Effect of trapped electrons on odd "drift" modes in a tokamak

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We investigated the odd "drift" modes in a tokamak, which were discussed earlier in a number of Coppi's papers. We show that Coppi's premises, that the odd and even modes are independent of one another, do not hold in general. For this reason the odd modes, just as the even modes, are sensitive to dissipation on trapped electrons, albeit to a lesser degree than the even ones. It is shown that one of the physical factors leading to the coupling of the odd and even modes is the compressibility of the plasma. It is also shown that odd modes are coupled with the even ones also if compressibility is neglected, providing that the number of wavelengths spanned by the torus is not an integer. The growth rates (or decrements) of the odd modes are small in comparison with those of the even modes; they can be substantial, however, in problems dealing with the buildup of odd modes by a group of fast ions in a two-component tokamak, since the growth rates of this buildup are also small. This is precisely the situation in the case of buildup of odd modes by fast ions, a buildup considered by Coppi and Bhadra. It is concluded that the theory of collective processes in a two-component tokamak, developed by Coppi and Bhadra, calls in general for a revision.

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1. INTRODUCTION

Coppi and Bhadra[1] have considered the excitation of odd "drift" modes by fast ions in a tokamak. The influence of the trapped electrons on these modes was neglected by them. This neglect was justified by qualitative arguments, advanced in Coppi's review, [2] that there is no relation between the even and odd modes in symmetrical magnetic traps.

The purpose of the present paper is to analyze the relation between the even and odd modes in a more rigorous and consistent matter than in Coppi's paper. [2] With an axially symmetric tokamak of round cross section as an example, we show that Coppi's premise[2] that the even and odd modes are independent do not hold in general. For this reason, the odd modes, just as the even ones, are sensitive to dissipation on trapped electrons, albeit to a lesser degree than the even ones. We shall show that one of the physical factors that lead to a coupling of the odd modes with the even ones and not accounted for by Coppi and Bharda[1,2] is the compressibility of the plasma. The odd modes are then coupled with the even ones even if an integer number of wavelengths is spanned by the length of the torus. We show in addition that the odd modes are coupled with the even ones also when compressibility is neglected, if an non-integer number of wavelengths is spanned by the torus.

The initial equations describing the coupling of the even and odd drifts modes in a tokamak are shown in Sec. 2. The calculation of the growth rates (or of the damping decrements) of the odd modes is carried out in Sec. 3. The role of mode coupling and the associated dissipation on the trapped electrons, in the problem of excitation of odd modes by a beam of fast ions is discussed in Sec. 4. The results are summarized in Sec. 5.

2. INITIAL EQUATIONS

1. *General relations that describe the perturbations.* The perturbations are assumed potential, with an electric field

$$\mathbf{E}=-\nabla\Psi. \tag{2.1}$$

The perturbation potential is represented in the form

$$\Psi=\exp[-i\omega t+ik_a a+i(m\theta-n\varphi)]\psi(\theta). \tag{2.2}$$

Here ω is the oscillation frequency, a is the "radial" coordinate that characterizes the distance between the running magnetic surface and the magnetic axis, θ and φ are cyclic coordinates in the minor and major azimuths of the torus. For details on these coordinates see, e.g., [3].

The quantity k_a denotes the radial wave number. We note that by representing the radial dependence of the perturbation in the form $\exp(ik_a a)$ we neglect by the same token the effect of the shear. This neglect is introduced to simplify the effect of interest to us, that of the coupling of modes of opposite parity. It will be made clear that when shear is taken into account, the coupling between modes of opposite parity should be even stronger than when shear is neglected.

The integer n is the wave number of the perturbation along the major azimuth of the torus. The integer m at constant $\psi(\theta)$ characterizes the poloidal dependence of the perturbation and in this case constitutes a wave number along the minor azimuth. We, however, will consider perturbations with $\psi(\theta)\neq$ const. Then the number m can generally speaking be set equal to any integer. We assume m to be such that the difference

$$s=m-nq \tag{2.3}$$

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is smaller in absolute magnitude than one-half,

$$|s|\leqslant 1/2. \tag{2.4}$$

In (2.3) we have designated by $q\equiv aB_s/RB_\theta$ the tokamak margin coefficient, B_s and B_θ are the toroidal and poloidal magnetic field, and R is the radius of the magnetic axis.

We represent the function $\psi(\theta)$ in the form of a Fourier series (cf. [4]):

$$\psi(\theta)=\sum_{l=0}^{\infty}\frac{1}{\sigma_l}(\psi_l^+\cos l\theta+\psi_l^-\sin l\theta). \tag{2.5}$$

Here $\sigma_l=1$ for $l\geqslant 1$ and $\sigma_0=2$. It is assumed that only the first few terms are significant in this series, so that actually the summation extends only to a certain $l=l_{\max}$. It is assumed also that $m\gg l_{\max}$.

The condition that must be satisfied by the perturbation potential Ψ is obtained by using the quasineutrality equation

$$n_i=n_e, \tag{2.6}$$

where n_e and n_i are the perturbations of the electron and ion densities, which are connected with the corresponding perturbations of the distribution functions by the known relations

$$n_e=\int f_e\,d\mathbf{v},\quad n_i=\int f_i\,d\mathbf{v}. \tag{2.7}$$

We proceed to find f_e, f_i, and correspondingly n_e and n_i.

