Quantum effects in the field of a nonmonochromatic electromagnetic wave

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(Submitted October 8, 1976)
Zh. Eksp. Teor. Fiz. 72, 865–874 (March 1977)

The effects of an external field consisting of two monochromatic electromagnetic waves of different frequencies propagating in the same direction on the quantum processes of photon emission by electrons, pair production, and elementary-particle decay are examined. Expressions for the probabilities of the processes are obtained, the results of numerical calculations of the probabilities for specific values of the invariant field-dependent parameters are presented, and the dependences of the probabilities on the characteristics of the external field are discussed.

PACS numbers: 12.20.Ds

1. INTRODUCTION

There has recently been considerable study of quantum processes in external electromagnetic fields in connection with the availability of lasers as sources of intense electromagnetic radiation. These effects are distinguished by their great variety. They include, for example, problems of atomic excitation and ionization, [1–6] photon emission by electrons and pair production effects, [7,8] elementary particle decays, [9] nuclear capture and breakup in the field of a laser beam, [10] and others. The effects of external fields on quantum phenomena have been most successfully and consistently traced for processes, such as photon emission by an electron, pair production, and elementary particle decays, that are not complicated by various side effects (e.g the effect of the Coulomb field of the nucleus in the case of the ionization of an atom). It is just these processes that we shall be concerned with here. Quantum effects of this type have been investigated in considerable detail for the case of a plane monochromatic wave and the case of a crossed field[7–9, 11–13]; for these fields, expressions have been obtained for the probabilities of the processes and various limiting cases have been investigated. Specific features of the dependence of the probabilities for the quantum processes on various characteristics of the field (e.g., on the wave frequency and polarization[8,11]) have already been noted in the cited papers and in a number of others.

Since interest in such studies will doubtless be even greater in the future, it seems desirable also to give a detailed treatment of the behavior of the quantum effects for other external-field configurations for which exact solutions of the Dirac and Klein-Gordon equations can be obtained. One of the external-field configurations of greatest practical interest is, for example, the field of two monochromatic waves of different frequencies propagating in the same direction. Explicit solutions of the Dirac and Klein-Gordon equations for such a model field can be obtained without difficulty by methods analogous to those given in[14]; we present such solutions below. The case of two waves propagating in the same direction provides the simplest model of a nonmonochromatic wave and can be used as an example for a detailed study of the general behavior of quantum effects in external fields of similar type.

We note that a light beam consisting of two coherent waves with different frequencies ω_1 and ω_2 has been produced experimentally (see, e.g., [15,16]). Multifrequency lasers have been described by a number of auththors.[17] A review of this subject will be found in Yariv's book.[18]

It should also be mentioned that some effects involving atoms in the field of a nonmonochromatic wave have already been treated in literature. An expression has been obtained for the probability for the ionization of an atom by the field of a nonmonochromatic wave.[3] Excitation effects and optical properties of atoms in the field of a beam consisting of two waves with different frequencies have also been treated, [4–6] and the passage of two-frequency laser light through a resonant medium has been investigated.[19] The resonance interaction of electrons and protons in the field of two waves was recently treated by Fedorov.[20]

A study of photon emission by an electron, pair production, and elementary-particle decays in the field of the waves of different frequencies propagating in the same direction accordingly seems timely. We have previously treated these effects for the simplest two-wave model, [11,12] in which it was assumed that both waves are linearly polarized with mutually perpendicular polarization vectors. Estimates of the probability for photon emission by electrons in the field of two waves have also been made by other investigators.[21,22] However, Lebedev[21] used a solution of the Klein-Gordon equation for the electron wave functions and thereby neglected important effects associated with the electron's spin, [23] and Zhukovskiĭ and Herrmann[22] considered only the case in which both waves are circularly polarized.

