# Interaction between neutrons and matter in the field of a strong electro-magnetic wave

D. F. Zaretskiĭ and V. V. Lomonosov

*I. V. Kurchatov Institute of Atomic Energy*
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It is shown that scattering of neutrons by the nuclei of ions in the light field of a laser can be accompanied by the absorption or emission of field quanta. As a result, additional satellites appear in the spectrum of scattered neutrons and differ in energy by an amount equal to the field quantum. It is also shown that similar satellites appear in compound-nucleus levels. The field strengths required to observe these effects are estimated.

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## INTRODUCTION

The study of the interaction between low-energy neutrons and the plasma produced by laser-beam irradiatiation (laser plasma) is of interest for two reasons: first, new plasma-diagnostics methods become available, and second, neutron scattering by the ions in the plasma should lead to a modification of the neutron spectrum.

We consider here the scattering of thermal and resonant neutrons by a multicharged ion that executes forced oscillations in the field of a strong electromagnetic wave. Such ions are produced in a laser plasma both by multiphoton ionization and by electron collisions.

In the case when the external field changes little over distances on the order of the ion dimension $a$ ($ka \ll 1$, $k$ is the wave vector), the interaction potential of the ion with this field can be expressed in the form

$$U=-\mathbf{E}_0\cos\omega t\sum_{i=1}^{z^*+1}e_i\mathbf{r}_i, \tag{1}$$

where $\mathbf{E}_0$ is the amplitude of the field strength and $\mathbf{r}_i$ is the radius vector; the summation in (1) is over all the charges $e_i$. Since the potential energy of the electron interaction with the nucleus depends solely on the relative distance $r_i^e = r_i - r_{\text{nuc}}$ between the electron and the nucleus, it is useful to express Eq. (1) in the form

$$U=-\mathbf{E}_0\cos\omega t\left[(Z-Z^*)e\mathbf{r}_{\text{nuc}}-e\sum_{i=1}^{z^*}\mathbf{r}_i^e\right]. \tag{1'}$$

Thus, the Hamiltonian that describes the motion of the ion nucleus in the field of the electromagnetic wave is of the form

$$H_{\text{nuc}}=\mathbf{p}_{\text{nuc}}^2/2AM-(Z-Z^*)e\mathbf{E}_0\mathbf{r}_{\text{nuc}}\cos\omega t, \tag{2}$$

where $AM$ is the mass of the nucleus and $\mathbf{p}_{\text{nuc}}$ is the momentum of the nucleus.

Correspondingly, the Hamiltonian of the neutron-plus-nucleus system in an external electromagnetic field is equal to

$$H_{\text{nuc}}+H_n=\frac{\mathbf{p}_{\text{nuc}}^2}{2AM}+\frac{\mathbf{p}_n^2}{2M}-(Z-Z^*)e\mathbf{E}_0\mathbf{r}_{\text{nuc}}\cos\omega t+V(|\mathbf{r}_n-\mathbf{r}_{\text{nuc}}|), \tag{3}$$

where $\mathbf{r}_n$ is the coordinate, and $\mathbf{p}_n$ is the momentum of the neutron; $V(|\mathbf{r}_n-\mathbf{r}_{\text{nuc}}|)$ is the neutron–nucleus interaction potential.

In the center-of-mass system the Hamiltonian (3) takes the form

$$H_{\text{cms}}=\frac{\mathbf{p}_{\text{rel}}^2}{2MA/(A+1)}+\frac{Z-Z^*}{A+1}e\mathbf{E}_0\mathbf{r}_{\text{rel}}\cos\omega t+V(|\mathbf{r}_{\text{rel}}|), \tag{4}$$

where $\mathbf{r}_{\text{rel}}=\mathbf{r}_n-\mathbf{r}_{\text{nuc}}$ and $\mathbf{p}_{\text{rel}}=\mathbf{p}_n-\mathbf{p}_{\text{nuc}}$.

It follows from (7) that the neutron that interacts with the ion nucleus acquires in the external field an effective charge equal to

$$e_{eff}=\frac{Z-Z^*}{A+1}e. \tag{5}$$

We examine below certain processes connected with the appearance of an effective charge in the neutron.