2. *Perturbation of the electron density.* We start with the drift kinetic equation for the electrons

$$\frac{df_e}{dt}=-\frac{e_eF_e}{T_e}\frac{d\Psi}{dt}-i(\omega-\hat{\omega}_{*e})\frac{e_eF_e}{T_e}\Psi+C(f_e). \tag{2.8}$$

Here F_e is the electron equilibrium distribution function, assumed to be Maxwellian with density $n_0(a)$ and temperature $T_e(a)$; e_e is the electron charge; C is the collision integral. The operator d/dt stands for

$$d/dt=-i\omega+\mathbf{v}_\parallel\nabla, \tag{2.9}$$

$\mathbf{v}_\parallel=v_\parallel\mathbf{e}_0$, where $v_\parallel$ is the particle velocity along the tokamak magnetic field $\mathbf{B}$; $\mathbf{e}_0=\mathbf{B}/B$ is a unit vector along $\mathbf{B}$. The symbol $\hat{\omega}_{*e}$ stands for the operator of the electron drift frequency

$$\omega_{*e}=\frac{k_bcT_e}{e_eB_s}\frac{\partial\ln n_0}{\partial a}\left[1+\eta_e\left(z_e-\frac{3}{2}\right)\right], \tag{2.10}$$

where $k_b\equiv m/a$ is the poloidal component of the wave vector, $\eta_e=d\ln T_e/d\ln n_0$, $z_e=M_ev^2/2T_e$, M_e is the electron mass, and v is the modulus of their total velocity.

We represent the solution of (2.8) in the form

$$f_e=f_e^{(1)}+f_e^{(2)}, \tag{2.11}$$

$$f_e^{(1)}=-e_eF_e\Psi/T_e, \tag{2.12}$$

where $f^{(2)}$ satisfies the equation

$$\left(\frac{\partial}{\partial\theta}-inq\right)f_e^{(2)}=\frac{qR}{v_\parallel}\left[i\omega f_e^{(2)}-i(\omega-\hat{\omega}_{*e})\frac{e_eF_e}{T_e}\Psi+C_e(f_e^{(2)})\right]. \tag{2.13}$$

Solving this equation by expansion in powers of $1/v_\parallel$, we obtain in the approximation required by us (cf. [5-7])

$$f_e^{(2)}=\begin{cases}He^{i(nq\theta-n\varphi)} & \text{for the trapped electrons}\\ 0 & \text{for the untrapped electrons}.\end{cases} \tag{2.14}$$

The equation for H takes here the form

$$\omega H-i\bar{C}_e(H)=(\omega-\hat{\omega}_{*e})\frac{e_eF_e}{T_e}\overline{\Psi e^{-inq\theta}}, \tag{2.15}$$

where the superior bar denotes averaging over the closed trajectory of the trapped particle, and

$$\bar{X}=\oint X\frac{d\theta}{v_\parallel}\Big/\oint\frac{d\theta}{v_\parallel}. \tag{2.16}$$

We introduce the functions

$$M_l^\pm(\lambda,s)=\oint\left\{\begin{matrix}\cos l\theta\\ \sin l\theta\end{matrix}\right\}\frac{e^{is\theta}}{(1-\lambda B)^{1/2}}d\theta, \tag{2.17}$$

$$L(\lambda)=\oint\frac{d\theta}{(1-\lambda B)^{1/2}}, \tag{2.18}$$

where $\lambda=\mu/E$, $E=v^2/2$ is the particle energy per unit mass, $\mu=v_\perp^2/2B$ is the magnetic moment, and $v_\perp$ is the modulus of the particle transverse velocity. Taking (2.17), (2.18), and (2.15) into account, we rewrite (2.15) in the form

$$\omega H-i\bar{C}_e(H)=(\omega-\hat{\omega}_{*e})\frac{e_eF_e}{T_eL}\sum_l\frac{1}{\sigma_l}(M_l^+\psi_l^++M_l^-\psi_l^-). \tag{2.19}$$

It will be useful to represent H in the form

$$H=\frac{e_eF_e}{T_e}\sum_l\frac{1}{\sigma_l}(h_l^+\psi_l^++h_l^-\psi_l^-), \tag{2.20}$$

where the functions $h_l^\pm(\lambda,E)$ satisfy the equations

$$\omega h_l^\pm-i\bar{C}_e(h_l^\pm)=(\omega-\hat{\omega}_{*e})M_l^\pm/L. \tag{2.21}$$

Thus, the total perturbed distribution function of the trapped electron is represented in the form

$$f_e=-\frac{e_eF_e}{T_e}\left[\Psi-e^{in(q\theta-\varphi)}\sum_l\frac{1}{\sigma_l}(h_l^+\psi_l^++h_l^-\psi_l^-)\right], \tag{2.22}$$

whereas for the untrapped electrons we have

$$f_e=-e_eF_e\Psi/T_e. \tag{2.23}$$

Representing the perturbed electron density in the form

$$n_e=\exp(-i\omega t+ik_aa+im\theta-in\varphi)\sum_l\frac{1}{\sigma_l}(n_{el}^+\cos l\theta+n_{el}^-\sin l\theta) \tag{2.24}$$

and taking into account (2.22), (2.23), and (2.7), we obtain

$$n_{ei}{}^{\pm}=-\frac{e_e n_0}{T_e}\left[\psi_l{}^{\pm}-\frac{1}{4\pi}\left\langle B_s\int d\lambda M_l{}^{\pm *}\sum_{l'}\frac{1}{\sigma_{l'}}(h_{l'}{}^{+}\psi_{l'}{}^{+}+h_{l'}{}^{-}\psi_{l'}{}^{-})\right\rangle\right] \tag{2.25}$$

where $M_l^{\pm *}$ is the complex conjugate of $M_l^{\pm}$, and the symbol $\langle\ldots\rangle$ stands for

$$\langle\ldots\rangle=\frac{2}{\sqrt{\pi}}\int_0^\infty\frac{dE}{T_e}\exp\left(-\frac{E}{T_e}\right)\left(\frac{E}{T_e}\right)^{1/2}(\ldots). \tag{2.26}$$

The values of $h_l^{\pm}$ were calculated by us previously.[6,7] The explicit form of these quantities will be used later on.

