbf{k})$ and general formulas for this quantity were given earlier.[8] Expressions that are valid at plasma-resonance frequencies were obtained for $\varepsilon_{np}^{\text{eff}}(\omega,\mathbf{k})$ in preceding papers.[2,9] Here we present the required results in a form convenient for calculations, and also for comparison with absorption due to the collision between electrons and heavy particles.

2. EFFECTIVE DIELECTRIC CONSTANT OF A MAGNETOACTIVE PLASMA AT PLASMA-RESONANCE FREQUENCIES

We consider first isotropic small-scale fluctuations of the electron density with a correlation scale l. The

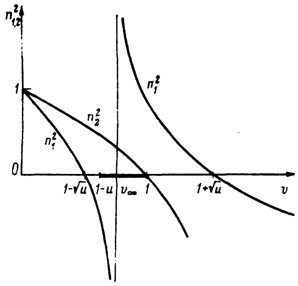

FIG. 1. Refractive indices of normal waves at $u<1$, $0<\vartheta\leq\pi/2$. The region $1-u<v<1$ is responsible for the resonant absorption.

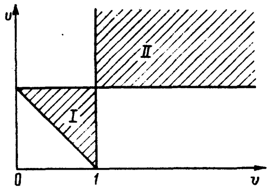

FIG. 2.

effective dielectric constant of a plasma with such inhomogeneities ($k_0 l\ll 1$, $k_{1,2}l\ll 1$) is of the form[8]

$$\varepsilon_{np}^{\text{eff}}(\omega)=\varepsilon_{np}(\omega)+\operatorname{Re}\xi_{np}(\omega)+i\operatorname{Im}\xi_{np}(\omega),$$

where $\varepsilon_{np}(\omega)$ is the average dielectric tensor of a randomly inhomogeneous plasma, and $\xi_{np}(\omega)$ is a tensor in the form

$$\xi_{np}(\omega)=\begin{vmatrix}\xi_{11} & \xi_{12} & 0\\ \xi_{21} & \xi_{22} & 0\\ 0 & 0 & \xi_{33}\end{vmatrix},\quad \xi_{11}=\xi_{22},\ \xi_{12}=-\xi_{21}. \tag{1}$$

The real part of the tensor $\xi_{np}(\omega)$ influences only the wave propagation velocity, but this influence will not be taken into account here. The anti-Hermitian part of the tensor is connected with the energy lost by the waves to excitation of the scattered ordinary and extraordinary waves. In contrast to the nonresonant case,[8] we are considering here the conditions under which the refractive index $n_1^2(\vartheta)\sim 1/N(\vartheta)$ of the extraordinary wave becomes infinite at $\vartheta=\vartheta_r$. The condition under which the equation $N(\vartheta)=0$ has real roots can be written in the form

$$a=(\eta-\varepsilon)/\varepsilon<-1. \tag{2}$$

The shaded regions of the (u, v) plane in Fig. 2 are those in which plasma resonance is possible, i.e., the equation $N(\vartheta)=0$ has real roots. The triangular region ($u<1$) corresponds to the frequencies $\omega_H\leq\omega\leq\omega_r$ at $\omega_H>\omega_0$ and to the frequencies $\omega_0\leq\omega\leq\omega_r$ at $\omega_H<\omega_0$, $\omega_r=(\omega_0^2+\omega_H^2)^{1/2}$. We consider here only the case $u<1$.

Taking the collisions into account, the components of the average tensor $\varepsilon_{np}(\omega)$ are

$$\varepsilon_{11}=\varepsilon=1-\frac{v}{1-u}-i\frac{s_1 v(1+u)}{(1-u)^2},$$

$$\varepsilon_{12}=-ig=-\varepsilon_{21},$$

$$g=\frac{vu^{1/2}}{1-u}+i\frac{2s_2 vu^{1/2}}{(1-u)^2},\quad \varepsilon_{33}=\eta=1-v-is_3 v,$$

