Nonlinear helicon resonance in an aluminum plate

V. I. Bozhko and E. P. Volskiĭ

Institute of Solid State Physics, USSR Academy of Sciences
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The frequency dependence of the first resonance of helicon standing waves in an aluminum plate at a constant magnetic field intensity has revealed the following: a shift of the resonance peak, asymmetry of the resonance curve and the appearance of signal extinction on the curve with increasing amplitude of the current that excites the helicon. The effect is observed at temperatures between 0.5 and 1.5 K in the fields of 5–30 kG, when the de Haas–van Alphen oscillations of the resonance frequency of the sample exceed the width of the resonance curve by a factor 5–10. The effect is the greatest at the maxima and minima of the resonance frequency as a function of the magnetic field intensity; it is practically absent for those magnetic field intensities for which the resonance frequency assumes values corresponding to a zero amplitude of the de Haas–van Alphen effect at increased temperature (4.2 K). The amplitude dependence of the position and shape of the helicon resonance is interpreted as the consequence of the nonlinear dependence of the magnetic moment of the metal on the instantaneous value of the helicon field intensity.

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The de Haas–van Alphen effect leads, as has been established,[1] to an oscillating dependence on the magnetic field of the helicon resonance frequencies in a metallic plate. In specially fabricated aluminum samples[2] characterized by the absence of a mosaic structure, we succeeded in obtaining a spread of the oscillation of the resonance frequency that exceeded by a factor of 5–10 the width of the resonance curve on the frequency scale. This allowed us to observe a very pronounced effect of the dependence of the position and shape of the resonance curve on the amplitude of the field exciting the helicons.

The experiments were conducted in the temperature range 0.5–4.2 K. A magnetic field up to 60 kG was generated by a superconducting coil with a switch for conversion to the short-circuit mode. The sample was a plate measuring 7×7×0.6 mm, made of aluminum with a room to liquid-helium resistance ratio 2×10^4.

Using first the method of crossed coils in the helicon generator regime,[3] we recorded the resonance frequency of the sample as a function of the magnetic field (Fig. 1). Then, fixing the magnetic field at a number of points within the period of a single oscillation, we plotted at each one the first resonances of the helicons in the plate in the case of symmetric excitation, as a function of the frequency of the exciting signal at different amplitudes in the exciting coil (Fig. 1). The strongest nonlinear effect is observed with a fixed field at the maximum or minimum oscillations of the resonance frequency (points of type 1 and 2 in Fig. 1). In this case, upon increase of the amplitude of the exciting current, the maximum of the resonance curve shifts toward a position corresponding to zero amplitude of the de Haas–van Alphen effect; the leading edge of the resonance curve becomes steeper, and interruptions of the signal appear with hysteresis, depending on the direction of the trace. When the maximum field of the helicon along the thickness of the plate (by estimate) is greater than about twice the period of oscillation of the de Haas–van Alphen effect, the resonance curve reaches a null position and again becomes symmetric. At points of type 3 in Fig. 1, the nonlinear effect is virtually absent.

The effect has been observed both on oscillations from zone III of the γ orbits as well as from orbits of zone II of the Fermi surface of aluminum, and is interpreted as a consequence of the nonlinear dependence of the magnetic moment on the instantaneous value of the helicon field. The ratio $\Delta f/\Delta f_0$ can serve as a quantitative measure of the nonlinear effect; here Δf_0 is the de Haas–van Alphen shift in the resonance frequency of the sample at some fixed magnetic field, and Δf is the nonlinear shift of the maximum of the resonance curve from its position at a vanishingly small amplitude of the helicon at the same magnetic field. The indicated ratio, if our interpretation is valid, should be a function of the quantity $b_0/\Delta B$, where b_0 is the mean amplitude of the helicon in the sample, and ΔB is the interval of the magnetic field between neighboring maxima or minima of the resonance frequency as a function of the magnetic field. The quantity ΔB varies in proportion to the square of the magnetic field, and the field b_0 is proportional to the Q of the resonance, i.e., to B and, of course, to the amplitude of the exciting current.

Thus, fixing the magnetic field at one and the same position on the oscillation of the resonance frequency of the sample, we must vary the current in the exciting coil in inverse proportion to the number of the oscilla-

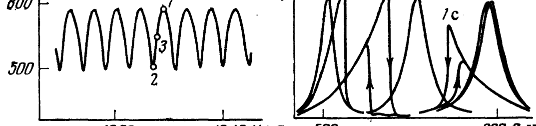

FIG. 1. Resonance frequency of the sample vs the magnetic field (single curve) and the resonance curves of the first number of the resonance of helicons at fixed magnetic field at points 1 and 2 at different values of the exciting current: a—$I=0.3$ mA, b—$I=3.0$ mA, c—$I=10$ mA, d—$I=100$ mA. When the exciting current was varied, the gain of the system was changed in the opposite direction by a corresponding amount.

