[1)]It is thereby assumed that the interaction with the thermostat is sufficiently weak, making it possible to introduce a system energy uniquely determined by the state of the system itself.

[1]G. F. Efremov, Zh. Eksp. Teor. Fiz. 51, 156 (1966) [Sov. Phys. JETP 24, 105 (1967)].
[2]G. F. Efremov, Zh. Eksp. Teor. Fiz. 55, 2322 (1969) [Sov. Phys. JETP 27, 1232 (1969)].
[3]G. F. Efremov, Izv. Vuzov (Radiofizika) 15, 1207 (1972).
[4]G. F. Efremov, Candidate's dissertation, Gor'kiĭ, 1973.
[5]R. L. Stratonovich, Zh. Eksp. Teor. Fiz. 39, 1647 (1960) [Sov. Phys. JETP 12, 1150 (1961).
[6]R. L. Stratonovich, Zh. Eksp. Teor. Fiz. 58, 1612 (1970) [Sov. Phys. JETP 31, 864 (1970)].
[7]R. L. Stratonovich, Izv. Vuzov (Radiofizika) 13, 1512 (1970).
[8]R. L. Stratonovich, Vestn. Mosk. Univ., fiz. astron. 5, 16 (1962).
[9]R. L. Stratonovich, Vestn. Mosk. Univ., fiz. astron. 14, 84 (1967).
[10]N. A. Krupennikov, Candidate's dissertation, Moscow State University, 1974.

Translated by P. J. Shepherd

Current fluctuations in semiconductors in the presence of a quantizing electric field

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(Submitted June 22, 1976)
Zh. Eksp. Teor. Fiz. 72, 248–256 (January 1977)

The current fluctuations in a semiconductor are investigated under the conditions for "Stark quantization". It is shown that the fluctuations may be anomalously large. The obtained dependence of the fluctuations on the parameters of the scattering system and on the width of the energy band allow us to reach definite conclusions about the nature of the energy dissipation and band structure of the semiconductor.

PACS numbers: 72.20.Dp, 71.70.Ej

1. INTRODUCTION

A number of articles devoted to the investigation of semiconductors in strong electric fields have recently appeared. If the electric field is sufficiently strong (and the allowed band is relatively narrow), in such a field the electron is able to reach the top of the allowed band in energy space before being scattered. In such a situation the electron may undergo periodic motion in the Brillouin zone between collision events, which leads to qualitatively new quantum effects which are not observable in weak fields. The general theory of kinetic phenomena in semiconductors in a strong electric field is developed in the articles by Bryksin and Firsov.[1,2] In the single-band approximation they obtained[1] an expression for the current in an arbitrary electric field and an equation for the distribution function on the basis of a diagram technique. A quantum transport equation is presented and also a number of specific physical situations[2] are investigated. Similar questions are considered by Levinson and Yasevichyute in[3], where the quantum kinetic equation is solved and the current is calculated for a model of scattering. It should be noted that the solution of the problem by Levinson and Yasevichyute[3] is of a less general nature than the solution by Bryksin and Firsov[1,2] since the authors of[3] confined their attention to the case of weak electron-phonon coupling and to a specific choice for the form of the electron band.

It is known that the fluctuation-dissipation theorem is valid for a system in thermodynamic equilibrium; according to this theorem the problem of fluctuations reduces to a calculation of the linear response of the system to an external perturbation. There is no such generalized theorem for nonequilibrium systems, and in each specific case the calculation of the fluctuations requires special consideration. The theory of fluctuations in nonequilibrium electron-phonon systems is given in in[4–6]. In these articles current fluctuations were investigated under the quasi-classical condition $\bar{\varepsilon} \gg \hbar\omega$, where $\bar{\varepsilon}$ denotes the average energy of the electron and ω denotes the frequency of the fluctuations. High-frequency fluctuations in electron-phonon systems were investigated in[7].

The present article is devoted to a calculation of the current fluctuations in semiconductors in a strong electric field such that quantum effects due to the appearance of the "Stark levels"[8] begin to exert influence on the quantum effects. The existence of these levels has been experimentally established.[9,10] As far as the authors know, fluctuations under the conditions for quantization of the electron longitudinal motion have not been hitherto investigated.

Earlier[6,7] a method was proposed for a calculation of the fluctuations, based on the equations of motion for the quantum analog of the microscopic distribution function. In particular, this method enabled one to introduce outside sources of fluctuations into the equation for the fluctuating part of the distribution function without making any kind of assumption except those which are used in the derivation of the corresponding kinetic equations.

