sion effect. The fact that resonance photons can be transmitted by a thick crystal is due to the unique feature of the suppression effect, namely, the fact that the effect depends on the thermal vibrations of the nuclei. To see this in its proper perspective, we note that, in typical cases of the Borrmann effect, the residual absorption factor connected with the thermal vibrations of the atoms is about 5% of the normal. If an analogous situation were to be characteristic for the suppression effect, then the residual factor would be 3 in our case, which would result in a reduction of the recoil-free component of the beam transmitted by the crystal by a factor of roughly 6–7, as compared with the corresponding quantity in the incident beam.

The authors are greatly indebted to Yu. Kagan and A. M. Afanas'ev for useful discussions, to K. P. Aleshin, M. A. Volkov, and I. B. Filippov for extensive technical assistance, to A. M. Artem'ev and E. P. Stepanov for placing at our disposal the equipment for the Mössbauer analysis of the scattered beam, and to V. V. Mostov for help in this research.

1) A similar spectrum shape was observed by Mössbauer *et al.*[10]

2) Mössbauer analysis was first used in connection with the resonance scattering of γ rays in[11,12].

3) For this polarization, the magnetic field vector oscillates at right-angles to the plane of scattering.

---

[1] A. M. Afanas'ev and Yu. Kagan, Zh, Eksp. Teor. Fiz. **48**, 327 (1965) [Sov. Phys. JETP **21**, 215 (1965)] Yu. Kagan and A. M. Afanas'ev, Zh. Eksp. Teor. Fiz. **49**, 1504 (1965) [Sov. Phys. JETP **22**, 1032 (1966)].

[2] V. V. Sklyarevskiĭ, G. V. Smirnov, A. N. Artem'ev, B. Shestak, and S. Kadechkova, Pis'ma Zh. Eksp. Teor. Fiz. **8**, 295 (1968) [JETP Lett. **8**, 181 (1968)].

[3] V. K. Voĭtovetskiĭ, I. L. Korsunskiĭ, and Yu. F. Pazhin, Pis'ma Zh. Eksp. Teor. Fiz. **8**, 563 (1968) [JETP Lett. **8**, 343 (1968)].

[4] V. K. Voĭtovetskiĭ, I. L. Korsunskiĭ, and Yu. F. Pazhin, Pis'ma Zh. Eksp. Teor. Fiz. **8**, 611 (1968) [JETP Lett. **8**, 376 (1968)].

[5] V. V. Sklyarevskiĭ, G. V. Smirnov, A. N. Artem'ev, R. M. Mirzababaev, B. Shestak, and S. Kadechkova, Pis'ma Zh. Eksp. Teor. Fiz. **11**, 531 (1970) [JETP Lett. **11**, 363 (1970)].

[6] V. K. Voĭtovetskiĭ, I. L. Korsunskiĭ, A. I. Novikov, and Yu. F. Pazhin, Pis'ma Zh. Eksp. Teor. Fiz. **11**, 149 (1970) [JETP Lett. **11**, 91 (1970)].

[7] G. V. Smirnov, V. V. Sklyarevskiĭ, and A. N. Artem'ev, Pis'ma Zh. Eksp. Teor. Fiz. **11**, 579 (1970) [JETP Lett. **11**, 400 (1970)].

[8] V. V. Sklyarevskiĭ, G. V. Smirnov, A. N. Artem'ev, R. M. Mirzababaev, and E. P. Stepanov, Zh. Eksp. Teor. Fiz. **64**, 934 (1973) [Sov. Phys. JETP **37**, 474 (1973)].

[9] G. V. Smirnov, N. A. Semioshkina, V. V. Sklyarevskiĭ, S. Kadechkova, and B. Shestak, Zh. Eksp. Teor. Fiz. **71**, 2214 (1976) [Sov. Phys. JETP **44**, 1167 (1976)].

[10] R. L. Mössbauer, H. E. Seelbach, B. Persson, M. Bent, and G. Longworth, Phys. Lett. A **28**, 94 (1968).

[11] A. N. Artem'ev, G. V. Smirnov, and E. P. Stepanov, Zh. Eksp. Teor. Fiz. **54**, 1028 (1968) [Sov. Phys. JETP **27**, 547 (1968)].