$$\varepsilon_{13}=\varepsilon_{23}=\varepsilon_{31}=\varepsilon_{32}=0, \tag{3}$$

$$s=\nu_{\text{eff}}/\omega.$$

We assume henceforth $s_1=s_2=s_3=0$. The general expression for $\varepsilon_{np}^{\text{eff}}(\omega,\mathbf{k})$, however, can be written in fact in the form (3), i.e., the scattering processes can be formally replaced by equivalent collisions with heavy particles. Retaining for these equivalent collisions the symbols s_1, s_2, and s_3, we have

$$\varepsilon_{11}^{\text{eff}}=\varepsilon_{\text{eff}}=\varepsilon-i\frac{vs_1(1+u)}{(1-u)^2},\quad \varepsilon_{12}^{\text{eff}}=-\varepsilon_{21}^{\text{eff}}=-ig_{\text{eff}},$$
$$g_{\text{eff}}=g+i\frac{2vs_2u^{1/2}}{(1-u)^2},\quad \varepsilon_{33}^{\text{eff}}=\eta-ivs_3, \tag{4}$$
$$s_1=s_2=\frac{\pi\sigma_N^2}{4}\frac{v\eta}{|\varepsilon|(\eta-\varepsilon)|a|^{1/2}},\quad s_3=\frac{\pi\sigma_N^2}{2}\frac{v}{(\eta-\varepsilon)|a|^{1/2}},$$
$$\sigma^2=\langle\Delta N^2\rangle,\quad \sigma_N^2=\sigma^2/\langle N\rangle^2$$

in the quasi-electrostatic approximation. [2] The wave theory yields the formulas[9]

$$s_1=s_2=\frac{\pi v\sigma_N^2}{2(\eta-\varepsilon)}\left[(1-\alpha)|a|^{1/4}\left(\frac{k_0l}{2}\right)^{1/2}\exp\left(-\frac{k_0^2l^2|a|}{8}\right)\times W_{1/4,1/4}\left(\frac{k_0^2l^2}{4\alpha}\right)\right],$$
$$s_3=\frac{\pi v\sigma_N^2}{2(\eta-\varepsilon)|a|^{1/2}}\exp\left(-\frac{k_0^2l^2|a|}{4}\right), \tag{5}$$

where $W_{\lambda,\mu}$ is the Whittaker function, which can be defined by the integral

$$W_{\lambda,\mu}(z)=\frac{z^{\mu+1/2}\exp(-z/2)}{\Gamma(\mu-\lambda+1/2)}\int_0^\infty e^{-zt}t^{\mu-\lambda-1/2}(1+t)^{\mu+\lambda-1/2}\,dt.$$

When writing down the expressions for s_1, s_2, and s_3 we have neglected the wave corrections, which are proportional to $(k_0l)^2$ and $(k_0l)^3$. If this is not done, then $s_1\neq s_2$. If the inequality

$$\tfrac{1}{4}k_0^2l^2|a|\ll 1,\quad k_0^2l^2/(1-\alpha)^{1/2}\ll 1,\quad (\alpha=1/|a|) \tag{6}$$

is satisifed, then expressions (5) go over to expressions (4). The quasi-static approximation is thus violated near the points $\varepsilon=0$ ($\vartheta_r\sim\pi/2$) and $\eta=0$ ($\vartheta_r\sim 0$).

It must be borne in mind that the conditions $\varepsilon=0$ and $\eta=0$ determine the limits of the resonance interval relative to the parameter $v(1<v<1-u)$, so that the violation of the conditions of the applicability of the quasi-electrostatic approximation takes place near the indicated boundaries. Formulas (4) and (5) can be derived from an analysis of the energy characteristics of an individual radiator immersed in a magnetoactive plasma. The field of such an isolated radiator is quasi-potential near a resonant conical surface. When the conditions (6) are satisfied, the decisive role is played by the loss to excitation of the quasi-electrostatic oscillations near the surface $\vartheta=\vartheta_r$. This circumstance is reflected in the expression for the effective dielectric tensor, which is independent, if the inequalities (6) holds, of the parameter k_0l that characterizes the increased accuracy of the electrostatic formulas of the wave theory near the boundaries of the resonant region.