In the present article the spatially homogeneous current fluctuations of a relatively strong transverse electric field $\mathbf{E}$ are calculated by this method. In this connection the investigation is valid for low as well as high frequencies (an exact criterion for the frequency range of the fluctuations will be indicated below). It is shown in this work that Stark quantization significantly changes the nature of the fluctuations whose intensity may appreciably exceed the intensity of the equilibrium fluctuations. The investigated fluctuations are found to depend on the parameters of the scattering system and on the width of the energy band which allows one to reach definite conclusions about the nature of energy dissipation and the band structure of the semiconductor.

2. THE HAMILTONIAN OF THE SYSTEM AND THE CURRENT OPERATOR

Let us consider an electron–phonon system with a weak interaction, located in a strong electric field $\mathbf{E}$. We assume that in equilibrium the carriers obey Boltzmann statistics. We take the interaction between electrons into account by the introduction of self-consistent fields $\delta\mathbf{E}(t)$ and $\delta\mathbf{H}(t)$ which are due to fluctuations of the charge density and in turn exert influence on the fluctuations of the distribution function. In the single-band approximation the Hamiltonian of such a system has the form

$$H=H_e+H_p+H_{ep}+H_c,$$

where H_e is the single-band Hamiltonian of the electrons in the electric field, H_p is the Hamiltonian of the phonon field, H_{ep} is the Hamiltonian characterizing the interaction between electrons and phonons, and H_c describes the interaction of the electrons with the self-consistent fields.

The single-band approximation is valid when one can neglect the effect of interband tunneling, i.e., in the case of narrow allowed bands and rather broad forbidden bands. In this approximation the eigenfunctions of an electron in the crystal in the presence of a uniform electric field $\mathbf{E}$ directed along the x axis are given by[8]

$$\varphi_\nu(\mathbf{r})=\int_{-\pi/a}^{\pi/a} A_n(\mathbf{k})\,\psi_{\mathbf{k}}(\mathbf{r})\,dk_x$$

with an energy

$$\varepsilon_\nu=eEan+\varepsilon(\mathbf{k}_\perp),$$

where

$$A_n(\mathbf{k})=\left(\frac{a}{2\pi}\right)^{1/2}\exp\left\{\frac{i}{eE}\int^{k_x}[\varepsilon_\nu-\bar\varepsilon(\mathbf{k})]\,dk_x\right\}$$

$$\varepsilon(\mathbf{k}_\perp)=\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a}\bar\varepsilon(\mathbf{k})\,dk_x,\quad \bar\varepsilon(\mathbf{k})=\varepsilon(\mathbf{k})-eEX(\mathbf{k}).$$

Here ν denotes the set of quantum numbers n, $\mathbf{k}_\perp$ (n is the label on the Stark level and $\mathbf{k}_\perp=\{k_y, k_z\}$), a is the smallest lattice vector in the direction of $\mathbf{E}$, e is the carrier charge, and $\varepsilon(\mathbf{k})$ denotes the energy of the Bloch state $\psi_{\mathbf{k}}(\mathbf{r})$. In what follows we shall assume that the crystal has a center of inversion. In this case $X(\mathbf{k})=0$.

In the second-quantization representation we have the following expressions for the components of the Hamiltonian H:

$$H_e=\sum_\nu \varepsilon_\nu a_\nu^+ a_\nu,\quad H_p=\sum_{\mathbf{q}}\hbar\omega_{\mathbf{q}}\left(b_{\mathbf{q}}^+ b_{\mathbf{q}}+\frac{1}{2}\right),$$

$$H_{ep}=\sum_{\nu,\nu',\mathbf{q}}\{C_{\mathbf{q}}\mathcal{I}_{\nu\nu'}^{\mathbf{q}} a_\nu^+ a_{\nu'} b_{\mathbf{q}}+C_{\mathbf{q}}^*\mathcal{I}_{\nu\nu'}^{-\mathbf{q}} a_\nu^+ a_{\nu'} b_{\mathbf{q}}^+\},$$

$$H_c=-e\delta E\sum_{\nu,\nu'}\langle\nu|\mathbf{r}|\nu'\rangle\, a_\nu^+ a_{\nu'},$$

where $a_\nu^+(a_\nu)$ and $b_\nu^+(b_\nu)$ denote the creation (annihilation) operators, respectively, for an electron in the state ν and for a phonon with wave vector $\mathbf{q}$, $\omega_{\mathbf{q}}$ denotes the phonon frequency, $C_{\mathbf{q}}$ denotes the matrix element of the electron–phonon interaction, and $\mathcal{I}_{\nu\nu'}^{\pm\mathbf{q}}=\langle\nu|e^{\pm i\mathbf{q}\cdot\mathbf{r}}|\nu'\rangle$.