3. *Perturbation of ion density.* To determine the perturbed ion density we assume

$$f_i=f_i^{(1)}+f_i^{(2)}. \tag{2.27}$$

It is assumed that the function $f_i^{(1)}$ satisfies a drift-kinetic equation of the form

$$\frac{d_if_i^{(1)}}{dt}=-\frac{e_iF_i}{T_i}\frac{d_i\Psi}{dt}-i(\omega-\hat{\omega}_{*i})\frac{e_iF_i}{T_i}\Psi, \tag{2.28}$$

$$d_i/dt=-i\omega+v_\parallel\nabla+v_{Di}\nabla, \tag{2.29}$$

where $\mathbf{v}_{Di}$ is the magnetic drift velocity of the ions and is determined by the relation

$$\mathbf{v}_{Di}=-\frac{(v_\perp^2/2+v_\parallel^2)}{\omega_{Bi}}[\nabla\ln B\times\mathbf{e}_0], \tag{2.30}$$

$\omega_{Bi}=e_iB/M_ic$ is the cyclotron frequency of the ions, $e_i=-e_e$ and M_i are the charge and mass of the ions, T_i and F_i are their temperature and the equilibrium distribution functions, and $\hat{\omega}_{*i}$ is the ion drift-frequency operator, defined by a relation analogous to (2.10).

The function $f_i^{(2)}$ takes into account the transverse inertion of the ions and their magnetic viscosity (effect of finite Larmor radius of the ions), and in accordance with[8], is assumed equal to

$$f_i^{(2)}=-\frac{e_i}{M_i}\frac{k_\perp^2\Psi}{\omega_{Bi}^2}(\omega-\omega_{pi}^*)F_i, \tag{2.31}$$

where $k_\perp^2=k_a^2+k_b^2$ is the square of the transverse wave vector, $\omega_{pi}^*=(k_bcT_i/e_iB_s)(d\ln p_{0i}/da)$ and $p_{0i}=n_0T_i$ is the drift frequency of the ion along the pressure gradient.

We assume that

$$\omega\gg v_\parallel\nabla\gg v_{Di}\nabla.$$

The solution of (2.28) can then be represented in the form (cf.[7]):

$$f_i^{(1)}=-\frac{e_iF_i}{T_i}\Psi+\frac{e_iF_i}{T_i}\frac{\omega-\hat{\omega}_{*i}}{\omega+iv_\parallel\nabla}\Psi+\frac{\omega-\hat{\omega}_{*i}}{\omega}\frac{M_i(v_\perp^2/2+v_\parallel^2)}{RT_i}F_i\frac{mc}{\omega aB_s}\left(\Psi\cos\theta-i\frac{a}{m}\frac{\partial\Psi}{\partial a}\sin\theta\right). \tag{2.32}$$

Here $(\omega+i\mathbf{v}_\parallel\nabla)^{-1}$ is an operator inverse to the operator $\omega+i\mathbf{v}_\parallel\nabla$; it takes into account the effects of the finite $\mathbf{v}_\parallel\nabla/\omega$. In the derivation of (2.32) we used the explicit expression for $\mathbf{v}_{Di}$ from[3].

The perturbed ion density, in analogy with (2.24), is represented in the form

$$n_i=\exp[-i\omega t+ik_aa+i(m\theta-n\varphi)]\sum_l\frac{1}{\sigma_l}(n_{il}{}^{+}\cos l\theta+n_{il}{}^{-}\sin l\theta). \tag{2.33}$$

Then, taking into account (2.7), (2.27), (2.31), and (2.32) we obtain

$$n_{il}{}^{\pm}=-\frac{e_in_0}{T_i}\psi_l{}^{\pm}+\frac{e_in_0}{T_i}\left(1-\frac{\omega_{ni}}{\omega}\right)\left[\frac{Z_l+Z_{-l}}{2}\psi_l{}^{\pm}\mp i\frac{Z_l-Z_{-l}}{2}\psi_l{}^{\mp}\right]+\left(1-\frac{\omega_{ni}}{\omega}\right)\frac{n_0}{R}\frac{mc\sigma_l}{\omega aB_s}\left[\psi_{l+1}^{\pm}+\psi_{l-1}^{\pm}\pm\frac{k_a}{k_b}(\psi_{l+1}^{\mp}-\psi_{l-1}^{\mp})\right]-\frac{e_in_0}{M_i}\frac{k_\perp^2}{\omega_{Bi}^2}(\omega-\omega_{ni})\psi_l{}^{\pm}. \tag{2.34}$$

Here and below we assume for simplicity that $\eta_i\equiv d\ln T_i/\partial\ln n_0=0$;

$$\omega_{ni}=(k_bcT_i/e_iB_s)\partial\ln n_0/\partial a,\quad Z_l=Z(\omega/k_lv_{Ti}),$$

where $Z(x)\equiv-i\sqrt{\pi}xW(x)$, $W(x)$ is the Kramp function (the probability integral of complex argument)[8], $v_{Ti}=(2T_i/M_i)^{1/2}$ is the thermal velocity of the ions, and the k_l stand for

$$k_l=(l+s)/qR,\quad l=0,1,2,\ldots \tag{2.35}$$

Since we plan to consider henceforth perturbations with $s\ll1$ and $(k_lv_{Ti}/\omega)^2\ll1$, we simplify expression (2.34). We recognize that under the assumptions made above we have

$$\tfrac{1}{2}(Z_l+Z_{-l})\approx1+l^2v_{Ti}^2/2q^2R^2\omega^2, \tag{2.36}$$

$$\tfrac{1}{2}(Z_l-Z_{-l})\approx2ilsv_{Ti}^2/q^2R^2\omega^2. \tag{2.37}$$

In this case (2.34) reduces to

$$n_{il}{}^{\pm}=-\frac{e_in_0}{T_i}\psi_l{}^{\pm}\left[\frac{\omega_{ni}}{\omega}+\frac{T_i}{T_e}\left(1-\frac{\omega_{ni}}{\omega}\right)\left(b_i-\frac{k_l^{02}c_s^2}{\omega^2}\right)\right]+\frac{n_0}{R}\frac{mc\sigma_l}{\omega aB_s}\left(1-\frac{\omega_{ni}}{\omega}\right)\left[\psi_{l+1}^{\pm}+\psi_{l-1}^{\pm}\pm\frac{k_a}{k_b}(\psi_{l+1}^{\mp}-\psi_{l-1}^{\mp})\right]\mp i\frac{4e_in_0}{T_e}\left(1-\frac{\omega_{ni}}{\omega}\right)\psi_l{}^{\mp}\frac{k_l^0k_sc_s^2}{\omega^2}. \tag{2.38}$$

Here $b_i=k_\perp^2T_e/M_i\omega_{Bi}^2$; $c_s^2=T_e/M_i$; $k_l^0=l/qR$; $k_s=s/qR$.