[12] A. N. Artem'ev, G. V. Smirnov, V. V. Sklyarevskiĭ, and E. P. Stepanov, Zh. Eksp. Teor. Fiz. **63**, 1390 (1972) [Sov. Phys. JETP **36**, 736 (1973)]

Translated by S. Chomet

Melting curve of hydrogen up to 10 kbar

V. V. Kechin, A. I. Likhter, Yu. M. Pavlyuchenko, L. Z. Ponizovskiĭ, and A. N. Utyuzh

Institute of High Pressure Physics, USSR Academy of Sciences

(Submitted July 7, 1976)
Zh. Eksp. Teor. Fiz. 72, 345–347 (January 1977)

The melting point of molecular normal hydrogen was measured by the piston-displacement method. The range of the investigation was extended beyond those used by others, to $P_m = 10$ kbar and $T_m = 112$ K. The molar volumes of the solid phase on the melting curve were determined experimentally.

PACS numbers: 64.70.Dv

Experimental data on the melting of hydrogen at high pressures are very skimpy. There is only one 1930 study, in which the melting curve of hydrogen was measured up to 5 kbar.[1] In the remaining studies[2,3] the pressure did not exceed 3.6 kbar. This large gap in the experimental material, compared with other substances,[4,5] is attributed to experimental difficulties faced when the hydrogen density is increased at high pressures[6] and to the well-known hydrogen brittleness of steel at temperatures above 120 K.

We present here the results of measurements of the melting point of hydrogen of purity 99.8% up to 10 kbar. The initial hydrogen had an equilibrium ortho-para composition at room temperature.1) The measurements were performed with a cylinder + piston type of apparatus, in which hydrogen precompressed to 2 kbar was fed through a capillary with the aid of a thermocompressor and a gas booster. The high-pressure vessel was thermostatically controlled with an automatic temperature regulator accurate to ±1°. During the time of the experiment, we measured the displacement L of the piston as a function of the applied load F at a fixed temperature. The melting was revealed by the discontinuity of

TABLE I. Temperature T_m, pressure P_m, and molar volume of the solid phase V_{ms} of molecular hydrogen on the melting curve.

Experiment	T_m, K	P_m, kbar	P_m^{smooth}, kbar	V_{ms}, cm³/mole	V_{ms}^{smooth}, cm³/mole
	77.4	5.0	5.2	14.4	14.25
I	84.0	6.2	6.0	13.8	13.81
	90.6	6.9	6.8	13.4	13.42
	99.0	8.0	8.0	12.8	12.94
II	77.4	5.2	5.2	(14.25)	14.25
	112.0	9.8	9.9	12.4	12.29

the piston displacement on the $L(F)$ plot. The melting pressure P_m was defined as the average of the values obtained when the load was increased and decreased. The pressure was determined accurate to ± 0.3 kbar.

The results of two measurements in a high-pressure vessel of beryllium bronze (experiment I) and in a vessel of ÉP-637 steel (experiment II) are shown in the second and third column of Table I. The fourth column lists the smoothed values of P_m, calculated with Simon's equation by least squares from our data:

$$P_m = a[(T_m/13.96)^c - 1], \tag{1}$$

where $a = 0.2972$ kbar, $c = 1.6991$. These constants correspond to $P_m = 54.2$ kbar at $T_m = 300$ °K. For comparison, we present calculations in accordance with the data of[1,3,5]. At $T = 300$ K they yield melting pressures 68.5, 66.6 and 56.1 kbar, respectively. Figure 1 shows the melting curve of hydrogen plotted in accordance with (1). The points up to 5 kbar were taken from[1], and the higher points from our measurements.

To determine the molar volumes in the beryllium-bronze vessel (experiment I) we measured, besides the volumes on the melting curve, also the compressibility of hydrogen gas at room temperature at pressures up to 8 kbar. These measurements have enabled us to determine the amount of hydrogen contained in the high-pressure vessel in this experiment, since the pressure dependence of the molar volume of hydrogen at room temperature is known.[7] The obtained molar volume V_m of the solid phase on the melting curve is listed in the fifth column of the table (experiment I). For the steel-vessel measurements (experiment II), the molar volume at $T = 112$ K, given in the fifth column, was determined from the ratio of the volumes at 112 and 77.4 K, under the assumption that $V_{ms} = 14.25$ cm³/mole at $T_m = 77.4$ K (see below). The accuracy with which the volumes were determined was ± 0.2 cm³/mole.