For a spatially homogeneous system the current operator is given by

$$\hat{\mathbf{J}}=\frac{e}{V}\sum_i\hat{\mathbf{v}}_i=\frac{e}{i\hbar V}\sum_i[\hat{\mathbf{r}}_i, H_i]. \tag{1}$$

where $\hat{\mathbf{v}}_i$, $\hat{\mathbf{r}}_i$, and H_i respectively denote the velocity, coordinate, and Hamiltonian operators for the ith electron, and V denotes the volume of the crystal (furthermore, we take $V=1$).

In the ν-representation of operators longitudinal and transverse with respect to the direction of the field $\mathbf{E}$, the coordinates have the form

$$x=-a\sum_\nu n a_\nu^+ a_\nu+\sum_{\mathbf{k}_\perp,\,n\neq n'} X_{n-n'}^{\mathbf{k}_\perp} a_n^+(\mathbf{k}_\perp)\,a_{n'}(\mathbf{k}_\perp), \tag{2}$$

$$\mathbf{r}_\perp=\sum_{\nu,\nu'} Y_{n-n'}^{\mathbf{k}_\perp\mathbf{k}_\perp'}\left(\frac{\partial}{\partial\mathbf{k}_\perp}\delta_{\mathbf{k}_\perp,\mathbf{k}_\perp'}\right) a_\nu^+ a_{\nu'}, \tag{3}$$

where

$$X_{n-n'}^{\mathbf{k}_\perp}=\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a}\frac{\varepsilon(\mathbf{k})}{eE}\exp\{i(n-n')ak_x\}\,dk_x,$$

$$Y_{n-n'}^{\mathbf{k}_\perp\mathbf{k}_\perp'}=i\int_{-\pi/a}^{\pi/a}A_n^*(k_x,\mathbf{k}_\perp)\,A_{n'}(k_x,\mathbf{k}_\perp)\,dk_x,\quad \mathbf{r}_\perp=\{y,z\}.$$

By calculating the corresponding commutators in formula (1), we obtain the following expression for the current $\hat{\mathbf{J}}$:

$$\hat{\mathbf{J}}=e\sum_{\mathbf{k}_\perp,\,n,\,n'}\mathbf{V}_{n-n'}^{\mathbf{k}_\perp} a_n^+(\mathbf{k}_\perp)\,a_{n'}(\mathbf{k}_\perp), \tag{4}$$

where

$$\mathbf{V}_{n-n'}^{\mathbf{k}_\perp}=\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a}\frac{1}{\hbar}\frac{\partial\varepsilon(\mathbf{k})}{\partial\mathbf{k}}\exp\{i(n-n')ak_x\}\,dk_x.$$

3. FLUCTUATIONS OF THE DISTRIBUTION FUNCTION

The problem of fluctuations is self-consistent. The fluctuating fields $\delta\mathbf{E}(t)$ and the current fluctuations $\delta\mathbf{J}(t)$ are related by Maxwell's equations. But in order to calculate $\delta\mathbf{J}(t)$ it is necessary to solve the equation for the fluctuations $\delta f_{\nu\nu'}(t)=a_\nu^+ a_{\nu'}-\langle a_\nu^+ a_{\nu'}\rangle$ of the distribution function. Therefore, first of all let us find the time de-

pendence of $\delta f_{\nu\nu'}$, assuming the field fluctuations $\delta\mathbf{E}$ to be given. To this end we write down the equation of motion for the operator $a_\nu^+ a_{\nu'}$,

$$i\hbar\frac{\partial a_\nu^+ a_{\nu'}}{\partial t}=[a_\nu^+ a_{\nu'}, H_e+H_{ep}+H_c]. \tag{5}$$

In the present article we consider only fluctuations $\delta\mathbf{J}_\perp=\{\delta J_y, \delta J_z\}$ which are transverse with respect to the direction of $\mathbf{E}$. And also we shall assume that $\varepsilon(\mathbf{k})=\varepsilon(k_x)+\varepsilon(\mathbf{k}_\perp)$ in order to not complicate the calculations. Taking account of the latter leads to a simplication of formula (4) and for $\delta\mathbf{J}_\perp$ we obtain

$$\delta\mathbf{J}_\perp(t)=e\sum_\nu\frac{1}{\hbar}\frac{\partial\varepsilon(\mathbf{k}_\perp)}{\partial\mathbf{k}_\perp}\delta f_{\nu\nu}(t). \tag{6}$$