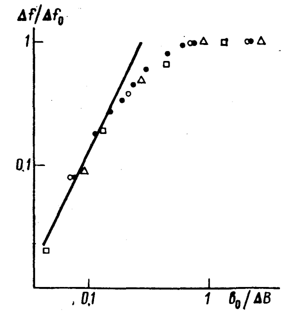

FIG. 2. Relative nonlinear shift of the maximum of the resonance curve of the sample ($\Delta f/\Delta f_0$) at fixed magnetic field, at points of type 2 in Fig. 1, vs. the ratio of the mean amplitude of the helicon to the period of oscillation ($b_0/\Delta B$). The scales of both axes are logarithmic. The straight line corresponds to the quadratic dependence. Δ—$T=0.8$ K, $B=9.00$ kG, ○—$T=0.8$ K, $B=11.80$ kG, □—$T=0.8$ K, $B=19.15$ kG, ●—$T=0.5$ K, $B=11.30$ kG.

tion, in order to obtain the same nonlinear effect. If we plot $\Delta f/\Delta f_0$ against $b_0/\Delta B$, then the points corresponding to different oscillation numbers should lie on one and the same curve.

The described scaling effect is illustrated in Fig. 2. The entire phenomenon is, on the whole, similar to the resonance of an oscillator with a nonlinear element, known in radio engineering and mechanics.[4] Nevertheless, it is useful, for our case of helicons with the de Haas–van Alphen effect, to describe it mathematically, albeit partially and approximately. We consider the propagation of a helicon wave in half-space, using a cartesian coordinate system x_1, x_2, x_3, such that the x_3 axis is normal to the surface of the metal. All the fields and the currents of the wave are functions only of a single coordinate, x_3. The wave propagation is determined by the two-dimensional magnetoresistance tensor[5,6]

$$\hat{\rho}=\rho_{12}\left\|\begin{matrix}\alpha_1 & 1\\ -1 & \alpha_2\end{matrix}\right\|,\quad \alpha_1,\alpha_2\ll 1. \tag{1}$$

The initial wave equations are of the form

$$\alpha_2\frac{\partial^2 h_1}{\partial x_3^2}+\frac{\partial^2 h_2}{\partial x_3^2}=\frac{4\pi}{c^2\rho_{12}}\frac{\partial b_1}{\partial t},\qquad \frac{\partial^2 h_1}{\partial x_3^2}-\alpha_1\frac{\partial^2 h_2}{\partial x_3^2}=-\frac{4\pi}{c^2\rho_{12}}\frac{\partial b_2}{\partial t}. \tag{2}$$

The component b_3 of the helicon field is equal to zero. The de Haas–van Alphen effect is taken into account by the relations

$$h_1=b_1-4\pi M_1(\mathbf{B}+\mathbf{b}),\quad h_2=b_2-4\pi M_2(\mathbf{B}+\mathbf{b}) \tag{3}$$

Let the oscillating magnetic moment **M** be determined by the γ orbits on a tube of zone III of the Fermi surface of aluminum. We confine ourselves to the simple situation in which the constant magnetic field is parallel to the fourfold axis, which coincides with the x_3 axis. The axes x_1 and x_2 are so chosen that the four tubes of the zone III, which make angles of 45° with the x_3 axis, lie in the planes x_1x_3 and x_2x_3, respectively. In this case, $M_1=M_1(b_1)$, $M_2=M_2(b_2)$ and the wave equations (2) reduce to the form

$$\begin{aligned}&\left(1-4\pi\frac{\partial M_1}{\partial b_1}\right)\frac{\partial^2 b_1}{\partial x_3^2}+\frac{4\pi}{c^2\rho_{12}}\frac{\partial b_2}{\partial t}=4\pi\frac{\partial^2 M_1}{\partial b_1^2}\left(\frac{\partial b_1}{\partial x_3}\right)^2,\\&\left(1-4\pi\frac{\partial M_2}{\partial b_2}\right)\frac{\partial^2 b_2}{\partial x_3^2}-\frac{4\pi}{c^2\rho_{12}}\frac{\partial b_1}{\partial t}=4\pi\frac{\partial^2 M_2}{\partial b_2^2}\left(\frac{\partial b_2}{\partial x_3}\right)^2.\end{aligned} \tag{4}$$

In the derivation of these equations, we have neglected the quantities α_1 and α_2. We note also that the derivatives of the magnetic moment in the coefficients of Eqs. (4) are taken not at zero but at some instantaneous value of the helicon field **b**.

We assume that the magnetic moment of one tube can be written down in the form

$$\mathbf{m}_i=m_0\mathbf{n}_i\sin\left(2\pi F_0/\mathbf{n}_i(\mathbf{B}+\mathbf{b})\right), \tag{5}$$

where $\mathbf{n}_i$ is a unit vector in the direction of the tube. The total magnetic moment is determined as the vector sum of terms of the type (5).