## 1. NONRESONANT SCATTERING

By solving the nonstationary Schrödinger equation with the Hamiltonian (4) outside the effective region of the nuclear forces, we obtain an expression for the wave function of the relative motion of the nucleus and neutron ($\hbar=c=1$):

$$\varphi_{\mathbf{p}}(\mathbf{r},t)=\exp(-i\mathbf{p}_{\text{rel}}\mathbf{r}+i\varepsilon_{\mathbf{p}}t)F_{\mathbf{p}}(t), \tag{6}$$

where

$$F_{\mathbf{p}}(t)=\exp\left\{-i\int dt'\left(\frac{e_{eff}\mathbf{A}(t')\mathbf{p}}{M}+\frac{e_{eff}^2\mathbf{A}^2(t')}{2M}\right)\right\}$$

the vector potential $\mathbf{A}(t)$ is determined from the relation $\mathbf{E}(t)=-\partial\mathbf{A}/\partial t$, $\varepsilon_{\mathbf{p}}=\mathbf{p}_{\text{rel}}^2/2MA(A+1)^{-1}$ is the energy of relative motion, and $MA/(A+1)\equiv\mu$ is the nucleus–neutron reduced mass. To calculate the elastic scattering of a slow neutron by a nucleus it is convenient to use the pseudopotential method[2]:

$$V(\mathbf{r})=-\frac{2\pi}{\mu}B\delta(\mathbf{r}), \tag{7}$$

where $B$ is the scattering amplitude of the slow neutron by the nucleus. For simplicity, we examine the case


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when the external electromagnetic wave is linearly polarized and the vector potential is of the form

$$\mathbf{A}(t)=\mathbf{a}\cos\omega t. \tag{8}$$

It is then easy to obtain from (6), (7), and (8) the cross section for neutron–nucleus scattering in an electromagnetic field

$$\frac{d\sigma_{pp'}}{d\Omega_{p'}}=|B|^2\sum_{s=-\infty}^{\infty}J_s^2(\Delta)\frac{|\mathbf{p}_s'|}{|\mathbf{p}|}\delta(\varepsilon_p+s\omega-\varepsilon_{p'})\,d\varepsilon_{p'}, \tag{9}$$

where $J_s(\Delta)$ is a Bessel function of order $s$, $\mathbf{p}_s'$ is the momentum after scattering and $\Delta=e_{\text{eff}}\,\mathbf{a}(\mathbf{p}-\mathbf{p}')/M\omega$. Thus, the neutron scattering cross section in the field of an electromagnetic wave constitutes an infinite sum of terms, each one of which satisfies the energy conservation law:

$$\varepsilon_{p'}=\varepsilon_p+s\omega. \tag{10}$$

The $s$-th term of the sum describes either the emission of $s$ photons of the electromagnetic wave, if $s>0$, or the absorption of $s$ photons when $s<0$. Integrating (9) with respect to $d\varepsilon_{p'}$, we obtain

$$\frac{d\sigma_{pp'}}{d\Omega_{p'}}=|B|^2\sum_{s=-\infty}^{\infty}J_s^2(\Delta)\frac{|\mathbf{p}_s'|}{|\mathbf{p}|}, \tag{11}$$

where

$$|\mathbf{p}_s'|=[2\mu(s\omega+\varepsilon_p)]^{1/2}.$$

It follows from the foregoing analysis that neutron scattering by the nucleus can be followed by absorption or emission of field quanta. Accordingly, the spectrum of the neutrons scattered by the ion nucleus acquires additional components (satellites). The intensity of the satellites is determined by the value of the parameter $\Delta$. If $\Delta\lesssim 1$, then only the nearest satellites with an energy shift $\pm\omega$ will have a noticeable intensity. The scattering cross section for each satellite is then determined by the expression

$$d\sigma_{pp'}^{\pm}/d\Omega\approx|B|^2\Delta_\pm^2|\mathbf{p}_\pm'|/4|\mathbf{p}|, \tag{12}$$

where

$$|\mathbf{p}_\pm'|=[2\mu(\varepsilon_p\pm\omega)]^{1/2},$$

$$\Delta_\pm=\frac{e_{eff}|\mathbf{E}_0||\mathbf{p}|}{M\omega^2}\left[\cos\theta_0-\left(\frac{\varepsilon_p\pm\omega}{\varepsilon_p}\right)^{1/2}(\cos\theta_0\cos\theta+\sin\theta_0\sin\theta\cos\varphi)\right];$$