It is clear from formula (6) that Eq. (5) can be used to determine $\delta\mathbf{J}_\perp$ only at $\nu=\nu'$. Evaluating the commutators on the right side of Eq. (5), we obtain the following equation for $a_\nu^+ a_\nu$:

$$\left(\frac{\partial}{\partial t}+\frac{e}{\hbar}\delta E_l\frac{\partial}{\partial k_l}\right)a_\nu^+ a_\nu+i\frac{e\delta E_x}{\hbar}\sum_{m\neq 0}X_m^{\mathbf{k}_\perp}(a_n^+ a_{n+m}-a_{n+m}^+ a_n)$$
$$=\frac{1}{i\hbar}\sum_{\nu',\mathbf{q}}\{C_\mathbf{q}(\mathcal{J}_{\nu\nu'}^{\mathbf{q}}a_\nu^+ a_{\nu'}b_\mathbf{q}-\mathcal{J}_{\nu'\nu}^{\mathbf{q}}a_{\nu'}^+ a_\nu b_\mathbf{q})+$$
$$+C_\mathbf{q}^*(\mathcal{J}_{\nu\nu'}^{-\mathbf{q}}a_\nu^+ a_{\nu'}b_\mathbf{q}^+-\mathcal{J}_{\nu'\nu}^{-\mathbf{q}}a_{\nu'}^+ a_\nu b_\mathbf{q}^+)\}. \tag{7}$$

Here and below $l=\{y, z\}$.

Let us derive the equation for $\delta f_\nu\equiv\delta f_{\nu\nu}$ according to the scheme proposed in[6,7]. Writing down the equations of motion for operators of the type $a_\nu^+ a_{\nu'}b_\mathbf{q}$ and linearizing these equations with respect to the fluctuating quantities, let us find the time dependence of the operator $a_\nu^+ a_{\nu'}b_\mathbf{q}$ to lowest order in the interaction constant $C_\mathbf{q}$. We can now write for the operator

$$\delta f_\nu^\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}\delta f_\nu(t)e^{-i\omega t}dt$$

the following equation which is linearized with regard to the fluctuations:

$$i\omega\delta f_\nu^\omega=\frac{e\delta E_x^\omega}{i\hbar}\left[\sum_{m\neq 0}X_m^{\mathbf{k}_\perp}(\langle a_n^+ a_{n+m}\rangle-\langle a_{n+m}^+ a_n\rangle)+A_\nu^\omega\right]$$
$$-\frac{e\delta E_l^\omega}{\hbar}\left[\frac{\partial f_\nu}{\partial k_l}+iB_l^\omega(\nu)\right]-\hat{\nu}_{ep}\left(\delta f_\nu^\omega-\frac{e\delta E_l^\omega}{\hbar\omega}\frac{\partial f_\nu}{\partial k_l}\right)+j_\nu^\omega, \tag{8}$$

where $f_\nu=\langle a_\nu^+ a_\nu\rangle$ and $j_\nu(t)$ is the operator of the extraneous current which appears in the equation as a consequence of taking account of the initial conditions upon solution of the equations of motion for operators of the type $a^+ a_\nu b_\mathbf{q}$. The form of j_ν coincides with the right side of Eq. (7), but the operators a_ν and $b_\mathbf{q}$ have a time dependence of the same type as for noninteracting particles:

$$a_\nu(t)=a_\nu(t_0)\exp\left\{-\frac{i\varepsilon_\nu(t-t_0)}{\hbar}\right\},\quad b_\mathbf{q}(t)=b_\mathbf{q}(t_0)\exp\{-i\omega_\mathbf{q}(t-t_0)\}.$$

In connection with the derivation of Eq. (8) the moment of time t_0 was chosen such that the inequality

$$\varepsilon(t-t_0)\gg\hbar. \tag{9}$$

was fulfilled.

Let us present the explicit form of $\mathbf{B}^\omega$ and $\hat{\nu}_{ep}$:

$$\mathbf{B}^\omega(\nu)=\frac{1}{\omega^2}\sum_{\nu',\mathbf{q}}(\mathbf{v}_{\mathbf{k}_\perp}-\mathbf{v}_{\mathbf{k}_\perp}')\{W_{\nu\nu'}^{\mathbf{q}-}(\omega)f_\nu+W_{\nu'\nu}^{\mathbf{q}-}(\omega)f_{\nu'}\},$$
$$\hat{\nu}_{ep}(\delta f)=\sum_{\nu',\mathbf{q}}\{W_{\nu\nu'}^{\mathbf{q}+}(\omega)\delta f_\nu^\omega-W_{\nu'\nu}^{\mathbf{q}+}(\omega)\delta f_{\nu'}^\omega\}.$$