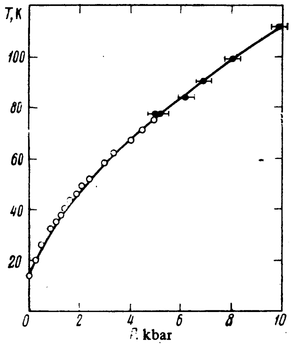

FIG. 1. Melting curve of molecular hydrogen: ● —our data, ○—data of[1].

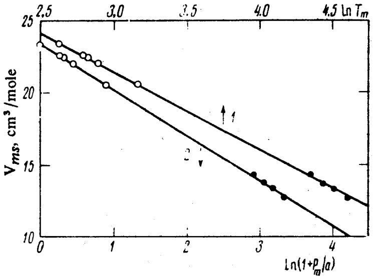

FIG. 2. Dependence of the molar volume of crystalline hydrogen on the temperature and pressure according to the melting curve. 1) Formula (2), 2) formula (3), ● —our data, ○ —data of[8–11].

Figure 2 (curve 1) shows our data (see the table, experiment I) on the temperature dependence of the molar volume of hydrogen in the form $V_{ms} = f(\ln T_m)$. The figure shows also the data of[8–11] on the functions $V_{ms}(T_m)$ of para-hydrogen and normal hydrogen, respectively, plotted in the same coordinates.

As seen from the figure, the plot is linear:

$$V_{ms} = A_1 - B_1 \ln T_m, \text{ cm}^3/\text{mole}, \tag{2}$$

where $A_1 = 37.28$ and $B_1 = 5.297$ cm³/mole. The dependence of the molar volume on the pressure can be used by using the Simon equation. Substituting T_m from (1) and (2), we get

$$V_{ms} = A_2 - B_2 \ln(1 + P_m/a), \text{ cm}^3/\text{mole}, \tag{3}$$

where $A_2 = A_1 - \ln 13.96 = 23.32$ and $B_2 = B_1/c = 3.117$ cm³/mole. This relation is also plotted in Fig. 2 (curve 2). The smoothed values of V_{ms} calculated from Eqs. (2) and (3) are given in the sixth column of the table.

In conclusion, the authors thank Academician L. F. Vershchagin for constant interest and support.

1) Each experiment took approximately 24 hours. Before and after the experiment, control measurements were made of the compressibility of the sample under pressure at liquid-nitrogen temperature. The values agreed within the limits of experimental accuracy. This can serve as an indication that, within the limits of experimental accuracy, the possible changes in the para-ortho composition are insignificant, and also that there was no leakage of the sample during the measurement cycle.

---

[1] F. Simon, M. Ruheman, and W. A. M. Edwards, Z. Phys. Chem. B6, 331 (1930).
[2] R. L. Mills and E. R. Grilly, Phys. Rev. 101, 1246 (1956).
[3] R. D. Goodwin, Cryogenics 2, 353 (1962).
[4] S. M. Stishov, Usp. Fiz. Nauk 96, 467 (1968) [Sov. Phys. Usp. 11, 816 (1969)].
[5] S. E. Babb, Rev. Mod. Phys. 35, 400 (1963).
[6] J. W. Stewart, Physics of High Pressures and the Condensed

Phase, ed. A. Van Itterbeek, North-Holland Publ. Co., Amsterdam, 1965.
[7] D. S. Tsiklis, V. Ya. Maslennikova, S. D. Gavrilov, A. N. Egorov, and G. V. Timofeeva, Dokl. Akad. Nauk SSSR 220, 1384 (1975).
[8] R. F. Dwyer, G. A. Cook, O. E. Berwaldt, and H. E. Nevis, J. Chem. Phys. 43, 801 (1965).
[9] E. Bartholomé, Z. Phys. Chem. B33, 387 (1936).
[10] R. D. Goodwin and H. M. Roder, Cryogenics 3, 12 (1963).
[11] K. Clusius and K. Weigand, Z. Phys. Chem. B46, 1 (1940).