$\theta_0$ is the angle between $\mathbf{E}_0$ and $\mathbf{p}$, while $\theta$ is the angle between $\mathbf{p}'$ and $\mathbf{p}$. Integrating (12) over the scattering angle $\theta$, we obtain

$$\sigma_{pp'}^{(\pm)}=\frac{\pi}{4}|B|^2\left(\frac{\varepsilon_p\pm\omega}{\varepsilon_p}\right)^{1/2}\xi^2\left(\cos^2\theta_0+\frac{1}{3}\left(\frac{\varepsilon_p\pm\omega}{\varepsilon_p}\right)\right), \tag{13}$$

where

$$\xi^2=(e_{eff}|\mathbf{E}_0|(2M\varepsilon_p)^{1/2}/M\omega^2)^2.$$

The ratio of the cross section for scattering with photon absorption to the elastic-scattering cross section is $(\varepsilon_p\ll\omega)$

$$\sigma_{pp'}^{+}/\sigma_{pp'}^{0}\approx{}^1/_{12}\,\xi^2(\omega/\varepsilon_p)^{3/2}. \tag{14}$$

In the case of thermal neutrons $(\varepsilon_p\approx 0.025)$ the value of the ratio (14) becomes noticable ($\approx 10\%$) in the infrared region of the spectrum 1 eV$\gtrsim\hbar\omega\gtrsim$0.1 eV, if the field strength is respectively in the range $10^9$ V/cm $\gtrsim|\mathbf{E}_0|\gtrsim 10^7$ V/cm $(e_{\text{eff}}\approx\frac{1}{2}e)$. The obtained estimate of the field strengths corresponds to values that can be obtained, for example, with neodymium-glass solid-state lasers or $CO_2$ gas lasers.

## 2. RESONANT SCATTERING

We examine the resonant scattering of a neutron by a single isolated compound-nucleus resonance of energy $E_n$. We assume that this resonance arises in the nucleus of an ion with charge $e(Z-Z^*)$ situated in a strong electromagnetic field of strength $\mathbf{E}(t)=\mathbf{E}_0\cos\omega t$. The transition of the neutron from a continuous spectrum to a bound state with energy $E_n$ is the result of a nuclear four-fermion interaction described by an operator $\hat{H}'$. The wave function of the continuous spectrum of the neutron + ion system in the field of a strong electromagnetic wave is described (in the c. m. s.) by Eq. (6).

To calculate the amplitudes of the resonant scattering of the neutron we use the Heitler method.[2] Let $B_n(t)$ be the amplitude of the neutron transition from the continuous spectrum to the bound state $E_n$, and let $C_{\mathbf{p}'}(t)$ be the amplitude of the resonant scattering. According to[2], the system of equations for these amplitudes is $(\hbar=c=1)$

$$\begin{aligned}i\dot{B}_n(t)&=\sum_{\mathbf{p}'}H'_{n\mathbf{p}'}C_{\mathbf{p}'}(t)\exp[i(\varepsilon_{p'}-E_n)t]F_{\mathbf{p}'}(t),\\ i\dot{C}_{p'}(t)&=H'_{\mathbf{p}'n}B_n(t)\exp[-i(\varepsilon_{p'}-E_n)t]F_{\mathbf{p}'}^{*}(t)+\delta(t)\delta_{\mathbf{p}'\mathbf{p}_0},\end{aligned} \tag{15}$$

where $\mathbf{p}_0$ is the initial momentum and $\mathbf{p}'$ is the momentum after scattering.

The function $F_{\mathbf{p}}(t)$ can be expanded in a Fourier series

$$F_{\mathbf{p}}(t)=\sum_{s=-\infty}^{\infty}J_s(\beta)e^{-is\omega t}, \tag{16}$$

where $s$ is either integer or zero, and the argument of the Bessel function $J_s(\beta)$ is

$$\beta=e_{eff}\mathbf{E}_0\mathbf{p}_0/M\omega^2. \tag{17}$$

In the following we limit ourselves to terms with $s=0$ and $\pm1$. This restriction is valid if $\beta\lesssim 1$. Then the system (15) takes in the Fourier representation the form

$$\begin{aligned}(E-E_n)B_n(E)&=\sum_{\mathbf{p}'}H'_{n\mathbf{p}'}[J_0(\beta')C_{\mathbf{p}'}(E)+J_1(\beta')C_{\mathbf{p}'}(E+\omega)+J_{-1}(\beta')C_{\mathbf{p}'}(E-\omega),\\ (E-\varepsilon_{p'})C_{\mathbf{p}'}(E)&=H'_{\mathbf{p}'n}[J_0(\beta')B_n(E)+J_1(\beta')B_n(E-\omega)\\ &\quad+J_{-1}(\beta')B_n(E+\omega)]+\delta_{\mathbf{p}'\mathbf{p}_0}.\end{aligned} \tag{18}$$