Here

$$W_{\nu\nu'}^{\mathbf{q}\pm}(\omega)=W_{\nu\nu'}^{\omega}(\mathbf{q})\pm W_{\nu\nu'}^{-\omega}(\mathbf{q}),\quad \mathbf{v}_{\mathbf{k}_\perp}=\frac{1}{\hbar}\frac{\partial\varepsilon(\mathbf{k}_\perp)}{\partial\mathbf{k}_\perp},$$
$$W_{\nu\nu'}^{\omega}(\mathbf{q})=\frac{\pi}{\hbar}|C_\mathbf{q}|^2|\mathcal{J}_{\nu\nu'}^{\mathbf{q}}|^2[(N_\mathbf{q}+1)\delta(\Delta_{\nu\nu'}^--\hbar\omega)+N_\mathbf{q}\delta(\Delta_{\nu\nu'}^+-\hbar\omega)],$$
$$|\mathcal{J}_{\nu\nu'}^{\pm\mathbf{q}}|^2=\delta_{\mathbf{k}_\perp\mathbf{k}_\perp'\mp\mathbf{q}_\perp}\left|\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a}\exp\left\{\frac{i}{eE}\left[(\varepsilon_\nu-\varepsilon_{\nu'})k_x-\int_{k_x}^{k_x}(\varepsilon(\mathbf{k}\mp\mathbf{q})-\varepsilon(\mathbf{k}))dk_x\right]\right\}dk_x\right|^2, \tag{10}$$

where $\Delta_{\nu\nu'}^\pm=\varepsilon_\nu-\varepsilon_{\nu'}\pm\hbar\omega_\mathbf{q}$ and $N_\mathbf{q}$ is the Planck distribution for the phonons. The expression for A_ν^ω has a rather cumbersome form and is therefore not given here. In addition, in the case considered below A_ν^ω is an even function of $\mathbf{k}_\perp$ and, as will be evident from the following, does not give any contribution to the current.

Equation (8) was obtained under the conditions

$$\Omega_E\gg\bar{\nu}_{ep}, \tag{11}$$
$$|\omega-n\Omega_E|\gg\bar{\nu}_{ep},\quad n=1, 2, \ldots, \tag{12}$$

where $\Omega_E=eEa/\hbar$ denotes the frequency of the electron vibrations in coordinate and momentum space and $\bar{\nu}_{ep}$ is the characteristic frequency of electron-phonon collisions. Condition (11) indicates that the quantization of the longitudinal motion is essential and the Stark levels are not washed out by the collisions, and criterion (12) indicates the range of fluctuation frequencies under consideration.

4. FLUCTUATIONS OF THE TRANSVERSE CURRENT

As is evident from formula (6), the antisymmetric part of the function δf_ν gives a contribution to the fluctuating current:

$$\delta f_a(n, \mathbf{k}_\perp)=\tfrac{1}{2}[\delta f(n, \mathbf{k}_\perp)-\delta f(n, -\mathbf{k}_\perp)].$$

We shall assume that the electrons are scattered by unpolarized optical phonons. Then in the cases investigated by Bryksin and Firsov[2] f_ν is an even function of $\mathbf{k}_\perp$ and therefore A_ν^ω and $\mathbf{B}^\omega(\nu)$ are even and odd functions of $\mathbf{k}_\perp$, respectively, and also

$$\sum_{\nu',\mathbf{q}}W_{\nu'\nu}^{\mathbf{q}+}(\omega)\left[\delta f_a^\omega(\nu')-\frac{ie\delta E_l^\omega}{\hbar\omega}\frac{\partial f_{\nu'}}{\partial k_l}\right]=0.$$

In addition the first term inside the square brackets in Eq. (8) vanishes. This follows from the fact that in the electric field space remains homogeneous and one can show that $\langle a_n^+(\mathbf{k}_\perp)a_{n'}(\mathbf{k}_\perp)\rangle$ only depends on the difference $n-n'$. Also bearing in mind that $X_m^{\mathbf{k}_\perp}=X_{-m}^{\mathbf{k}_\perp}$, one can easily see that the indicated quantity vanishes.