Here we shall give a detailed treatment of the effects mentioned above for a two-wave model under fairly general assumptions about the polarizations of the waves. We shall also develop a method of calculating the probabilities for quantum effects of this type, our method being a generalization to the two-wave model of

 0038-5646/77/4503-0452$02.40

the method proposed by Nikishov and Ritus[7] for calculating the same quantities for a monochromatic wave. The proposed method can be easily generalized to the case of three or more waves; this is of interest in connection with multifrequency lasers. In the calculations we use the same metric and representation of the γ matrices as in our previous papers.[11,12]

2. PHOTON EMISSION BY AN ELECTRON AND PAIR PRODUCTION

We choose the field potential in the form

$$A_\mu(\varphi_1\varphi_2)=A_{1,\mu}(\varphi_1)+A_{2,\mu}(\varphi_2), \tag{1}$$

where

$$A_{1,\mu}(\varphi_1)=a_{1,\mu}\cos\varphi_1+\delta_1 a_{2,\mu}\sin\varphi_1,$$
$$A_{2,\mu}(\varphi_2)=b_{1,\mu}\cos\varphi_2+\delta_2 b_{2,\mu}\sin\varphi_2, \tag{2}$$
$$\varphi_i=k_i x \quad (i=1,\ 2),$$

treating the superposition of coherent waves in accordance with Yariv.[18] This is evidently just the case of greatest interest, since it is the one for which the effects will be maximum. The values of the parameters δ_i corresponding to linear and right- and left-hand circular polarizations are $\delta_i=0$ and $\delta_i=\pm1$, respectively. Further, in the case of linear polarization we shall assume that the polarization vectors are parallel to the x axis. Thus, if $\delta_1=\delta_2=0$ we have two linearly polarized waves with parallel polarizations (we have previously treated the case of perpendicular polarizations[11,12]).

The values $|\delta_i|<1|$ for the polarization parameters correspond to elliptically polarized waves; in this case, however, we shall assume for simplicity that the axes of the polarization ellipses are parallel. We thus see that our field model (1) is fairly general and encompasses a wide range of wave polarizations. This enables us to make a detailed study of the behavior of the quantum effects as functions of the polarization states of the external field.

With our choice of metric we have

$$a_1a_1=a_2a_2=-a^2, \qquad b_1b_1=b_2b_2=-b^2.$$

An exact solution of Dirac's equation for potential (1) is given by the formula

$$\psi_p(x)=e^{-ipx}F_1(\varphi_1)F_2(\varphi_2)F(\varphi_1\varphi_2)u(p)/(2p_0)^{1/2}. \tag{3}$$

Here

$$F_i(\varphi_i)=\left[1+\frac{e}{2(pk_i)}\hat{k}_i\hat{A}_i\right]\exp\left\{-i\left[\frac{e}{(k_ip)}(p\tau_{1i})\sin\varphi_i - e\delta_i\frac{(p\tau_{2i})}{(pk_i)}\cos\varphi_i-\frac{e^2\tau_{ii}^2}{8(k_ip)}(1-\delta_i^2)\sin 2\varphi_i\right]\right\}, \tag{4a}$$

$$F(\varphi_1\varphi_2)=\exp\left\{-i\left[\frac{e^2ab}{2(k_1p-k_2p)}\sin(k_1x-k_2x)(1+\delta_1\delta_2) +\frac{e^2ab}{2(k_1p+k_2p)}\sin(k_1x+k_2x)(1-\delta_1\delta_2)\right]\right\}. \tag{4b}$$

We have used the following notation: $\tau_{i1}=a_i$, $\tau_{i2}=b_i$ $(i=1,2)$.

For the simplest two-wave model[11,12] the solution of Dirac's equation has the form (3) with $F(\varphi_1\varphi_2)\equiv1$ and $\delta_i=0$. In the case of the more general model (1), the function $F(\varphi_1\varphi_2)$ describes a peculiar frequency correlation: only one sine term appears in the argument of the exponential for the case of circularly polarized waves, but both sine terms appear if the polarization of even one of the waves deviates from circularity.

The method for calculating the matrix elements for the quantum processes using the wave function (3) must be discussed in somewhat more detail. Let us first consider the case of photon emission by an electron. The matrix element for this process is given, for example, by formulas (5) of[11] with expression (3) substituted for the electron wave functions $\psi_{p'}$ and ψ_p. On making this substitution we find that the matrix element M_{if} for the process is expressed as an integral involving several functions of the variables φ_1 and φ_2, and these functions can be treated in two ways; first, by introducing the frequency ratio $\varkappa=k_2/k_1$ and noting that $\varphi_2=\varkappa\varphi_1$, we can treat the integrand in formula (5) of[11] as a function of the single variable φ_1. We shall assume that the wave frequencies are incommensurable; then the integrand will not be periodic and can be expanded in a Fourier integral. Second, the same functions occurring in the integrand can be regarded as functions of the two independent variables φ_1 and φ_2; in this case the functions will be separately periodic in each of the variables φ_1 and φ_2, and can therefore be expanded in a double Fourier series. As will be evident from what follows, both these representations of the functions turn out to be very useful in calculating the matrix elements for the corresponding processes.[1)]