4. *Quasineutrality equation.* Taking into account Eq. (26) and the ensuing relation

$$n_{el}{}^{\pm}=n_{il}{}^{\pm} \tag{2.39}$$

and using Eqs. (2.25) and (2.38), we obtain for $\psi_l^{\pm}$ the equation

$$\hat{R}_l\psi_l{}^{\pm}-\frac{1}{4\pi}\left\langle B_s\int d\lambda M_l{}^{\pm *}\sum_{l'=0}^{\infty}\frac{1}{\sigma_{l'}}(h_{l'}{}^{+}\psi_{l'}{}^{+}+h_{l'}{}^{-}\psi_{l'}{}^{-})\right\rangle-\sigma_l\left(1-\frac{\omega_{ni}}{\omega}\right)\frac{\varepsilon\Omega}{\omega}\left[\psi_{l+1}^{\pm}+\psi_{l-1}^{\pm}\pm\frac{k_a}{k_b}(\psi_{l+1}^{\mp}-\psi_{l-1}^{\mp})\right]\pm i\frac{4k_l^0k_sc_s^2}{\omega^2}\left(1-\frac{\omega_{ni}}{\omega}\right)\psi_l{}^{\mp}=0. \tag{2.40}$$

Here

$$\hat{R}_l=1-\frac{\omega_{ne}}{\omega}+\left(1-\frac{\omega_{ni}}{\omega}\right)\left(b_i-\frac{k_l^{02}c_s^2}{\omega^2}\right), \tag{2.41}$$

$\omega_{ne}=-T_e\omega_{ni}/T_i$ is the drift frequency of the electron along the density gradient, $\Omega = k_b c T_e / e_i a B_s$ is a quantity on the order of ω_{ni} or ω_{ne}, and $\varepsilon = a/R$ is the ratio of the minor running radius of the tokamak to the curvature radius of the magnetic axis. For k_l in (2.41) we can put approximately $k_l = l/qR$ (since we assume that $|s| \ll 1$).

We introduce the notation

$$A_l = \psi_l^+, \quad B_l = \psi_l^-, \tag{2.42}$$

so that the quantities A_l characterize the even part of the potential $\psi(\theta)$, and B_l the odd part. According to (2.40), these quantities are connected by the relations

$$\hat{R}_l A_l - \frac{1}{4\pi}\left\langle B_* \int d\lambda\, M_l^{+*} \sum_{l'} \frac{1}{\sigma_{l'}} (h_{l'}^+ A_{l'} + h_{l'}^- B_{l'}) \right\rangle$$
$$-\sigma_l \left(1 - \frac{\omega_{nl}}{\omega}\right) \frac{\varepsilon\Omega}{\omega} \left[A_{l+1} + A_{l-1} + \frac{k_a}{k_b}(B_{l+1} - B_{l-1}) \right]$$
$$+4i \frac{k_l^0 k_s}{\omega^2} c_s^2 \left(1 - \frac{\omega_{ni}}{\omega}\right) B_l = 0, \tag{2.43}$$

$$\hat{R}_l B_l - \frac{1}{4\pi}\left\langle B_* \int d\lambda\, M_l^{-*} \sum_{l'} \frac{1}{\sigma_{l'}} (h_{l'}^+ A_{l'} + h_{l'}^- B_{l'}) \right\rangle$$
$$-\sigma_l \left(1 - \frac{\omega_{ni}}{\omega}\right) \frac{\varepsilon\Omega}{\omega} \left[B_{l+1} + B_{l-1} - \frac{k_a}{k_b}(A_{l+1} - A_{l-1}) \right]$$
$$-4i \frac{k_l^0 k_s}{\omega^2} c_s^2 \left(1 - \frac{\omega_{ni}}{\omega}\right) A_l = 0. \tag{2.44}$$

Equations (2.43) and (2.44) are in fact the sought equations with the aid of which we shall investigate the coupling of modes of different parity.

5. *Limiting case* $\varepsilon \to 0$ *and* $s \to 0$. From (2.43) and (2.44) it follows that Coppi's premises,[1,2] that there is no relation between the even and odd modes, are valid if $\varepsilon \to 0$ and $s \to 0$. Actually, putting $\varepsilon = 0$ in (2.43) and (2.44) and recognizing that

$$\lim_{s\to 0} M_l^- = 0, \quad \lim_{s\to 0} h_l^- = 0, \tag{2.45}$$

we reduce (2.43) and (2.44) to the form

$$\hat{R}_l A_l - \frac{1}{4\pi}\left\langle B_* \int d\lambda\, M_l^{+*} \sum_{l'} \frac{1}{\sigma_{l'}} h_{l'}^+ A_{l'} \right\rangle = 0, \tag{2.46}$$

$$\hat{R}_l B_l = 0. \tag{2.47}$$

It is seen that Eq. (2.46) contains only the A_l, whereas (2.47) only B_l, thus indicating that there is no coupling between the even and odd modes.

It is also seen that in accordance with the premises of[1,2] the odd modes, in contrast to the even ones, are not sensitive to effects due to trapped electrons.