After everything that has been said, we obtain the following equation for $\delta f_a^\omega(\nu)$:

$$[i\omega+\tau_{\mathbf{k}_\perp}^{-1}(\omega)]\delta f_a^\omega(\nu)$$
$$=-\frac{e\delta E_l^\omega}{\hbar}\left[\left(1-\frac{i}{\omega\tau_{\mathbf{k}_\perp}(\omega)}\right)\frac{\partial n(\mathbf{k}_\perp)}{\partial k_l}+iB_l^\omega(\nu)\right]+j_a^\omega(\nu), \quad (13)$$

where

$$\tau_{\mathbf{k}_\perp}^{-1}(\omega)=\sum_{\nu',\mathbf{q}} W_{\nu\nu'}^{\mathbf{q}+}(\omega), \quad j_a^\omega(\nu)=\frac{1}{2}[j^\omega(n,\mathbf{k}_\perp)-j^\omega(n,-\mathbf{k}_\perp)].$$

We note that $\tau_{\mathbf{k}_\perp}(\omega)$ and $\mathbf{B}^\omega$ do not depend on n since n appears in $W_{\nu\nu'}^\omega$ in the form $n-n'$ but summation is carried out over n'. Here the notation $n(\mathbf{k}_\perp)=f_\nu$ is introduced since f_ν does not depend on the quantum number of the Stark level as a consequence of the above mentioned property of spatial homogeneity.

Equation (13) is easily solved and we obtain the following expression for the fluctuations of the transverse current:

$$\delta J_l^\omega=\sigma_{lm}^\omega\delta E_m^\omega+I_l^\omega. \quad (14)$$

Here

$$I_l^\omega=e\sum_\nu v_{\mathbf{k}_\perp}^l j_a^\omega(\nu)[i\omega+\tau_{\mathbf{k}_\perp}^{-1}(\omega)]^{-1}$$

are the outside fluctuating currents and σ_{lm} is the conductivity tensor $(l,m=y,z)$ which is given by[1)]

$$\sigma_{lm}^\omega=\frac{ie^2}{\hbar\omega}\sum_\nu v_{\mathbf{k}_\perp}^l\frac{\partial n(\mathbf{k}_\perp)}{\partial k_m}$$
$$-\frac{ie^2}{\hbar\omega^2}\sum_{\nu,\nu',\mathbf{q}}\left[\frac{v_{\mathbf{k}_\perp}^l v_{\mathbf{k}_\perp}^m}{i\omega+\tau_{\mathbf{k}_\perp}^{-1}}+\frac{v_{\mathbf{k}_\perp}^l v_{\mathbf{k}_\perp'}^m}{i\omega+\tau_{\mathbf{k}_\perp'}^{-1}}\right]n(\mathbf{k}_\perp)W_{\nu\nu'}^{\mathbf{q}-}(\omega). \quad (15)$$

Now let us go on to the calculation of the correlation functions $\langle I_l^\omega I_m^{\omega'}\rangle$ since it is just these quantities which are observable, and not the outside currents $I_l(t)$. The average values of quantities of the type

$$a_\nu^+ a_{\nu'} b_{\mathbf{q}} a_\mu^+ a_{\mu'} b_{\mathbf{q}'}^+\delta(\Delta_{\nu\nu'}^- -\hbar\omega)\delta(\Delta_{\mu\mu'}^+ -\hbar\omega'). \quad (16)$$

are needed in order to determine the correlation functions of the outside currents. In Eq. (16) all of the operators are specified at a certain moment of time t_0 (the choice of t_0 was mentioned above). Neglecting the nondiagonal elements $f_{\nu\nu'}=\langle a_\nu^+ a_{\nu'}\rangle$ of the density matrix, which are small in comparison with the diagonal elements in terms of the parameter $\bar{\nu}_{ep}/\Omega_E$, we obtain the average value of expression (16)

$$\delta(\omega+\omega')\frac{1}{\hbar}\delta_{\mathbf{q},\mathbf{q}'}\delta_{\nu',\mu}\delta_{\nu,\mu'}(N_{\mathbf{q}}+1)n(\mathbf{k}_\perp)\delta(\Delta_{\nu\nu'}^- -\hbar\omega). \quad (17)$$

Then with (17) taken into consideration we have

$$\langle I_l^\omega I_m^{\omega'}\rangle^c=\delta(\omega+\omega')\frac{e^2}{2\pi}\sum_{\nu,\nu',\mathbf{q}}\left[\frac{v_{\mathbf{k}_\perp}^l v_{\mathbf{k}_\perp}^m}{\omega^2+\tau_{\mathbf{k}_\perp}^{-2}}+\frac{v_{\mathbf{k}_\perp}^l v_{\mathbf{k}_\perp'}^m}{\omega^2+\tau_{\mathbf{k}_\perp'}^{-2}}\right]n(\mathbf{k}_\perp)W_{\nu\nu'}^{\mathbf{q}+}(\omega), \quad (18)$$

where

$$\langle I_l(t)I_m(t')\rangle^c=\frac{1}{2}[\langle I_l(t)I_m(t')\rangle+\langle I_m(t')I_l(t)\rangle].$$