3. ABSORPTION COEFFICIENTS OF NORMAL WAVES

Assuming the fluctuations of the electron density to be small enough, we neglect the influence of the anti-Hermitian part of the tensor $\varepsilon_{np}^{\text{eff}}$ on the refractive indices of the waves in the plasma. The absorption coefficients are in this case linear functions of $\operatorname{Im}\varepsilon_{np}^{\text{eff}}$. [10] Introducing the notation $n_{\text{eff}}=\mu-i\varkappa$, we obtain

$$(n_{1,2}^{\text{eff}})^2=n_{1,2}^2-i\gamma_{1,2},$$
$$\gamma_{1,2}=\frac{2aAn_{1,2}^4+(aB+bA)n_{1,2}^2+(cA+aC)}{A(B+2An_{1,2}^2)}, \tag{7}$$
$$n_{1,2}^2=\frac{-B\pm(B^2-4AC)^{1/2}}{2A}$$

are the refractive indices in the unperturbed plasma, while the quantities a, b, c, A, B, and C are defined by the relations

$$A_{\text{eff}}=\varepsilon_{\text{eff}}\sin^2\chi+\eta_{\text{eff}}\cos^2\chi=A+ia,$$
$$B_{\text{eff}}=-\varepsilon_{\text{eff}}\eta_{\text{eff}}(1+\cos^2\chi)-(\varepsilon_{\text{eff}}^2-g_{\text{eff}}^2)\sin^2\chi=B+ib, \tag{8}$$
$$C_{\text{eff}}=\eta_{\text{eff}}(\varepsilon_{\text{eff}}^2-g_{\text{eff}}^2)=C+ic,$$
$$a/A\ll 1,\quad b/B\ll 1,\quad c/C\ll 1,\quad B^2-4AC\neq 0,$$

χ is the angle between the propagation directions of the wave and the magnetic field.

For the quantities μ and $\varkappa$ we obtain

$$\mu^2=\tfrac{1}{2}[n^2+(n^4+\gamma^2)^{1/2}],\quad \varkappa^2=\tfrac{1}{2}[(n^4+\gamma^2)^{1/2}-n^2]. \tag{9}$$

If $n^2\gg\gamma$, then $\mu^2\approx n^2$ and $\varkappa\approx\gamma/2n$. As $\eta\to 0$ (at the edge of the resonance region) we have $n_0^2\to 0$, so that the case $n^2\ll\gamma$ is possible, and then $\mu\sim\varkappa\approx(\gamma/2)^{1/2}$.

We write down the expressions for A, B, C, a, b, and c in explicit form

$$A=\varepsilon\sin^2\chi+\eta\cos^2\chi,\quad B=-\varepsilon\eta(1+\cos^2\chi)-(\varepsilon^2-g^2)\sin^2\chi,\quad C=\eta(\varepsilon^2-g^2),$$
$$a=-v\left[\frac{1+u}{(1-u)^2}s_1\sin^2\chi+s_3\cos^2\chi\right], \tag{10}$$
$$b=v\left\{\left[s_3\varepsilon+\frac{s_1\eta(1+u)}{(1-u)^2}\right](1+\cos^2\chi)+\frac{2\sin^2\chi}{(1-u)^2}[s_1\varepsilon(1+u)+2s_2gu^{1/2}]\right\},$$
$$c=-v\left\{\frac{2\eta}{(1-u)^2}[s_1\varepsilon(1+u)+2s_2gu^{1/2}]+s_3(\varepsilon^2-g^2)\right\}.$$

The general formulas for $\varkappa_{1,2}$ at an arbitrary angle χ are cumbersome, and we consider therefore the particular cases of longitudinal and transverse wave propagation at $u\ll 1$.