Apart from a constant, Eq. (2.47) has a solution

$$B_l = \delta_{lN}, \quad N = 1, 2, 3, \ldots, \tag{2.48}$$

corresponding to eigenfriequencies ω_n that satisfy the equation

$$\hat{R}_N(\omega_N) = 0, \tag{2.49}$$

i.e., according to (2.41),

$$1 - \frac{\omega_{ne}}{\omega_N} + \left(1 - \frac{\omega_{ni}}{\omega_N}\right)\left(b_i - \frac{k_N^2 c_s^2}{\omega_N^2}\right) = 0. \tag{2.50}$$

In the approximations $b_i \ll 1$ and $(k_N c_s/\omega)^2 \ll 1$ assumed above, this means that

$$\omega_N = \omega_{ne}[1 + (1 + T_i/T_e)(k_N^2 c_s^2/\omega_{ne}^2 - b_i)]. \tag{2.51}$$

These are the "drift" waves, first considered by Radakov and Sagdeev as $b_i \to 0$, and then by others[10-12] for finite b_i. In the cited studies[9-12] they investigated also "drift" ("universal") instabilities due to the interaction of the resonance electrons with these waves. The magnetic field is assumed in this case homogeneous, so that the effect of the trapping of the resonance electrons, an effect typical of the tokamak geometry, was neglected. The growth rate of the perturbations, $\gamma = \mathrm{Im}\,\omega$, turned out to be of the order of

$$\gamma \approx \left[\left(1 + \frac{T_i}{T_e}\right)\left(b_i - \frac{k_N^2 c_s^2}{\omega_{ne}^2}\right) - \frac{\eta_e}{2}\right] \frac{\omega_{ne}^2}{k_N v_{Te}}, \tag{2.52}$$

where $v_{Te} = (2T_e/M_e)^{1/2}$ is the thermal velocity of the electrons.

The influence exerted on the drift instability by the trapping of the resonant electrons in toroidal traps was first considered by Coppi *et al.*,[13,14] who have shown that this effect leads to a decrease of the growth rate (2.52) by an amount on the order of $\varepsilon^{-1}(\omega/k_N v_{Te})^2$, so that when this effect is taken into account we have

$$\gamma \approx \left[\left(1 + \frac{T_i}{T_e}\right)\left(b_i - \frac{k_N^2 c_s^2}{\omega_{ne}^2}\right)\right] \frac{\omega_{ne}^4}{\varepsilon (k_N v_{Te})^3}. \tag{2.53}$$

(We have put here for simplicity $\eta_e = 0$.)

We shall show that the coupling of the odd drift modes with the even ones and the associated interaction of the odd modes with the trapped electrons leads to a growth (or damping) of the odd modes with a growth rate (decrement) much larger than (2.53).

3. CALCULATION OF THE GROWTH RATE (DAMPING DECREMENT) OF THE ODD MODES

We put $s \to 0$ in (2.43) and (2.44). We then obtain

$$[\hat{Q}_{ll'}(\omega) + \hat{q}_{ll'}(\omega)]A_{l'} + \hat{P}_{ll'}(\omega) B_{l'} = 0, \tag{3.1}$$

$$[\hat{R}_{ll'}(\omega) + \hat{r}_{ll'}(\omega)]B_{l'} + \hat{t}_{ll'}(\omega) A_{l'} = 0. \tag{3.2}$$

Summation over repeated indices is implied, and the newly introduced symbols denote

$$\hat{Q}_{ll'}(\omega) = \hat{R}_l \delta_{ll'} - \frac{1}{4\pi}\left\langle B_* \int d\lambda\, M_l^{+(0)*} \hbar_{l'}^+ \right\rangle, \tag{3.3}$$

$$\hat{q}_{ll'}(\omega) = -\left(1 - \frac{\omega_{ni}}{\omega}\right)\frac{\varepsilon\Omega}{\omega}(\delta_{l+1,l'} + \delta_{l-1,l'})\sigma_l, \tag{3.4}$$

$$\hat{p}_{ll'}(\omega) = -\hat{t}_{ll'} = -\sigma_l \left(1 - \frac{\omega_{ni}}{\omega}\right)\frac{\varepsilon\Omega}{\omega}\frac{k_a}{k_b}(\delta_{l+1,l'} - \delta_{l-1,l'}), \tag{3.5}$$

$$\hat{R}_{ll'}(\omega) = \hat{R}_l \delta_{ll'}; \quad \hat{r}_{ll'}(\omega) = \hat{q}_{ll'}(\omega), \tag{3.6}$$

$$M_l^{+(0)} = M_l^+|_{s\to 0} = \oint \frac{\cos l\theta}{(1 - \lambda B)^{1/2}} d\theta; \quad \hbar_l^+ = \frac{h_l^+}{\sigma_l}. \tag{3.7}$$

We assume the operators $\hat{Q}$ and $\hat{R}$ to be of zero orders, and $\hat{q}$, $\hat{p}$, $\hat{r}$, and $\hat{t}$ of first order in ε.

In zero order it follows from (3.1) and (3.2) that

$$\hat{Q}_{ll'}(\omega^{(0)})A_{l'}^{(0)}=0;\quad \hat{R}_{ll'}(\omega^{(0)})B_{l'}^{(0)}=0, \tag{3.8}$$

where the zero superscript demotes the zero order.

We consider an odd mode, putting (cf. (2.48), (2.49), (2.51))

$$A_l^{(0)}=0,\quad B_l^{(0)}=\delta_{lN},\quad \omega^{(0)}=\omega_N. \tag{3.9}$$

We then obtain in first order from (3.1) and (3.2)

$$\hat{Q}_{ll'}^{(0)}A_{l'}^{(1)}+\hat{p}_{ll'}B_{l'}^{(0)}=0, \tag{3.10}$$

$$\omega^{(1)}\frac{\partial\hat{R}_{ll'}^{(0)}}{\partial\omega}B_{l'}^{(0)}+\hat{R}_{ll'}^{(0)}B_{l'}^{(1)}+\hat{r}_{ll'}B_{l'}^{(0)}=0. \tag{3.11}$$

Here $\hat{R}_{ll'}^{(0)}=R_{ll'}(\omega^{(0)})$, $Q_{ll'}^{(0)}=\hat{Q}_{ll'}(\omega^{(0)})$, and the superscripts 1 denote increments of first order. Multiplying both halves of (3.11) by $B_l^{(0)}$, summing over l, and taking (3.8) into account, we obtain

$$\omega^{(1)}=0, \tag{3.12}$$

i.e., the correction to the first-order frequency is equal to zero. It is therefore necessary to calculate the second-order frequency correction $\omega^{(2)}$, and this calls for considering the second order perturbation theory. In second order it follows from (3.2) that

$$\hat{R}_{ll'}^{(0)}B_{l'}^{(2)}+\frac{\partial\hat{R}_{ll'}^{(0)}}{\partial\omega}\omega^{(2)}B_{l'}^{(0)}+\hat{r}_{ll'}B_{l'}^{(1)}+\hat{t}_{ll'}A_{l'}^{(1)}=0. \tag{3.13}$$