Formulas (15) and (18) are valid for both low and high frequencies (the exact criterion for the range of ω is given by inequality (12)). These formulas simplify considerably for low-frequency fluctuations ($\omega\ll\bar{\varepsilon}$). And since in an experiment very often just the low-frequency fluctuations are measured, it makes sense to present the expressions for σ_{lm} and $\langle I_l I_m\rangle$ for the case of small ω:

$$\sigma_{lm}^\omega=-\frac{e^2}{\hbar}\sum_\nu v_{\mathbf{k}_\perp}^l\frac{\partial n(\mathbf{k}_\perp)}{\partial k_m}[i\omega+\tau_{\mathbf{k}_\perp}^{-1}(\omega=0)]^{-1}, \quad (19)$$

$$\langle I_l I_m\rangle_\omega=\frac{e^2}{\pi}\sum_\nu v_{\mathbf{k}_\perp}^l v_{\mathbf{k}_\perp}^m n(\mathbf{k}_\perp)\frac{\tau_{\mathbf{k}_\perp}^{-1}(\omega=0)}{\omega^2+\tau_{\mathbf{k}_\perp}^{-2}(\omega=0)}, \quad (20)$$

The kinetic equation for $n(\mathbf{k}_\perp)$ was utilized in the derivation of Eqs. (19) and (20). Thus, for example, taking account of the kinetic equation in (8) as $\omega\to 0$ gives the equation

$$\left\{\mathbf{B}(\nu)-\frac{1}{\omega}\hat{\nu}_{ep}\left(\frac{\partial n(\mathbf{k}_\perp)}{\partial\mathbf{k}_\perp}\right)\right\}_{\omega\to 0}=0,$$

as a result of which Eq. (8) takes a simpler form. Starting from this equation for the function δf_a, one can easily obtain formula (19). Analogous simplifications also occur in connection with the derivation of expression (20) for the correlation function.

The equation for $n(\mathbf{k}_\perp)$ is derived under the same assumptions that were made for the derivation of Eq. (8) from the averaged equation (7) and in the stationary case has the form

$$\sum_{\nu',\mathbf{q}}[W_{\nu\nu'}^{\mathbf{q}+}(\omega=0)n(\mathbf{k}_\perp)-W_{\nu'\nu}^{\mathbf{q}+}(\omega=0)n(\mathbf{k}_\perp')]=0. \quad (21)$$

Equation (21) and its solution were first found by Bryksin and Firsov.[1,2]

5. THE FLUCTUATION-DISSIPATION THEOREM AND OTHER EXAMPLES

In this section we investigate the consequences of the above derived formulas for the conductivity tensor and for the correlation functions of the outside currents. Up till now the fundamental condition on the entire investigation was criterion (11), which determines the magnitude of the constant applied electric field. But the width ΔE of the allowed band implicitly enters into all expressions. And different physical situations are possible depending on the relationship between the quantities ΔE and $\hbar\Omega_E$, and also depending on the relationship between ΔE_x and $\Delta E_\perp$ (ΔE_x denotes the width of the band along the field and $\Delta E_\perp$ denotes the width transverse to the field). A number of such situations were investigated by Bryksin and Firsov,[2] who found the distribution function $n(\mathbf{k}_\perp)$ for each case.

In the limit of quantizing fields the electrons are only scattered inside a single Stark sub-band and[2]

$$n(\mathbf{k}_\perp)=C\exp\left\{-\frac{\varepsilon(\mathbf{k}_\perp)}{T}\right\}, \quad (22)$$

where C is a constant determined from the normalization condition, and T is the temperature of the lattice. The function (22) corresponds to thermodynamic equilibrium of the system. In this case the fluctuation-dissi-

pation theorem, which can be represented in the form

$$\frac{\langle I_l I_m\rangle_\omega^c}{\mathrm{Re}\,\sigma_{lm}^\omega}=\frac{\hbar\omega}{2\pi}\mathrm{cth}\frac{\hbar\omega}{2T} \tag{23}$$

should be satisfied. It is not difficult to obtain formula (23) if only the terms with $n'=0$ are kept in the summation over n' in Eqs. (15) and (18) and also if the relationship

$$n(\mathbf{k}_\perp)(N_q+1)\delta(\varepsilon(\mathbf{k}_\perp)-\varepsilon(\mathbf{k}_\perp')-\hbar\omega_q\pm\hbar\omega)$$
$$=n(\mathbf{k}_\perp')N_q e^{\pm\hbar\omega/T}\delta(\varepsilon(\mathbf{k}_\perp)-\varepsilon(\mathbf{k}_\perp')-\hbar\omega_q\pm\hbar\omega)$$

is taken into consideration.