We now introduce a certain characteristic amplitude of the helicon b_0 such that $b_1=b_0f_1(x_3,t)$, $b_2=b_0f_2(x_3,t)$, where f_1 and f_2 are wave functions, with rms times or coordinates on the order of unity. Calculating the derivatives of the components of the total magnetic moment of all four tubes, expanding them in a series in $b_0/\Delta B$, where ΔB is the period of the oscillations in the direct field, and retaining only the square terms of the expansion, we obtain the following approximate wave equations:

$$\begin{aligned}&\frac{\partial^2 f_1}{\partial x_3^2}+\varkappa^2\frac{\partial f_2}{\partial t}=\varepsilon\left[2f_1\left(\frac{\partial f_1}{\partial x_3}\right)^2+f_1^2\frac{\partial^2 f_1}{\partial x_3^2}\right],\\&-\varkappa^2\frac{\partial f_1}{\partial t}+\frac{\partial^2 f_2}{\partial x_3^2}=\varepsilon\left[2f_2\left(\frac{\partial f_2}{\partial x_3}\right)^2+f_2^2\frac{\partial^2 f_2}{\partial x_3^2}\right],\end{aligned} \tag{6}$$

where

$$\varkappa^2=\frac{4\pi}{c^2\rho_{12}(1+4\pi\chi)},\qquad \varepsilon=\frac{4\pi\chi}{1+4\pi\chi}\left(\frac{2\pi b_0}{\Delta B}\right)^2,$$
$$\chi=\frac{4\pi m_0F_0}{B^2}\cos\frac{2\pi F_0\sqrt{2}}{B}. \tag{7}$$

The equations (6) describe only the initial state of the nonlinear behavior of the helicon, when its amplitude is significantly smaller than the de Haas–van Alphen period. If the nonlinear parameter here is $\varepsilon\ll 1$, then the approximate solution (6) can be obtained by the substitution

$$\begin{aligned}&f_1=\cos(\omega t-kx_3)+\varepsilon g_1(\omega t-kx_3),\\&f_2=\sin(\omega t-kx_3)+\varepsilon g_2(\omega t-kx_3),\end{aligned} \tag{8}$$

where $g_1(s+2\pi)=g_1(s)$, $g_2(s+2\pi)=g_2(s)$. In first-order approximation in ε we obtain the dispersion relation

$$k^2=\frac{\omega\varkappa^2}{1+\alpha\varepsilon},\qquad \alpha=-\frac{1}{4}, \tag{9}$$

and the solution of (6) in the form

$$\begin{aligned}&f_1=\cos(\omega t-kx_3)+\beta\varepsilon\cos 3(\omega t-kx_3),\\&f_2=\sin(\omega t-kx_3)-\beta\varepsilon\sin 3(\omega t-kx_3),\end{aligned} \tag{10}$$

where $\beta=1/24$. Thus, in the given approximation, the deviation of the wave from sinusoidal and of its polarization from circular are quite insignificant.

We write the condition of symmetric resonance of the standing waves in a plate of thickness $2d$ in the form

$$\int_{-d}^{d} k(x_3)\,dx_3=(2n-1)\pi, \quad n=1,2,3\ldots \tag{11}$$

Here we consider a steady-state standing wave in a plate, with a certain wave-amplitude distribution that is symmetric relative to the symmetry plane of the plate. The dependence of k on x_3 is determined by the relation (9), where $\varepsilon\equiv\varepsilon(x_3)$.

In the approximation considered, we can substitute in (9) and (11) $\varepsilon=\varepsilon_m\cos^2 k_1 x_3$, where k_1 is the value of k in the linear regime. As a result, we obtain the following relation for the frequency of the first resonance:

$$\omega_{1r}=\omega_{1rl}\left(1+\frac{\alpha\varepsilon}{2}\right), \quad \omega_{1rl}=\frac{\pi c^2}{16d^2}\rho_{12}(1+4\pi\chi), \tag{12}$$

where ω_{1rl} is the resonance frequency in the linear regime.

The value of the nonlinear shift in the resonance frequency is equal to

$$\Delta\omega_{rn}=\frac{\alpha}{2}\frac{\pi c^2\rho_{12}}{16d^2}4\pi\chi\left(\frac{2\pi b_m}{\Delta B}\right)^2, \tag{13}$$

where b_m is the maximum amplitude in the middle of the plate. In the transition of the magnetic field from minimum to maximum χ, the shift (13) changes sign, but retains the same value. A significant asymmetry is noted experimentally in the nonlinear shift of the resonance frequency (curves 1b and 2b in Fig. 1), which can be explained apparently by the nonsinusoidal dependence of the magnetic moment on the field. This asymmetry is quite marked on the left-hand $f(H)$ curve of Fig. 1 and is not taken into account in the calculation (5) above.

We now compare numerically the experimentally observed nonlinear shift in the resonance frequency with that computed from Eq. (13). For curves 1b and 2b in Fig. 1, the shift in the maximum relative to the curves 1a and 2a amounts to 5 and 27 Hz, respectively. Substituting in (13) the values obtained from the left-hand $f(H)$ curve of Fig. 1, as well as the value $b_m=7$ G, calculated for an excitation current of 3 mA, starting from the known parameters of the excitation coil and the Q of the resonance, we obtain a value of the nonlinear resonance shift equal to 4.5 Hz. All the calculations were carried out accurate to 10%. Taking into account the rather approximate character of the foregoing analysis, the agreement with experiment for curve 1b should be regarded as satisfactory. Curve 2b is severely distorted and the above approximation is scarcely adequate in this case.

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