Multiplying both halves of this equation by $B_l^{(0)}$ and summing over l, we obtain

$$\omega^{(2)}=-\frac{B_l^{(0)}\hat{r}_{ll'}B_{l'}^{(1)}+B_l^{(0)}\hat{t}_{ll'}A_{l'}^{(1)}}{B_l^{(0)}(\partial\hat{R}_{ll'}^{(0)}/\partial\omega)B_{l'}^{(0)}}. \tag{3.14}$$

We are interested only in the imaginary part of the increment to the frequency, $\operatorname{Im}\omega^{(2)}$. Recognizing that according to (3.11) we have $\operatorname{Im}B_l^{(1)}=0$, we get from (3.14)

$$\operatorname{Im}\omega^{(2)}=-\frac{B_l^{(0)}\hat{t}_{ll'}\operatorname{Im}A_{l'}^{(1)}}{B_l^{(0)}(\partial\hat{R}_{ll'}^{(0)}/\partial\omega)B_{l'}^{(0)}}. \tag{3.15}$$

Taking into account the explicit form of the quantities in the right-hand sides of this equation, we get

$$\frac{\operatorname{Im}\omega_N^{(2)}}{\omega_{ne}}=-\left(1+\frac{T_i}{T_e}\right)\frac{\varepsilon\Omega}{\omega_{ne}}\frac{k_a}{k_b}(\operatorname{Im}A_{N+1}^{(1)}-\operatorname{Im}A_{N-1}^{(1)}). \tag{3.16}$$

We now proceed to calculate $\operatorname{Im}A_{N\pm1}^{(1)}$. We note that according to (3.10) and (3.19) we have

$$\hat{Q}_{ll'}^{(0)}A_{l'}^{(1)}=-\hat{p}_{lN}. \tag{3.17}$$

Substituting here the explicit form of the matrix $\hat{Q}_{ll'}$ from (3.3) and recognizing that according to (2.41) and (2.50) we have

$$\hat{R}_l(\omega_N)=\left(1+\frac{T_i}{T_e}\right)\frac{c_s^2}{\omega_{ne}^2}\frac{N^2-l^2}{q^2R^2}, \tag{3.18}$$

we obtain the following equation for $\tilde{A}_l^{(1)}\equiv A_l^{(1)}/\varepsilon$:

$$\frac{(N^2-l^2)c_s^2}{q^2R^2\omega_{ne}^2}\left(1+\frac{T_i}{T_e}\right)\tilde{A}_l^{(1)}=\sigma_l\left(1+\frac{T_i}{T_e}\right)\frac{\Omega}{\omega_{ne}}\frac{k_a}{k_b}(\delta_{l,N-1}-\delta_{l,N+1})$$
$$+\frac{1}{4\pi}\left\langle B_s\int d\lambda\, M_l^{+*}h_{l'}^{+}\right\rangle\tilde{A}_{l'}^{(1)}. \tag{3.19}$$

We shall solve this equation by successive approximations in the contribution of the trapped particles (i.e., in the parameter $\varepsilon^{1/2}$, see, e.g., [6,7]). In the highest order we have

$$\tilde{A}_l^{(1)0}=\sigma_l\frac{\Omega q^2R^2\omega_{ne}}{(N^2-l^2)c_s^2}\frac{k_a}{k_b}(\delta_{l,N-1}-\delta_{l,N+1}), \tag{3.20}$$

this means that

$$\tilde{A}_{N\pm1}^{(1)0}=\frac{\Omega q^2R^2\omega_{ne}}{c_s^2}\frac{k_a}{k_b}\frac{\sigma_{N\pm1}}{2N\pm1}. \tag{3.21}$$

At $l=N$, Eq. (3.20) is not valid. We obtain $A_N^{(1)0}$ from the equation

$$\left\langle B_s\int d\lambda\, M_N^{+*}h_{l'}\right\rangle\tilde{A}_{l'}^{(1)0}=0. \tag{3.22}$$

When (3.21) is taken into account, this yields

$$\tilde{A}_N^{(1)0}=-\frac{\Omega q^2R^2\omega_{ne}}{c_s^2}\frac{k_a}{k_b}\left\langle B_s\int d\lambda\, M_N^{+*}g_N\right\rangle\Big/\left\langle B_s\int d\lambda\, M_N^{+*}h_N\right\rangle.$$

The quantity g_N introduced here is defined by the relation

$$g_N=h_{N-1}/(2N-1)+h_{N+1}/(2N+1) \tag{3.23}$$

and according to (2.21) it satisfies the equation

$$\omega g_N-i\hat{C}(g_N)=(\omega-\hat{\omega}_{*e})G_N/L, \tag{3.24}$$

$$G_N=M_{N+1}^{+}/(2N+1)+M_{N-1}^{+}/(2N-1). \tag{3.25}$$

Next, putting $\tilde{A}_l^{(1)}=\tilde{A}_l^{(1)0}+\delta\tilde{A}_l^{(1)}$, we obtain from (3.19)

$$\frac{(N^2-l^2)c_s^2}{q^2R^2\omega_{ne}^2}\left(1+\frac{T_i}{T_e}\right)\delta\tilde{A}_l^{(1)}=\frac{1}{4\pi}\left\langle B_s\int d\lambda\, M_l^{+*}h_{l'}\right\rangle\tilde{A}_{l'}^{(1)0}.$$