Now let us investigate the case when $\hbar\Omega_E\ll\Delta E$. Then we have

$$n(\mathbf{k}_\perp)=C \tag{24}$$

for the scattering of electrons by unpolarized optical phonons ($\omega_q=\omega_0$). [2] It is assumed that $\hbar\omega_0\ll\Delta E_x$, $\Delta E_\perp$ and $\Delta E_x\sim\Delta E_\perp\sim\Delta E$. This is the so-called case of total warming-up. It is of no interest due to the fact that for frequencies $\omega\gg\bar{\nu}_{ep}$ (see Eq. (15) we have

$$\mathrm{Im}\,\sigma_{lm}^\omega=0.$$

This corresponds to the fact that the plasma frequency tends to zero, and for low frequencies, as is evident from Eq. (19), $\sigma_{lm}^\omega=0$ and the ratio

$$\theta\equiv\langle I_l I_m\rangle_\omega^c/\mathrm{Re}\,\sigma_{lm}^\omega$$

tends to infinity. This implies that the fluctuations in the system are anomalously large. In order to determine θ it would apparently be necessary to determine $n(\mathbf{k}_\perp)$ to the next order in $\hbar\omega_0/\Delta E$. But θ is easy to calculate if the band is asymmetric: $\Delta E_x\ll\Delta E_\perp$. In this case[2]

$$n(\mathbf{k}_\perp)=C\exp\left\{-\frac{\varepsilon(\mathbf{k}_\perp)}{T^*}\right\}, \tag{25}$$

where

$$T^*=\frac{(\Delta\bar{E}_x)^2}{\hbar\omega_0}\mathrm{cth}\frac{\hbar\omega_0}{2T},\quad (\Delta\bar{E}_x)^2=\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a}\varepsilon^2(k_x)\,dk_x.$$

We note that in spite of the Maxwellian form of the function (25), for arbitrary frequencies T is not replaced by T^* in formula (23), but for low-frequency fluctuations

$$\theta=T^*/\pi. \tag{26}$$

If we assume that $\hbar\omega_0\ll T$, we obtain an extremely simple expression for θ:

$$\theta=\frac{2}{\pi}\frac{(\Delta\bar{E}_x)^2}{(\hbar\omega_0)^2}T. \tag{27}$$

From the last formula it is quite clear that the fluctuations in the investigated system are considerably more intensive than the equilibrium thermal fluctuations for $\Delta E_x>\hbar\omega_0$.

Thus, as follows from formulas (26) and (27), the investigation of fluctuations in strong electric fields allows one to investigate the band structure of semiconductors, and the simplicity of the results significantly offsets the experimental difficulties.

In conclusion we further note that in the present work the problem of a linear response has been solved and the results can be used in problems where there is a weak alternating electric field in addition to a strong constant field.

The authors express their gratitude to V. V. Bryksin for helpful discussions and also to S. V. Gantsevich and R. Katilyus for a discussion of the results of this work.

[1)]One can show that all remaining components of the conductivity tensor vanish except σ_{xx}.

[1]V. V. Bryksin and Yu. A. Firsov, Fiz. Tverd. Tela (Leningrad) **13**, 3246 (1971) [Sov. Phys. Solid State **13**, 2629 (1972)].
[2]V. V. Bryksin and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. **61**, 2373 (1971) [Sov. Phys. JETP **34**, 1272 (1972)].
[3]I. B. Levinson and Ya. Yasevichyute, Zh. Eksp. Teor. Fiz. **62**, 1902 (1972) [Sov. Phys. JETP **35**, 991 (1972)].
[4]Sh. M. Kogan and A. Ya. Shul'man, Zh. Eksp. Teor. Fiz. **56**, 862 (1969) [Sov. Phys. JETP **29**, 467 (1969)].
[5]S. V. Gantsevich, V. L. Gurevich, and R. Katilyus, Zh. Eksp. Teor. Fiz. **57**, 503 (1969) [Sov. Phys. JETP **30**, 276 (1970)].
[6]P. M. Tomchuk and A. A. Chumak, Institute of Physics, Ukrainian SSR Academy of Sciences, Preprint No. 9, 1971.
[7]P. M. Tomchuk and A. A. Chumak, Ukr. Fiz. Zh. **18**, 1625, 1822 (1973).
[8]E. O. Kane, J. Phys. Chem. Solids **12**, 181 (1959).
[9]S. Maekawa, Phys. Rev. Lett. **24**, 1175 (1970).
[10]R. W. Koss and L. M. Lambert, Phys. Rev. **B5**, 1479 (1972).

Translated by H. H. Nickle