1) *Longitudinal propagation* ($\chi=0$). The resonance condition can be written in the form of the inequality $1-u\leq v<1$. At values of v in this interval, longitudinal propagation of the extraordinary wave is impossible ($n_1^2<0$). For the ordinary wave we have

$$n_2^2=\varepsilon+g\approx g\approx u^{1/2},\quad n_2^4\approx u+2\varepsilon u^{1/2},\quad v\approx 1,$$
$$g\gg\varepsilon,\quad a=-s_3,\quad b=2(\varepsilon s_3+\eta s_1),\quad c=g^2s_3-2\eta(s_1\varepsilon+2s_2gu^{1/2}).$$

From (7) we obtain

$$\gamma_2\approx s_1,\quad \varkappa_2=s_1/2n_2=s_1/2u^{1/4}. \tag{11}$$

2) *Transverse propagation* ($\chi=\pi/2$)

$$A=\varepsilon,\quad B\approx u,\quad C=-\eta u,\quad a=-s_1,$$
$$b=\varepsilon s_3+\eta s_1+2\varepsilon s_1+4us_2,\quad c=s_3u-2\varepsilon s_1-4\eta gu^{1/2}s_2.$$

For the ordinary wave we have

$$n_2^2(\pi/2)\approx\eta,\quad \gamma_2(\pi/2)\approx s_3,\quad \varkappa_2=s_3/2\eta^{1/2}\ \text{at}\ s_3\ll\eta\ \text{and}$$
$$\varkappa_2=(s_3/2)^{1/2}\ \text{at}\ \eta\ll s_3.$$

For the extraordinary wave

$$n_1^2(\pi/2)\approx -u/\varepsilon,\quad \gamma_1(\pi/2)=us_1/|\varepsilon|^2,$$
$$\varkappa_1(\pi/2)=\left(\frac{u}{|\varepsilon|}\right)^{1/2}\frac{s_1}{2|\varepsilon|}.$$

Let us compare the absorption of the normal waves, using s_1 and s_2 in the quasi-electrostatic approximation (formulas (4)):

$$\varkappa_2(\pi/2)/\varkappa_1(\pi/2)=|\varepsilon|^{5/2}/2\eta^{3/2}u^{1/2}. \tag{12}$$

The ratio $\varkappa_2/\varkappa_1$ is small near one end of the resonance interval ($\varepsilon \to 0$) and large near the other ($\eta \to 0$).

4. ABSORPTION IN A PLASMA WITH INHOMOGENEITIES STRETCHED ALONG THE MAGNETIC FIELD

Assume that the correlation function of the fluctuations of the electron density is given by

$$B(x,y,z)=\sigma^2\exp\left\{-\frac{x^2+y^2}{l^2}-\frac{z^2}{L^2}\right\},$$
$$\Phi(\mathbf{p})=\pi^{3/2}\sigma^2l^2L\exp\left\{-\frac{p_x^2+p_y^2}{4}l^2-\frac{p_z^2}{4}L^2\right\}.$$

The transverse scale l is assumed, as before, to be small in comparison with the wavelength: $k_0l \ll 1$ and $k_{1,2}l \ll 1$. The scale of the inhomogeneity along the z axis ($\mathbf{z}_0 \parallel \mathbf{H}$) is assumed to be large (so that $k_0L \gg 1$). In this case, we cannot neglect in the calculation of $\varepsilon_{ij}^{\mathrm{eff}}(\omega, \mathbf{k})$ the spatial dispersion due to the inhomogeneity of the medium. However, to avoid cumbersome calculations, we stipulate beforehand that we are interested only in the absorption coefficients of the normal waves.

Assuming the anti-Hermitian increments to the average tensor $\varepsilon_{np}(\omega)$ to be small enough, we can replace the dispersion equation for the radial wave in an inhomogeneous plasma

$$\det|k^2\delta_{ij}-k_ik_j-k_0^2\varepsilon_{ij}^{\mathrm{eff}}(\omega,\mathbf{k})|=0 \tag{13}$$

by the approximate expression

$$\det|k^2\delta_{ij}-k_ik_j-k_0^2\varepsilon_{ij}^{\mathrm{eff}}(\omega,\mathbf{k}_{00})|=0, \tag{14}$$

where $\mathbf{k}_{00}$ is the wave vector of the unperturbed wave, the damping of which we wish to calculate. In other words, we should be in the possession of expressions for $\varepsilon_{ij}^{\mathrm{eff}}(\omega,\mathbf{k})$ for the concrete values of $\mathbf{k}=\mathbf{k}_{1,2}$.