Substituting the obtained values of $A_l^{(1)0}$, we obtain for $l\neq N$

$$\delta\tilde{A}_l^{(1)}=\frac{1}{4\pi(N^2-l^2)}\frac{\Omega}{\omega_{ne}}\left(1+\frac{T_i}{T_e}\right)^{-1}\left(\frac{qR\omega_{ne}}{c_s}\right)^4\frac{k_a}{k_b}\left\{\left\langle B_s\int d\lambda\, M_l^{+*}g_N\right\rangle\right.$$
$$\left.-\left\langle B_s\int d\lambda\, M_l^{+*}h_N\right\rangle\left\langle B_s\int d\lambda\, M_N^{+*}g_N\right\rangle\Big/\left\langle B_s\int d\lambda\, M_N^{+*}h_N\right\rangle\right\}. \tag{3.26}$$

It follows therefore that

$$\delta\tilde{A}_{N-1}^{(1)}-\delta\tilde{A}_{N+1}^{(1)}=\frac{\Omega}{4\pi\omega_{ne}}\left(1+\frac{T_i}{T_e}\right)^{-1}\left(\frac{qR\omega_{ne}}{c_s}\right)^4\frac{k_a}{k_b}\left\{\left\langle B_s\int d\lambda\, G_N^{*}g_N\right\rangle\right.$$
$$\left.-\left\langle B_s\int d\lambda\, G_N^{*}h_N\right\rangle\left\langle B_s\int d\lambda\, M_N^{+*}g_N\right\rangle\Big/\left\langle B_s\int d\lambda\, M_N^{+*}h_N\right\rangle\right\}. \tag{3.27}$$

With the aid of (3.16) and (3.27) we obtain

$$\operatorname{Im}\frac{\omega_N^{(2)}}{\omega_{ne}}=\frac{\varepsilon^2}{4\pi}\left(\frac{q^2R^2\Omega\omega_{ne}}{c_s^2}\right)^2\left(\frac{k_a}{k_b}\right)^2\operatorname{Im}\left\{\left\langle B_s\int d\lambda\, G_N^{*}g_N\right\rangle\right.$$
$$\left.-\left\langle B_s\int d\lambda\, G_N^{*}h_N^{+}\right\rangle\left\langle B_s\int d\lambda\, M_N^{+*}g_N\right\rangle\Big/\left\langle B_s\int d\lambda\, M_N^{+*}h_N\right\rangle\right\}. \tag{3.28}$$

Using our earlier results,[6,7] we get

$$\frac{1}{4\pi}\,\mathrm{Im}\left\{\left\langle B_s\int d\lambda\, G_N{}^{*} g_N\right\rangle-\left\langle B_s\int d\lambda\, G_N{}^{*} h_N{}^{+}\right\rangle\left\langle B_s\int d\lambda\, M_N{}^{+*} g_N\right\rangle \times\left\langle B_s\int d\lambda\, M_N{}^{+*} h_N\right\rangle^{-1}\right\}=-\chi_N\left[\frac{\nu_e}{\omega_{ne}}\ln\left(\frac{128\varepsilon\omega_{ne}}{\nu_e}\right)\right]^{1/2} \times\left[\left(1-\frac{\omega_{ne}}{\omega_N}\right)I_3-\frac{\omega_{ne}\eta_e I_6}{\omega_N}\right]; \tag{3.29}$$

$$\chi_N=\frac{1}{\pi}\left(\frac{4N}{4N^2-1}-\alpha_N\right)^2,\quad \alpha_N=\left\langle B_s\int d\lambda\frac{M_N G_N}{L}\right\rangle\Big/\left\langle B_s\int d\lambda\frac{M_N{}^2}{L}\right\rangle. \tag{3.30}$$

Here I_3 and I_6 are integrals calculated in our paper[6]:

$$I_3=\langle z^{-3/4}[1+G(\sqrt{z})]^{1/2}\rangle=1.61,$$
$$I_6=\langle z^{-3/4}(z-3/2)[1+G(\sqrt{z})]^{1/2}\rangle=-1.07.$$

The symbol $\langle\cdots\rangle$ stands for (cf. (2.26))

$$\langle\ldots\rangle=\frac{2}{\sqrt{\pi}}\int_0^\infty z^{1/2}e^{-z}(\ldots)\,dz. \tag{3.31}$$

The function $G(x)$ is of the form

$$G(x)=\frac{1}{x\sqrt{\pi}}e^{-x^2}+\left(1-\frac{1}{2x^2}\right)\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt; \tag{3.32}$$

ν_e is the frequency of the electron collisions, defined by the relation

$$\nu_e=\omega_{pe}{}^2 e^2 M_e\,\Lambda/(2T_e)^{3/2}; \tag{3.33}$$

Λ is the Coulomb logarithm, $\omega_{pe}^2=4\pi n_0 e^2/M_e$ is the electron plasma frequency. It is assumed that $\omega>\nu_e/\varepsilon$. The integration with respect to λ in (3.30) is carried out from $1/B_{\max}$ to $1/B_{\min}$.

Substituting (3.29) in (3.28), we obtain ultimately

$$\mathrm{Im}\,\omega_N=1.61\varepsilon^2(k_a/k_b)^2(q^2R^2\omega_{ne}\Omega/c_s{}^2)^2\chi_N[\nu_e\omega_{ne}\ln(128\varepsilon\omega_{ne}/\nu_e)]^{1/2} \times[(1+T_i/T_e)(b_i-k_N{}^2c_s{}^2/\omega_{ne}{}^2)-0.66\eta_e]. \tag{3.34}$$