In the course of solving the system (18), sums of the form

$$\sum_{\mathbf{p}'}|H_{n\mathbf{p}'}|^2\xi(E-\varepsilon_{p'})J_0(\beta')J_{\pm1}(\beta'),$$

$$\xi(E-\varepsilon_{p'})=(E-\varepsilon_{p'}+i\delta)^{-1}$$

arise. Integrating over the angles, we verify that, at least for the $S$-neutrons, all terms of this type vanish.

As a result we obtain

$$C_{p'}(E)=H_{p'n}H_{np'}\xi(E-\varepsilon_{p'})\left\{\sum_{s=0,\pm1}J_0(\beta')J_s(\beta)\frac{\xi(E+s\omega-\varepsilon_{p_0})}{E-\bar{E}_n+i\tilde{\gamma}_n(E)/2}\right.$$
$$+\sum_{s=0,1}J_1(\beta')J_s(\beta)\cdot\frac{\xi(E+\omega(s-1)-\varepsilon_{p_0})}{E-\bar{E}_n-\omega+i\tilde{\gamma}_n(E-\omega)/2}$$
$$\left.+\sum_{s=0,-1}J_{-1}(\beta')J_s(\beta)\cdot\frac{\xi(E+\omega(s+1)-\varepsilon_{p_0})}{E-\bar{E}_n+\omega+i\tilde{\gamma}_n(E+\omega)/2}\right\}, \qquad (19)$$

where $\bar{E}_n-E_n$ and $\tilde{\gamma}_n/2$ are respectively the real and imaginary parts of the quantity

$$\sum_{p'}\sum_{s=0,\pm1}J_s^2(\beta')|H_{np'}|^2\xi(E+s\omega-\varepsilon_{p'}).$$

In the following we shall disregard the shift of the compound-nucleus level.

Of greatest interest in the resonance amplitude (19) are those terms which correspond to scattering with emission or absorption of a single field quantum. After the singular multiplier $\xi(x)$ are separated, the corresponding amplitudes can be expressed in the form (20)

$$C_{p'}(E)=U_{p'}\xi(E-\varepsilon_{p'})\xi(E+s\omega-\varepsilon_{p_0});$$
$$U_{p'}(\varepsilon_{p'})=J_0(\beta')J_1(\beta)\frac{H_{p'n}H_{np_0}}{\varepsilon_{p'}-\bar{E}_n+i\tilde{\gamma}_n(\varepsilon_{p'})/2}$$
$$+J_0(\beta')J_{-1}(\beta)\frac{H_{p'n}H_{np_0}}{\varepsilon_{p'}-\bar{E}_n+i\tilde{\gamma}_n(\varepsilon_{p'})/2},\quad \varepsilon_{p'}=\varepsilon_{p_0}-\omega; \qquad (20)$$
$$U_{p'}(\varepsilon_{p'})=J_0(\beta')J_{-1}(\beta)\frac{H_{p'n}H_{np_0}}{\varepsilon_{p'}-\bar{E}_n+i\tilde{\gamma}_n(\varepsilon_{p'})/2}$$
$$+J_0(\beta')J_1(\beta)\frac{H_{p'n}H_{np_0}}{\varepsilon_{p'}-\bar{E}_n+i\tilde{\gamma}_n(\varepsilon_{p'})/2},\quad \varepsilon_{p'}=\varepsilon_{p_0}+\omega.$$