Taking these considerations into account, employing expression (3) for the electron wave function, and making use of the conservation laws, we can express the matrix element for photon emission by an electron in the form

$$M_{if}=-ie\,(2p_0'\cdot2p_0\cdot2k_0')^{-1/2}\int\bar{u}(p')\left\{\hat{e}''C_{00}+e\left[\frac{\hat{a}_1\hat{k}_1\hat{e}''}{2(p'k_1)}+\frac{\hat{e}''\hat{k}_1\hat{a}_1}{2(pk_1)}\right]C_{10} + e\delta_1\left[\frac{\hat{a}_2\hat{k}_1\hat{e}''}{2(p'k_1)}+\frac{\hat{e}''\hat{k}_1\hat{a}_2}{2(pk_1)}\right]C_{1'0}+e\left[\frac{\hat{b}_1\hat{k}_2\hat{e}''}{2(p'k_2)}+\frac{\hat{e}''\hat{k}_2\hat{b}_1}{2(pk_2)}\right]C_{01} + e\delta_2\left[\frac{\hat{b}_2\hat{k}_2\hat{e}''}{2(p'k_2)}+\frac{\hat{e}''\hat{k}_2\hat{b}_2}{2(pk_2)}\right]C_{01'}\right\}\cdot u(p)(2\pi)^4\delta(sk_1+q-q'-k')\,ds, \tag{5}$$

where

$$e''=e'-k'(e'k_i)/(k_ik'),$$

p and p' (q and q') are the electron momenta (quasimomenta[7]) in the initial and final states, and k' and e' are the momentum and polarization of the emitted photon. The quasimomentum

$$q_\mu=p_\mu+\frac{e^2a^2}{4(pk_1)}(1+\delta_1^2)k_{1,\mu}+\frac{e^2b^2}{4(pk_2)}(1+\delta_2^2)k_{2,\mu}$$

satisfies the condition $q^2=m_*^2$ with

$$m_*=m\left[1+\frac{1}{2}\left(\frac{ea}{m}\right)^2(1+\delta_1^2)+\frac{1}{2}\left(\frac{eb}{m}\right)^2(1+\delta_2^2)\right]^{1/2}.$$

We have also used the following notation in (5):

$$C_{nk}(s)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\cos^n\varphi_1\cos^k\varkappa\varphi_1\exp[iB(\varphi_1,\varkappa\varphi_1)+is\varphi_1]d\varphi_1. \quad (6)$$

To obtain the functions $C_{nk}(s)$ with primed subscripts one must replace $\cos\varphi_i$ by $\sin\varphi_i$ on the right in (6). The function $B(\varphi_1\varphi_2)$ has the form

$$B(\varphi_1\varphi_2)={}^1/_2\gamma_-\sin(\varphi_1-\varphi_2)(1+\delta_1\delta_2)+{}^1/_2\gamma_+\sin(\varphi_1+\varphi_2)(1-\delta_1\delta_2)$$
$$+\sum_{i=1,2}\{|\tau_{ii}|[\alpha_i^1\sin\varphi_i-\alpha_2^1\delta_i\cos\varphi_i]+\beta_i(1-\delta_i^2)\sin 2\varphi_i\}, \quad (7)$$

where

$$\alpha_i^1=e\left[\frac{(n_ip')}{(p'k_1)}-\frac{(n_ip)}{(pk_1)}\right],\quad \beta_1=\frac{e^2a^2}{8}\left[\frac{1}{(p'k_1)}-\frac{1}{(pk_1)}\right]$$
$$\gamma_\pm=e^2ab\left[\frac{1}{p'k_1\pm p'k_2}-\frac{1}{pk_1\pm pk_2}\right]. \quad (8)$$

Here the n_i are unit vectors parallel to the vectors a_i. To obtain α_i^2 and β_2 one must make the substitutions $k_1\to k_2$ and $a\to b$ in (8).