It is easy to verify that the task of obtaining $\mathrm{Im}\,\varepsilon_{ij}^{\mathrm{eff}} \times(\omega,\mathbf{k})$ at fixed $\mathbf{k}$ is much simpler than in the arbitrary case (in particular, owing to the inequality $n_1^2(\vartheta) \gg n_2^2(\vartheta)$, which is valid in the resonance region for all propagation angles). Equation (14) gives the previous expressions (7)–(9) for the refraction and attenuation coefficients, except that the tensor components themselves are generally speaking dependent on the propagation angle χ. Thus, it follows from the general formulas of electrodynamics of randomly inhomogeneous media that[9]

$$s_1=s_2=\frac{\pi v\sigma_N^2k_0^2l^2|a|^2}{8(\eta-\varepsilon)}\exp\left\{-\frac{k_0^2l^2|a|}{4}\right\},$$
$$s_3=\frac{\pi v\sigma_N^2k_0^2l^2}{4(\eta-\varepsilon)}[|\varepsilon|+n_2^2(\chi)\cos^2\chi]\exp\left\{-\frac{k_0^2l^2|a|}{4}\right\}. \tag{15}$$

These formulas were written for an ordinary wave propagating at an angle χ to the magnetic field. Account was taken of only the contribution made to $\mathrm{Im}\,\varepsilon_{ij}^{\mathrm{eff}}$ by the scattering into the extraordinary waves. Formulas (15) together with the expressions for $\varkappa$ determine the absorption of the ordinary wave in a plasma with anisotropic inhomogeneities.

5. ABSORPTION IN A LINEAR LAYER

We have so far considered a statistically homogeneous infinite plasma. Bearing wave propagation in the ionosphere in mind, we assume that the average plasma concentration depends linearly on the coordinates. We assume for simplicity that the wave propagates along ∇N and at an angle χ to the magnetic field. This assumption makes it possible to disregard the refraction of the propagating waves. In addition, we neglect the refraction of the scattered waves.

The energy of the scattered extraordinary waves goes over practically completely into heat in a layer $v_\infty < v \le 1+u^{1/2}$. Only a small fraction of this energy can leave this layer, via multiple scattering processes, in the form of random ordinary waves. At $u \ll 1$, the dimensions of the absorbing resonant layer are small ($1-u < v < 1$). Thus, at $\mu_0 = -N^{-1}dN/dz \sim 10^{-5}$ m^{-1} and $u = 10^{-1}$ we have $\Delta z = u/\mu = 10$ km.

Let us find the integrated absorption for the ordinary wave in a resonant layer when the wave propagates at an angle $\chi = 0$ to the magnetic field, in the case of small-scale fluctuations

$$\varkappa_2=\frac{1}{2u^{1/2}}\int_{z(\varepsilon=0)}^{z(\eta=0)}s_1(z)\,dz. \tag{16}$$

Substituting the expression for s_1 from (5), recognizing that at $u \ll 1$ we have

$$|a|\sim u/|\varepsilon|,\ v\sim 1,\ d\varepsilon/dz\approx\mu_0,\ \eta-\varepsilon\approx u,$$

and introducing a new integration variable by means of the formula $k_0^2l^2|a|/4 = t$, we get

$$\varkappa_2=\pi\sigma_N^2/6\mu_0u^{1/2}. \tag{17}$$

Analogously, in the case of elongated inhomogeneities we have

$$\varkappa_2=\pi\sigma_N^2/4\mu_0u^{1/2}. \tag{18}$$

The last expression can be obtained from the results of other authors.[6,7]