It follows from the obtained expressions that the amplitudes of neutron scattering with excitation of a compound-nucleus level acquire in strong electromagnetic fields additional poles (satellites) that are separated in energy from the principal pole by an amount equal to $\pm\omega$ (if only the nearest satellites are taken into account). The corresponding cross section is of the form ($\beta\ll1$, $\gamma_n\ll\omega$)

$$\sigma_\pm(\varepsilon_p)=g\pi\lambda^2\frac{\beta^2}{4}\frac{\gamma_n(\varepsilon_p)\gamma_n(\varepsilon_p\pm\omega)}{(\varepsilon_p\pm\omega-E_n)^2+\gamma_n^2(\varepsilon_p\pm\omega)/4}, \qquad (21)$$

where

$$\gamma_n(\varepsilon_p)=2\pi\sum_{p'}|H_{np'}|^2\delta(\varepsilon_p-\varepsilon_{p'}),$$

$g$ is a statistical factor.

We present an estimate of the electromagnetic field strengths required to make the cross sections $\sigma_\pm$ of the order of 10% of the elastic scattering cross section $\sigma_0$. If $\varepsilon_p\sim\omega\sim1$ eV, then the value of $\beta=e_{eff}\mathbf{E}_0 p/M\omega^2$ turns out to be of the order of unity when $e_{eff}|\mathbf{E}_0|\sim10^9$ V/cm. Accordingly if $\varepsilon_p\sim100$ eV, the indicated field strength may be lowered by an order of magnitude, i.e., we need $e_{eff}|\mathbf{E}_0|\approx10^8$ V/cm. As follows from the obtained equations, the required field strength decreases like $\omega^2$ with decreasing energy of the field quantum. We can therefore pick a resonance ($\gamma_n\sim0.01$ eV) such that satellites appear in the field of a $CO_2$ laser with a field strength on the order of $10^7$ V/cm.

Satellites with an energy $E_n+\omega$ are of particular interest. If the level $E_n$ lies below zero in energy, then the quantity $-|E_n|+\omega$ may turn out to be positive. A shift of a negative level to the region of positive energies can be detected by observing the sharp growth in the dependences of the radiative-capture of fission cross sections for low energy neutrons on the frequency and on the electromagnetic field strength.

## 3. CONCLUSION

The effects of the inelastic interaction of neutrons with a crystal lattice in the field of a laser light wave have been studied previously.[3] In the case of very strong field of intensity $\gtrsim10^6$ V/cm we must deal as a rule with a laser plasma. Hence the study of the interaction of neutrons with ions in the field of a strong electromagnetic wave is of independent interest. The estimates given above show that a noticeable effect of inelastic scattering of neutrons by ion nuclei arises in fields much weaker than the presently attained limits. In accordance with the estimates, the range of field strengths of interest in observing the effect is $10^9$ V/cm $\gtrsim|\mathbf{E}_0|\gtrsim10^7$ V/cm, and the range of the field-quantum energies is 1 eV $\gtrsim\hbar\omega\gtrsim0.1$ eV. The possibility of "extracting" a negative level of a compound nucleus into the region of positive energies is of particular interest.

The possibility of observing the effects discussed here depends on the following basic factors: the values of $\Delta$, the values of the Doppler broadening of the satellite, and the ratio of the durations of the neutron and laser pulses (duty factor). As shown above, in the laser fields attainable in practice the parameter $\Delta$ is of the order of unity even in the case of scattering of thermal neutrons. The electromagnetic field strength can be decreased by increasing the energy of the incident neutrons. The pulse durations of the thermal neutron sources are not less than $10^{-4}$ sec. The duration of the laser pulse with the required field strength can be of the order of $10^{-9}$ sec (neodymium glass) or $10^{-4}$ sec ($CO_2$). Thus, during the laser pulse, the number of effectively used neutrons incident on a unit surface of the target is $10^{-5}$ in the first case and of the order of unity in the second. Besides, the total number of neutrons inelastically scattered by the plasma ions depends also on the diameter of the laser-beam focus. These estimates show that the effects examined in this paper can be observed in principle with pulsed thermal neutron sources of the orders of $10^{12}$ neut/cm$^2$ per pulse. To observe these effects it is also necessary that the plasma ion temperature not be excessive. In fact, the satellites can be resolved if their Doppler broadening is smaller than $\hbar\omega$: $(kTE_n/A)^{1/2}<\hbar\omega$; if $A\sim100$ and $E_n\sim\hbar\omega$, then $kT<100$ eV for the neodymium laser and $kT<30$ eV for the $CO_2$ laser.

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