Now we make use of the adiabatic hypothesis, i.e., we assume, as is usually done, that $A(\varphi_1\varphi_2)\to 0$ as $|\varphi_i|\to\infty$.[14,25] This can be assured by assuming that the fields $A_i(\varphi_i)$ in (1) contain factors of the type $\exp(-\alpha|\varphi_i|)$, where α is an indefinitely small quantity.[14] Taking this into account,[2)] we can establish the following important relation among the functions $C_{ij}(s)$ (see[7,11]):

$$[s-2\beta_1(1-\delta_1^2)-2\beta_2(1-\delta_2^2)]C_{00}+\alpha_1^1C_{10}+\delta_1\alpha_2^1C_{1'0}$$
$$+\alpha_1^2C_{01}+\delta_2\alpha_2^2C_{01'}+4\beta_1(1-\delta_1^2)C_{20}$$
$$+4\beta_2(1-\delta_2^2)C_{02}+\gamma C_{11}+\delta_1\delta_2\gamma C_{1'1'}=0, \quad (9)$$

where

$$\gamma=e^2ab\left[\frac{1}{(p'k_1)}-\frac{1}{(pk_1)}\right]. \quad (9')$$

Relation (9) turns out to be very useful in calculating specific expressions for the probabilities for the quantum effects, and we shall make use of it in what follows. We also note that it generalizes the corresponding expression for the case of a single monochromatic wave.[7,11]

We emphasize once more that up to now our treatment has been based on the integral expression (5) for the matrix element, so that k_2 is a function of k_1, i.e., $k_2=\varkappa k_1$, the formulas (6)—(9). As we noted above, however, there is another possible way to represent M_{if}. Thus, if we regard the functions in expression (3) as depending on the two independent variables φ_1 and φ_2, then, since the functions are separately periodic in each of the variables φ_i, we can introduce the quantities $C_{nk}(s_1s_2)$, which depend on two integers s_1 and s_2, in place of the quantities (6):

$$C_{nk}(s_1s_2)=\frac{1}{(2\pi)^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\cos^n\varphi_1\cos^k\varphi_2e^{iB(\varphi_1\varphi_2)}\exp(is_1\varphi_1+is_2\varphi_2)d\varphi_1 d\varphi_2. \quad (10)$$

In this case we obtain an expansion of the matrix element M_{if} in a double Fourier series in s_1 and s_2 in place of the Fourier integral expansion (5). Since the two expansions are equivalent, we can obtain the following relation between the coefficient by comparing (6) and (10):

$$C_{nk}(s)=\sum_{s_1,s_2}C_{nk}(s_1s_2)\delta(s_1+s_2\varkappa-s). \quad (11a)$$

As a consequence of this, the integration in (5) is replaced by a summation in accordance with the scheme

$$\int ds\,\delta(sk_1+\ldots)\to\sum_{s_1,s_2}\delta(s_1k_1+s_2k_2+\ldots). \quad (11b)$$

Of course we must also make the substitution $C_{nk}(s)\to C_{nk}(s_1s_2)$.

The advantage of using the integral representation (5) of the matrix element is associated with the fact that in calculating $|M_{if}|^2$ it is useful to employ relation (9), which is also obtained on the basis of such a representation. After performing the necessary mathematical operations and obtaining the required values of $|M_{if}|^2$ we must use (11a) and (11b) to pass from an integration over ds to a summation over s_1 and s_2 in the expressions for the probabilities of the processes. We note that on squaring the matrix element we obtain a summation over new variables s_1' and s_2' in addition to the summation over s_1 and s_2. However, the same arguments as were used previously[11] easily show that $s_1=s_1'$ and $s_2=s_2'$, so that we actually still have only a double summation over s_1 and s_2. This summation, unlike the one in the case of a monochromatic wave,[7] taken over both positive and negative values of the s_i. This is due to the fact that in the case of interaction with the field of a nonmonochromatic wave the trend of the quantum effects may be associated, in particular, with both the absorption from one of the waves and the emission into the other wave of a definite number of quanta.

The probability for photon emission by an electron is determined (after summing over the electron spin states) by formula (28) of[11], in which[3)]