In particular, for $N=1$ we have

$$\alpha_N\approx 0.18,\quad \chi_N\approx 0.42.$$

It is seen that the growth rate increases with k_a. The value of k_a, however, cannot be too large, since we have used above perturbation theory with respect to a small parameter proportional to k_a. An estimate of the maximum permissible k_a and $k_a^{\max}$ at which formula (3.34) is still valid can be obtained from the condition that the difference $\Delta\omega$ between the zeroth-approximation eigenfrequencies is of the order of the real frequency correction necessitated by the coupling of the modes. Using for $\Delta\omega$ the estimate $\Delta\omega\sim k_N^2 c_s^2\varepsilon^{1/2}/\omega_{ne}^2$ and for $\mathrm{Re}\,\omega^{(2)}$ the estimate (see (3.14))

$$\mathrm{Re}\,\omega^{(2)}\sim\varepsilon^{3/2}(q^2R^2\Omega^2/c_s{}^2)(k_a/k_b)^2,$$

we obtain

$$(k_a^{\max}/k_b)^2\sim(\varepsilon^2 b_i)^{-1}. \tag{3.35}$$

In the derivation of (3.35) we have put $b_i\sim k_N^2c_s^2/\omega_{ne}^2$. According to (3.34), this value of k_a corresponds to the growth rate

$$\gamma_{\max}\approx[(1+T_i/T_e)(b_i-k_N{}^2c_s{}^2/\omega_{ne}{}^2)-0.66\eta_e](\nu_e\omega_{ne})^{1/2} \tag{3.36}$$

Thus, the upper limit of the growth rate (damping) of the odd modes coincides with the growth rate (decrement) of the even modes. To obtain an estimate of $\gamma_{\min}$ we recall that in our case

$$\omega_{ne}{}^2q^2R^2/c_s{}^2\gg 1.$$

Putting also $k_a\sim k_b$, we get

$$\gamma_{\min}\sim\varepsilon^2\gamma_{\max}. \tag{3.37}$$

From a comparison of (3.37) with (2.53) it is seen that the odd-mode growth rate obtained by us and due to the coupling of these modes with the even ones via the compressibility effect is large in comparison with the growth rate of the uncoupled-mode approximation when

$$(\nu_e/\omega_{ne})^{1/2}>(\omega_{ne}/\varepsilon k_N v_{Te})^3. \tag{3.38}$$

Since, according to[11], the ratio ω_{ne}/k_N is bounded from above by the Alfven velocity $c_A=B_s/(4\pi M_i n_0)^{1/2}$,

$$\omega_{ne}/k_N<c_A, \tag{3.39}$$

it follows that (3.38) is satisfied for all the permissible ω_{ne}, if

$$(\nu_e qR/c_A)^{1/2}>(M_e/M_i\beta\varepsilon^2)^{1/2}. \tag{3.40}$$

For the example of the thermonuclear reactor, considered in[15], when $n_0\approx 3\cdot 10^{14}$ cm^{-3}, $T_e\approx T_i\approx 15$ keV, $B_0=4\cdot 10^4$ G, $R=10^3$ cm, $\varepsilon=\frac{1}{5}$, and $q=2$, the left-hand side of the inequality (3.40) is of the order of 10^{-1} as against 10^{-2} on the right side. This means that the growth rate obtained by us is much larger than the one that follows from the uncoupled-mode approximation.

4. ROLE OF DISSIPATION ON THE TRAPPED ELECTRON IN THE PROBLEM OF EXCITATION OF ODD "DRIFT" MODES BY FAST IONS

Coppi and Bhadra[1] have found that a group of fast electron ("beam") of density n_b and temperature T_b excites odd "drift" modes with a growth rate on the order of

$$\gamma_b\approx\omega_{ne}\Delta_b,\quad \Delta_b\approx\frac{n_b}{n_0}\frac{T_e}{T_b}\varepsilon^{-1}. \tag{4.1}$$

It was assumed in[1] that this instability can have a bearing on the problem of the two-component tokamak.[16] Coppi and Bhadra[1] applied their results to an example with a two-component tokamak having the parameters

$$n_b/n_0\approx 5\cdot 10^{-2},\quad T_b/T_e\approx 20. \tag{4.2}$$

If we take by way of an estimate $\varepsilon=\frac{1}{4}$, then we obtain from (4.1) and (4.2)

$$\Delta_b\approx 10^{-2}. \tag{4.3}$$

We compare the beam growth rate (4.1) with the growth rate (or decrement (3.37)) obtained in Sec. 3. At $|\eta_e| \approx 1$ we get from (3.37) the estimate

$$\gamma_e = \omega_{ne}\Delta_e, \quad \Delta_e \approx \varepsilon^2 (\nu_e/\omega_{ne})^{1/2}. \tag{4.4}$$

Recognizing that according to Coppi and Bhadra[1] the characteristic wave numbers $k_\perp$ of the perturbations excited by the beam are of the order of the reciprocal Larmor radius of the fast ions,

$$k_\perp \rho_b \approx 1, \quad \rho_b \approx (T_b/M_b)^{1/2}/\omega_B, \tag{4.5}$$

and putting $k_\perp \approx k_b$, we obtain from (4.4)

$$\Delta_e \approx \varepsilon^2 (T_b/T_e)^{1/4} (\nu_e a/c_s)^{1/2}. \tag{4.6}$$

Following[17], we put $T_e = 5$ keV, $n \approx 10^{14}$ cm^{-3}. Here $\nu_e \approx 10^4$ sec^{-1} and $c_s \approx 5\times10^7$ cm/sec. Putting also $a \approx 10^2$ cm, we obtain from (4.6)

$$\Delta_e \approx 2\cdot10^{-2}. \tag{4.7}$$

In the example considered by us, the growth rate of the odd mode, due to collisional dissipation by trapped electrons, is comparable with the beam growth rate, and even exceeds it.

5. CONCLUSION

It follows from the foregoing analysis that, in contrast to the previously held concepts,[1,2] odd drift modes in a tokamak are sensitive to collisional dissipation by trapped electrons. This is attributed to coupling of the odd and even modes, a coupling not accounted for in[1,2]. This coupling is due to the compressibility of the plasma, and also to the fact that the number of wavelengths spanned by the length of the torus is in general not an integer.

The growth rates calculated by us for the odd modes are quadrating in the coupling parameters. They are therefore smaller than the growth rates of the even modes. These growth rates (or decrements) can, however, be significant in problems involving the buildup of odd modes by a group of fast ions, since the growth rates of such a buildup are also small. The estimates given above show that this is precisely the situation in the case considered by Coppi and Bhadra.[1] This means that the theory of collective processes developed by them needs, generally speaking, to be revised.

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