Translated by J. G. Adashko

Equation of state in the critical region with inclusion of non-asymptotic terms

A. T. Berestov

All-Union Research Institute for Physicotechnical and Radio-Engineering Measurements

(Submitted July 9, 1976)
Zh. Eksp. Teor. Fiz. 72, 348–353 (January 1977)

The leading nonasymptotic terms in the Migdal equation of state are determined by the ϵ-expansion method to order ϵ^2. The contribution of the additional terms is related to the susceptibility of the system and is proportional to $\chi^{-\Delta/\gamma}$. The equation of state is determined by the expression $H\chi^{(\beta+\gamma)/\gamma} = \varphi_0(M\chi^{\beta/\gamma}) + c\varphi_1(M\chi^{\beta/\gamma})\chi^{-\Delta/\gamma}$. The functions $\varphi_0(m)$ and $\varphi_1(m)$ have a simple form and are determined by formulas (3.5) and (4.4).

PACS numbers: 64.10.+h

1. INTRODUCTION

Substantial progress in the description of critical phenomena has recently been achieved. Using a method developed by himself, Wilson[1] has found the first terms of the ε-expansion for the critical indices to order ε^2 ($\varepsilon = 4 - d$; d is the dimensionality of space). It was found that the coefficients of ε and ε^2 are small and for $\varepsilon = 1$ give values of the indices that are close to the experimental values. This circumstance served as a stimulus for the determination of other universal quantities too. The first three terms of the ε-expansion of the equation of state of the Ising model were determined by Avdeeva and Migdal, and also by Brézin, Wallace and Wilson.[2]

However, it is essential to note that the results of[2] are applicable only in the immediate vicinity of the critical point. As we move away from it there arise additional terms,[3] a knowledge of which is important for the following reasons. First, they give the possibility of determining whether the experimental investigations are being carried out sufficiently close to the critical point, and, secondly, they give the possibility of describing the substance in a wide region about the critical point. We shall discuss these questions in more detail.

The techniques of present-day experiments in the study of critical phenomena make it possible to obtain reliable results (with error ~ 0.1–1% for the specific heat, for example) near the transition point, down to values of $t = (T - T_c)/T_c$ of the order of 10^{-4}–10^{-5},[4] where T is the temperature. On the other hand, since it is known that the theory is asymptotic in character, the fitting of the experimental data by the corresponding power laws, which define the critical indices, is usually carried out for $|t| \lesssim 10^{-2}$–10^{-1}. The best critical-index values obtained in different experiments for different substances frequently do not coincide (cf.[4]), even though current theory requires them to be universal. The principal inaccuracy in their determination is evidently associated with systematic errors—in particular, with the use of the pure-power, asymptotic laws in the whole range of the fitting. But it is completely unclear beforehand whether any particular region of measurements is asymptotic. The natural criterion for applicability of the asymptotic laws in the analysis of the experimental data is that the nonasymptotic terms be small compared with the experimental error. Strictly speaking they should be, to use the terminology of mathematical statistics, insignificant, i.e., systematic (albeit, possibly, small) deviations of the experimental points from the asymptotic dependences should be absent. Thus, taking nonasymptotic terms into account should lead to a more rigorous appraisal of the degree of universality of the experimentally obtained quantities.

The other topic in which the determination of nonasymptotic terms can play an important role is the description of the behavior of matter, e.g., a liquid and a dense gas, in a wide interval of the parameters of state. In such an approach, as was noted in[5], the universal scaling equation of state in the critical region can be used as the "zeroth" approximation. It is found that in practically the whole region $t < 1$ and $|\Delta\rho| = |(\rho - \rho_c)/\rho_c| < 1$ the discrepancies between the asymptotic dependences and the experimental data are not large and can, apparently, be described by the introduction of non-asymptotic terms. The important point here is that these turn out to be universal to the same degree as the asymptotic equation of state.[3,6]

The first results in the determination of the nonas-