$$N(s_1s_2)=|C_{00}|^2[2(p'e'')(pe'')+(p'p)]$$
$$+2\lambda_1(a\,\mathrm{Re}\,C_{00}C_{10}^*+b\,\mathrm{Re}\,C_{00}C_{01}^*)$$
$$+2\lambda_2(a\delta_1\,\mathrm{Re}\,C_{00}C_{1'0}^*+b\delta_2\,\mathrm{Re}\,C_{00}C_{01'}^*)$$
$$+e^2r_1(a^2|C_{10}|^2+2ab\,\mathrm{Re}\,C_{10}C_{01}^*+b^2|C_{01}|^2)$$
$$+e^2r_2(a^2\delta_1^2|C_{1'0}|^2+2ab\delta_1\delta_2\,\mathrm{Re}\,C_{1'0}C_{01'}^*$$
$$+b^2\delta_2^2|C_{01'}|^2)+4e^2(n_1e'')(n_2e'')$$
$$\times[a^2\delta_1\,\mathrm{Re}\,C_{10}C_{1'0}^*+ab(\delta_1\,\mathrm{Re}\,C_{1'0}C_{01}^*$$
$$+\delta_2\,\mathrm{Re}\,C_{10}C_{01'}^*)+b^2\delta_2\,\mathrm{Re}\,C_{01}C_{01'}^*]. \quad (12)$$

Here we have used the notation

$$\lambda_i=-2e(pe'')(n_ie'')+{}^1/_2(k_1k')\alpha_i^1,$$
$$r_i=2(n_ie'')^2+\frac{1}{2}\frac{(k_ik')^2}{(p'k_i)(pk_i)}. \quad (13)$$

In what follows we shall drop the arguments of the functions C_{nk} for brevity in calculating the probabilities of the various processes.

Formula (12) determines the probability for photon emission with allowance for the polarization states of the photon. If we are not interested in the photon polarization, we must sum over e' in (12). This gives

$$N(s_1s_2)=-2m^2|C_{00}|^2+e^2\left[2+\frac{(k_1k')^2}{(p'k_1)(pk_1)}\right]R. \quad (14)$$

Here

$$R=-(a^2\delta_1{}^2+b^2\delta_2{}^2)\,|C_{00}|^2+a^2|C_{10}|^2+a^2\delta_1{}^2|C_{1'0}|^2 +b^2|C_{01}|^2+b^2\delta_2{}^2|C_{01'}|^2-\mathrm{Re}\,C_{00}C_{20}^*a^2(1-\delta_1{}^2) -\mathrm{Re}\,C_{00}C_{02}^*b^2(1-\delta_2{}^2)+2\mathrm{Re}\,(C_{10}C_{01}^*-C_{00}C_{11}^*)\,ab +2\mathrm{Re}\,(C_{1'0}C_{01'}^*-C_{00}C_{1'1'}^*)\,ab\delta_1\delta_2. \tag{15}$$

As is evident from (14), we have succeeded in obtaining an expression for the square of the matrix element for the two-wave model whose form is analogous to that of the expression obtained for the case of a single wave.[7] There appears a new quantity R, which depends on the variables describing the external field and generalizes the corresponding quantities derived earlier[7, 11] for the case of a single monochromatic wave. For example, R reduces at once for the case of $b=0$ to known expressions.[7, 11]

The probabilities for processes in the field (1) depend on invariants of the type[7]

$$x_1=ea/m,\quad x_2=eb/m,\quad \chi_i=(k_ip)\,x_i/m^2\quad (i=1,\ 2).$$

As in earlier papers,[8, 11–13] we shall be interested here in values of these parameters of the order of unity. This region of parameter values corresponds to a nonlinear dependence of the probabilities for quantum processes on the characteristics of the external field, and that, in particular, is why it is of interest to examine this parameter region for the two-wave model (1). The method of calculating the probabilities is the same as for the model treated earlier.[11] It is not difficult to obtain expressions for the invariant quantities (8), on which the photon emission probability depends, in terms of the integration variables by analogy with[11], and we shall not give them here. The same values of the parameters x_i and χ_i were used in the present calculations as in the earlier ones.[11]

The results of the calculations are presented in Fig. 1. As before,[8, 11] the probability $\overline{W}_\gamma$ for emission of a photon by an electron was calculated for various polarizations of the waves and, as is evident from the results, $\overline{W}_\gamma$ depends significantly on the wave polarizations, this dependence becoming stronger with increasing values of the x_i.