Comparing (17) and (18), we see that in the case of longitudinal wave propagation the integral absorption is practically independent of the choice of the model of the inhomogeneities. We consider now transverse wave propagation ($\chi = \pi/2$):

$$\varkappa_2 = 2^{-1/2} \int_{z(\varepsilon=0)}^{z(\eta=0)} s_3^{1/2}\, dz \tag{19}$$

(we can assume $s_3 \ll \eta$ in the region of importance for the integration). Substituting here s_3 from (5), we obtain

$$\varkappa_2 = 2(\pi u)^{1/2} \sigma_N / 5\mu_0, \quad \sigma_N = (\sigma_N^2)^{1/2}, \tag{20}$$

whereas in the case of elongated inhomogeneities we have at $\chi = \pi/2$

$$\varkappa_2 = (\pi u)^{1/2} k_0 l \sigma_N / 3\mu_0. \tag{21}$$

Thus, in transverse propagation the absorption due to the small-scale fluctuations greatly exceeds the absorption due to the elongated inhomogeneities. When the angle χ changes from 0 to $\pi/2$, the role of the large-scale inhomogeneities, elongated along the magnetic field, decreases as transverse propagation is approached.

Let us compare the resonant absorption due to wave scattering with the absorption determined by the collisions between the electrons and the heavy particles. In the comparison, it is most convenient to use formulas (4) and (5) for the equivalent number of collisions ν_{eq}:

$$\frac{\nu_{eq}}{\omega} = \frac{\pi\sigma_N^2}{4} \frac{v\eta}{|\varepsilon|(\eta-\varepsilon)|a|^{1/2}} \tag{22}$$

(we have taken here the expression for $s_1 = s_2$ in the electrostatic approximation). From (22) at $v \approx 1$, $\eta \approx 0.05$, $|a| \approx 1$, $\varepsilon \approx -0.1$, $\omega = 2\pi \cdot 6 \cdot 10^6$ rad/sec we obtain $\nu_{eq} \approx 10^8 \sigma_N^2$ sec^{-1}. At $\sigma_N^2 \approx 10^{-2}-10^{-3}$ we have $\nu_{eq} \sim (10^6-10^5)$ sec^{-1}. Thus, the equivalent number of collisions is large enough. In particular, for the F-layer of the ionosphere, where the effective number of collisions is $\nu_{eq} \sim 10^3$ sec^{-1}, the absorption due to the statistical transformation of the energy in scattering by electron-density inhomogeneities $\sigma_N^2 \sim 10^{-2}-10^{-3}$ is decisive.

6. HIGH-FREQUENCY HEATING IN THE PROBLEM OF PRODUCING A HIGH-TEMPERATURE PLASMA

Heating of a collisionless high-temperature plasma by high-frequency fields is coming into ever increasing use. [11,12] Customary plasma heating is at the frequencies of the ion cyclotron harmonics and the lower hybrid resonance ($\omega = (\omega_H \Omega_H)^{1/2}$). Alfvén waves and shock waves are also regarded as sources of plasma heating to thermonuclear-reaction temperatures. As a rule, different instabilities in the plasma lead to the appearence of irregular inhomogeneities of the electron density. At this stage, interest attaches to the presently considered resonant mechanism of absorption of normal waves in the course of the transformation of their energy into the energy of scattered waves that are trapped in a localized region by the propagation conditions. Knowledge of the effective dielectric constant of such a plasma, a quantity providing the most substantial information on inhomogeneous plasma, makes it possible to calculate the heat released in the plasma at any field configuration.

In the present article we have discussed resonant absorption at $u < 1$. For laboratory-plasma heating, resonance at $u > 1$ may be of interest. In this case it is the ordinary wave which is resonant, and none of the formulas given above are suitable. From the fundamental point of view, however, the situation changes little and can be investigated with the aid of the effective dielectric constant. In particular, the components of the tensor ε_{ij}^{eff} in the quasi-static approximation for small-fluctuations differ at $u < 1$ only in sign from the case $u > 1$. [2]

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