The probability $\overline{W}_p$ for pair production by a photon of momentum l is given by formula (32) of[11]. We obtain the value of $\overline{N}(s_1s_2)$ for this case from formulas (12)—(15) after making the substitutions $k'\to -l$ and $p\to -p$ and changing the sign of $N(s_1s_2)$.[7] Figure 2 shows the calculated values of W_p for circularly polarized waves. As in the case of a single monochromatic wave,[8, 11] $\overline{W}_p$ has a nonmonotonic frequency dependence, which is due to the presence of a threshold for the pair production process.[8] The nonmonotonic character of the x dependence of $\overline{W}_p$ is more strongly expressed for the two-wave model (1) considered here than for the case of a single monochromatic wave of the same polarization.[11] It should also be noted that the x dependence of $\overline{W}_p$ is stronger in the region $x\lesssim 0.5$ for model (1) than for the model treated earlier.[11]

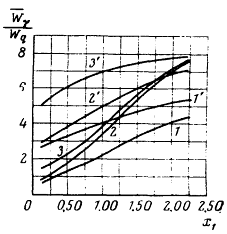

FIG. 1. Photon emission probability $\overline{W}_\gamma$ calculated for $\chi_1=1$ and $\chi_2=1.2$; for $\delta_1=\delta_2=0$ (curves 1 and 1′), $\delta_1=1$ and $\delta_2=0$ (2 and 2′), and $\delta_1=\delta_2=1$ (3 and 3′); and for $x_2=0.5$ (1, 2, and 3) and $x_2=1.5$ (1′, 2′, and 3′).

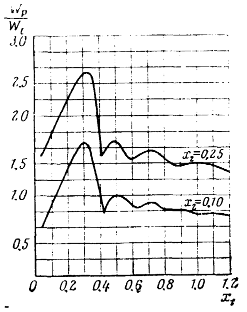

FIG. 2. Pair production probability $\overline{W}_p$ calculated for $\chi_1=1$, $\chi_2=1.2$, and $\delta_1=\delta_2=1$.

3. ELEMENTARY PARTICLE DECAYS

It is of interest not only to investigate the effect of an external field on the photon emission and pair production processes, but also to investigate the effect of such a field on elementary particle decays. Problems of this type have been repeatedly examined, both for various constant external fields, and for the field of a laser wave.[8, 9, 12, 24] In particular, it is of interest to investigate particle decay processes in external fields in order to explore the effects of various characteristics of such fields (frequency, polarization, etc.) on parity violating processes. We have previously used the simplified two-wave model[11] to investigate elementary particle decays.[12] Here we investigate pion and muon decay processes, using the more general two-wave model (1).

The simplest process of this type is the $\pi\to\mu(e)+\nu$ decay. Using this decay as an example, it has been shown[8] that the external field of a monochromatic electromagnetic wave may either accelerate or retard particle decays, depending on the specific mass relationships. We have confirmed this effect for a particular two-wave model.[12] It is therefore of interest to examine this effect for the field (1), too. After performing calculations similar to those presented in[12], we obtain the following expression, in place of expression (18) of of[12], for the square of the matrix element for the $\pi\to\mu(e)+\nu$ decays;

$$|M_{if}|^2=|C_{00}|^2(m^2-m'^2)+e^2\frac{(k_1l)}{(q'k_1)}R -e\frac{1}{(q'k_1)}I_1(k_1)-e^2\frac{(k_1l)}{(q'k_1)}j_{12}'\,I_2. \tag{16}$$

Here

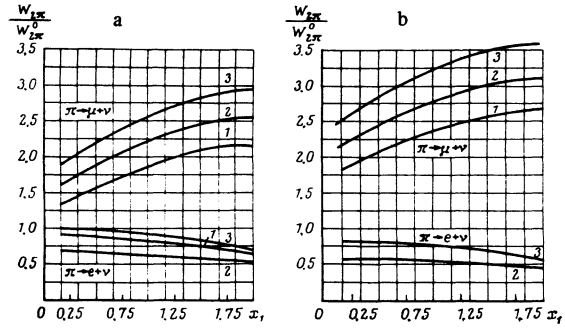

FIG. 3. Probabilities for the decays $\pi \to \mu + \nu$ and $\pi \to e + \nu$ calculated for $\chi_1 = \chi_2 = 1$; for $\delta_1 = \delta_2 = 0$ (curves 1), $\delta_1 = \delta_2 = 1$ (curves 2), and $\delta_1 = \delta_2 = -1$ (curves 3); and for $x_2 = 0.5$ (plot a) and $x_2 = 1.5$ (plot b).

$$I_1(k_i) = (2\,\mathrm{Im}\, C_{00}C_{10}^{*}a + 2\,\mathrm{Im}\, C_{00}C_{01}^{*}b)\, j_1(k_i) + (2\,\mathrm{Im}\, C_{00}C_{1'0}^{*}a\delta_1 + 2\,\mathrm{Im}\, C_{00}C_{01'}^{*}b\delta_2)\, j_2(k_i),$$
$$I_2 = 2\,\mathrm{Im}\, C_{10}C_{1'0}^{*}a^2\delta_1 + (2\,\mathrm{Im}\, C_{10}C_{01'}^{*}\delta_2 + 2\,\mathrm{Im}\, C_{01}C_{1'0}^{*}\delta_1)\, ab + 2\,\mathrm{Im}\, C_{01}C_{01'}^{*}b^2\delta_2. \tag{17}$$

The rest of the notation in (16) and (17) is the same as in the previous paper.[12]

The probability for $\pi \to \mu(e) + \nu$ decays is given by formula (20) of[12], in which, however, $N_{2\pi}(s_1 s_2)$ now has the form

$$N_{2\pi}(s_1s_2) = (1-\mu)\,|C_{00}|^2 + uR + 2(u+1)P[x_1\,\mathrm{Im}\, C_{10}C_{00}^{*}\sin\varphi_0 - x_1\delta_1\,\mathrm{Im}\, C_{1'0}C_{00}^{*}\cos\varphi_0 + x_2\,\mathrm{Im}\, C_{01}C_{00}^{*}\sin\varphi_0 - x_2\delta_2\,\mathrm{Im}\, C_{01'}C_{00}^{*}\cos\varphi_0]$$
$$-2u[x_1^2\delta_1\,\mathrm{Im}\, C_{10}C_{1'0}^{*} + x_1x_2\delta_2\,\mathrm{Im}\, C_{10}C_{01'}^{*} + x_1x_2\delta_1\,\mathrm{Im}\, C_{01}C_{1'0}^{*} + x_2^2\delta_2\,\mathrm{Im}\, C_{01}C_{01'}^{*}]. \tag{18}$$

It is not difficult to obtain expressions for the invariants (8) in terms of the parameters of the problem and the integration variables, much as analogous expressions were obtained in[12]. We give only the expression for E_s[12]: in place of Eq. (15) of[12], we have

$$\frac{E_s^2}{m^2} = 1 + \frac{1}{2}x_1^2(1+\delta_1^2) + \frac{1}{2}x_2^2(1+\delta_2^2) + \frac{2s_1\chi_1}{x_1} + \frac{2s_2\chi_2}{x_2}. \tag{19}$$

The results of numerical calculations of the decay probabilities are presented, together with the values of x_i and χ_i used in the calculations, in Fig. 3, a and b. As in the case of a single monochromatic wave,[8,12] so in the case of model (1) the probabilities for $\pi \to \mu(e) + \nu$ decays depend substantially on the polarization characteristics of the external field. The difference in the dependences of the total probabilities for the $\pi \to \mu + \nu$ and $\pi \to e + \nu$ decays on the δ_i as the latter vary within the limits $-1 \leq \delta_i \leq 1$ should also be noted. And finally we note that, as is evident from the results of the calculations, turning on the external field also increases the probability for $\pi \to \mu + \nu$ decay and reduces that for $\pi \to e + \nu$ decay when the field is described by model (1). This confirms the corresponding effect established for the model fields investigated earlier[8,12] and shows that this is a fairly general effect.

In a similar manner one can also obtain expressions for $|M_{if}|^2$ for more complicated decays such as $\mu^{\pm} \to e^{\pm} + \nu + \tilde{\nu}$ and $\pi^{\pm} \to \pi^0 + e^{\pm} + \nu$. Thus, for the $\mu^{\pm} \to e^{\pm} + \nu + \tilde{\nu}$ decay we obtain the following expression for $N(s_1 s_2)$ in place of expression (6) of[12]:

$$N(s_1s_2) = |C_{00}|^2\delta(l) + e^2Rh_0(k_i) + eh_1(k_i)I_1(k_i) + e^2h_2(k_i)\,j_{12}'(k_i)\,I_2 + 2e^2h_3(k_i)\,j_{12}(k_i)\,I_2. \tag{20}$$

Here, as above, we retain the notation of our previous paper.[12]

One can treat the $\pi^{\pm} \to \pi^0 + e^{\pm} + \nu$ decay in a similar manner. The expression for $|M_{if}|^2$ has the form of (23) of[12], but with R replaced by $|C_{00}|^2$ and the L_i, by the following expressions:

$$L_1 = 2\,\mathrm{Re}\, C_{00}(C_{10}a + C_{01}b)^{*}t_1 + 2\,\mathrm{Re}\, C_{00}(C_{1'0}a\delta_1 + C_{01'}b\delta_2)^{*}t_2,$$
$$L_2 = a^2(|C_{10}|^2 + |C_{1'0}|^2\delta_1^2) + b^2(|C_{01}|^2 + |C_{01'}|^2\delta_2^2) + 2ab(\mathrm{Re}\, C_{10}C_{01}^{*} + \mathrm{Re}\, C_{1'0}C_{01'}^{*}\delta_1\delta_2),$$
$$L_3 = 2\,\mathrm{Im}\, C_{00}(C_{10}af_1 + C_{1'0}af_2\delta_1 + C_{01}bf_1 + C_{01'}bf_2\delta_2)^{*},$$
$$L_4 = 2\,\mathrm{Im}\, C_{10}(C_{1'0}a\delta_1 + C_{01'}b\delta_2)^{*}a + 2\,\mathrm{Im}\, C_{01}(C_{1'0}a\delta_1 + C_{01'}b\delta_2)^{*}b. \tag{21}$$

Here, too, we have retained the notation of[12].

The probabilities for the $\mu^{\pm} \to e^{\pm} + \nu + \tilde{\nu}$ and $\pi^{\pm} \to \pi^0 + e^{\pm} + \nu$ decays are given by formulas similar to those presented in[12]. In this case, however, these probabilities involve a fivefold integration, and this makes it substantially more difficult to calculate them numerically. More powerful computing facilities will therefore be required for calculating the probabilities for these decays, although one may expect such calculations to become quite feasible in the future. In concluding, the author thanks V. V. Lomonosov for discussing a number of problems treated here.

[1)] We note that an integral representation of the matrix element M_{if} has also been used for the case of a constant crossed field.[24]

[2)] We also note that this assumption assures that sufficient conditions for the expansion of the matrix element M_{if} in the Fourier integral (5) will be satisfied.

[3)] In what follows we shall retain the notation of our previous papers[11,12] wherever possible.

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Translated by E. Brunner

Multiple scattering of light in an inhomogeneous medium near the critical point. II. Spectral composition of scattered radiation

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(Submitted June 18, 1976)
Zh. Eksp. Teor. Fiz. **72**, 875–887 (March 1977)

The spectral composition of light after multiple scattering in an inhomogeneous medium is examined. The number of components, the positions of their centering points, the half widths, the relationships between the components, and the characteristics of the components in the spectra are determined. Conditions for mode conservation and decay during multiple scattering are formulated. The "field" and temprature dependence of the properties of the single- and double-scattering spectra near the critical point are described. The dependence of these spectra on the transferred wave vector is also discussed. An experiment designed to investigate multiparticle correlations in a condensed medium is outlined.

PACS numbers: 42.20.–y

Multiple scattering of light near the critical point has been attracting increasing attention among researchers. The theory of integrated multiple scattering of light in the critical region has now been developed in considerable detail.[1–3] Thus, among the new and interesting results of these studies is the prediction[1,2] and discovery[4,5] of a nontrivial dependence of the doubly-scattered intensity on the fourth power of the linear size of the scattering volume; the gradual reduction of the exponent in the temperature dependence of doubly scattered radiation[1,6] as the critical point is approached is in agreement with theoretical predictions based on the theory of scaling transformations[7]; the role of external factors in the description of critical opalescence has been elucidated[8–11]; the properties of the scattering indicatrix in the critical region has been investigated[1,2,9,11]; and so on. Considerable success has also been achieved in the study of the depolarization of critical opalescence.[12,13,4–6]

Higher-order scattering effects are just as important for the spectra of critical opalescence. Unfortunately, a theory of multiply scattered spectra, capable of predicting and explaining their experimentally accessible properties, is not as yet available. In this paper, we solve this problem for the liquid-vapor critical point. In addition to the study of temperature dependence, which is now traditional in the investigation of critical opalescence, we have here the further possibility of examining the "field" properties connected with departures from the homogeneity of the medium under the influence of external factors (for example, the gravitational field). This was used in our earlier work on the physical properties of such inhomogeneous media near the critical point in the single-scattering approximation.[8,9,14]

We shall devote particular attention to the number of components in spectra of different order, the position of the centering points, the half-widths, and the relationships between the components in the spectra, as well as the temperature, "field," and angular dependence of the multiply-scattered spectra. The general

 0038-5646/77/4503-0457$02.40